Module 9: Recurrence Relations

1 Given the recurrence relation $a_{n+1} = 3a_n - 2a_{n-1}$ and initial values $a_0 = 0$ and $a_1 = 1$, what is a_5?

2 Find p_6 for $p_n = 4p_{n-1} - 2n - 3$ and $p_0 = 2$.

Module 10: Induction Methods

For questions 3 through 5, provide the following information:

- What is the base case? If the problem requires strong induction, you might need several base cases.
- What is the induction hypothesis? If the problem requires strong induction make sure you include that in your induction hypothesis.
- What needs to be proved to complete the induction step?
- For an extra challenge, complete the proof.

3 For all natural numbers n, the following equality holds:

$$11 + 19 + 27 + \cdots + (8n + 3) = n(4n + 7).$$

4 For all natural number n, 3 evenly divides $7^n - 4^n$.

5 For all integers n larger than 2 the following inequality holds: $2^n > 2n$

Module 11: Recursive Structures

6 The following algorithm computes the GCD of two integers. The function `remainder(n,m)` calculates the remainder of the division n divided by m $(n = qm + r)$.

Algorithm 1 GCD

```
procedure GCD(n,m)
r := remainder(n,m)
if r == 0
  return m
else
  GCD(m,r)
```

(a) If you execute the command GCD(21,99) how many times, in total, will the function GCD be called?

(b) What are the inputs of each of those calls?

(c) Will GCD(n,m) always equal GCD(m,n)?

(d) If $n > m$ which will call GCD more times, GCD(n,m) or GCD(m,n)?

7 Evaluate the following recursive algorithm at n = 6 and n = 7. Explain, in your own words, what this algorithm does.

Algorithm 2 Double Factorial

```
procedure DFactorial(n)
input:  a non-negative integer n

if n = 1 or 0 return 1
r := DFactorial(n - 2)
  return (r*n)
```

8 Given the recursive algorithm in this pseudocode.

What is the output of `wild(8)`? How many times, in total, is `wild` called?

Module 12: Simple Recurrence Relations

Algorithm 3 a wild function

```
procedure WILD(n)
input:  a positive integer n.

if n < 3
  return 1
if n ≥ 3
    t:  = wild (n - 1)
    r:  = wild (n - 2)
    if n is odd
      return r+t
    if n is even
      return r*t
```

9 For the recurrence relation, $d_n = -d_{n-1} + 6d_{n-2}$, answer the following questions.

a. What is the characteristic equation?

b. What are the roots of the characteristic equation?

c. If $d_0 = 1$ and $d_1 = 4$, find the closed form expression for d_n. (Solve the recurrence relation.)

d. What is d_{10}? What is d_{14}?