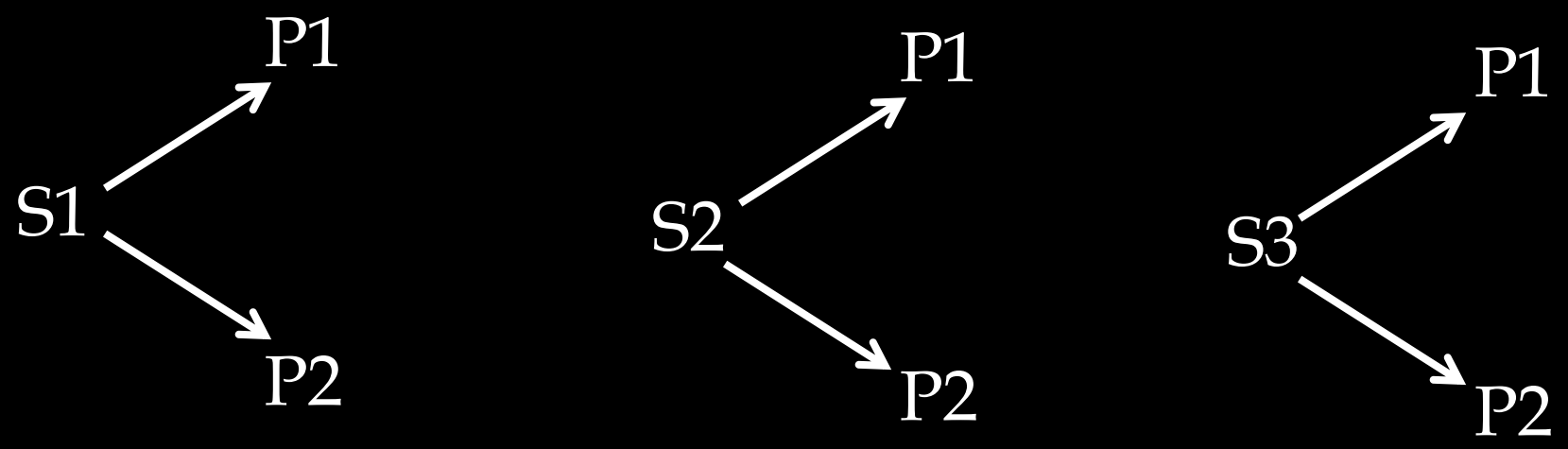
Permutation
&
Combination

Fundamental Principal of counting

- **Rule of product :** If there are **‘m’** ways to do a process and there are **‘n’** ways to do another, then total number of ways of doing both process is given by **‘m x n’**

• If there are 3 shirts and 2 pants then in how many ways a person can dress up for a seminar?



Directly, Total ways = 3 * 2 = 6 ways

- **Rule of addition :** If there are **‘m’** ways to do a process and there are **‘n’** ways to do another and we can not do both at the same time, then there are ‘**m + n**’ ways to choose one of the actions.

• If there are 3 formal shoes and 2 casual shoes then in how many ways we can choose a footwear for a party.

F1, F2, F3, C1, C2

F1 or F2 or F3 or C1 or C2

Total ways = 3 + 2 = 5 ways

Question: Let us assume you have 3 shirts, 4 pants, 3 shoes and 2 sandals to wear. Find in how many ways you can decide an outfit.

1. **Shirt – Pants – Shoes**
2. **Shirt – Pants – Sandal**

(3x4x3) + (3x4x2) = 60

Note : Multiplication ------- “ **AND**” (Stages)

Addition---------- “ **OR**” (Choice)

Difference between Permutation and Combination

Permutation : Arrangement
: Order matters

Combination : Selection
: Order doesn't matters

Permutations and Combinations

Number of permutations (order matters) of n things taken r at a time:

$$P(n,r) = \frac{n!}{(n-r)!}$$

Number of combinations (order does not matter) of n things taken r at a time:

$$C(n,r) = \frac{n!}{(n-r)!r!}$$

Question: If suppose we have 3 objects A, B, C then find no. of ways in which any 2 items can be selected.

Answer:

1. AB (BA)
2. BC (CB)
3. CA (AC)

Question: If suppose we have 3 objects A, B, C then find no. of ways to arrange any 2 items.

Answer:

1. AB
2. BA
3. BC
4. CB
5. CA
6. AC

Practice Question

1. In how many ways can we select a team of 4 players out of 15 eligible players.

[A] 1365

[B] 1455

[C] 1295

[D] 1525

2. In a class there are 6 boys and 5 girls. In how many ways can a group of 5 members to be formed by selecting 3 boys and 2 girls.

[A] 350

[B] 300

[C] 250

[D] 200

3. In how many ways 3 VIPs can be seated in 3 seats of first row of a function.

[A] 3

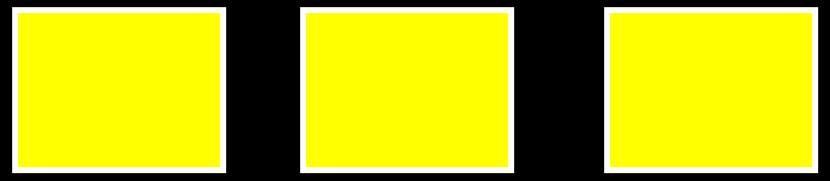
[B] 4

[C] 5

[D] 6

Note : Number of ways of arranging ‘n’ different items in a row = n !

In previous question,



3 x 2 x 1 = 3! = 6 ways

4. In how many ways 5 medals of different games can be arranged in a shelf.

[A] 100

[B] 110

[C] 120

[D] 150

5. Suppose you have to choose a 3 letter password. First letter is an alphabet, followed by a number and last one is an special character. There are 5 special character available. Find no. of ways to choose password.

[A] 1050

[B] 1200

[C] 1300

[D] 1560

Problems on Numbers

6. How many 2 digit numbers can be made from the digits 1, 2, 3 and 4 without repetition?

[A] 24

[B] 18

[C] 12

[D] 6

7. How many 4 digit numbers are possible with the digits 1, 2, 3, 6, 7, 8 and 9 without repetition?

[A] 720

[B] 480

[C] 840

[D] 320

8. How many 4 digit numbers are possible with the digits 1, 2, 3, 6, 7, 8 and 9 if repetition is allowed?

[A] 2401

[B] 820

[C] 343

[D] 729

9. How many 4 digit numbers can be made from the digits 7, 8, 5, 0, and 4 without repetition?

[A] 70

[B] 96

[C] 84

[D] 48

10. How many 3 digit numbers greater than 400 can be made with the digits 2, 3, 4, 0, 5, 6 (digits cannot be repeated)?

[A] 119

[B] 59

[C] 120

[D] 60

11. How many 3 digit numbers between 200 and 700 can be made with the digits 1, 3, 4, 0, 5, 6 (digits cannot be repeated) ?

[A] 80

[B] 120

[C] 60

[D] None of these

12. How many 3 digit number can be formed with the digits 5, 6, 2, 3, 7 and 9 which are divisible by 5 and none of its digit is repeated?

[A] 12

[B] 16

[C] 20

[D] 100

13. How many 4 digit number can be formed with the digits 0, 1, 2, 3, 4, 5, 6 which are divisible by 5 and none of its digit is repeated?

[A] 120

[B] 100

[C] 220

[D] 320

14. How many 4 digit odd number can be formed with the digits 0, 1, 2, 3, 4, 5, 6 if none of its digit is repeated?

[A] 120

[B] 100

[C] 220

[D] 300

15. How many 4 digit even number can be formed with the digits 0, 1, 2, 3, 4, 5, 6 if none of its digit is repeated?

[A] 120

[B] 420

[C] 220

[D] 200

16. Find the no of 3 digit numbers such that at least one of the digit is 6 (with repetitions)?

[A] 252

[B] 345

[C] 648

[D] 560

Problems on Words:

17. In How many different ways the letters of the word EQUATION can be arranged ?

[A] 7!

[B] 8!

[C] 9!

[D] 6!

18. In How many different ways the letters of the word EQUATION can be arranged, if it starts with letter Q ?

[A] 7!

[B] 8!

[C] 9!

[D] 6!

19. In How many different ways the letters of the word EQUATION can be arranged, if it starts with consonants?

[A] 7!

[B] 8!

[C] 2*7!

[D] 3*7!

20. In How many ways the word OPTICAL be arranged such that all vowels are together?

[A] 720

[B] 820

[C] 2160

[D] 1000

21. In How many ways the word OPTICAL be arranged such that all vowels are never together?

[A] 720

[B] 1000

[C] 2160

[D] 4320

22. In How many ways the word MANPOWER be arranged such that all vowels are together?

[A] 3! 6!

[B] 2! 7!

[C] 3! 5!

[D] 4! 4!

23. In How many ways letters of word PRAISE be arranged such that all consonants are together?

[A] 3! 4!

[B] 4! 4!

[C] 3! 5!

[D] 4! 5!

24. In How many ways letters of word PREVIOUS be arranged such that all vowels always come together?

[A] 1440

[B] 2880

[C] 4320

[D] 840

25. In how many ways can the letters of word FLEECED be arranged?

[A] 410

[B] 880

[C] 840

[D] 1260

26. Find the total arrangement of the letters of the word “MISSISSIPPI?

[A] 34650

[B] 32540

[C] 28450

[D] 24560

27. In how many different ways can the letter of the word “ELEPHANT” be arranged so that E’s are never together?

[A] 5040

[B] 15120

[C] 20160

[D] 35280

28. Find the total arrangement of the letters of the word “INVISIBILITY” such that all ‘I’ always come together.

[A] 8!

[B] 8!*5!

[C] 8!*5

[D] 7!*5!

29. In how many ways can the letters of the word "MACHINE" be arranged so that the vowels may occupy only odd positions?

[A] 4*7!

[B] 576

[C] 288

[D] 4 * 4!

30. Find the rank of the word “CHASM” if all the words can be formed by permuting the letters of this word without repetition are arranged in dictionary order.

[A] 24

[B] 31

[C] 32

[D] 30

31. Find the rank of the word “JAIPUR” if all the words can be formed by permuting the letters of this word without repetition are arranged in dictionary order.

[A] 241

[B] 122

[C] 123

[D] 242

31. Find the rank of the word “INDIA” if all the words can be formed by permuting the letters of this word without repetition are arranged in dictionary order.

[A] 41

[B] 42

[C] 45

[D] 46

32. Find the rank of the word “GOOGLE” if all the words can be formed by permuting the letters of this word without repetition are arranged in dictionary order.

[A] 78

[B] 84

[C] 85

[D] 88

Problems on Combination (Group Formation)

33. In how many ways a group of 4 men and 3 women be made out of a total of 8 men and 5 women?

[A] 720

[B] 700

[C] 120

[D] 360

34. There are 8 men and 7 women. In how many ways a group of 5 people can be made such that the particular woman is always to be included?

[A] 860

[B] 1262

[C] 1001

[D] 1768

35. There are 4 men and 3 women. In how many ways a group of three people can be formed such that there is at least 1 women in the group.

[A] 40

[B] 20

[C] 34

[D] 31

36. In a group of 6 boys and 5 girls, 5 students have to be selected. In how many ways it can be done so that at least 2 boys are included.

[A] 124

[B] 526

[C] 154

[D] 431

37. A box contains ten balls out of which 3 are red and rest blue. In how many ways can a random sample of six balls be drawn so that at most 2 red balls are included.

[A] 105

[B] 189

[C] 168

[D] 175

38. In a party there are 12 persons. How many handshakes are possible if every person handshake with every other person?

[A] 66

[B] 24

[C] 72

[D] 68

Circular arrangements

n distinct objects --------- Linear---------n!
n distinct objects---------- Circular----- (n-1)!

Note: In circle there is symmetry and hence there is no starting and end point, so when we need to arrange n distinct objects around a circle 1st object will break the symmetry (specify the position) and it can be done in 1 way and rest (n-1) objects can be arranged in (n-1)! Ways

Circular arrangement of n objects= 1 x (n-1)!= (n-1)!

If there is a difference between Clockwise and anti-Clockwise arrangement , and if

1. We need to arrange r objects out of n objects then **= nPr/r**

2. We need to arrange all n distinct objects **=nPn/n = n!/n = (n-1)!**

If there is no difference between Clockwise and anti-Clockwise arrangement (like in case of Garlands, Bead and Necklace etc.) , and if

1. We need to arrange r objects out of n objects then = **nPr/2r**

2. We need to arrange all n distinct objects = **nPn/2n = n!/2n = (n-1)!/2**

39. In how many ways 5 Americans and 5 Indians be seated along a circular table, so that they occupy alternative positions

[A] 5! 5!

[B] 6! 4!

[C] 4! 5!

[D] 4! 4!

40. A meeting of 20 delegates is to be held in a hotel. In how many ways these delegates can be seated around a circular table if 3 particular delegates always seat together.

[A] 17! 3!

[B] 18! 3!

[C] 17! 4!

[D] None

41. How many triangles can be formed by joining the vertices of hexagon?

[A] 20

[B] 12

[C] 24

[D] 10

42. How many diagonals can be formed by joining the vertices of hexagon?

[A] 10

[B] 12

[C] 9

[D] 8