Quantitative Macroeconomics Project

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Question 1

Proof of proposition 3, Harenberg and Ludwig (2015)

The proof is by guessing and verifying. As all households are ex-ante indentical, we guess that:

$$a_{2,t+1} = s(i-\tau)w_t = s(1-\tau)(1-\alpha)Y_tC_tK_t^{\alpha}$$

If this is correct, then the equilibrium dynamics are given by

$$K_{t+1} = a_{2,t+1} = s(1-\tau)(1-\alpha)Y_tC_tK_t^{\alpha}$$

$$\text{as} \;\; K_{t+1} = \frac{(k_{t+1})}{(Y_{t+1}(1+\lambda)}$$

$$\text{we} \quad \text{get} \quad K_{t+1} = \frac{1}{(1+g)(1+\lambda)}s(1-\tau)(1-\alpha)C_tK_t^{\alpha}$$

To verify

$$K_{t+1} = \frac{1}{(1+g)(1+\lambda)}s(\tau)(1-\tau)(1-\alpha)C_tK_t^{\alpha}$$

notice that our guess for

$$a_2, t+1$$

implied that:

$$C_{1,t} = (1-s)(1-\tau)(1-\alpha)Y_tC_tK_t^{\alpha}$$

$$C_{1,2,t+1} = (s)(1-\tau)(1-\alpha)Y_tC_tK_t^{\alpha}(\alpha)(C_{t+1})(\rho)_{t+1}(k_{t+1}^{\alpha-1}) + (1-\alpha)Y_{t+1}C_{t+1}K^(\alpha)_{t+1}(\lambda)(\eta_{i,2,t+1}) + \tau(1+\lambda(1-\eta_{i,2,t+1})$$

where we used the budget constraint.

employing K_{t+1}

$$K_{t+1} = \frac{1}{(1+g)(1+\lambda)}s(\tau)(1-\tau)(1-\alpha)C_tK_t^{\alpha}$$

we get

$$C_{i,2,t+1} = (\alpha(\rho_{t+1}(1+\lambda) + (1-\alpha)(\lambda)(\eta_{i,2,t+1} + \tau(1+(\lambda)(1-\eta_{i,2,t+1}))Y_{t+1}C_{t+1}K^(\alpha)_{t+1}$$

Next notice that the first-order condition of household maximization gives

$$1 = (\beta)E_t[\tfrac{c_{1,t}(1+r_{t+1})}{c_{i,2,t+1}}]$$

$$1 = (\beta)E_t[\tfrac{c_{1,t}^{\alpha}C_{t+1}\rho_{t+1}K_{t+1}^{\alpha-1}}{(\alpha)(rho_{t+1})(1+\lambda)+(1-\alpha)(\lambda\eta_{i,2,t+1}+\tau(1+\lambda(1-\eta_{i,2,t+1}))Y_{t+1}C_{t+1}K_{t+1}^{\alpha}}]$$

$$1 = \tfrac{\beta(1-s)}{s}\Phi$$

Where Φ is defined in

$\mathrm{S}(\tau) = \frac{\beta\Phi(\tau)}{1+\beta\Phi(\tau)} \leq \frac{\beta}{1+\beta}$

Equation $\Phi(\tau) = E_t[\frac{1}{1+\frac{1-\alpha}{\alpha(1+\lambda)\rho_{t+1}}(\lambda\eta_{i,2,t+1}+\tau(1+\lambda(1-\eta_{i,2,t+1}))}]$ immediately follows.

Since the problem is concex, the solution is unique. As for the upper bound on Φ observe that $\Phi = 1$ for $\lambda = 0$.

For λ>0,$\Phi = E_t[\frac{1}{1+x}]$ for $x = \frac{1-\alpha}{(\alpha)(rho_{t+1})(1+\lambda)}(\lambda\eta_{i,2,t+1} + \tau(1 + \lambda(1 - \eta_{i,2,t+1}))$

Our assumptions ensure that $x \leq 0$, hence $\Phi = E_t[\frac{1}{1+x}]\leq 1$,

which implies that s$\leq \frac{\beta}{1+\beta}$

Question 2

Steps of algorithm to solve:

1. Guess p_{s10} p_{si1} given this and K_t we have law of motion of k.
2. Now we can solve policy function by using Euler equation by employing a unit root solver.
3. We can simulate forward find k, c_t, c_{t+1} for a given T=50,000
4. Run regressions to find psi when X=k, y=k (t-1]
5. Now we can solve for Household problem —ψ_4-ψ_0—$\geq\epsilon$ until convergence.

Question 3

The value of the variable saving from Matlab was 0.0293.

The savings grid was as follows:

$-0.68676 \quad -0.41211 \quad -0.13746 \quad 0.13719 \quad 0.41185$

Complex Variant of Krussell-Smith Algorithm

For the Complex Variant I roughly tried to use the same five steps outlined above as in the Simple Krussel-Simple case. Except in this case the saving variable now 5 dimensions. The policy function x_t is five dimensions along with other policies. However I was unable to reproduce the results. I was attempting to generate a five node grid for the state variables of the household (j,a,η,z,k).