Quantitative Macroeconomics - Question 1

1. Compute the steady state. Choose z to match an annual capital-output ratio of 4, and an investment-output ratio of .25.

   The consumer maximization problem can be represented as the following:

$$\max E_o \left[\sum_{t=0}^{\infty} \beta_t u(ln(c_t)) \right]$$

$$c_t + i_t = y_t \tag{1}$$

$$y_t = k_t^{1\theta}(zh_t)^\theta \tag{2}$$

$$i_t = k_{t+1} + (1-\delta)k_t \tag{3}$$

   Therefore, we can define the Lagrangian function as the following:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta_t u(ln(c_t)) + \lambda(k_t^{1-\theta}(zh_t)^\theta - c_t - k_{t+1} + (1-\delta)k_t)$$

   From F.O.C conditions we can derive the following equations:

$$\frac{\partial \mathcal{L}}{\partial c_t} = \frac{\beta^t}{c_t} - \lambda_t = 0$$

$$\frac{\partial \mathcal{L}}{\partial k_{t+1}} = -\lambda_t + \lambda_{t+1}[(zh)^\theta(1-\theta)k_{t+1}^{-\theta} + (1-\delta)] = 0$$

   Therefore, we can get the steady state equilibrium solving the F.O.C equations:

$$k = \left[\frac{(1-\theta)(zh)^\theta}{\frac{1}{\beta} - 1 + \delta} \right]^{\frac{1}{\theta}}$$

   Since, $\frac{\delta k}{y} = 0.25$ and $\frac{k}{y} = 4$. We can derive z using the budget constraints: $c = y - \delta k$ and Production Function: $y = k^{1-\theta}(zh)^\theta$.
   Where,

$$z = \left[\frac{k^\theta(\frac{1}{\beta} - 1 + \delta)}{(1-\theta)h^\theta} \right]^{\frac{1}{\theta}}$$

   . Normalizing $y = 1$, from $y = k^{1-\theta}(zh)^\theta$ we get:

$$z = \left[\frac{\frac{1}{\beta} - 1 + \delta}{1-\theta} \right]^{\frac{1-\theta}{\theta}} \frac{1}{h}$$

   Outputs at Steady State are reported in Table 1.

Table 1: Question 1

	Value ,
Z Steady State	1.63,
Capital Steady State	4.00,
Output Steady State	1.00,
Consumption Steady State	0.75,
Investment Steady State	0.25,

Table 2: Question 2

	Value ,
Z New Steady State	3.26,
Capital New Steady State	8.00,
Output New Steady State	2.00,
Consumption New Steady State	1.50,
Investment New Steady State	0.50,

2. Double permanently the productivity parameter z and solve for the new steady state. Outputs at new steady state are reported in Table 2.

3. Compute the transition from the first to the second steady state and report the time path for savings, consumption, labor and output. The transition graph is reported in Figure 1.

4. Unexpected shocks. Let the agent believe productivity z doubles once and for all periods. However, after 10 periods, surprise the economy by cutting the productivity back to its original value. Compute the transition for savings, consumption, labor and output. Figure 2 reports the transition path.

5. Can taxes explain differences in the speed of transition to steady-state?
   (a) Add a permanent capital tax. Recompute the new SS, and the transitions.
   A permanent capital tax will change the capital at steady state such that:

$$k^* = (\frac{1-\theta}{\frac{1}{\beta}+\delta+\tau-1})^{\frac{1}{\delta}} zh$$

   Therefore, we can clearly observe this will change the output, consumption and savings in the economy. This will eventually lead to a change in the transition towards the steady state depending on the magnitude of the tax introduced. We have assumed a tax rate of 4 percent.

   Output at steady state after capital tax and output at new steady state after capital tax are reported in Table 3. The transition instead are:
   Transition of capital with permanent capital tax 6.39
   Transition of consumption with permanent capital tax 0.67

Figure 1: Transition graph

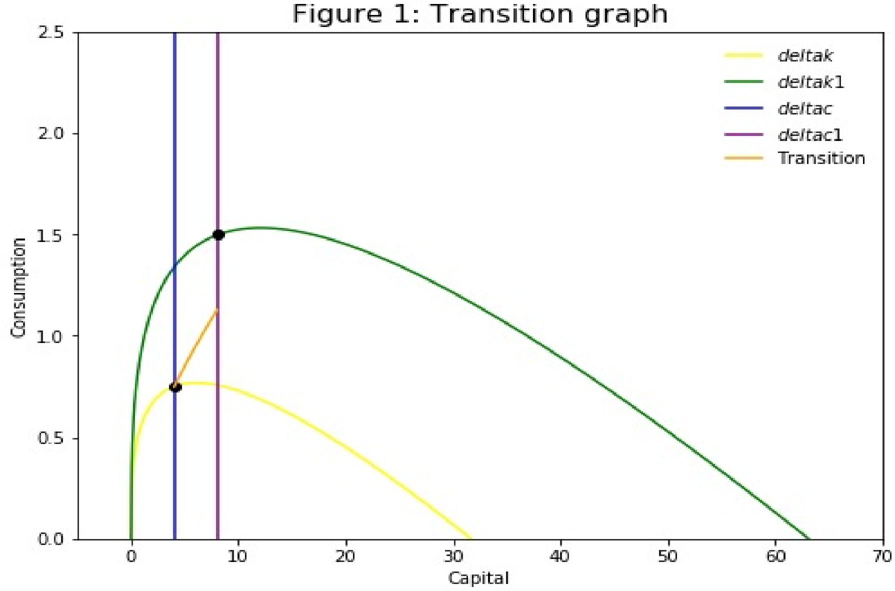

Figure 2: Effect of shock to z

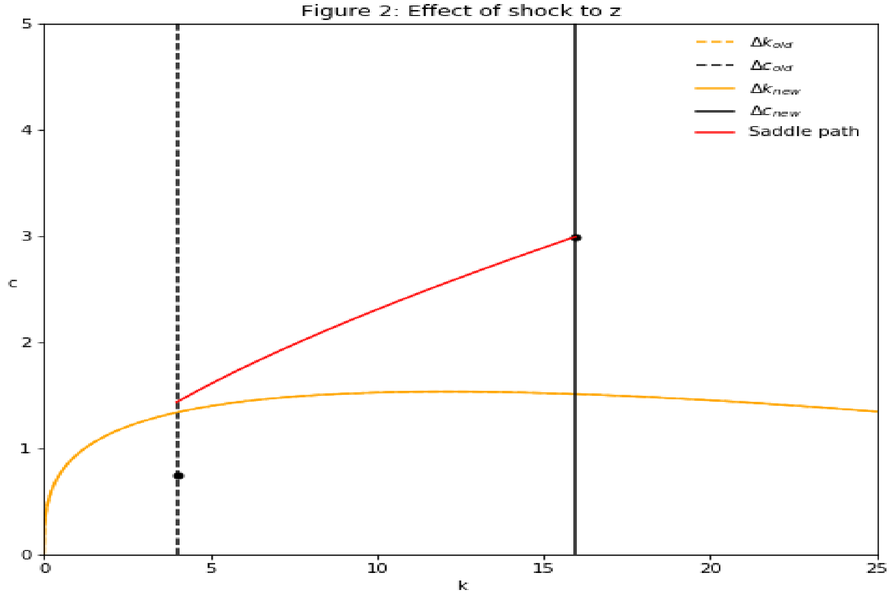

Table 3: Question 5

	Value ,
Capital Old Steady State after tax	1.85,
Consumption Old Steady State after tax	0.50,
Output Old Steady State after tax 0.61	0.61,
Capital per output Old Steady State after tax	3.02,
Investment per output Old Steady State after tax	0.19,
Capital New Steady State after tax	4.44,
Consumption New Steady State after tax	1.19,
Output New Steady State after tax 0.61	1.47,
Capital per output New Steady State after tax	3.02,
Investment per output New Steady State after tax	0.19,

(b) Add a permanent consumption tax. Recompute the new SS, and the transitions. The effect of a tax on consumption only changes the budget constraint where,

$$(1-\tau) * c_t + i_t = y_t$$

Hence, if we re-estimate the Euler equation since, the tax is the same across all periods the steady state will remain the same.

Quantitative Macroeconomics - Question 2

1. Consider the case that each of these two countries are closed economies.
   (a) Write the equilibrium of a closed economy. We can start from the firm's problem:

$$\max_{K^d,H^d} Z(H^d)^{1-\theta}H^{d\theta} - wH^d - rK^d$$

The $FOCK^d$ and $FOCK^d$ solved with respect to r and w are:

$$r = z(1-\theta)(K^d)^{-\theta}(H^d)^{\theta}$$

$$w = z\theta(K^d)^{1-\theta}(H^d)^{\theta-1}$$

The household maximization problem is instead:

$$\max_{c,h,k} \frac{c^{1-\sigma}}{1-\sigma} - k\frac{h^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}$$

$$c = \lambda(w(h)\eta)^{1-\phi} + rk^{\eta} \tag{1}$$

We got two different Euler equations, one for type low and one for type high, i.e.:

$$(1-\phi)c^{-\sigma}\lambda(wh\eta)^{-\phi}w\eta - kh^{\frac{1}{\nu}} \tag{2}$$

With market clearing conditions we are sure that by Walras' law markets clear. Lastly, we need to put into $FOCK^d$ and $FOCK^d$:

$$K = k_h + k_l \tag{3}$$

$$H = \eta_h h_h + \eta_l h_l \tag{4}$$

   (b) Solve the economy.
   Table 1 reports Country A static equilibria. Instead, Table 2 reports Country B statistic equilibria.

Table 1: Country A

	value ,
Consumption Low type	1.6184.564.894,
Consumption High type	0.5099073724,
Labor supply Low type	0.42375146939,
Labor supply High type	0.12407676791,
Wage	0.55848305863,
Return on capital	0.44542218441,

Table 2: Country B

	value ,
Consumption Low type	1.0722.311.779,
Consumption High type	0.86260899599,
Labor supply Low type	0.31443604022,
Labor supply High type	0.12407676791,
Wage	0.59454997057,
Return on capital	0.40551256761,

2. Consider the case of union economy.

   (a) Write the equilibrium of the union economy. Since we have an open economy the Budget constrain of the household problem has changed to :

$$c = \lambda(w_l(h_l)\eta_l)^{1-\phi} + r_l k^\eta + r_{-l}(kmax_l - k_l) \tag{5}$$

   The subscript l refers to one country, while the subscript $-l$ refers to the other one. Therefore, FOC optimal condition for A:

$$r_A \eta (k)^{\eta-1} = r_B \tag{6}$$

   Using the above new conditions, we solved the problem with all equations for both countries A and B.

   (b) Solve the economy for a given set of parameters. Table 3 reports Country A and Country B union equilibria.

Table 3: Union Economy: Country A and B

	value ,
Consumption A Low type	1.3673.915.239,
Consumption A High type	0.60475777127,
Labor supply A Low type	0.45486009399,
Labor supply A High type	0.10623269917,
Wage A	0.52474920808,
Return on capital A	0.48905666181,
Consumption B Low type	0.96948520942,
Consumption B High type	0.79289707788,
Labor supply B Low type	0.38662898896,
Labor supply B High type	0.33399294846,
Wage B	0.59827198546,
Return on capital B	0.40173425641,
Capital A Low tipe	0.65537921849,
Capital A High type	0.65545202183,
Capital B Low tipe	0.37049353543,
Capital B High type	0.54288352332,

3. Discuss how you would choose the optimal progressive taxation of labor income for these economies? Write the planners problem and an algorithm to solve it.

   Let:

$$U(c,k,h) = \frac{c^{1-\sigma}}{1-\sigma} - k\frac{h^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}$$

   We would choose a social welfare function with Pareto weights for both countries A and B, where the social planner will maximize the following problem:

$$\max_{c_t,h_t,k_t,\tau} \sum_t \beta^t [\sum_i \lambda^i_{A,B} u^i_{A,B}(c^i_t, k^i_t, h^i_t, \tau)]$$

   s.t.

$$c_t + g_t + K_t = F(K_t, H_t) + (1-\delta)K_t \tag{7}$$

   Where τ=tax rate, $K_t = K_A + K_B$, $H_t = H_A + H_B$, $g_t = \textit{goverment}$ expenditure for both countries, and $\lambda_i = \textit{Pareto}$ weight for all i. To solve the problem, we obtain the optimal conditions and solve by using the GEKKO algorithm in Pynthon.