CS564 Foundations of Machine Learning
Assignment: 4

Mukuntha N S (1601CS27)
R J Srivatsa (1601CS35)
Shikhar Jaiswal (1601CS44)

November 7, 2019

1 Assignment Description

Designing and implementing a Hidden Markov Model (HMM) based Part-of-Speech (POS) tagger using Viterbi algorithm. We try to assign the most appropriate sequence of part-of-speech tags to each word in a sentence for the **Brown Corpus** .

2 Procedure

Installation

Install the following dependencies either using pip or through conda in a Python 3.5+ environment:
- jupyter
- numpy
- pandas

```
python3 -m pip install jupyter numpy pandas
```

or alternatively,

```
conda install -c anaconda jupyter numpy pandas
```

Running The Notebook

To run the program, head to the directory $Assignment3/Q1$. Please note that the **dataset should be present in the same directory** as the jupyter notebook. Use the following command to run the notebook:

```
jupyter notebook Q1.ipynb
```

3 Discussion

The primary objective of the assignment is to assign the most appropriate sequence of part-of-speech tags to each word in a sentence. The following sub-sections contain the explanation for individual code snippets. For a detailed look at the output, please refer the notebook and the individual files provided.

Notebook Code

Pre-processing and Visualization

Import the Python dependencies, and check the data samples and their values and pre-process the corpus.

```
# Import the required libraries.
import re
import math
import random
import collections
import operator
import numpy as np
import pandas as pd
from sklearn.model_selection import KFold
from collections import defaultdict

random.seed(11)
np.random.seed(11)
```

```
def parse_sentence(sentence):
    '''
    Function for parsing the words and tags from the
    sentences of the input corpus.
    '''
    word_tag_pairs = sentence.split(" ")
    words = []
    tags = []

    for i, word_tag in enumerate(word_tag_pairs):
        word, tag = word_tag.strip().rsplit('/', 1)
        words.append(word)
        tags.append(tag)

    return words, tags

# Parse the sentences into a list.
parsed_sentences = []

with open('./Brown_train.txt', 'r') as file:
    sentences = file.readlines()

    for sentence in sentences:
```

```
        sentence = sentence.strip()
        parsed_sentences.append(parse_sentence(sentence))
```

```
def get_vocab(X_train, Y_train):
    '''
    Function for building the vocabulary from the training set of
    words and tags.
    '''
    vocabulary2id = dict()
    tag2id = dict()
    vocabulary2id['UNK'] = 0

    for sent in X_train:
        for word in sent:
            if word not in vocabulary2id.keys():
                vocabulary2id[word] = len(vocabulary2id)

    for sent in Y_train:
        for tag in sent:
            if tag not in tag2id.keys():
                tag2id[tag] = len(tag2id)

    return vocabulary2id, tag2id

def get_word_tag_counts(X_train, Y_train, vocabulary2id, tag2id):
    '''
    Function for calculating the counts pertaining to the
    individual word tags.
    '''
    wordcount = defaultdict(int)
    tagcount = defaultdict(int)
    tagpaircount = defaultdict(int)
    tagtriplecount = defaultdict(int)

    for sent in X_train:
        for word in sent:
            wordcount[word] += 1

    for sent in Y_train:
        for tag in sent:
            tagcount[tag] += 1

    for sent in Y_train:
        for i in range(len(sent) - 1):
            tagpaircount[sent[i], sent[i + 1]] += 1

    for sent in Y_train:
        for i in range(len(sent) - 2):
```

```
            tagtriplecount[sent[i], sent[i + 1], sent[i + 2]] += 1

    return wordcount, tagcount, tagpaircount, tagtriplecount
```

HMM Code

We build the initial HMM using word and tag counts, and make use of transition and emission probabilities for the purposes of tackling evaluation, decoding and learning problems.

```
# Token to map all out-of-vocabulary words (OOVs).
UNK = "UNK"
# Index for UNK
UNKid = 0
epsilon = 1e-100
array, ones, zeros, multiply, unravel_index = np.array, np.ones, np.zeros, np.multiply,
                                              np.unravel_index

class HMM:
    def __init__(self, state_list, observation_list, transition_proba = None,
                 observation_proba = None, initial_state_proba = None,
                 smoothing_obs = 0.01, transition_proba1 = None, prob_abs = 0.00001):
        '''
        Builds a Hidden Markov Model.
        * state_list is the list of state symbols [q_0...q_(N-1)]
        * observation_list is the list of observation symbols [v_0...v_(M-1)]
        * transition_proba is the transition probability matrix
            [a_ij] a_ij,a_ik = Pr(Y_(t+1)=q_i|Y_t=q_j,Y_(t-1)=q_k)
        * observation_proba is the observation probablility matrix
            [b_ki] b_ki = Pr(X_t=v_k|Y_t=q_i)
        * initial_state_proba is the initial state distribution
            [pi_i] pi_i = Pr(Y_0=q_i)
        '''
        # Number of states.
        self.N = len(state_list)
        # Number of possible emissions.
        self.M = len(observation_list)
        self.prob_abs = prob_abs
        self.omega_Y = state_list
        self.omega_X = observation_list

        if transition_proba1 is None:
            self.transition_proba1 = zeros( (self.N, self.N), float)
        else:
            self.transition_proba1 = transition_proba1

        if transition_proba is None:
            self.transition_proba = zeros( (self.N, self.N, self.N), float)
        else:
```

```
            self.transition_proba=transition_proba

        if observation_proba is None:
            self.observation_proba = zeros( (self.M, self.N), float)
        else:
            self.observation_proba = observation_proba

        if initial_state_proba is None:
            self.initial_state_proba = zeros( (self.N,), float )
        else:
            self.initial_state_proba = initial_state_proba

        # Build indexes, i.e., the mapping between token and int.
        self.make_indexes()
        self.smoothing_obs = smoothing_obs

    def make_indexes(self):
        '''
        Function for creating the reverse table that maps
        states/observations names to their index in the probabilities
        array.
        '''
        self.Y_index = {}

        for i in range(self.N):
            self.Y_index[self.omega_Y[i]] = i

        self.X_index = {}

        for i in range(self.M):
            self.X_index[self.omega_X[i]] = i

    def get_observationIndices(self, observations):
        '''
        Function for returning observation indices,
        and dealing with OOVs.
        '''
        indices = zeros( len(observations), int )
        k = 0

        for o in observations:
            if o in self.X_index:
                indices[k] = self.X_index[o]
            else:
                indices[k] = UNKid

            k += 1
```

```
        return indices

    def data2indices(self, sent):
        '''
        Function for extracting the words and tags and returning a
        list of indices for each.
        '''
        wordids = list()
        tagids  = list()

        for couple in sent:
            wrd = couple[0]
            tag = couple[1]

            if wrd in self.X_index:
                wordids.append(self.X_index[wrd])
            else:
                wordids.append(UNKid)

            tagids.append(self.Y_index[tag])

        return wordids, tagids

    def observation_estimation(self, pair_counts):
        '''
        Function for building the observation distribution where
        observation_proba is the observation probablility matrix.
        '''
        # Fill with counts.
        for pair in pair_counts:
            wrd = pair[0]
            tag = pair[1]
            cpt = pair_counts[pair]
            # For UNK.
            k = 0

            if wrd in self.X_index:
                k = self.X_index[wrd]

            i = self.Y_index[tag]
            self.observation_proba[k, i] = cpt

        # Normalize.
        self.observation_proba = self.observation_proba + self.smoothing_obs
        self.observation_proba = self.observation_proba /
                                 self.observation_proba.sum(axis = 0).reshape(1, self.N)

    def transition_estimation(self, trans_counts):
```

```python
        '''
        Function for building the transition distribution where
        transition_proba is the transition matrix with:
        [a_ij] a[i, j] = Pr(Y_(t+1) = q_i | Y_t = q_j, Y_(t-1) = q_k)
        '''
        # Fill with counts.
        for triple in trans_counts:
            i = self.Y_index[triple[2]]
            j = self.Y_index[triple[1]]
            k = self.Y_index[triple[0]]
            self.transition_proba[k, j, i] = trans_counts[triple]

        # Normalize.
        self.transition_proba = self.transition_proba /
                                self.transition_proba.sum(axis = 0).reshape(self.N, self.N)

    def transition_estimation1(self, trans_counts):
        '''
        Function for building the transition distribution where
        transition_proba is the transition matrix with:
        [a_ij] a[i,j] = Pr(Y_(t+1)=q_i|Y_t=q_j)
        '''
        # Fill with counts.
        for pair in trans_counts:
            i = self.Y_index[pair[1]]
            j = self.Y_index[pair[0]]
            self.transition_proba1[j, i] = trans_counts[pair]

        # Normalize.
        self.transition_proba1 = self.transition_proba1 /
                                 self.transition_proba1.sum(axis = 0).reshape(1, self.N)

    def init_estimation(self, init_counts):
        '''
        Function for building the initial distribution.
        '''
        # Fill with counts.
        for tag in init_counts:
            i = self.Y_index[tag]
            self.initial_state_proba[i] = init_counts[tag]

        # Normalize.
        self.initial_state_proba = self.initial_state_proba / sum(self.initial_state_proba)

    def supervised_training(self, pair_counts, trans_counts, init_counts, trans_counts1):
        '''
        Function for training the HMM's parameters.
        '''
```

```
        self.observation_estimation(pair_counts)
        self.transition_estimation(trans_counts)
        self.transition_estimation1(trans_counts1)
        self.init_estimation(init_counts)

    def viterbi(self, observations):
        if len(observations) < 2:
            return [hmm.Y_index[z] for z in observations]

        nSamples = len(observations)
        # Number of states.
        nStates = self.transition_proba.shape[0]
        # Scale factors (necessary to prevent underflow).
        c = np.zeros(nSamples)
        # Initialise viterbi table.
        viterbi = np.zeros((nStates, nStates, nSamples))
        # Initialise viterbi table.
        viterbi1 = np.zeros((nStates, nSamples))
        # Initialise the best path table.
        psi = np.zeros((nStates, nStates, nSamples))
        best_path = np.zeros(nSamples)
        idx0 = self.X_index[observations[0]]
        idx1 = self.X_index[observations[1]]
        viterbi1[:, 0] = self.initial_state_proba.T * self.observation_proba[idx0, :].T

        # Loop through the states.
        for s in range (0, nStates):
            for v in range (0, nStates):
                viterbi[s, v, 1] = viterbi1[s, 0] * self.transition_proba1[s, v] *
                                   self.observation_proba[idx1, v]

        psi[0] = 0;

        # Loop through time-stamps.
        for t in range(2, nSamples):
            idx = self.X_index[observations[t]]
            # Loop through the states.
            for s in range (0, nStates):
                for v in range (0, nStates):
                    self.transition_proba[np.isnan(self.transition_proba)] = self.prob_abs
                    trans_p = viterbi[:, s, t-1] * self.transition_proba[:, s, v]

                    if (math.isnan(trans_p[0])):
                        trans_p[0] = 0

                    psi[s, v, t], viterbi[s, v, t] = max(enumerate(trans_p),
                                                         key = operator.itemgetter(1))
                    viterbi[s, v, t] = viterbi[s, v, t] * self.observation_proba[idx, v]
```

```
    cabbar = viterbi[:, :, nSamples - 1]
    best_path[nSamples - 1] = unravel_index(cabbar.argmax(), cabbar.shape)[1]
    best_path[nSamples - 2] = unravel_index(cabbar.argmax(), cabbar.shape)[0]

    # Return the best path, number of samples and psi.
    for t in range(nSamples - 3, -1, -1):
        best_path[t] = psi[int(round(best_path[t + 1])),
                           int(round(best_path[t + 2])), t + 2]

    return best_path

def fwd_bkw(self, observations):
    observations = x
    self = hmm
    nStates = self.transition_proba.shape[0]
    start_prob = self.initial_state_proba
    trans_prob = self.transition_proba1.transpose()
    emm_prob = self.observation_proba.transpose()

    # Forward part of the algorithm.
    fwd = []
    f_prev = {}

    for i, observation_i in enumerate(observations):
        f_curr = {}
        for st in range(nStates):
            if i == 0:
                # Base case for the forward part.
                prev_f_sum = start_prob[st]
            else:
                prev_f_sum = sum(f_prev[k] * trans_prob[k][st] for k in range(nStates))

            f_curr[st] = emm_prob[st][self.X_index[observation_i]] * prev_f_sum

        fwd.append(f_curr)
        f_prev = f_curr

    p_fwd = sum(f_curr[k] for k in range(nStates))

    # Backward part of the algorithm.
    bkw = []
    b_prev = {}

    for i, observation_i_plus in enumerate(reversed(observations[1:] + [None,])):
        b_curr = {}
        for st in range(nStates):
            if i == 0:
```

```
                    # Base case for backward part.
                    b_curr[st] = 1.0
                else:
                    b_curr[st] = sum(trans_prob[st][l] *
                                     emm_prob[l][self.X_index[observation_i_plus]]
                                     * b_prev[l] for l in range(nStates))

            bkw.insert(0,b_curr)
            b_prev = b_curr

        p_bkw = sum(start_prob[l] * emm_prob[l][self.X_index[observations[0]]] * b_curr[l]
                    for l in range(nStates))

        # Merging the two parts.
        posterior = []

        for i in range(len(observations)):
            posterior.append({st: fwd[i][st] * bkw[i][st] / p_fwd for st in range(nStates)})

        assert abs(p_fwd - p_bkw) < 1e-6
        return fwd, bkw, posterior
```

```
def make_counts(X, Y):
    '''
    Function for building the different count tables to train a HMM.
    Each count table is a dictionary.
    '''
    c_words = dict()
    c_tags = dict()
    c_pairs= dict()
    c_transitions = dict()
    c_inits = dict()
    c_transitions1 = dict()

    for sent in zip(X, Y):
        sent = list(zip(*sent))
        for i in range(len(sent)):
            couple = sent[i]
            wrd = couple[0]
            tag = couple[1]

            # Word counts.
            if wrd in c_words:
                c_words[wrd] = c_words[wrd] + 1
            else:
                c_words[wrd] = 1

            # Tag counts.
```

```
            if tag in c_tags:
                c_tags[tag] = c_tags[tag] + 1
            else:
                c_tags[tag] = 1

            # Observation counts.
            if couple in c_pairs:
                c_pairs[couple] = c_pairs[couple] + 1
            else:
                c_pairs[couple] = 1

            if i >= 1:
                trans1 = (sent[i - 1][1], tag)
                if trans1 in c_transitions1:
                    c_transitions1[trans1] = c_transitions1[trans1] + 1
                else:
                    c_transitions1[trans1] = 1

            if i > 1:
                trans = (sent[i - 2][1], sent[i - 1][1], tag)
                if trans in c_transitions:
                    c_transitions[trans] = c_transitions[trans] + 1
                else:
                    c_transitions[trans] = 1
            else:
                if tag in c_inits:
                    c_inits[tag] = c_inits[tag] + 1
                else:
                    c_inits[tag] = 1

    return c_words,c_tags,c_pairs, c_transitions, c_inits, c_transitions1
```

Testing and Analysis

We perform 3-fold cross validation along with accuracy, precision, recall and f-score metrics.

```
# Build the test and training sets of sentences.
kf = KFold(n_splits = 3, shuffle = False)
parsed_sentences = np.asarray(parsed_sentences)
scores = []
scores1 = []
y_pred_idx = []
y_pred_idx1 = []
y_test_idx = []
y_test_idx1 = []

for train_index, test_index in kf.split(parsed_sentences):
    train_data = parsed_sentences[train_index]
```

```python
    test_data = parsed_sentences[test_index]
    X_train = [a[0] for a in train_data]
    Y_train = [a[1] for a in train_data]
    X_test = [a[0] for a in test_data]
    Y_test = [a[1] for a in test_data]

    # Build the vocabulary and word counts.
    vocabulary2id, tag2id = get_vocab(X_train, Y_train)
    wordcount, tagcount, tagpaircount, tagtriplecount = get_word_tag_counts(X_train,
                                    Y_train, vocabulary2id, tag2id)
    cwords, ctags, cpairs, ctrans, cinits, ctrans1 = make_counts(X_train, Y_train)

    state_list = list(ctags.keys())
    observation_list = [a[0] for a in sorted(vocabulary2id.items(), key = lambda x: x[1])]
    hmm = HMM(state_list = state_list, observation_list = observation_list,
              transition_proba = None, observation_proba = None, initial_state_proba = None,
              smoothing_obs = 0.4, prob_abs = 0)
    hmm.supervised_training(cpairs, ctrans, cinits, ctrans1)

    for x, y_true in zip(X_test, Y_test):
        for i in range(len(x)):
            if x[i] not in vocabulary2id.keys():
                x[i] = 'UNK'

        pred_idx = hmm.viterbi(x)
        y_pred = np.asarray([state_list[int(round(i))] for i in pred_idx])
        y_true = np.asarray(y_true)
        y_pred_idx += np.asarray([tag2id[lab] for lab in y_pred], dtype = np.int32).tolist()
        y_test_idx += np.asarray([tag2id[lab] for lab in y_true], dtype = np.int32).tolist()
        scores += (y_pred == y_true).tolist()

    x, y_true = X_train[0], Y_train[0]

    for x, y_true in zip(X_test, Y_test):
        for i in range(len(x)):
            if x[i] not in vocabulary2id.keys():
                x[i] = 'UNK'

        pred_probs = hmm.fwd_bkw(x)
        pred_idx = [max(probs.items(), key=lambda x: x[1])[0] for probs in pred_probs[2]]
        y_pred = np.asarray([state_list[int(round(i))] for i in pred_idx])
        y_true = np.asarray(y_true)
        y_pred_idx1 += np.asarray([tag2id[lab] for lab in y_pred], dtype = np.int32).tolist()
        y_test_idx1 += np.asarray([tag2id[lab] for lab in y_true], dtype = np.int32).tolist()
        scores1 += (y_pred == y_true).tolist()

prec, rec, fscore, _ = precision_recall_fscore_support(y_test_idx, y_pred_idx,
                                                       average = 'macro')
```

```python
prec1, rec1, fscore1, _ = precision_recall_fscore_support(y_test_idx1, y_pred_idx1,
                                                          average = 'macro')

print('Overall Accuracy and Scores:')
print('Only Forward-Backward Accuracy: {}, Precision: {}, Recall: {}, FScore: {}'.format(
    np.asarray(scores1).mean(), prec1, rec1, fscore1))
print('Viterbi Accuracy: {}, Precision: {}, Recall: {}, FScore: {}'.format(
    np.asarray(scores).mean(), prec, rec, fscore))
```

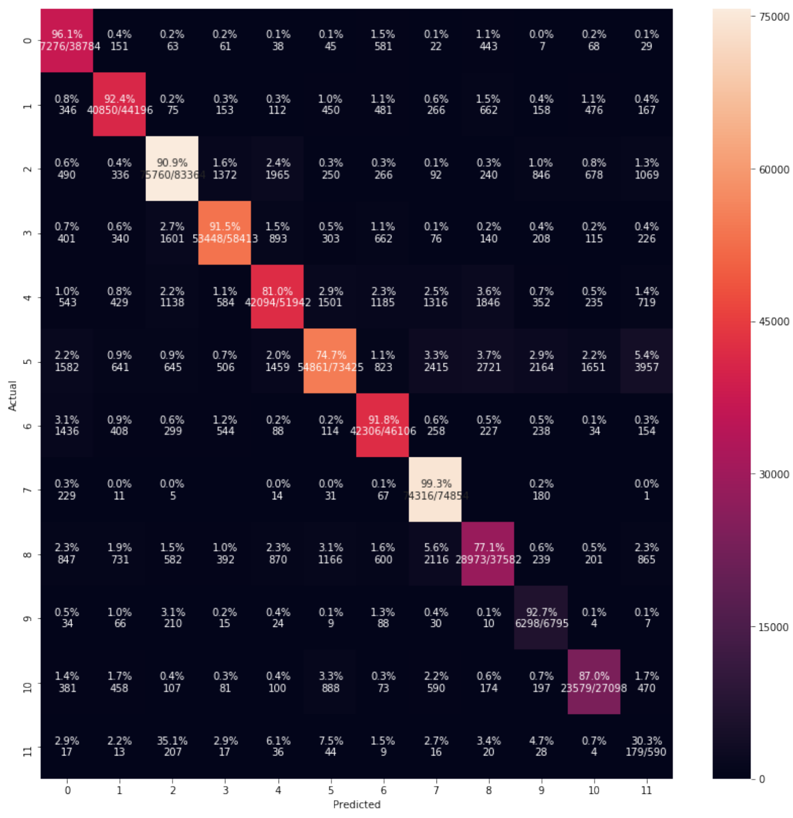
```
Output:
Overall Accuracy and Scores:
Only Forward-Backward Accuracy: 0.7866681150107981, Precision: 0.7522332251027066,
                      Recall: 0.7410914676078869, FScore: 0.7181261096051349
Viterbi Accuracy: 0.8836249353308209, Precision: 0.7957061479967794,
        Recall: 0.837373245121495, FScore: 0.8017269798075152
```

```python
y_pred_idx = np.asarray(y_pred_idx)
y_test_idx = np.asarray(y_test_idx)

def get_scores(Y_test_idx, y_pred_test, average='weighted'):
    acc = (y_pred_test == Y_test_idx).mean()
    prec, rec, fscore, _ = precision_recall_fscore_support(Y_test_idx, y_pred_test,
                                                            average=average)
    return acc, prec, rec, fscore

cm = confusion_matrix(y_pred_idx, y_test_idx)
accuracy = (cm.astype('float') / cm.sum(axis=1)[:, np.newaxis]).diagonal()
_, precision, recall, f1_scores = get_scores(y_pred_idx, y_test_idx, None)

for tag, tagid in tag2id.items():
    print('TAG: {} - Accuracy: {}, F1: {}, Precision: {}, Recall: {}'.format(tag,
        accuracy[tagid], f1_scores[tagid], precision[tagid], recall[tagid]))

# Confusion Matrix
cm
```

```
Output:
array([[37276,   346,   490,   401,   543,  1582,  1436,   229,   847,
           34,   381,    17],
       [  151, 40850,   336,   340,   429,   641,   408,    11,   731,
           66,   458,    13],
       [   63,    75, 75760,  1601,  1138,   645,   299,     5,   582,
          210,   107,   207],
       [   61,   153,  1372, 53448,   584,   506,   544,     0,   392,
           15,    81,    17],
       [   38,   112,  1965,   893, 42094,  1459,    88,    14,   870,
           24,   100,    36],
```

```
        [   45,    450,    250,    303,   1501, 54861,    114,     31,   1166,
             9,    888,     44],
        [  581,    481,    266,    662,   1185,    823, 42306,     67,    600,
            88,     73,      9],
        [   22,    266,     92,     76,   1316,   2415,    258, 74316,   2116,
            30,    590,     16],
        [  443,    662,    240,    140,   1846,   2721,    227,      0, 28973,
            10,    174,     20],
        [    7,    158,    846,    208,    352,   2164,    238,    180,    239,
          6298,    197,     28],
        [   68,    476,    678,    115,    235,   1651,     34,      0,    201,
             4,  23579,      4],
        [   29,    167,   1069,    226,    719,   3957,    154,      1,    865,
             7,    470,    179]])
```

Plot



Additionally please refer *Q1.ipynb* for the outputs.