

Mesh Smoothing

C.B. Hovey

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Let the subject point $\mathbf{p} \in \mathbb{R}^{n_{\text{sd}}}$ have coordinates (x) in 1D, (x, y) in 2D, and (x, y, z) in 3D. The subject point connects to n neighbor points \mathbf{q}_i for $i \in [1, n]$ through n edges. In Fig. 1, for example, the point \mathbf{p} connects for four neighbors.

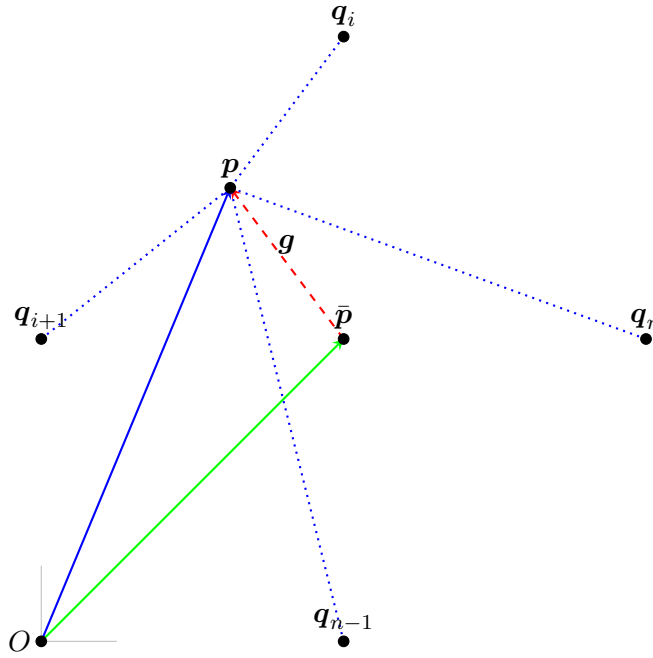


Figure 1: Subject node \mathbf{p} with edge connections (dotted lines) to neighbor nodes \mathbf{q}_i with $i \in [1, n]$ (without loss of generality, the specific example of $n = 4$ is shown). The average position of all neighbors of \mathbf{p} is denoted $\bar{\mathbf{p}}$, and the gap (dashed line) from $\bar{\mathbf{p}}$ to \mathbf{p} is gap \mathbf{g} .

Let $\bar{\mathbf{p}}$ denote the average position of all neighbors of \mathbf{p} and be defined as

$$\bar{\mathbf{p}} := \frac{1}{n} \sum_{i=1}^n \mathbf{q}_i. \quad (1)$$

Let the gap vector \mathbf{g} be defined as originating at $\bar{\mathbf{p}}$ and terminating at \mathbf{p} , such that

$$\mathbf{g} := \mathbf{p} - \bar{\mathbf{p}}, \quad \text{since} \quad \bar{\mathbf{p}} + \mathbf{g} = \mathbf{p}. \quad (2)$$

Let $\lambda \in \mathbb{R}^+ \subset (0, 1)$ be a scaling factor for the gap \mathbf{g} . Then we seek to iteratively update the position of \mathbf{p}^k at the k^{th} iteration by an amount $\lambda \mathbf{g}^k$ to \mathbf{p}^{k+1} as

$$\mathbf{p}^{k+1} := \mathbf{p}^k - \lambda \mathbf{g}^k, \quad \text{since} \quad (3)$$

$$\bar{\mathbf{p}} = \mathbf{p} - \mathbf{g} \quad \text{when} \quad \lambda = 1. \quad (4)$$

We typically select $\lambda < 1$ to avoid overshoot of the update. Following are two iterations for $\lambda = 0.1$ and initial positions $\mathbf{p} = 1.5$ and $\bar{\mathbf{p}} = 0.5$ (given two neighbors, one at 0.0 and one at 1.0, that never move), a simple 1D example:

Table 1: Two iteration update of a 1D example.

k	$\bar{\mathbf{p}}$	\mathbf{p}^k	$\mathbf{g}^k = \mathbf{p}^k - \bar{\mathbf{p}}$	$\lambda \mathbf{g}^k$
0	0.5	1.5	1.0	0.1
1	0.5	1.5 - 0.1 = 1.4	0.9	0.09
2	0.5	1.4 - 0.09 = 1.31	0.81	0.081