Module 13: Counting by Bijections and Products of Sets

1 A certain model of pickup truck is available in five exterior colors, three interior colors, and three interior styles. In addition, the transmission can be either manual or automatic, and the truck can have either two-wheel or four-wheel drive. How many different versions of the pickup truck can be ordered?

2 A local charity organization must elect a president, vice president, and treasurer from its 30 members. How many ways can this be done?

3 Which of the following functions maps an $11-$to-1 correspondence from the set $\{1, 2, 3, \ldots, 44\}$ to the set $\{1, 2, 3, 4\}$? Note that option B uses the floor function and option C uses the ceiling function.

A. $f(x) = \binom{x}{3}$ B. $f(x) = \lfloor \frac{x}{11} \rfloor$ C. $f(x) = \lceil \frac{x}{11} \rceil$ D. $f(x) = \frac{x}{11}$

4 Which of the following functions maps an $8-$to-1 correspondence from $\{0, 1, 2, 3, \ldots, 55\}$ to $\{0, 1, \ldots, 6\}$?

A. $f(x) = \sin(\pi x)$ B. $f(x) = x \bmod 7$ C. $f(x) = x \bmod 8$ D. $f(x) = \lceil \frac{x}{7} \rceil$

5 Suppose that $B = \{0, 1, 2, \ldots 8\}$. If $f : A \to B$ is 3-to-1 mapping of A onto B, what is $|A|$?

Module 14: Counting with Permutations and Combinations

6 The local branch of a national business chain has 27 employees. If they all shake each others hand at the company picnic, at least how many handshakes occurred?

7 Consider a standard deck of 52 playing cards. If you are unfamiliar with playing cards, check out this explanation of playing cards.

    a. How many ways can five cards be drawn if their order is important?

    b. How many five card hands exist where the order doesn't matter?

8 At a university there are 10 faculty members in the mathematics department and 12 faculty members in the computer science department.

(a) How many ways are there to form a committee from three members of the mathematics department.

(b) How many ways are there to select a department head, a graduate chair, and under-graduate chair for the computer science faculty. One person can not occupy multiple positions.

(c) How many ways are there to form a committee from one member of the computer science department and 2 members of the mathematics department.

(d) How many ways are there to form a committee from one faculty member of the mathematics department and two faculty members of the computer science department if one of these faculty members is selected as the spokesperson of the committee.

9 A coin is flipped 20 times and the results are recorded. How many outcomes have exactly 9 heads?

10 Each student at State University has a student ID number consisting of five digits (the first digit is nonzero, and the digits can be repeated) followed by two of the letters A, B, C, and D (letters cannot be repeated). How many different student numbers are possible?

Module 15: Counting with Multisets

11 There are 250 pieces of candy in a jar and each one has a color, either red, orange, yellow, green, blue, or purple. The jar contains 37 red, 48 blue, and 55 yellow. How many pieces of candy are either blue, green, purple, or orange?

12 Twenty coins are flipped. How many outcomes have at least 4 heads facing up?

13 How many ways are there to place 6 identical objects into 3 different bins?

14 There are 15 varieties of donuts sold at a bakery. How many ways are there to select a dozen donuts?

15 A program is asked to ping a server exactly 100 times in the course of a seven day week. How many different schedules are there for the program to ping the server if it must do at least 20 on each weekend day (Saturday and Sunday)? A schedule consists of the number of pings the program does on each of the seven days of the week, for example, Mon: 15, Tue: 0, Wed: 15, Thu: 15, Fri: 5, Sat: 25, Sun: 25.

16 How many ways can we add three non-negative integers such that they sum to 9? (order matters)

Module 16: Generating Permutations and Combinations

17 Write the following 5-tuples in lexicographic order.

- $(5, 1, 4, 3, 2)$
- $(4, 3, 2, 2, 2)$
- $(5, 2, 1, 2, 1)$
- $(4, 3, 3, 5, 1)$

18 Write the next 5 permutations in lexicographic order immediately following $(2, 3, 4, 1)$.

19 Write the following 4-subsets of {a, b, c, d, e, f, g} in lexicographical order.

- {c, e, f, g}
- {a, d, g, e}
- {c, f, e, a}
- {g, f, e, b}

Module 17: Advanced Counting Techniques

20 A college has 1105 students. This semester 320 students are taking a math course, 405 of the students are taking a history class, and 250 of the students taking a math class are not enrolled in a history class.

  (a) Construct a Venn diagram, and write the appropriate numbers in the four regions

  (b) How many of the students are taking math and history?

  (c) How many students are taking history but not math?

  (d) How many students are taking math or history?

  (e) How many students are taking neither math nor history?

21 Computer science students were surveyed about which operating systems they have used recently. Results showed that 70 students have used Windows, 62 have used macOS, and 16 have used Linux recently. Of these, 24 used both Windows and macOS, 7 used Windows and Linux, and 10 used macOS and Linux recently. Furthermore, 3 students report using all three operating systems recently. How many students reported using at least one of the operating systems recently?

22 What is the coefficient of $x^6 y^4$ in $(x - 2y)^{10}$?

23 How many cards must be drawn from a standard deck of 52 playing cards to guarantee:

  a. at least two have the same suite?

  b. at least 5 have the same suit?