Mesh Smoothing

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Let the subject point $\boldsymbol{p} \in \mathbb{R}^{n_{\mathrm{sd}}}$ have coordinates (x) in 1D, (x, y) in 2D, and (x, y, z) in 3D. The subject point connects to n neighbor points $\boldsymbol{q}_i$ for $i \in [1, n]$ though n edges. In Fig. 1, for example, the point $\boldsymbol{p}$ connects for four neighbors.

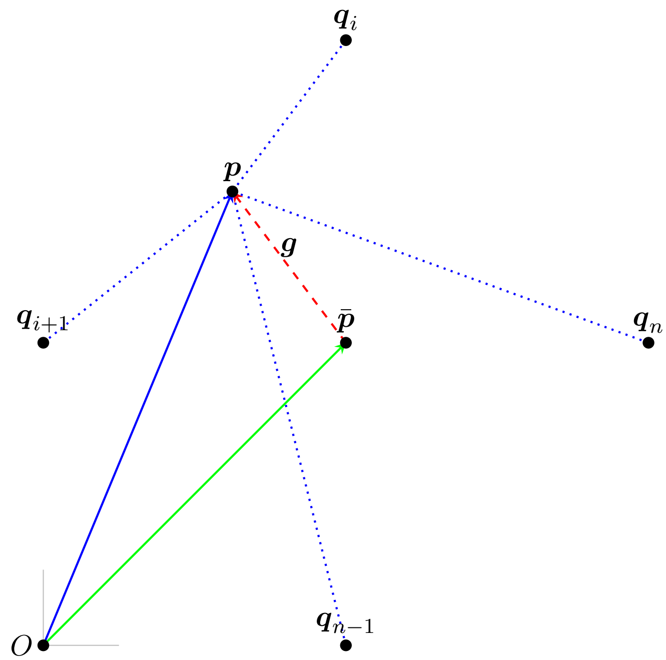

Figure 1: Subject node $\boldsymbol{p}$ with edge connections (dotted lines) to neighbor nodes $\boldsymbol{q}_i$ with $i \in [1, n]$ (without loss of generality, the specific example of $n = 4$ is shown). The average position of all neighbors of $\boldsymbol{p}$ is denoted $\bar{\boldsymbol{p}}$, and the gap (dashed line) from $\bar{\boldsymbol{p}}$ to $\boldsymbol{p}$ is gap $\boldsymbol{g}$.

Let $\bar{\boldsymbol{p}}$ denote the average position of all neighbors of $\boldsymbol{p}$ and be defined as

$$\bar{\boldsymbol{p}} \coloneqq \frac{1}{n} \sum_{i=1}^{n} \boldsymbol{q}_i. \tag{1}$$

Let the gap vector $\boldsymbol{g}$ be defined as originating at $\bar{\boldsymbol{p}}$ and terminating at $\boldsymbol{p}$, such that

$$\boldsymbol{g} \coloneqq \boldsymbol{p} - \bar{\boldsymbol{p}}, \quad \text{since} \quad \bar{\boldsymbol{p}} + \boldsymbol{g} = \boldsymbol{p}. \tag{2}$$

Let $\lambda \in \mathbb{R}^+ \subset (0, 1)$ be a scaling factor for the gap $\boldsymbol{g}$. Then we seek to iteratively update the position of $\boldsymbol{p}^k$ at the k^{th} iteration by an amount $\lambda \boldsymbol{g}^k$ to $\boldsymbol{p}^{k+1}$ as

$$\boldsymbol{p}^{k+1} \coloneqq \boldsymbol{p}^k - \lambda \boldsymbol{g}^k, \qquad \text{since} \tag{3}$$

$$\bar{\boldsymbol{p}} = \boldsymbol{p} - \boldsymbol{g} \qquad \text{when } \lambda = 1. \tag{4}$$

We typically select $\lambda < 1$ to avoid overshoot of the update. Following are two iterations for $\lambda = 0.1$ and initial positions $\boldsymbol{p} = 1.5$ and $\bar{\boldsymbol{p}} = 0.5$ (given two neighbors, one at 0.0 and one at 1.0, that never move), a simple 1D example:

Table 1: Two iteration update of a 1D example.

k	$\bar{\boldsymbol{p}}$	$\boldsymbol{p}^k$	$\boldsymbol{g}^k = \boldsymbol{p}^k - \bar{\boldsymbol{p}}$	$\lambda \boldsymbol{g}^k$
0	0.5	1.5	1.0	0.1
1	0.5	1.5 - 0.1 = 1.4	0.9	0.09
2	0.5	1.4 - 0.09 = 1.31	0.81	0.081