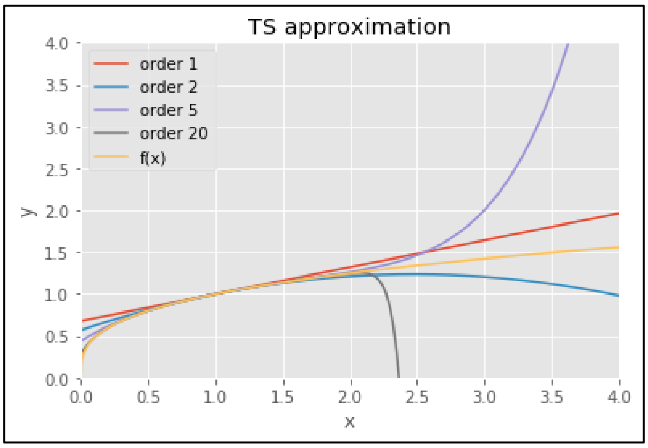
Quantitative Macroeconomics
Homework 2

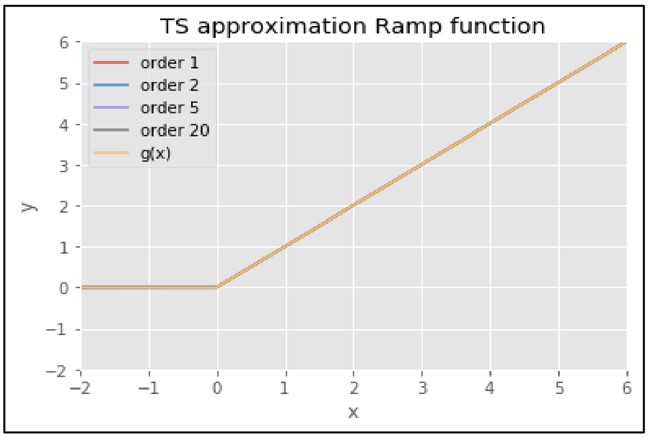
Question 1. Function Approximation: Univariate

1.



The function clearly indicates that as we move away from the original function the errors increase. Additionally, as higher orders are taken the error will increase. The approximation error is at a minimum when x=1.

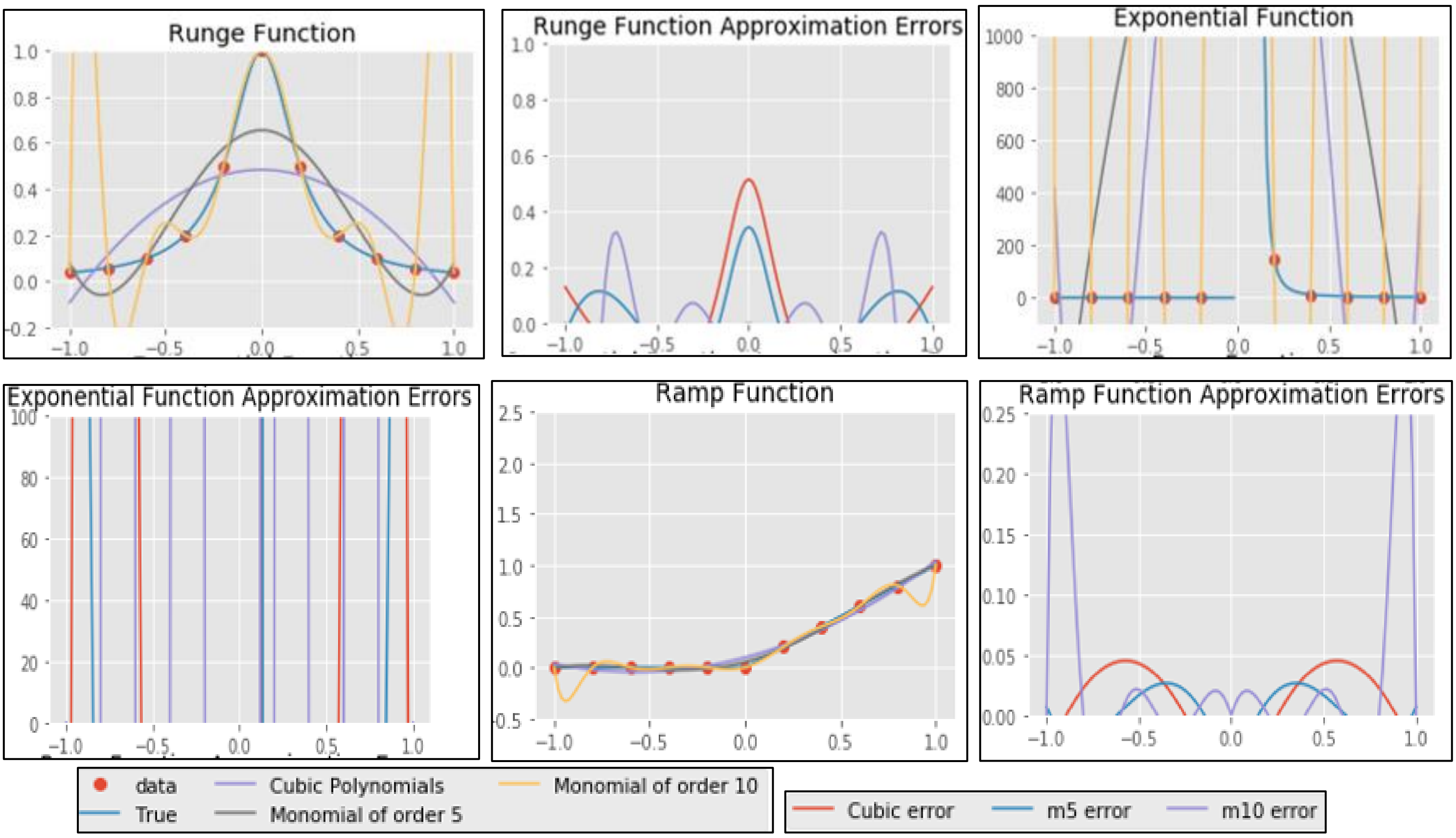
2.



The ramp function shows that when the function is approximated close to 2 there is no ramp. Within the positive part of the domain the Taylor approximations fit the ramp function perfectly.

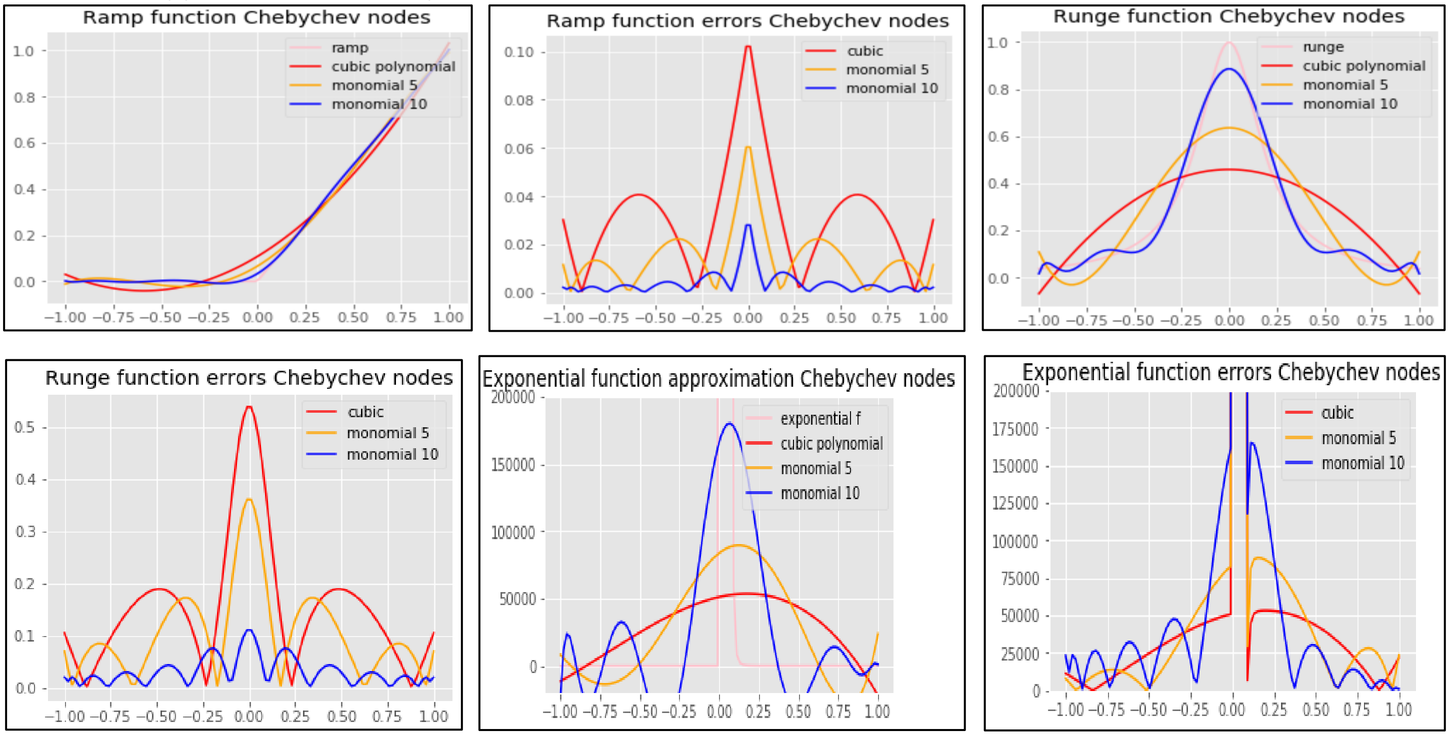
3(a).

Function Approximation: Evenly-Spaced nodes & monomials



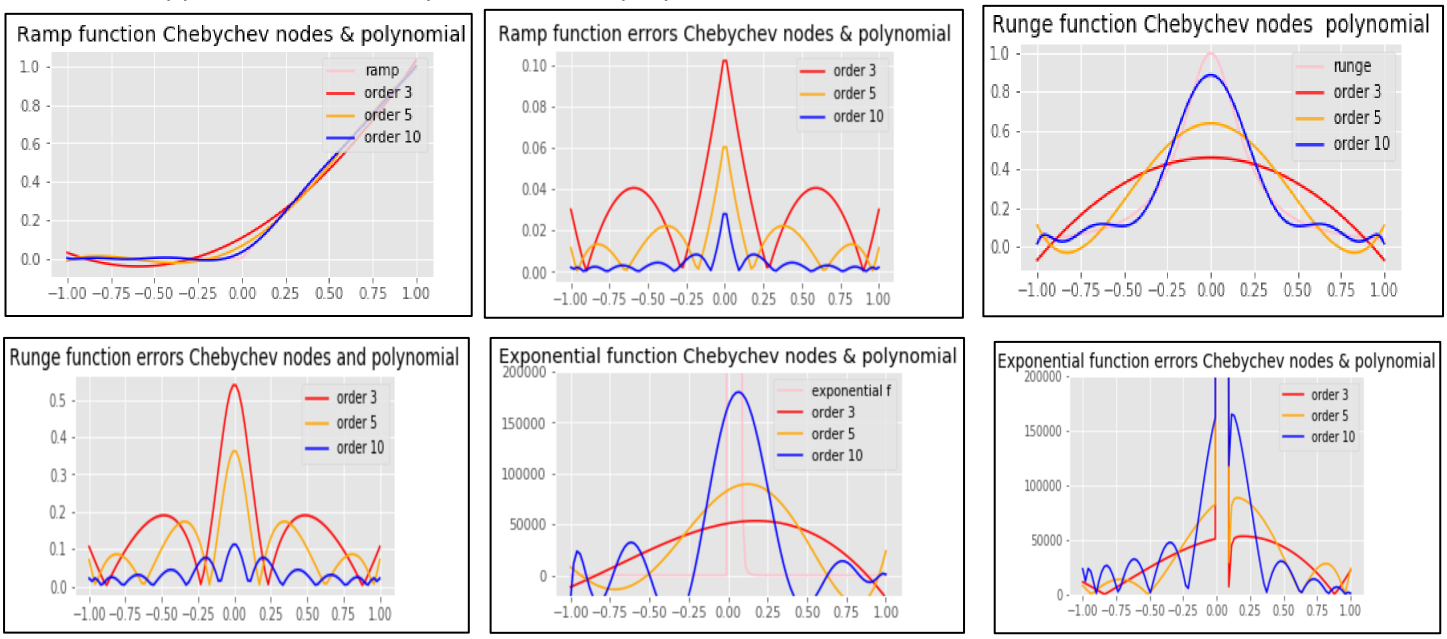
3(b).

Function Approximation: Chebyshev nodes



3(c).

Function Approximation: Chebyshev nodes & polynomial



The graphs estimated with Chebyshev nodes & polynomials, do not differ significantly from the case of approximates solely from Chebyshev nodes. However, it appears to reduce extreme oscillations and errors at the centre.

Question 2. Function Approximation: Multivariate

$$f(k,h) = ((1-\alpha)k^{\frac{\sigma-1}{\sigma}} + (\alpha)h^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}}$$

1. Show that σ is the elasticity of substitution:

The elasticity of substitution is defined as:

$$\sigma_{k,h} = \left(\frac{\partial \ln\left(\frac{k}{h}\right)}{\partial \ln\left(MRS_{k,h}\right)}\right)$$

$$MRS = \left(\frac{\frac{\partial f(k,h)}{\partial h}}{\frac{\partial f(k,h)}{\partial k}}\right)$$

$$\frac{\partial f(k,h)}{\partial h} = \left(\frac{\sigma}{\sigma-1}\right)\left(\left(\frac{\sigma-1}{\sigma}\right)(\alpha)h^{\left(\frac{\sigma-1}{\sigma}-1\right)}\right)^{\left(\frac{\sigma}{\sigma-1}-1\right)}$$

$$\frac{\partial f(k,h)}{\partial k} = \left(\frac{\sigma}{\sigma-1}\right)\left(\left(\frac{\sigma-1}{\sigma}\right)(1-\alpha)k^{\left(\frac{\sigma-1}{\sigma}-1\right)}\right)^{\left(\frac{\sigma}{\sigma-1}-1\right)}$$

$$MRS = \left(\frac{\frac{\partial f(k,h)}{\partial h}}{\frac{\partial f(k,h)}{\partial k}}\right) = \frac{\left(\frac{\sigma}{\sigma-1}\right)\left(\left(\frac{\sigma-1}{\sigma}\right)(\alpha)h^{\left(-\frac{1}{\sigma}\right)}\right)^{\left(\frac{\sigma}{\sigma-1}-1\right)}}{\left(\frac{\sigma}{\sigma-1}\right)\left(\left(\frac{\sigma-1}{\sigma}\right)(1-\alpha)k^{\left(-\frac{1}{\sigma}\right)}\right)^{\left(\frac{\sigma}{\sigma-1}-1\right)}} = \frac{(\alpha)h^{\left(\frac{-1}{\sigma}\right)}}{(1-\alpha)k^{\left(\frac{-1}{\sigma}\right)}}$$

Next, undertake a logarithmic transformation of the MRS in order to compute the elasticity of substitution.

$$MRS = \frac{(\alpha)}{(1-\alpha)}\left(\frac{h}{k}\right)^{\left(\frac{-1}{\sigma}\right)}$$

$$\ln(MRS) = ln\left(\frac{(\alpha)}{(1-\alpha)}\right) + ln\left(\frac{1}{\sigma}\right)\left(\frac{k}{h}\right)$$

Finally, get the derivative of this function with respect to $ln\left(\frac{h}{k}\right)$ to obtain:

$$\sigma_{k,h} = \text{elasticity of substituion}$$

2. Compute labor share for an economy with that CES production function assuming factor inputs face competitive markets:

$$\text{Labor Share} = \frac{hw}{f(k,h)}$$

$$\text{W}= \alpha h^{\frac{-1}{\sigma}}\left[(1-\alpha)k^{\frac{\sigma-1}{\sigma}} + \alpha h^{\left(\frac{\sigma-1}{\sigma}\right)}\right]^{\frac{1}{\sigma-1}}$$

$$\therefore \frac{hw}{f(k,h)} = \frac{\alpha h^{\frac{\sigma-1}{\sigma}}}{\left[(1-\alpha)k^{\frac{\sigma-1}{\sigma}} + \alpha h^{\frac{\sigma-1}{\sigma}}\right]^{\sigma}}$$