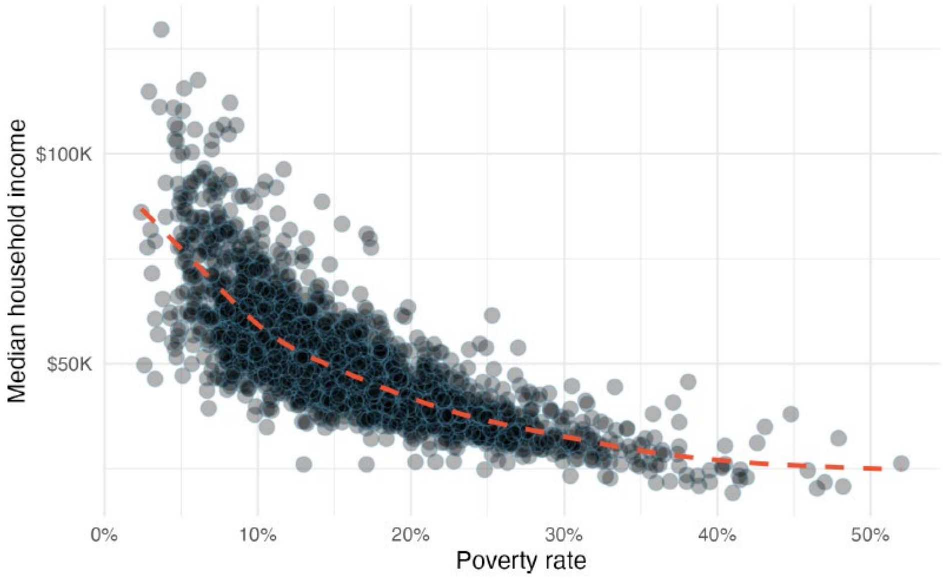
Interactive Notes Chapter 5 – Exploring Numerical Data
Math 217
Saidi

5.1 Scatterplots for paired data

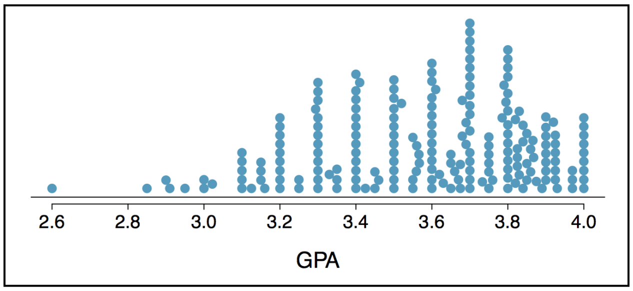
Use the `county` dataset to compare borrowers' median household incomes and their respective poverty rates. A statistical model has been fitted to the data to show the regression curve (dashed red line).

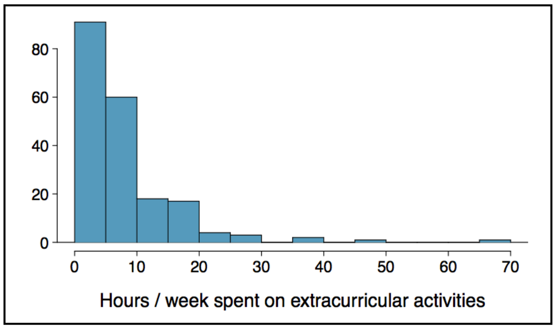


Quantitative Data Distributions

Dotplot – stacks of data points on a horizontal line

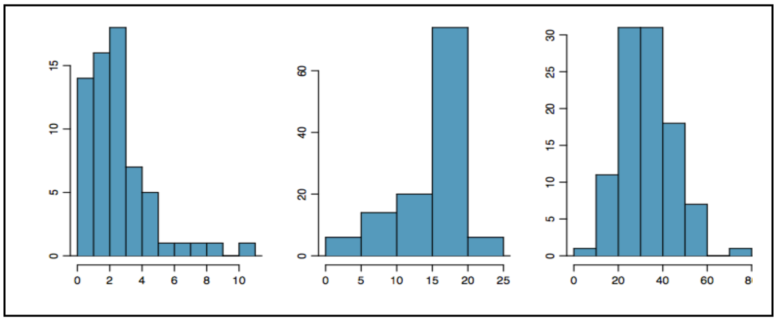
Histogram – show data density with bars; vertical axis is "frequency"





The **shape** may be described as **symmetric (bell-shaped), right-skewed, or left-skewed:**

Right Skewed Left Skewed Bell-Shaped

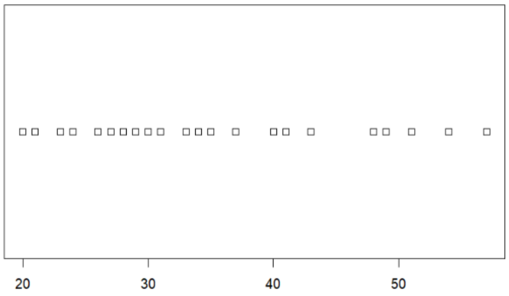


An **outlier** is a value that is distinctly far outside the range of the other values in the dataset.

Dotplots to see distributions of quantitative variables

Create a list of data about the dendritic segments and use stripchart to make a dotplot using R code

~~~~
```{r}
segment <- c(23, 30, 54, 28, 31, 29, 34, 35, 30, 27, 21,
43, 51, 35, 51, 49, 35, 24, 26, 29, 21, 29, 37, 27, 28,
33, 33, 23, 37, 27, 40, 48, 41, 20, 30, 57)
stripchart(segment)
```
~~~~



5.4 Variance and standard deviation

How much variability there is in a dataset is an important aspect to study.
Is all the data grouped closely together? Is the data spread out over a wide range? Are there a few observations that are FAR from the rest of the group?

x_i = one observation value $\bar{x}$= mean of all values

Using the mean, x as the reference point or center, we find the distance of each point x_i from the mean. These values $(x_i - \bar{x})$ are called the deviations from the mean and can be negative as well as positive. These deviations always mathematically add up to a zero since the negative ones can be shown to exactly cancel out the positive ones.

Σ Summation

Fact: $\sum(x_i - \bar{x}) = 0$ **for any data set.**

$x_i - \bar{x}$ = how far that one observation is from mean

$\sum(x_i - \bar{x})$

Thus, averaging these deviations does not tell us anything useful about the data spread. On the other hand, squaring these deviations makes them all positive and if we then average these, it will result in a useful measure of spread. It turns out that it is better to divide the sum of these squared deviations by $(n-1)$, rather than n, to give us what we will later call 30 Descriptive Statistics: Graphical and Numerical Summaries an "unbiased" measure of the true variation. This division by $(n-1)$ may also be justified by the fact that only $(n-1)$ of these deviations are actually independently determined, since their total is always fixed at zero. The standard deviation is a useful alternate measure of spread:

Standard Deviation

The ***standard deviation*** for a quantitative variable measures the spread of the data

$$s = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n-1}}$$

- Sample standard deviation: s
- Population standard deviation: σ ("sigma")

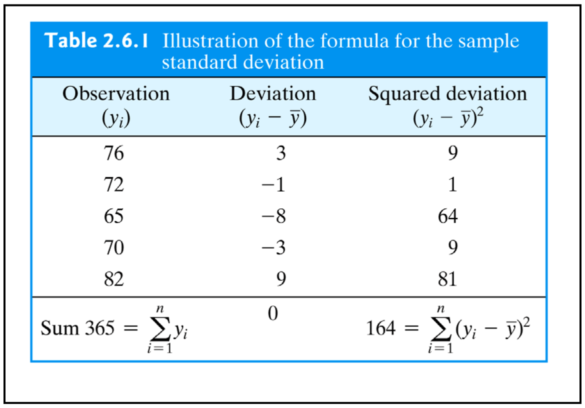


Table 2.6.1 Illustration of the formula for the sample standard deviation

Observation (y_i)	Deviation $(y_i - \bar{y})$	Squared deviation $(y_i - \bar{y})^2$
76	3	9
72	−1	1
65	−8	64
70	−3	9
82	9	81
Sum 365 = $\sum_{i=1}^{n} y_i$	0	164 = $\sum_{i=1}^{n} (y_i - \bar{y})^2$

Visualizing the Standard Deviation

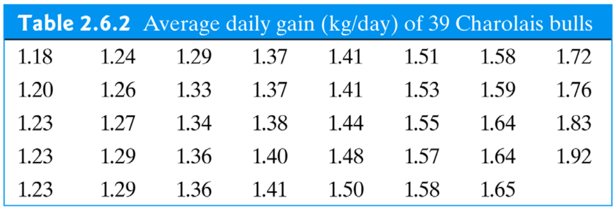
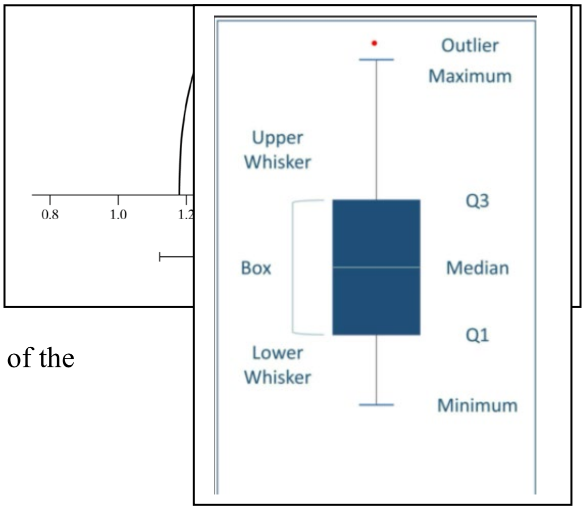


Table 2.6.2 Average daily gain (kg/day) of 39 Charolais bulls

1.18	1.24	1.29	1.37	1.41	1.51	1.58	1.72
1.20	1.26	1.33	1.37	1.41	1.53	1.59	1.76
1.23	1.27	1.34	1.38	1.44	1.55	1.64	1.83
1.23	1.29	1.36	1.40	1.48	1.57	1.64	1.92
1.23	1.29	1.36	1.41	1.50	1.58	1.65	



For this data set, the graph to the right shows a smoothed histogram and boxplot of 39 daily gain measurements. The shaded area contains about 50% of the observations.

5.5 Box plots, quartiles, and the median

5-Number Summary – $Min, Q_{1,} M, Q_{3,} Max$

A boxplot is better for describing skewed distributions or those w/ strong outliers
Quartile – every 25% of the data
Outlier – shown as upper or lower points above or below the boxplot

Min – the smallest valued observation
Q1 – the 25th percentile of observations (the lower quartile)
M – the 50th percentile of observations (the median)
Q3 – the 75th percentile of observations (the upper quartile)
Max – the largest valued observation

EXAMPLE: Use the data about sodium in boxed cereal to

1. List the sd, mean, and 5-number summary
2. Create a boxplot
3. Determine if there are any potential outliers in the cereal sodium data using the formula and the boxplot.
4. Find the IQR
5. Create a histogram and boxplot

0	340	70	140	200	180
210	150	100	130	140	180
190	160	290	50	220	180
200	210				

Here is the code in R:

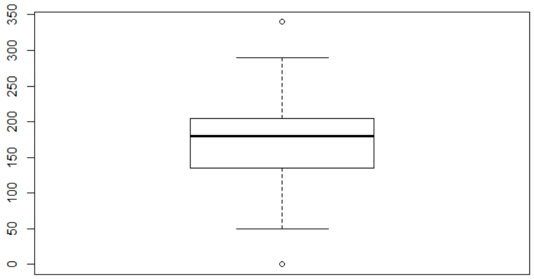
```
sodium <- c(0,    340,  70,   140,  200,  180,  210,  150,  100,  130,  140,  180,  190,
      160,  290,  50,   220,  180,  200,  210)

sd(sodium) # will give the standard deviation
mean(sodium)
median(sodium)
summary(sodium)  #give the mean and 5-number summary
IQR(sodium)
hist(sodium, main = "Sodium in Cereal (g)", ylab = "Sodium (g)")
boxplot(sodium, main = "Sodium in Cereal (g)", ylab = "Sodium (g)")
```

OUTPUT

```
> sodium <-c(0,340,70,140,200,180,210,150,100,130,140,180,190,160,290,50,220,180,200,210)
> sd(sodium) # will give the standard deviation
[1] 77.26237
> mean(sodium)
[1] 167
> median(sodium)
[1] 180
> summary(sodium)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max. 
    0.0   137.5   180.0   167.0   202.5   340.0 
> IQR(sodium)
[1] 65
```

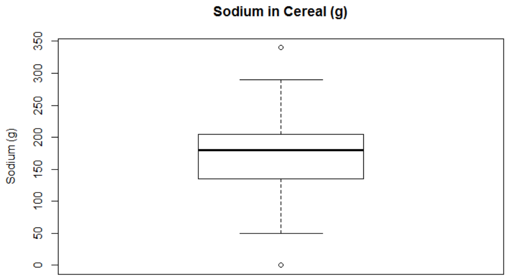
```
boxplot(sodium) #create a boxplot
```



Notice there are both upper and lower outliers. You should try using the formula to prove these are outliers:

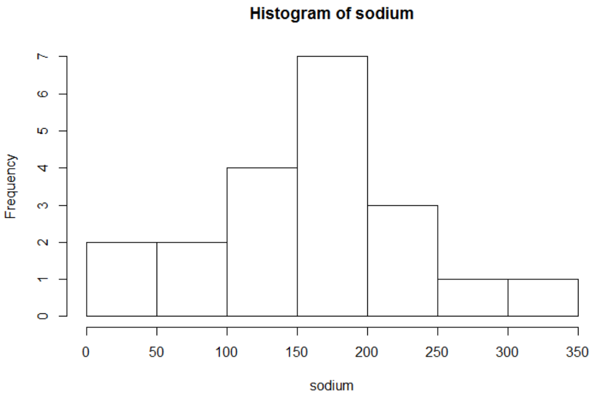
ADD TITLE AND Y-AXIS LABEL

```
boxplot(sodium, main = "Sodium in Cereal (g)", ylab = "Sodium (g)")  # If you are interested in putting a title and y-axis label
```



CREATE A HISTOGRAM

```
hist(sodium)
```



There is a default number of breaks for a base histogram (function “hist”). If you want to change the number of breaks, you simply write the code:

```
hist(sodium, breaks = 20)
```

Try playing with changing the number of breaks to see how that changes the histogram’s shape (i.e. distribution). If you have questions on how “breaks” works, try the code:

```
?hist
```

This will provide more information about how breaks are calculated.

5.6 Robust statistics

Robust statistics are values are **not heavily influenced by outliers**. The **median and IQR** are called robust statistics because extreme observations have little effect on their values: moving the most extreme value generally has little influence on these statistics. On the other hand, the **mean and standard deviation are more heavily influenced by changes in extreme observations**. The choice of center affects the types of statistical analysis you will need to choose.

5.7 Transforming data

When data are very strongly skewed, we sometimes transform them so they are easier to model.

A scatterplot of the population change from 2010 to 2017 against the population in 2010 is shown in Figure 5.11. In this first scatterplot, it's hard to decipher any interesting patterns because the population variable is so strongly skewed (left plot). However, if we apply a log10 transformation to the population variable, as shown in Figure 5.11, a positive association between the variables is revealed (right plot). In fact, we may be interested in fitting a trend line to the data when we explore methods around fitting regression lines in Chapter 7.

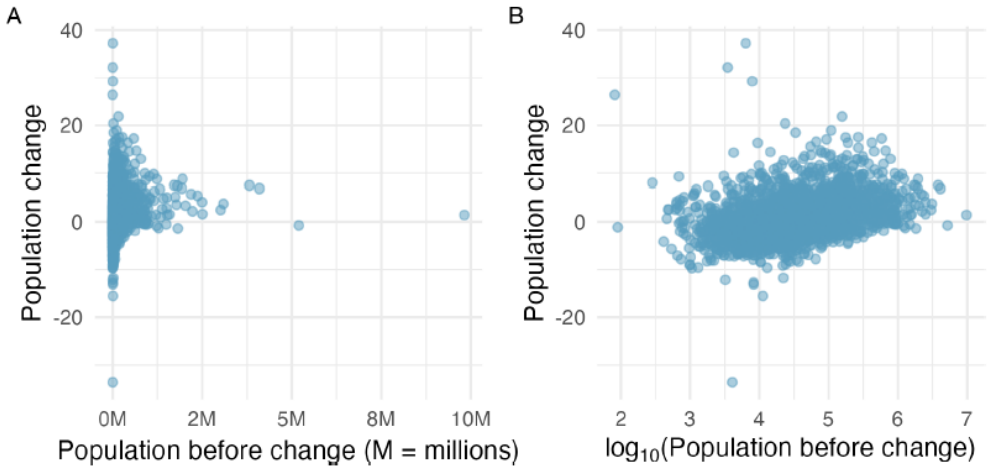
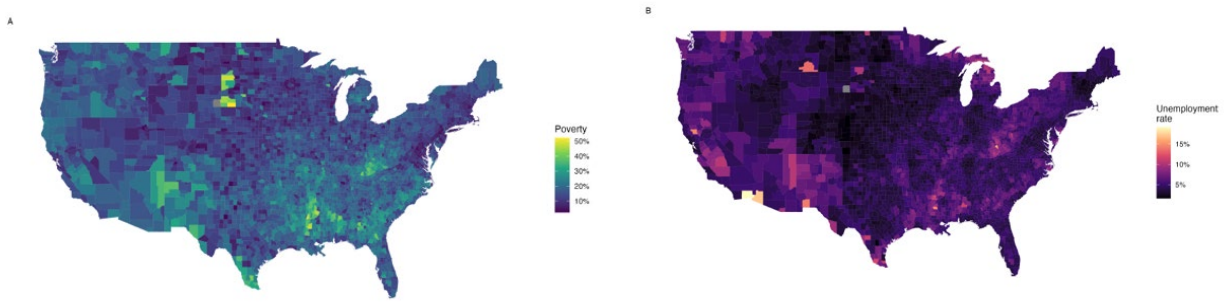


Figure 5.11: Plot A: Scatterplot of population change against the population before the change. Plot B: A scatterplot of the same data but where the population size has been log-transformed.

Transformations other than the logarithm can be useful, too. For instance, the square root $\sqrt{original\ observation}$ and inverse $\left(\frac{1}{original\ observation}\right)$ are commonly used by data scientists. Common goals in transforming data are to see the data structure differently, reduce skew, assist in modeling, or straighten a nonlinear relationship in a scatterplot.

5.8 Mapping data

For **geographic data**, create an **intensity map**, where colors are used to show higher and lower values of a variable. Figures 5.12 and 5.13 show intensity maps for **poverty rate in percent (poverty)** and **unemployment rate in percent (unemployment_rate)**. The color key indicates which colors correspond to which values. The intensity maps are not generally very helpful for getting precise values in any given county, but they are very helpful for seeing geographic trends and generating interesting research questions or hypotheses.



Homework Chapter 5

1. Review section 5.9 (the chapter review)
2. Suggested problems from textbook section 5.10 exercises: 10, 11 – 25 odd only
3. Suggested tutorials: Tutorial 1 – Lessons 1, 2, and 3

   ○ **Visualizing categorical data**

   ○ **Visualizing numerical data**

   ○ **Summarizing with statistics**