Fragility, Rescues, and Stability in Financial Networks*

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Abstract

This paper presents a coalitional rescue framework in financial networks, where banks' potential losses from failures incentivize them to rescue each other and a welfare-losses-minimizing government contributes to rescues via bailout transfers. Endogenous rescues reverse fundamental insights into financial networks and provide general insights into how interconnected agents behave differently against each other's fragility and how the government can reduce welfare-losses via a combination of ex-ante policies and ex-post transfers. Surprisingly, for any particular bank, the government's bailout decision is independent of contagious effects (or the whole network structure) and depends only on the direct costs and benefits of its rescue (or local connectivity). Therefore, rescues occur *selectively*. Nevertheless, a network-neutrality result holds: In any financial network (under a mild condition), the government always prevents certain types of failures. Despite the network-neutrality in rescue decisions, banks' contributions in rescues and the welfare losses vary dramatically depending on the network structure. I characterize welfare-losses-minimizing networks and introduce a new concept: "self-contained networks", which applies to different settings beyond financial networks.

Keywords: bail-ins, bailouts, coalition formation, financial contagion, financial networks, financial stability, interbank regulations, rescue mergers.

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Contents

1 Introduction

The underlying causes and consequences of risks associated with interconnectedness of financial institutions that I call *banks* henceforth, have been extensively studied in the literature.[1] Even though the risks of interconnectedness have attracted considerable attention, the studies on how banks and the government react against failures are limited.[2] Understanding the actions of banks and the government against failures is particularly important because rescue of distressed banks has repeatedly occurred over the last two centuries and played a key role in restoring financial stability[3]. Furthermore, the discussions on how to eliminate excessive bailout costs to the society are at the forefront in today's highly interconnected financial system, and recent policies in advanced economies show policymakers' increased interest in this problem.[4]

In this paper, I model financial stability and contagion in a framework where banks and the government transfer resources to distressed banks to prevent costly bank failures (i.e., rescue distressed banks). In particular, I develop a generalized rescue framework that allows banks and the government for *selective rescues*, meaning that banks and the government can prevent failures of any set of banks and let any set of banks fail, depending on the benefits and costs of rescues. Even though modeling the formation of rescue coalitions on the top of an already complex financial contagion model makes the analysis highly complex, the findings provide simple and clear insights into the actions of the government and banks in crisis times. Moreover, the findings reverse various fundamental insights into financial networks and provide

[1]Following the early contributions by Allen and Gale [8], Freixas, Parigi, and Rochet [40], Rochet and Tirole [68], and Kiyotaki and Moore [59]; there is a growing literature in financial networks including Acemoglu, Ozdaglar, and Tahbaz-Salehi ([1] and [2]), Allen and Babus [5], Allen, Babus, and Carletti [7], Caballero and Simsek [18], Cabrales, Gottardi and Vega-Redondo [20], Chang and Zhang [23], Elliott, Golub, and Jackson [35], Elliott, Hazell, and Georg [33], Erol [36], Erol and Vohra [37], Ibragimov, Jaffee and Walden [52], Farboodi [39], Gai and Kapada [41], Amini and Minca [10], Glasserman and Young [48], Galeotti, Ghiglino and Goyal [44], Battiston et al. [16], Cabrales, Gale, and Gottardi [19], Gofman [49], Corbae and Gofman [22], Jackson and Pernoud [54], Minca and Sulem [63], Nier et al. [64], Wang [76], Demange [26], Diebold and Yilmaz [28], Kuzubaş, Ömercikoğlu, and Saltoğlu [60], Gai, Haldane, and Kapadia [42], Cohen-Cole, Patacchini and Zenou [21], Eisenberg and Noe [32], Upper and Worms [74], Duffie and Wang [29], Duffie and Zhu [30]. Other types of difficulties can emerge in distressed times such as freezes in interbank lending, repo market or OTC market. See Di Maggio and Tahbaz-Salehi [31], Acharya, Gale and Yorulmazer [4], and Gorton and Metrick [50] for detailed discussions on such market freezes.

[2]Leitner [61], Rogers and Veraart [67], and Bernard, Capponi, and Stiglitz [17] are the related studies on bank rescues and contagion.

[3]See the Appendix for a history of bank rescues and practices of rescues by coalitions.

[4]See Section 12 for a discussion on the recent developments in regulations.

general insights into broader economic problems such as how the network properties affect collective actions of agents against each other's fragility and how the social-planner can improve the welfare via implementing ex-ante regulatory policies on the network structure and ex-post selective transfers.

In a nutshell, the model is as follows. Each bank holds exogenously given primary assets (e.g., interest earning loans) and external liabilities (e.g., deposits), and the financial network is defined over exogenously given interbank obligations. First, a negative shock hits the primary asset of a bank. A bankruptcy (or failure) occurs when a bank is insolvent, i.e., when its total assets are lower than its total liabilities. A failure that is not prevented, causes a drop in the value of the failing bank's assets, called *liquidation costs* (or losses on assets). A failure, if not stopped, causes losses to counterparties and can cause further failures due to counterparty exposures. The rescue formation occurs as follows. Following the financial shock, a welfare-losses-minimizing government makes non-negative bailout transfers to banks, and following the government transfers, banks form rescue coalitions. Lastly, the payments are realized after the formation of coalitions by banks. Similar to the government bailouts, coalition formation by banks is also defined in a generalized way that the coalitions can involve some banks but exclude some others. In particular, the economy after the coalition formation can be any partition of the initial economy (e.g., a partition including rescue mergers, coalitions of multiple banks that represent recapitalization by consortia of banks, and banks that are not involved in any coalition). Government transfers, which occur before the banks' transfers to each other, are designed to support banks' contributions in rescues. Therefore, the connectivity structure that affects how much the banks contribute in rescues might in turn affect the rescue decision of the government as well. So, there might be cases in which the government does not need to make any bailout transfers when banks have incentives to rescue each other, or cases in which the government is the only rescuer when banks have no incentives for transferring resources to each other, or both the government and banks can contribute in rescues at some degrees. In this paper, I do not model the government's commitment[5] (not to rescue) issue. In Section 12, I discuss how the government's commitment issue can be avoided in this framework under already existing regulatory practices. Lastly, it is a perfect information model. Nevertheless,

[5]Government's commitment issue might be thought of as follows. If banks know that the government has the incentives to prevent failures even without banks' contribution, then the government's threat not to rescue distressed banks is non-credible. In such cases, banks can avoid contributing in rescues. Even though it is a related aspect of the problem, it is avoidable under current financial regulations.

various insights from the paper are robust to the perfect information setting. In particular, a main contribution of the paper is showing some network-neutrality results hold, which implies that the government does not need to have the information on the whole network structure when deciding on bailouts. I discuss the potential implications of imperfect information in Section 6.1. The contributions of the paper are as follows.

1.1 Contributions and Related Literature

There are only a few other studies (Rogers and Veraart [67], Leitner [61], and lastly Bernard, Capponi and Stiglitz [17] that is a simultaneous and independent work to this paper) focusing on a rescue problem in a connected financial system.

First of all, the rescue framework developed in this paper allows for *selective* rescues, which makes it fundamentally different from the rescue models in related studies. The existing studies simplify the problem and focus on the issue of preventing the initial failure, which would then prevent the contagious effects of it if there is any. This fundamental difference play a key role in all the results throughout this study. Surprisingly, the rescue of any distressed bank is independent from its ultimate contagious benefits, and depends only on its direct costs and benefits (or some local connectivity properties). In other words, the rescue of a bank is not designed to prevent contagion, but designed to prevent only the direct costs of its failure. Therefore, the government's bailout decisions are independent of the whole network structure. The reason is straightforward: Government bailouts and coalitions by banks occur in a way that rescues can always exclude a single bank or similarly include an additional bank. This insight contradicts the basic assumption of the rescue formation framework used in other models. Moreover, it is practically important that knowing only the local network properties of the initially distressed banks would be enough for the government's bailout plan, regardless of the potential contagion scenario. Consequently, studying endogenous rescues in a generalized framework not only reverse fundamental insights from the plain vanilla contagion models such as in Acemoglu, Ozdaglar, Tahbaz-Salehi [1] and Elliott, Golub, Jackson [35], but also provide novel insights that cannot be explained under any other existing model focusing on a related problem.

Secondly, the results clearly classify which type of failures are prevented and which type of failures occur. Theorem 1 shows that a certain class of failures is always prevented in any given network, under mild conditions. In particular, as long as each banks' primary assets (e.g., interest earning loans) are greater than its interbank assets, then the government always prevents all of the *liquidation-driven-failures*, which are the domino failures that would occur due to the liquidation costs of the *shock-driven* failures (that are the failures due to the financial shock). This happens regardless of the network structure and banks' contributions in rescues. The reason behind not letting *liquidation-driven* failures occur is that such banks are solvent against the financial shock, meaning that they would not default only due to the financial shock; however, some reversible welfare losses from the shock-driven failures trigger these types of domino failures. Since these banks are otherwise solvent against the shock, and their failure would cause further reversible welfare losses, preventing such failures as early as possible, starting from the initial *liquidation-driven* failures, is always preferred by the government to letting some liquidation-driven failures occur. As a result, a network-neutrality result holds: Even though rescues are evaluated one-by-one, in any financial network (under the mild condition described above), the government prevents every liquidation-driven failure. However, banks' contributions to any type of rescues (both for liquidation-driven and shock-driven failures) vary dramatically depending on the network structure, which ultimately affects the government transfers and welfare losses.

In certain networks, banks internalize potential losses from failures in a way that they behave more inclusively and contribute to rescue of each other in a less selective way. A more inclusive rescue behavior by banks decreases bailout costs and reduces welfare losses, and in some cases leads the government to intervene to prevent more *shock-driven* failures. Accordingly, as shown in Theorem 1, *shock-driven* failures are prevented selectively since preventing these failures can be costly both for the government and banks, because such banks are insolvent against the financial shock and rescuers should absorb the losses from the financial shock that are irreversible as opposed to the liquidation costs. Then, the rescue of shock-driven failures occur selectively, and from the government's perspective, knowing only the local network properties of these banks would be enough for the government's ex-post reaction plan. With these intuitions in hand, selective rescues challenge the well-known concepts of *too-big-to-fail* or *too-interconnected-to-fail*: A welfare-losses-minimizing government lets a systemically-important-bank hit by a large shock fail, and instead makes transfers to the rest of the system, if needed.

Following the first main result of the paper (Theorem 1) on the ex-post actions of the government, in the remaining part of the paper, I focus on the ex-ante government policies that can reduce the welfare-losses (i.e., regulations). Accordingly, the third main contribution of the paper is analyzing how different network properties affect failures, transfers, and welfare-losses in distressed times. The network structure, financial shock, and liquidation costs altogether determine banks' incentives to contribute in rescues, but the aggregate capital in the system can restrict banks' ability to save each other. Under such counteracting forces, the second main finding of the paper is the characterization of the welfare-losses-minimizing networks, under large liquidation costs. The assumption of large liquidation costs is relaxed throughout the study, before and after this result. The result shows that whenever the liquidation costs are large, the welfare-losses-minimizing networks are non-clustered and formed under intermediate levels of counterparty exposures. This guarantees that the losses from failure of any subset of banks are internalized at sufficient levels by the other banks in the system. In such cases, banks always care about the fragility of any set of banks in the system and transfer resources whenever needed, which minimizes the bailout costs to the government and welfare losses. Following the characterization result, I focus on three well-known network properties: (i) integration (level of counterparty exposures), (ii) diversification (number of counterparties), and (iii) clustering (overall network structure). The analysis in the remaining part of the paper clearly identifies and separates the impacts of these three different network properties on welfare losses.

Another contribution of the paper is that it brings a novel interconnectivity concept on the table, called "self-contained networks", which can be applied to different settings beyond financial networks. Self-contained networks refers to networks in which agents care about the fragility of every "subset" of agents in the system and, hence, use the available resources to prevent the fragility of any set of agent. Section 8 provides a discussion on self-contained networks beyond financial networks.

Some other differences from the related studies are as follows. Rogers and Veraart [67] and Leitner [61] provide the initial steps towards understanding the private rescue mechanism, without any government bailouts in their models. Rogers and Veraart [67] show that the incentives for rescues emerge in the presence of bankruptcy costs, and then give results on how rescue of initial failures occur in canonical network structures. Proposition 5 in the Appendix corresponds to their main finding. Leitner [61] focuses on the ex-ante optimal size of clusters in a rescue model. Leitner shows that ex-ante optimal networks may not be optimal ex-post. In Leitner's setting, under no government intervention, the capacity constraints of banks is the main factor behind this result. Moreover, Leitner's model does not capture integration, diversification, and liquidation cost channels that are standard in the financial contagion models such as in Elliott, Golub, Jackson [35] and Acemoglu, Ozdaglar, Tahbaz-Salehi [1], which makes that study different from a rescue analysis in a standard financial network model.[6] Lastly, Bernard, Capponi, and Stiglitz [17], in a simultaneous and independent work to mine, compare welfare losses in ring and complete networks while considering whether a government's threat not to bailout is credible or not. Their main focus is the government's commitment issue and how diversification of counterparty exposures affects welfare losses

Besides these studies, Walther and White [75] study the optimal policy for mandatory bail-ins in a model where there are information frictions between banks, without any network analysis. In addition to these, correlated portfolio choices that leads to excessive risk-taking and moral hazard issues have attracted considerable attention in the literature. Erol [36] builds a network formation model and studies the moral hazard problem in financial networks. Among others, Acharya and Yorulmazer [3], Gale and Vives [43], Gennaioli, Shleifer, and Vishny [47], Dávila and Walther [24], Altinoglu and Stiglitz [9], and Jackson and Pernoud [54] focus on portfolio selection and risk-taking. In this study, I do not model the network externalities due to the correlated asset portfolios of banks. Investigating the formation of rescues under an endogenous portfolio selection model is left for future research. Lastly, relevant to the theoretical contributions of this study, Galeotti, Golub, and Goyal [45] study the design of optimal interventions in network games where individuals' incentives are affected by their network neighbors' actions, and Elliott and Golub [34] study the public good provision in networks.

The remainder of the paper is organized as follows. In Section 2, I discuss the rescue mechanism in a simple example and compare the plain vanilla contagion model with the rescue model. In Sections 3, 4, and 5, I introduce the model and rescue formation framework. Section 6 provides the first main result showing which type of failures are prevented in a financial network. Section 7 provides the characterization result. Section 8 introduces the concept of self-contained networks. Sections 9, 10, and 11 investigate how the network properties (clustering, diversification, and integration) affect welfare losses. Section 12 discusses the government's commitment issue. Section 13 discusses welfare-losses-minimizing vs welfare-maximizing networks. Section 14 concludes.

[6] More specifically, Leitner (2005) assumes that a bank receives a positive payoff from its investment only if all other banks it is connected to invest, which creates a "path dependency" property between banks. In Leitner's setting, consider any two banks that are directly or indirectly connected to each other, then these two banks either fail (do not invest) together or do not fail (invest). Therefore, under Leitner's "path dependency" assumption, a ring network, a complete network, a star network, and any network that banks have a "path dependency" are identical ex-ante and ex-post. Leitner's model does not capture integration, diversification, and liquidation cost channels again due to the "path dependency" assumption.

2 A Simple Example: Plain Vanilla Contagion Model vs Rescue Model

Banks' and the government's motives behind rescues can be explained as follows. A single shock[7] environment is representing a case where banks are affected by the financial shock at different degrees, and hence, can contribute to each others' rescue. A bank that fails cannot pay back to its creditors in full. Eventually, the realized liquidation costs are borne by the creditors of a failed bank, which consists of external creditors (e.g., depositors) and other banks (i.e., counterparties). Moreover, a failure might lead to further failures via domino effects and, hence, the initial losses can affect banks beyond the counterparties of a failed banks. Consequently, the overall network structure of counterparty exposures determines the potential contagion and the set of banks that would bear the losses from failures. The willingness to prevent the liquidation costs incentivize banks for transferring resources to each other. As a result, in the presence of failure risks, rescues by banks are similar to the *private provision of public goods*.[8] All banks in the system, including non-counterparties of a distressed bank, weakly benefit from any avoided bankruptcy; however, rescuing a distressed bank requires capital transfers that are costly for the contributors.

The motive behind government bailouts is slightly different from banks' motive behind rescues, and the difference can be explained as follows. The government makes bailout transfers under the following tradeoff: The social cost of a distress is increasing in both realized liquidation costs (failures are costly) and government's bailout transfers (bailouts are costly). Then, a distress-costs minimizing government has willingness to eliminate any avoidable welfare losses via bailout transfers as long as it is less costly for society to do so. On the other hand, a bank has willingness to contribute in rescues only if it would affect its own financial well-being. In addition to the incentives problem, the aggregate level of capital in the system can also restrict banks' ability to save each other. Consequently, the rescue problem has two facets: incentives and capabilities.

Next, I provide an example and compare the plain vanilla contagion model with

[7]The single shock environment is standard in the financial contagion literature and enough to reveal the insights into the rescues in the presence of counterparty risks. One can think this as a significant drop in a given bank's assets values compared to other banks. On the other hand, multiple shocks environment is closely related to the common asset holdings case where multiple banks' assets deteriorate simultaneously. In a multiple shock environment, as the number of banks that is hit by the negative shock rises, the rescue capabilities of banks would decrease.

[8]For surveys on games on networks, see Jackson [53] and Jackson and Zenou [55].

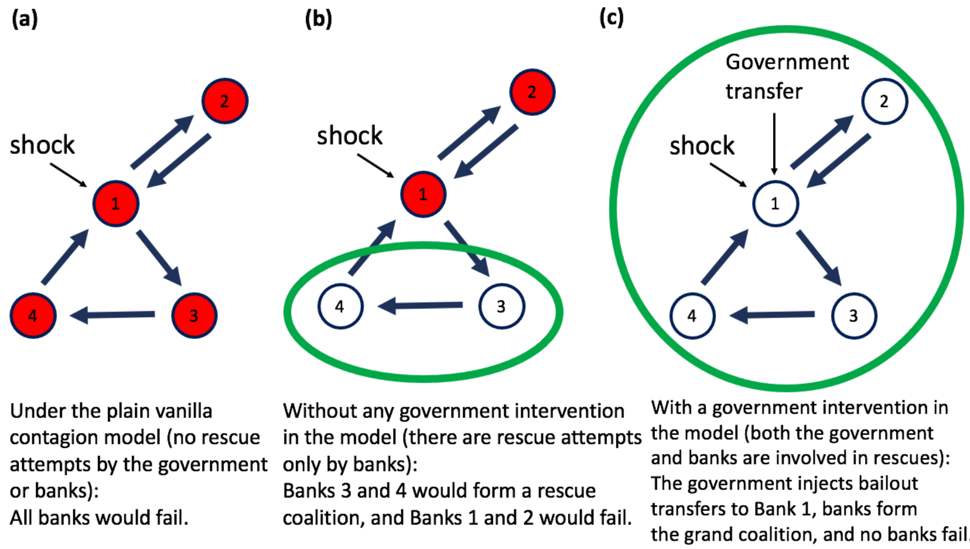

Figure 1: Potential Failures and Rescues under two different rescue models in Example 2. Panel (a). Plain vanilla contagion model. Panel (b). A model of rescues only by banks (with no government in the model). Panel (c). A model of rescues by banks and the government.

rescue models (with/without government assistance). Figure 1 illustrates a comparison of rescue models, which is based on Example 2 in the Appendix. In Example 2, there is a system-wide contagion risk as shown in Figure 1. Consequently, if there were no rescue attempts (under a plain vanilla contagion model), then all banks would fail, as shown in panel (a) in Figure 1. In this example, the losses due to failure of banks 1 and 2 are mostly borne by external creditors of these two banks and are not internalized by banks 3 and 4 at sufficient levels. Therefore, if we consider a model where there is no government intervention, then there would exists no equilibrium outcome in which bank 1 or bank 2 is rescued. As shown in part (b) in Figure 1, at equilibrium banks 1 and 2 would fail and banks 3 and 4 would form a coalition, since they prefer to absorb the losses from the failures of banks 1 and 2 instead of rescuing them. Lastly, when we consider the government assistance in the model, then the equilibrium outcome is the rescue of all banks. A distress-costs-minimizing government assists in a way that the grand coalition is formed and all banks are rescued. This example shows that banks can transfer resources selectively, and government assistance can reduce welfare losses in certain cases. Even though the bailout transfers help preventing all failures in this example, there might be cases where the government also behaves selectively.

3 A Model of Interconnectivity and Contagion

3.1 The Financial Network and Timing of Events

The financial network model is a version of the model introduced by Elliott, Golub, and Jackson [35]. There is a set $N = \{1, ..., n\}$ of banks. Each bank i is endowed with an exogenously given primary asset that has value p_i (e.g., interest-earning loans). In addition, each bank is endowed with exogenously given external liabilities and interbank obligations. The external liabilities of each bank i is denoted by l_i and capture a bank's obligations to its external creditors (e.g., depositors) or other obligations such as operational expenses (e.g., wages or tax). The interbank obligations are represented as *claims*[9] that banks hold in each other. For exogenously given $0 < C_{ij} < 1$, bank i is a creditor of bank j, and bank i claims C_{ij} portion of the total assets of bank j when payments are realized. The claims are such that $C_{ii} = 0$ for all $i \in N$ (a bank holds no interbank claims in itself), $C_{ji} \geq 0$ for all $i \neq j$ and $\sum\limits_{j \in N} C_{ji} < 1$ for all $i \in N$. As a result, $\sum\limits_{j \in N} C_{ji} < 1$ portion of assets of bank i are claimed by other banks. Then, $\sum\limits_{j \in N} C_{ji}$ captures the aggregate exposure of other banks to bank i, and play a key role in potential losses of other banks from the failure of bank i, which is defined in Section 3.2. The interdependencies among banks can be represented as a weighted directed graph, where the C matrix is an $n \times n$ matrix called the *claims matrix*. Finally, each bank i is owned by a single distinct shareholder such that each shareholder only holds the shares of a single bank. This implies that there exist no cross-equity holdings and no conflict of interest in rescue decisions among shareholders of a bank. Given these specifications of the model, a financial network is represented by (C, F) where C is the claims matrix representing the network characteristics and F is the bank characteristics including the information on primary assets, external liabilities, and liquidation costs that is introduced in Section 3.2.

Before introducing the government bailouts and the rescue formation framework, I introduce the contagion framework that is same as in Elliott, Golub, and Jackson [35] in Section 3.2. As it becomes clear at the end of Section 5, the analysis builds on

[9] The way of modeling the interbank contracts here is different than the standard way of modeling the debt contracts among banks. The claims represented in ratios might be thought of as a mapping from "face values of debt contracts" to "ratios of the face values of debt contracts to total assets". Then, the only difference from the standard way of modeling is that the claims create linearities in interbank contracts, which provide a tractable model of contagion and have no further implications on results except level effects. Figure 5 in the Appendix depicts the interdependencies in balance sheets.

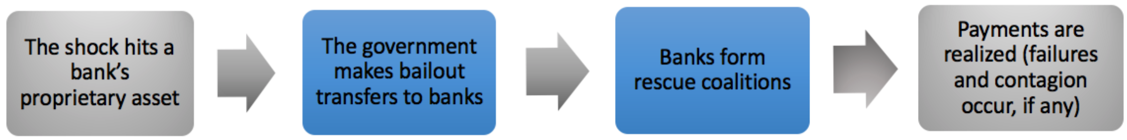

Figure 2: Timing of events

the combination of these three parts. Figure 2 shows the timing of events.[10] After the shock hits, first the government makes bailout transfers, and then banks form rescue coalitions. Following the rescue formation, payments to both external and internal creditors are realized simultaneously, and failures occur, if any.

3.2 Financial Shock, Potential Failures and Contagion Framework

The plain vanilla contagion framework is as follows. An exogenously given negative shock $s \in [0, p_r]$ hits the primary asset of a single bank, denoted by bank r. When the payments are realized, any bank that has total assets less than total liabilities becomes insolvent, and bankruptcy (or default) occurs. A bankruptcy in the model refers to an insolvency. Bankruptcy is costly. The primary asset of a bank drops at some rate in case of bankruptcy. Exogenously given $0 \leq \zeta \leq 1$ captures the ratio of liquidation[11] costs the value of primary assets. In other words, $1 - \zeta$ is the recovery rate of a primary asset's full value in a bankruptcy situation. ζ is common for all banks, $\zeta_i = \zeta \ \forall i \in N$. Lastly, β_i captures the liquidation costs of bank i in nominal terms, which can differ accross banks. Correspondingly, the bank hit by the negative shock, denoted by bank r, has liquidation costs equal to $\beta_r = (p_r - s)\zeta$. Any other bank $i \neq r$ has liquidation costs equal to $\beta_i = p_i\zeta$. Lastly, if a bank defaults, then the shareholder of that bank is wiped out and receives nothing, whereas both external and internal creditors of the failed bank are rationed in proportion to total assets of

[10] In the model, there is no specific notation used for the timing of events. Figure 2 provides a useful picture of the ordering of the events, which is as implied by the model.

[11] Different from the bankruptcy related liquidation-costs as described here, there might be some additional channels such as sales of assets at distressed-prices to solve liquidity problems of banks. Adding such channels to the model would enter as an additional force for solving liquidity issues, but will not affect the rescue mechanism studied in this paper that focuses on banks' and the government's attempts to prevent liquidation-costs (or welfare losses) associated with bankruptcies. Endogenous portfolio choices and additional fire-sale effects under a rescue model are interesting and left for future research. See Shleifer and Vishny [70] for an overview of the research on fire sales in finance and macroeconomics.

that bank with equal seniority. A failure can cause further failures via counterparty exposures.

4 Government Assistance in Rescues

The government's transfers and rescue formation by banks occur under perfect information (Section 6.1 includes a discussion on imperfect information). The government makes non-negative transfers to each bank after the shock hits and prior the formation of coalitions by banks and realization of payments between banks. The vector $\mathbf{t} \geq 0$ denotes the non-negative government assistance to each bank $i \in N$. Then, the vector of primary assets after the shock and the realization of the government assistance is given by $\mathbf{p} = [p_1 + t_1, ..., p_r - s + t_r, ..., p_n + t_n]'$.

4.1 The Social Costs of Financial Distress

Recall that liquidation costs are equal to $\beta_r = (p_r - s)\zeta$ for the bank hit by the shock, and $\beta_i = p_i\zeta$ for any other bank $i \neq r$. The government aims to minimize the total cost of a distress to the society as defined below

Definition 1 *The distress-costs in $(C, F, \boldsymbol{t})^M$ is equal to $H(C, F, \boldsymbol{t})^M = (\sum_{i \in N^M} b_i) + \eta \sum_{i \in N} t_i$ where $\eta > 0$, and $b_i = \beta_i$ if $i \in (C, F, \boldsymbol{t})^M$ defaults and 0 otherwise, and $(C, F, \boldsymbol{t})^M$ is the financial network after the rescue formation.*

The properties of the financial network after the rescue formation, $(C, F, \mathbf{t})^M$, is given in Section 5. The cost function above captures an important aspect of the rescue problem. The capital transfers between banks are less costly for society than the government bailouts. $\eta > 0$ captures the relative inefficiency of bailout transfers.[12] There is a simple reasoning behind this feature of the cost function. The government has the ability to allocate its resources to maximize social welfare and can provide funds to services such as health care, education; and similarly, the government can also provide funds to the financial system if it is beneficial for society to do so (e.g., loan guarantees). As a result, even if the banks make the rescue transfers at the first place, the government can always compensate banks' contributions via providing funds back to banks if it is social welfare improving. However, when the government makes the rescue transfers at the first place, then the government is not able to force

[12]One can add banks' transfers to the cost function and a parameter less than $\eta > 0$, which would capture inefficiency of banks' transfer compared to bailouts, but to simplify the notation, I only consider bailout transfers in the cost function and consider banks' transfers to each other costless.

banks to transfer resources to the government back to be used for other government services. Therefore, private rescues are always preferred for society than the government bailouts. Private rescues always being preferred to government bailouts is a standard view in existing resolution regimes.[13] I normalize $\eta = 1$. One can consider different values for η, but given that the other component of the cost function is the realized liquidation costs, $\sum_{i \in \aleph^M} b_i = \zeta \sum_{i \in \aleph^M} (p_i - s_i)$, the liquidation cost parameter ζ technically captures η as well. For η taking values different than 1, then ζ might be thought of as relative liquidation costs.

For any given financial network (C, F), $\mathbf{t}$ is a distress-costs-minimizing government-assistance if $\nexists \mathbf{t}'$ such that:

$$H(C, F, \mathbf{t}')^{M^*_{(C,F,\mathbf{t}')}} < H(C, F, \mathbf{t})^{M^*_{(C,F,\mathbf{t})}},$$

where $M^*_{(C,F,\mathbf{t})}$ and $M^*_{(C,F,\mathbf{t}')}$ are equilibrium set of coalitions that minimize the distress costs in $(C, F, \mathbf{t})$ and $(C, F, \mathbf{t}')$, respectively. A government is a distress cost minimizer if it implements the distress-costs-minimizing government assistance for any given network (C, F). Correspondingly, I consider that the government is a distress-costs-minimizer.

In this rescue framework where first the government makes bailout transfers and then banks transfer resources to each other, there is no government's commitment issue[14]. Even though the credibility of government's commitment not to rescue some distressed banks is a related aspect of the problem, it is avoidable under the existing financial regulations. Section 12 discusses how the government's commitment issue can be avoided in practice. In this model with no such issue, the government assistance and contributions of banks are complementary. The government makes transfers only if banks are not adequately incentivized for rescues or when banks do not have adequate capital to be used for rescue transfers.

Lastly, the cost function above only captures the welfare losses and the paper focuses on how to minimize welfare losses in distressed times. Section 13 involves a discussion on welfare-maximizing networks vs welfare-losses-minimizing networks.

5 Formation of Rescue Coalitions

The economy after the formation of coalitions is a partition of the initial economy. A bank is not necessarily involved in a coalition and can remain single after the

[13] Hoggarth, Reidhill and Sinclair [51] and White and Yorulmazer [77] discuss bank resolution concepts in detail.

[14] See Bernard, Capponi, and Stiglitz [17] for a model of government's commitment issue.

formation of coalitions. A coalition might include two banks, referring to a rescue merger, or more than two banks, referring to a multi-bank rescue consortium. A coalition might be thought of as pooling the available capital for rescues, and a coalition formed by multiple banks represents capital transfers to distressed banks by a group of banks. For instance, the formation of the grand coalition in the model, which includes the shocked bank, does not represent a case where all banks become a single bank; rather, it represents the case in which all banks contribute to prevent the failure of the shocked bank at some degrees. The coalition framework also allows for formation of multiple rescue coalitions at the same time, where each bank is involved in at most one coalition. Coalitions are defined in a way that the total assets and total liabilities of the members of a given coalition are summed up, and unchanged for the banks that are not involved in any coalition.[15]

Let $M = \{m_1, ..., m_n\}$ be the set of rescue coalitions formed, and $\phi \subseteq N$ is the set of banks that are involved in any rescue coalition. The set N^M denotes the banks after the coalitions are formed, and it is a partition of the initially given set of banks N. Then, after the government transfers and coalition formation by banks, the network is denoted by $(C, F, \mathbf{t})^M$.

Let V_i be the total assets of bank $i \in N$ and L_i be the total liabilities of bank $i \in N$ initially. The financial network after coalitions are formed is defined as follows:

Definition **2** *$(C, F, \boldsymbol{t})^M$ is the financial network after a set of banks $\phi \subseteq N$ form the set of coalitions $M = \{m_1, ..., m_n\}$ in a given financial network (C, F). $(C, F, \boldsymbol{t})^M$ has the following properties:*

i) $N^M = (N \setminus \phi) \cup M$,
ii) $p_j^M = p_j + t_j$, $l_j^M = l_j$, $\beta_j^M = \beta_j \ \forall j \in N \setminus \phi$,
iii) $V_j^M = V_j$, $L_j^M = L_j \ \forall j \in N \setminus \phi$,
iv) $p_{m_k}^M = \sum_{k \in m_k} (p_k + t_k)$, $l_{m_k}^M = \sum_{k \in m_k} l_k$, $\beta_{m_k}^M = \sum_{k \in m_k} \beta_k \ \forall m_k \in M$,
v) $V_{m_k}^M = \sum_{k \in m_k} V_k$, $L_{m_k}^M = \sum_{k \in m_k} L_k \ \forall m_k \in M$.

The payments are realized following the government transfers and coalition formation. As shown in the Appendix, the total assets, total liabilities, and shareholders' equities after the formation of coalitions is given by the following equations:

[15]Formation of coalitions requires restructuring of the *claims matrix*. Lemma 2 in the Appendix shows that there exists a unique way of restructuring the claims which satisfies the properties in Definition 2 for every given primary asset vector $\mathbf{p}$. The restructured claims are set according to the result in Lemma 2. Besides its uniqueness property, the restructuring rule given in Lemma 2 is a natural way of restructuring the interbank obligations.

$$\mathbf{V^M} = (I - C^M)^{-1}(\mathbf{p^M} + \mathbf{t^M} - \mathbf{b^M}),$$

$$L_i^M = (\sum_{j \in N^M} C_{ji}^M V_i^M) + l_i^M,$$

$$e_i^M = max\{0, V_i^M - L_i^M\},$$

where $\mathbf{p^M} = [p_1^M + t_1^M, , ..., p_k^M + t_k^M]'$ is the vector of primary assets of coalitions after the shock and government transfers, and $\mathbf{b}$ is the vector of realized liquidation costs such that $b_m^M = \beta_m^M$ if $m \in N^M$ defaults and 0 otherwise. As one can see, $p_{m_i}^M$ in Definition 2 captures government transfers but not the shock; however, the vector $\mathbf{p^M}$ above captures both transfers and the shock. I use the same notation for $\mathbf{p^M}$ in order to avoid heavy notation.[16] As shown in the Appendix, by some further algebra, shareholders' equities can be rewritten as:

$$\mathbf{e^M} = max\{0, A^M(\mathbf{p^M} - \mathbf{b^M}) - \mathbf{l^M}\}. \tag{1}$$

where A^M is the dependency matrix (after the coalition formation) that gives us the flow of payments between banks when payments are realized. On the other hand, before any shock hits, the following condition holds for the initial economy:

$$\mathbf{e} = max\{0, A(\mathbf{p} - \mathbf{l})\}, \tag{2}$$

where A_{ij} is the dependency of bank i to bank j and $A_{ij}p_j$ is the total interbank asset of bank i in bank j.

5.1 Rescue Formation Game

I use a non-cooperative game setting, but one can define a cooperative game and reach the same results and insights from the model. The results are robust to the coalition formation game selection.[17] After the government transfers, $\mathbf{t}$, each bank announces

[16]Coalitions are formed after a bank, which is known by every other bank and the government, is hit by the shock but before payments are realized and, so, before the shock propagates into the system. Therefore, the given definition of coalitions that does not take the shock into account is the appropriate way of defining the pooling of available capital for rescues. As a result, each $p_{m_i}^M$ in Definition 2 does not capture the shock. However, in order to avoid heavy notation, I used the same superscript as in Definition 2 for the vector of $\mathbf{p^M}$ after the coalitions are formed that captures the shock as well.

[17]See Appendix for the discussion on a cooperative game setting.

the coalitions in which it accepts to be involved. Given all strategies, coalitions are formed simultaneously where each bank is involved in at most one coalition. Following the formation of coalitions, failures and contagion occurs if any, and the payments are realized and the banks involved in a coalition share the payoff of the coalition. Sharing the payoff of a coalition can be thought of as sharing the contributions in rescues. Banks share the payoff under a sharing rule. In a multi-bank multi-coalition framework, a refinement of the simplest possible sharing rule is required, otherwise some unintended inefficient solutions might arise.[18] More specifically, the refinement property ensures that all else constant, if a collective action taken by a set of agents increase their total payoffs, then all agents in this group should weakly benefit from taking such an action. Alternatively, one can define a cooperative game environment with some refinements with similar properties.

The solution concept is Strong Nash Equilibrium (SNE), which requires stability against deviations by every conceivable coalition. Thus, an equilibrium is strong if there exists no coalition, taking the actions of its complement as given, that can deviate in a way that benefits all the members of the coalition. Then, a partition is an equilibrium outcome if there exists no coalition that deviates where each bank in the deviating coalition receives weakly higher payoff after deviation.

Formally, the rescue formation game is a simultaneous move game $\Gamma = ((S_i)_{i\in N}; (f_i)_{i\in N})$ consisting of set of banks $N = \{1, ..., n\}$, a strategy set S_i for each bank $i \in N$, and a payoff function $f_i : \prod_{i\in N} S_i \to \mathbb{R}$ for each bank $i \in N$. A particular strategy $s_i \in S_i$ represents the set of coalitions that bank i has willingness to be involved. The strategy set of bank i is $S_i = P(\{T \cup i \mid T \subseteq P(N \setminus i)\})$ where $P(N \setminus i)$ is the set of subsets of $N \setminus i$ and $P(\{T \cup i \mid T \subseteq P(N \setminus i)\})$ is the set of subsets of $\{T \cup i \mid T \subseteq P(N \setminus i)\}$. Given the strategy profiles, a bank is involved in at most one coalition. A coalition m_k is formed if $\{m_k\} \subseteq s_j$ for all $j \in m_k$. If there exist ties such that there exists a bank $j \in m_k, m_l$ where $\{m_l\} \subseteq s_i$ for all $i \in m_l$ and $\{m_k\} \subseteq s_i$ for all $i \in m_k$, then such ties are broken in a way that the distress-costs are minimized. The tie-breaking property that allows us to choose the distress-cost-minimizing equilibrium might be thought of as the government's coordination role. $s^* \in \prod_{i\in N} S_i$ is a SNE if and only if $\forall G \subseteq N$ and $\forall s_G \in \prod_{i\in G} S_i$, there exists an agent $i \in G$ such that $f_i(s^*) \geq f_i(s_G, s^*_{N\setminus G})$. Next, I define payoffs. For given $(C, F, \mathbf{t})^M$ and the sharing rule below, each bank $i \in N$ receives a payoff denoted by e_i^M. The properties of the sharing rule, including the refinement discussed above, are as follows.

Let Δ be a class of sharing rules. Any sharing rule $\delta \in \Delta$ satisfies the following

[18] See Ray and Vohra [66] for a related coalition formation problem where payoff to a player depends on the actions of all the other agents.

properties:

(i) the total payoff of a coalition is shared among its members. Formally, $f_i(s_i, s_{-i}) = \alpha_i^{m_k} e_{m_k}^M$ where $\sum_{i \in m_k} \alpha_i^{m_k} = 1$ for all $m_k \in M$ (equivalently $\sum_{i \in m_k} f_i(s_i, s_{-i}) = e_{m_k}^M$ for all $m_k \in M$)

(ii) all else constant, if the superset of some coalitions results in a weakly higher payoff than their separate sum, then any member involved in one of these coalitions receives weakly higher individual payoff if the superset is formed. Formally, consider any given three partitions M,M' and M'' of a given economy and consider coalitions $m_j \in M''$, $m_k \in M'$, $m_l \in M$, where $m_k \cup m_l = m_j$. If $e_{m_j}^{M''} \geq e_{m_k}^{M'} + e_{m_l}^{M}$, then $f_i^{m_j}(M'') \geq f_i^{m_k}(M')$ for all $i \in m_j \cap m_k$ and $f_i^{m_j}(M'') \geq f_i^{m_j}(M)$ for all $i \in m_j \cap m_l$. This holds for any such triple $((m_l, M), (m_k, M'), (m_j, M''))$ for any given economy.

I choose any $\delta \in \Delta$ and fix it as the sharing rule. For given set of coalitions, the contagion algorithm given in the Appendix applies for finding the set of failures in the network $(C, F, \mathbf{t})^M$ after the coalitions are formed.

6 Which Type of Failures are Prevented?

Let $\aleph_T$ be the set of potential failures under the plain-vanilla contagion scenario, with no rescue attempts, following a shock to bank r in a financial network (C, F). Let $\aleph_1$ be the set of *shock-driven* failures that includes only the banks that would fail at the first step of the contagion. Any bank $j \in \aleph_1$ defaults due to the shock (even for zero liquidation costs, $\zeta = 0$). Let $\aleph_T \setminus \aleph_1$ be the set of *liquidation-driven* failures. Any bank $j \in \aleph_T \setminus \aleph_1$ would not default if liquidation costs were zero, $\zeta = 0$.

Theorem 1 is the first main result. It shows that *liquidation-driven* failures (the failures that would occur after the *shock-driven* failures) never occurs at equilibrium, as long as primary assets of each bank is larger than its interbank assets.

Theorem 1 *In any given financial network (C, F) where the primary asset of each bank is weakly larger than its interbank assets ($p_i \geq \sum_{j \neq i} A_{ij} p_j$ holds for all $i \in N$), at equilibrium:*

(i) there exists no liquidation-driven-failure (the government always prevents all potential liquidation-driven-failures, regardless of banks' contributions in rescues),

(ii) all potential failures, including the shock-driven failures, are prevented iff $s \leq s^*(C, \zeta)$, where s^* is increasing in ζ.

First of all, in this selective rescue framework, the rescue decision for any particular bank does not depend on the ultimate contagious benefits of its rescue, and depends only on the direct costs and benefits of the rescue. The reason is that coalitions can always exclude a single bank (or similarly include an extra bank), and the decision whether to rescue a particular bank or not only depends on its direct costs and benefits. Even under this selective rescue framework, Theorem 1 shows that the banks and the government always prevent the liquidation-driven failures, under the mild condition that requires each bank's primary assets being larger than its interbank assets. The reason is that these banks are fundamentally sound against the shock but fragile due to the reversible liquidation costs that are the welfare losses absorbed by banks and depositors. The government has incentives to prevent any welfare losses that are reversible as long as it is less costly to do so. Therefore, the cheapest way of eliminating such welfare losses fully is stopping the liquidation-driven failures as early as possible, if there is any. The result holds regardless of the potential distress scenario, or the ratio of liquidation costs, ζ, that can take any value between 0 and 1, or the network structure (as long as the mild interbank obligations condition described above is satisfied). The only assumption here is that the ratio of liquidation costs, ζ, is common for all banks (the actual level of liquidation costs can differ). If the recovery ratio ζ is different for different banks, then the benefits and costs of each rescue might be different, which might lead to more selective rescue decisions. For instance, if the liquidation costs of a particular bank is small compared to other banks, then the benefits for rescuing that particular bank would be small compared to the costs of its rescue. The cost of a rescue is weakly increasing in liquidation costs for the other banks, because rescuers of a bank should absorb the counterparty losses of that bank as well, which includes a part of the liquidation costs from other failures. In such cases, banks with relatively higher liquidation costs are more likely to be rescued, which would be the additional insight from relaxing the assumption of common ratio of liquidation costs. Under common ratio of liquidation costs, Theorem 1 is a network-neutrality result showing that certain types of failures are always prevented in any given financial network, as long as non-interbank assets of each bank is larger than its interbank assets.

Different from the rescue of the liquidation-driven failures, part (ii) of Theorem 1 shows that all potential failures, including the shock-driven-failures, are prevented whenever the shock is sufficiently small. So, Theorem 1 implies that against large shocks, the government and banks can implement an alternative rescue plan and transfer resources to prevent only some of the shock-driven failures or only the liquidation-driven failures. The threshold level of shock for rescuing all banks is increasing in ζ. As the potential welfare losses from any failure increase, rescues occur less selectively.

From the government's perspective, in deciding whether to prevent some shock-driven failures, it is enough to consider the local connectivity properties to understand the costs and benefits of the rescue of each shock-driven failure. Then, each "subset" of shock-driven failures is evaluated separately, which implies that each shock-driven failure is also evaluated one-by-one. The increased bailout costs to the government to prevent an additional shock-driven failure depends on how much the counterparties would contribute to the rescue, which is determined by the losses (in case the failure occurs) that would be absorbed by the immediate counterparties of that bank. On the other hand, the increased benefits from preventing an additional failure is the liquidation costs that would be eliminated via that particular bank's rescue.

6.1 A Discussion on Imperfect Information

Imperfect information is an important challenge during crisis times. In order to overcome challenges, governments and banks analyze the assets and liabilities of distressed banks in detail before engaging in rescue actions. As an example, Lehman Brothers had involved in various negotiations with the government and other banks before declaring its bankruptcy, or similarly other SIFIs have been involved in negotiations with various other institutions and ended up in certain coalitions. In a contagion framework, not only the information about counterparties, but the exposures of counterparties and so on is required to understand the potential contagion. Nevertheless, in this generalized rescue formation framework, the rescue decisions for a particular bank is independent from the contagious effects of its failure, regardless of any parameter in the model. Even though the rescue decisions are independent from contagious effects, this does not mean that the indirect counterparties would not contribute to rescues. Both the direct and indirect counterparties decide whether to transfer resources to a particular distressed bank or not. In light of this rescue framework and Theorem 1, knowing the structure of the network fully is not required to decide on rescues. The local connectivity information (the information on how much the counterparties depend on the shocked bank and how much the shocked bank depends on its counterparties) is sufficient to decide whether to prevent the shock-driven failures or let some of them fail but rescue their counterparties. Following the rescue decision for each shock-driven failure, which is based on the local connectivity properties only, then the government finalizes its rescue decisions. Consequently, not knowing the whole network structure does not affect the bailout decisions and ultimate failures; however, such information asymmetries can lead to lower contributions by the banks that would be indirectly affected from the failures, and hence, can increase the government bailouts. The implications of imperfect information is an interesting research avenue, and analyzing such frictions are left for future research. The goal of this paper is to investigate network properties that can be implemented ex-ante, which would help to overcome the issues related to the information asymmetries in distressed times.

7 Characterization of Distress-Costs-Minimizing Networks under Large Losses on Assets

In this part, I characterize the distress costs minimizing connectivity under heterogeneity in both bank characteristics and network characteristics. The analysis is done under the assumption of large liquidation costs, $\zeta = 1$.[19] As in the previous part, this assumption is relaxed also throughout the rest of the paper.

In this part, I also consider a random shock environment. There exists a shock s, which hits a randomly selected bank and can be either small or large with probabilities below. The large shock s_L represents the case that banks do not have adequate capital to avoid all potential failures.

$$s := \begin{cases} s_S \in [0, N(1-l)] & \text{with probability } q \\ s_L \in (N(1-l), 1] & \text{with probability } 1-q \end{cases}$$

For the constrained optimization, the class of networks Ω in this part is defined as follows:

$$\psi(C, \overline{F}) \in \Omega := \begin{cases} C_{ii} = 0 & \forall i \in N \\ C_{ij} \geq 0 & \forall i, j \in N \\ p_i - l_i > 0 & \forall i \in N \\ \sum_{i \in N} (p_i - l_i) < min\{p_1, ..., p_n\} & \forall i \in N \\ v_i - l_i > 0 & \forall i \in N \end{cases}$$

The last two parts guarantees that for large shocks banks are not capable of rescuing all distressed banks and initially all banks are solvent.

Given these specifications, the constrained optimization problem is as follows:

$$\min_{\psi(C,\overline{F})} H(\psi(C, \overline{F}, \mathbf{t})^{M^*_{\psi(C,\overline{F})}}) \text{ subject to } \psi(C, \overline{F}) \in \Omega.$$

[19] Acemoglu, Ozdaglar, and Tahbaz-Salehi [1] explain this situation as follows: "...Furthermore, during bankruptcy, the liabilities of the institution may be frozen and its creditors may not immediately receive payment, leading to effectively small recovery rates." (Acemoglu, Ozdaglar, Tahbaz-Salehi [1]).

Then, an interbank network $\psi^*(C,\overline{F}) \in \Omega$ is distress-cost-minimizing if $H(\psi^*(C,\overline{F},\mathbf{t})^{M^*_{\psi^*(C,F)}}) \leq H(\psi(C,\overline{F})^{M^*_{\psi(C,\overline{F})}})$ for every interbank network $\psi(C,\overline{F}) \in \Omega$, where $M^*_{\psi(C,\overline{F},\mathbf{t})}$ is the equilibrium set of coalitions that minimizes the distress costs in $\psi(C,\overline{F})$. The characterization result is as follows.

THEOREM **2** *Under large liquidation costs ($\zeta = 1$), a financial network $(C,F) \in \Omega$ is a distress-costs-minimizing network iff (C,F) satisfies the following properties:*

- $(1 - A_{kk})\, p_k = \sum\limits_{j \in N\setminus k} (v_j - l_j)$ *for each singleton $k \in N$ (or equivalently $\sum\limits_{i \in N\setminus k}\sum\limits_{j \in N\setminus k} A_{ij}p_j = \sum\limits_{j \in N\setminus k} l_j$) (each bank holds an intermediate level of interbank liabilities)*
- $\sum\limits_{i \in K}\sum\limits_{j \in K} A_{ij}p_j \leq \sum\limits_{i \in K} l_i$ *for all $K \subset N$ (no clustering of interbank exposures)*

The characterization result shows that there are two properties of distress-costs-minimizing networks: (i) there exists no-clustering of interbank exposures among any subset of banks, and (ii) each bank has an intermediate level of interbank liabilities.

In such networks, banks behave non-selectively in rescues, and always prevent all failures without any government assistance as long as they collectively have enough capital. Secondly, whenever the government finds assisting the rescue of the shocked bank costly against large shocks, then banks absorb the losses from the failure of the shocked bank without any government assistance. The motive behind the *non-selective* rescues by banks can be explained as follows. First, the integration level (the ratio of total interbank liabilities of a bank to its total assets) determines the immediate counterparties' potential losses if a bank fails. On the other hand, clustering is the property that determines how the potential losses from failure of some banks are transmitted to the rest of the system. Accordingly, the characterization result itself provides a novel definition of non-clustering: Non-clustering is not a "0 or 1" property, but it is a function of bank and network characteristics. In particular, it has the following property: "any subset of banks is connected to the remaining part of the system at sufficiently high levels". In clustered networks, losses from failures in a cluster are absorbed by the banks in that cluster and their depositors. Contrarily, in non-clustered networks, losses are always transmitted to the remaining parts of the network. This happens regardless of the location of the initial distress. In other words, integration is a "dependency (and so, loss transmission) property at individual bank level", whereas clustering is a "dependency (and so, loss transmission) property at subset of banks level". In networks satisfying the two properties given in Theorem

7, the potential losses are transmitted to the others in a way that banks always care about the fragility of every set of banks.

A feature of the distress-costs minimizing networks given in Theorem 7 is that they are contagious, except certain cases (e.g., complete network). However, it should be clarified that the contagiousness property is not the reason that motives the non-selective rescues. For instance, in the Example in Section 2, the network is contagious but banks 3 and 4 would not rescue banks 1 and 2 without the government assistance. The main motive behind non-selective rescues described in Theorem 7 follows from the non-clustering property. In such networks, potential losses from the failure of any *subset* of banks are sufficiently internalized by the remaining banks in the system.

In order to clarify this more, let me explain it in a ring network example without any government intervention in the model. Consider that the liquidation costs are sufficiently large, and a large enough shock hits which makes the network potentially contagious. In a contagious ring network where there is a risk of system-wide failures, banks can form a coalition and prevent only a subset of potential failures. In this way, they can prevent the system-wide contagion. For example, consider a set of banks that form a large enough coalition that excludes the shocked bank and some additional banks closest to the shocked bank. Such a coalition would stop contagion, and all banks excluded from the coalition would fail, whereas the banks involved in the coalition would remain solvent. So, the contagion would be partially prevented. However, in such a network, banks do not only stop the contagion, but have incentives to eliminate all failures as long as the shock is sufficiently small. The reason for such strong incentives follows from the fact that any "subset" of initial failures would cause sufficiently high losses for the remaining banks.

As a result, integration and clustering properties together provide a subset level dependency property. Nevertheless, clustering (as well as diversification) might have different implications on welfare losses whenever the loss transmission mechanism does not work effectively when liquidation costs are small, which are discussed in Section 9 and Section 10. Next, I introduce a novel concept called "self-contained networks" in light of the characterization result, that can be applied beyond financial networks.

Lastly, to illustrate the heterogeneity in Theorem , I study the star network as a special case (in the Appendix). A star network can be a distress-costs-minimizing network for certain exogenously given primary assets and liabilities. The claims (in ratio) that the core bank holds in each periphery bank is higher than the claims that each periphery bank holds in the core bank. This guarantees that the connectedness of each periphery bank to the rest of the system is at sufficient levels.

8 Self-Contained Networks

This paper brings a new interconnectivity concept on the table, called "*self-contained networks*", while providing an answer to the following question:

> "In which network structures does the financial system require minimum external assistance (i.e., government bailouts) to minimize welfare losses in distressed times?"

A self-contained network is such that there exists no non-equilibrium outcome that would prevent a weakly larger set of failures with a lower amount of government assistance than an equilibrium outcome in that network. In other words, self-contained networks are the ones in which banks transfer resources to each other in exactly the same way as if there is a government that is freely allocating the resources of banks to each other, under any distress scenario. Therefore, in such networks, government intervention is not needed as long as there is high enough aggregate capital in the system, and such incentives of banks make these networks *self-contained.* Whenever the resources of banks are not enough to save each other, then the government intervenes. In other words, self-contained networks trigger a collective behavior against fragility in any part of the system, including any single individual fragility or a system-wide fragility. Formally:

Definition **3** *For exogenously given set of banks, primary assets, and external liabilities; a financial network* (C, F) *is self-contained if for any given equilibrium outcome* $(C, F, \boldsymbol{t})^M$ *that is formed after any given shock, there exists no non-equilibrium outcome* $(C, F\boldsymbol{t'})^{M'}$ *where* $(\aleph_T)^{M'} \subseteq (\aleph_T)^M$ *and* $\sum\limits_{i \in N} t_i > \sum\limits_{i \in N} t'_i$.

The characterization result under the assumption of large liquidation costs, Theorem 7, gives us the properties of self-contained financial networks.

Corollary **1** *For large losses on asset,* $\zeta = 1$, *any distress-costs-minimizing network given in Theorem 7 is a self-contained network.*

A more strict definition of self-containedness in this setup would focus on completely eliminating the government assistance in preventing failures. In order to achieve such a system, one should also have control over exogenously given primary assets and external liabilities. If regulators implement a dual policy of interbank regulations and minimum capital requirements, then it is possible to eliminate the need for government bailouts via increasing the bank capital up to a threshold level. In the presence of rescue incentives, a bank's capital not only works as a safety net for itself but works as a safety net for the whole system. Therefore, if the counterparty exposures are regulated effectively, then the required level of minimum capital can be set at a lower level for each bank.

In a broader perspective, a general question for networked markets is as follows:

> "In a given economic system, how should the interconnectivity be so that the system is *self-contained* where agents use the available resources in the most effective way to mitigate the risks that one or more agents face and, hence, minimize the need for external assistance against any fragility?"

As an example, one can consider the resolution of European sovereign debt crises, where countries had different levels of indebtedness and the financially sound countries had different incentives to contribute to the rescue plans. Aftermath of the crisis, the Euro area countries established the European Stability Mechanism (ESM) in 2012 to provide loans to Euro Area Member States facing financial difficulties. The foundation of ESM helped Euro zone countries to fight with distress as a coalition in a more systematic way. Another related example is the rescue of distressed firms in production economies, e.g., rescues in auto-industry in Japan and US or recent rescues plans to avoid catastrophic impacts of COVID-19 throughout the world. One other example is the recent discussions on the role of coordination between countries for building a safer globalization in the future. From a social networks perspective, one can consider the inequalities in the society and discriminatory behaviors of certain groups of agents. Building a society that would endogenously eliminate any discriminatory behaviors has always been a major concern for societies.

From the insights provided here, an answer to the question above is as follows. A system is *self-contained* if all agents care the fragility of any subset of agents in the system and help each other whenever needed. This can happen when selective behaviors of agents are fully eliminated. Eliminating the selectivity in resource transfers requires agents being connected in specific ways, so that every *subset* of agents have an essential role in the system and any subset of failures becomes too-costly-to-fail for the remaining. Analyzing self-contained networks in different environments requires different models. Finding answers to these questions is essential for ex-ante design of policies to achieve a socially efficient solutions. This paper provides a first step to understand the properties of self-contained networks. On the other hand, understanding how rescues would be formed in different environments and under which conditions self-contained networks would be endogeneously formed is important for both social and economic networks, and they are left for future research.

9 Clustering

In this section, I investigate further impacts of clustering under a specific network structure. In this part, I relax the assumption of large liquidation costs.

Let a financial network $(\overline{c}, k, F)$ be an *islands-connected* financial network such that the network consists of $\frac{N}{k}$ symmetric isolated islands denoted by the set of $J = \{J_1, ..., J_{\frac{N}{k}}\}$, and there exists k banks in each isolated island $J_i \in J$. Banks in each isolated island form complete connections and banks in different islands have no connections. Let $\overline{c}$ be the integration level of each bank, which is the ratio of total interbank liabilities to total asset for each bank. Then, in an *islands-connected* financial network $(\overline{c}, k, F)$:

$$(\overline{c}, k, F) := \begin{cases} \sum\limits_{i \in N} C_{ij} = \overline{c} & \forall j \in J_t, \forall J_t \in J \\ C_{ij} = \frac{\overline{c}}{k-1} & \forall i, j \neq i \in J_t, \forall J_t \in J \\ C_{ii} = 0 & \forall i \in N \\ C_{ij} = 0 & \forall i \in J_t, j \notin J_t, \forall J_t \in J \\ p_i = 1 & \forall i \in N \\ l_i = l < 1 & \forall i \in N \end{cases}$$

Next, I provide a result on clustering. In this result, I abstract away from the divisibility problem and rule out the cases for cluster sizes k where $\frac{N}{k}$ is not an integer, because not ruling out those cases would add no further insight to the comparative statics result below. Moreover, the definition can easily be relaxed to capture non-identical-size clusters as well, which would again have no implications for the result.

Let the number of isolated islands, $\frac{N}{|J|}$ in a network be the measure of clustering in it such that clustering is increasing in the number of isolated islands.

Proposition 1 *In an islands-connected network* $(\overline{c}, k, F)$*,*

(i) for $s > s^*(\overline{c}, k, l, \zeta)$*,* $\zeta < \zeta^*(\overline{c}, k, l, s)$*, and* $k < k^*(\overline{c}, s, l, \zeta)$*, all banks in the island that involves the shocked bank fail,*

(ii) $s^*(\overline{c}, k, l, \zeta)$ *is increasing in both* k *(size of each isolated island) and* ζ *(liquidation costs),*

(iii) there exists $0 < \zeta^{**}(\overline{c}, l, N, s) \leq 1$ *such that for* $\zeta < \zeta^{**}(\overline{c}, l, N, s)$ *and for* $s > s^{**}(\overline{c}, l, N)$*, welfare losses are weakly increasing in* k *(for sufficiently small liquidation*

costs and sufficiently large shock, welfare losses are weakly decreasing in clustering),

(iv) there exists $0 \leq \zeta^{***}(\overline{c}, l) < 1$ *such that for* $\zeta \geq \zeta^{***}(\overline{c}, l)$*, there exists no such* $s^*(\overline{c}, k, l, \zeta)$ *described in part (i). Then, for* $\zeta \geq \zeta^{***}(\overline{c}, l)$*, welfare losses are weakly decreasing in* k *(for sufficiently large liquidation costs, welfare losses are always increasing in clustering, regardless of the shock level).*

As discussed in the previous section, clustering (the overall network structure) plays the key role in selectivity of rescues. In more clustered networks, a failure more quickly depletes the capital of direct counterparties, and whenever the losses are greater than what the counterparties can absorb, then these excess losses are absorbed by the depositors. On the other hand, in less clustered networks, the number of immediate counterparties that can absorb the losses is larger. In light of this intuition, part (iv) of Proposition 1 shows that for sufficiently large liquidation costs, lower clustering always reduces the welfare losses. This can be explained as follows. Whenever the liquidation costs are sufficiently large, the government always have the incentives for rescuing the banks in the distressed-island, and the larger the island size, the weakly higher the banks' contributions in rescues (strictly higher in some cases). This happens regardless of the other network properties and contagiousness of the network. Interestingly, whenever the liquidation costs are small and rescuing the shock-driven failures becomes costly against a large shock, then the impact of clustering can be reversed. For small liquidation costs, an increase in cluster size (up to a threshold level) increases the welfare losses, because in this case, all banks in the distressed-island default if the rescues are costly for the government and the banks in the distressed island do not have enough capital to rescue themselves. The lower the liquidation costs, the weaker the rescue incentives. Consequently, for sufficiently small liquidation costs and sufficiently large shock, the highest level of clustering, which is all banks staying singleton, is the welfare-losses-minimizing network, because for such low levels of liquidation costs, the incentives for rescues disappear.

10 Diversification

In this section, I investigate the implications of diversification. I focus on a specific class of networks, called d-ring lattices. Figure 3 is an illustration of a sequence of d-ring lattices that can be ordered from a ring network ($d = 1$) to complete network ($d = N - 1$). So, a sequence of d-ring lattices captures the idea of changes in the number of connections of banks (diversification) while keeping the clustering and the total level of interbank exposures of a bank same accross networks.

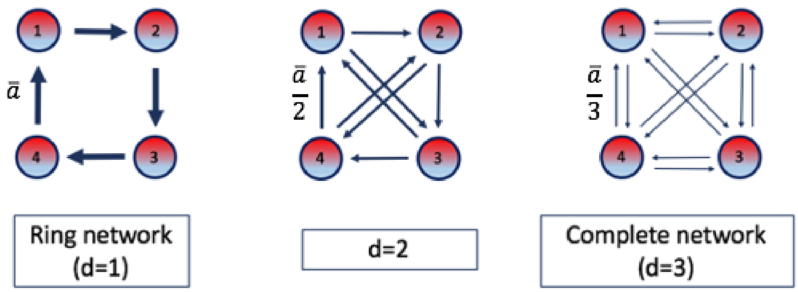

Figure 3: A sequence of $d-$ring lattices with four banks

As shown in the Appendix, under the plain vanilla contagion model, when payments are realized, a flow from bank i to bank j is given by $A_{ji}p_i$ where A_{ji} is the dependency of bank j to bank i and $A = \hat{C}(I-C)^{-1}$ matrix is a column stochastic matrix, called the *dependency matrix*. In this part, I use the A matrix instead of the C matrix to define integration.[20]

A d-ring lattice $(\overline{a}, d, \overline{F})$ is a non-clustered network with properties below, where $i - \{k\}_{k \leq N-1}$ refers to the k^{th} node preceding the node i when the nodes are ordered on a circle.

$$(A, d, F) := \begin{cases} A_{ii} = 1 - a & \forall i \in N, \\ A_{ij} = \frac{\overline{a}}{d} & \forall \{i, j : i - d \leq j \leq i - 1\}, \\ A_{ij} = 0 & otherwise. \end{cases}$$

In a given d-ring lattice $(\overline{a}, d, \overline{F})$, the integration level of each bank is equal to $\overline{a} = \sum_{j \in N} A_{ji}$ (the sum of dependencies of other banks to bank i) for each bank $i \in N$. Moreover, each bank is the creditor of the closest d number of preceding banks and the borrower of the closest d number of following banks in any given $(\overline{a}, d, \overline{F})$. In addition, $A_{ij} = \frac{\overline{a}}{d}$ for all (i, j) such that $A_{ij} > 0$, which capture the homogeneity in the dependencies of counterparties. For a given d-ring lattice $(\overline{a}, d, \overline{F})$, the diversification is increasing in d for given integration $\overline{a}$. The diversification level of $d = 1$ refers to the ring network in which each bank i is the single borrower of the bank following bank i, and $d = N - 1$ refers to the complete network in which every pair of distinct

[20]In a d-ring lattice defined over the C matrix, counterparty j of bank i and counterparty k of bank i might have infinitesimal differences in their dependencies to i such that $A_{ji} \neq A_{ki}$, even though there is full symmetry in the network and $C_{ji} = C_{ki}$. Such infinitesimal differences create excessive complications in the proof for non-large liquidation costs, without changing any insights. Therefore, I use this symmetric definition of integration, which becomes the most appropriate way of thinking about symmetric dependencies.

nodes is connected by a pair of links.

Let $\psi^R(\overline{a}, \overline{F})$ be a class of d-ring lattices where a given network $(\overline{a}, d, \overline{F}) \in \psi^R(\overline{a}, \overline{F})$ has integration level $\overline{a}$, diversification level d, and bank characteristics $\overline{F}$. The networks in $\psi^R(\overline{a}, \overline{F})$ only differ in diversification level, all else equal.

Proposition **2** *In a sequence of a d-ring lattices $\psi^R(\overline{a}, \overline{F})$, all else same,*

(i) for $s > s^*(\overline{a}, d, l, \zeta)$, $\zeta < \zeta^*(\overline{a}, d, l, s)$, and $d < d^*(\overline{a}, s, l, \zeta)$, the shocked-bank and its all counterparties (banks from $r+1$ to $r+d+1$) fail,

(ii) $s^*(\overline{a}, d, l, \zeta)$ is increasing in d (diversification) and ζ (liquidation costs),

(iii) there exists $0 < \zeta^{**}(\overline{a}, l, N, s) \leq 1$ such that for $\zeta < \zeta^{**}(\overline{a}, l, N, s)$ and for $s > s^{**}(\overline{a}, l, N)$, welfare losses are weakly increasing in d (for sufficiently small liquidation costs and large shock, welfare losses are weakly increasing in diversification),

(iv) there exists $0 \leq \zeta^{***}(\overline{a}, l, N) < 1$ such that for $\zeta \geq \zeta^{***}(\overline{a}, l, N)$, there exists no such $s^*(\overline{a}, d, l, \zeta) \leq p_r$ as described in part (i). Then, for $\zeta \geq \zeta^{***}(\overline{a}, l, N)$, welfare losses are identical in any d-ring lattice $(\overline{a}, d, \overline{F}) \in \psi^R(\overline{a}, \overline{F})$ (for sufficiently large liquidation costs, diversification plays no role in welfare losses, regardless of the shock level).

Diversification determines the set of banks that directly absorb the shock and the set of banks that directly absorb the costs of a failure. Therefore, it plays a key role in potential contagiousness of the network. Different from the clustering property, diversification is a bank-level property. Even though it has an impact on the contagiousness, it does not always affect how much banks contribute in rescues because rescues are independent from contagious effects of a failure. On the other hand, diversification can play a role in the rescue of the shock-driven failures in some cases, because it determines the set of shock-driven failures and the set of banks that the shock is absorbed by. As shown in Proposition 2, whenever liquidation costs are small relative to the shock, the government and remaining banks let shock-driven failures occur. As diversification rises (up to a threshold level), a higher number of unprevented shock-driven failures occur, which increases the welfare losses. As a result, if liquidation costs are sufficiently small and the shock is sufficiently large, then diversification always increases the welfare losses. On the other hand, as the liquidation costs increase, rescue of shock-driven failures occur less selectively and diversification becomes less relevant. If the liquidation cost is sufficiently high, then diversification plays no role in welfare losses, regardless of the shock level.

This result have some similarities with the result on clustering in Section 9. Nevertheless, as one can see by comparing these two results, for sufficiently large liquidation costs, diversification has no role in welfare losses and lower clustering is always welfare improving. The reason why there is such a difference in the ultimate affects of these two properties is that for large liquidation costs, any d-ring lattice always satisfies the property of "non-clustering (of the potential losses) at subset level"; however, this property is not satisfied in isolated-islands network case.

This result is related to the results in Acemoglu, Ozdaglar, Tahbaz-Salehi [1]. In their framework, for different shock and integration levels, complete network (high diversified) and ring network (low diversified) have different implications in terms of resilience and stability. Similarly, Elliott, Golub, Jackson [35] shows that diversification play a key role in contagiousness of a network. Proposition 2 reverses the insights from plain vanilla contagion models on the impact of diversification. Lastly, it worths here also to note that as shown in the proof of Proposition 2, the rescue decision for the bank hit by the shock is independent from diversification (similarly it holds for clustering in Proposition 1). On the other hand, the analyses in related studies build on how network characteristics (e.g., diversification, clustering) affect the rescue decision for the initial failure, and hence the welfare losses. Under the differences in rescue mechanisms, the analysis here builds on deeper counteracting forces affecting the selective rescue actions of banks and the government.

11 Integration

In this section, I investigate the implications of integration on welfare losses. I study integration in d-ring lattice networks (and define it over the C matrix). I also consider that the liquidation costs are large. For small liquidation costs, the insights would be similar to the ones from diversification and clustering. However, as discussed previously, the implications of diversification and clustering differ from each other whenever the liquidation costs are large. Therefore, instead of providing the similar insights for small liquidation costs for integration as well, I provide a result on large liquidation costs. Proposition 3 shows how the implications of integration differs from both diversification and clustering under large liquidation costs.

PROPOSITION **3** *Under large liquidation costs, $\zeta = 1$, in a given d-ring lattice network, there exists a threshold level of $c^*(s)$ such that*

(i) distress-costs are minimized at $c = c^*(s)$,

(ii) for $c < c^*(s)$, distress-costs are weakly decreasing in c, and

(iii) for $c > c^*(s)$, the distress-costs are weakly increasing in c.

where $c^*(s)$ is identical in all d-ring lattices $(\overline{a}, d, \overline{F})$.

The integration level determines how much direct losses the counterparties of a distressed bank would bear in case of a failure. As the counterparty exposures rise, potential losses of counterparties from failures increase and, therefore, counterparties have incentives for higher amounts of rescue transfers. However, it is important to recall that a counterparty can absorb losses up to its capital (or shareholder's equity). If the potential losses of a counterparty exceeds its capital, then the extra losses are absorbed by depositors. With these intuitions in hand, Proposition 3 shows that intermediate levels of counterparty exposures minimize the distress costs to the society, under large liquidation costs. It can explained as follows. First, weak interbank ties (low integration) leads to the lack of rescue incentives by banks, so lower their contribution in rescue of the shocked bank. On the other hand, against large shocks, rescuing all shock-driven failures become too costly and the government prefers to let shocked-bank fail and make transfers to the rest, if needed. In such a case, if the remaining banks' potential losses from the failure of the shocked bank exceed the sum of their capital, then government bailouts are always required, which increases the welfare costs. Consequently, the results imply that interbank liabilities entail a trade-off: strong ties among banks increase the rescue incentives and reduces the distress-costs up to a threshold level, but it increases the distress-costs if it is higher than that threshold level.

12 A Discussion on How to Avoid Government's Commitment Issue

If banks know that the government has the incentives to prevent failures even without banks' contribution, then the government's commitment not to rescue distressed banks is non-credible. Even though government's commitment issue is a related aspect of the rescue problem, it is avoidable under existing regulations of "mandatory bail-in"[21] power of the government, which is implemented in various financial systems after 2007-08 period. Bank of England in the UK has the power of mandatory bail-ins [14]. Similarly, the European Single Resolution Mechanism enables the EU resolution authorities to implement mandatory bail-in power as a resolution method, and the Dodd-Frank Act enacted in 2010 in the US ensures that both shareholders and unsecured creditors bear the losses from a failure in accordance with the priority of claims provisions.

[21]See Walther and White [75] for a model of mandatory bail-ins. For further discussion on bail-ins, see Thomas and Salord [72].

Under a mandatory bail-in regime, the counterparties of a bank can be paid back as if the bank defaults, even if the failure does not occur but the bank faces a failure risk. Therefore, the bail-in power of the government would eliminate any commitment issue, because if banks decide not to contribute in rescues, then the government can use its bail-in power for any bank in the system in a way that would be equivalent to the rescue actions by direct/indirect counterparties.[22] One might also think whether the government can make rescue contributions mandatory for the banks that it cannot use its bail-in power on. As opposed to the previous case, there exists no such policy in place since it is harder to apply such a policy in the absence of a direct tool as the "bail-in power" that would enable the government to force banks to make mandatory transfers. So, I rule out any such cases in the analysis.

Lastly, under such mandatory bail-in regimes, one might ask why banks would form networks with some desired properties (that I show throughout the paper) in the first place, instead of not forming such networks to avoid contributions in rescues in crisis times. Even after these regulations, banks continue to form connections via bilateral or multilateral contracts in the financial system[23]. Having said that, in cases where banks might not have willingness to form the networks with desired properties, then regulations might help the formation of such networks, similar to capital and liquidity regulations. However, since the rescue mechanism here is a risk sharing property, and since bank's contributions also complement government's bailout transfers, in certain cases banks might have the incentives to form such networks to guarantee their own rescue against any individual fragility. Endogenous formation of counterparty exposures under a rescue formation model is an interesting research direction and left for future research.

13 A Discussion on Welfare Maximizing Networks vs Distress-Costs Minimizing Networks

In this paper, I study the role of the network on welfare losses in distressed times. However, one can think that the properties of counterparty exposures that reduce the

[22]This mechanism always works effectively because banks' contributions are capped by their potential gains from eliminating the welfare losses on their interbank assets.

[23]e.g., overnight or long-term loans, repo transactions, syndicated loans, swaps and other derivative contracts, collateralized debt obligations (CDOs), and asset-backed securities. In the first half of 2009, gross credit exposures of dealer banks in the over-the-counter (OTC) market were more than 3.7 trillion dollars (BIS [12]). As of the first quarter of 2019, the value of total claims of financial institutions among each other has exceeded 9.1 trillion dollars (BIS [13])

welfare losses in crisis times might not be welfare maximizing because the interbank exposures can contribute to social welfare in non-crisis times. Modelling the welfare-enhancing impacts of interconnectivity in non-crisis times is important for regulatory purposes, however, the results throughout the paper help us to make predictions on how welfare-maximizing networks and welfare-losses-minimizing networks can differ. The welfare benefits of contracts between banks is that banks can make direct profits via these contracts or can have access to additional liquidity that would increase their loan supply to non-financial firms or households and enhance their profitability indirectly. So, instead of the overall network structure, the level of counterparty exposures (how much a bank can borrow from other banks in total) becomes relevant. The overall structure of counterparty exposures (who borrows from whom) is less likely to affect welfare enhancing benefits of counterparty exposures. For instance, it is not likely that diversification would create a benefit for banks in terms of welfare gains. The diversification and clustering of interbank exposures are risk-related properties instead of non-risk-related gains from exposures. On the other hand, when one considers common asset holdings of banks, then diversification and clustering of primary asset holdings (not the interbank assets) might have various implications, which is not studied in this paper and left for future research. As a result, a regulation that would allow banks to borrow and lend at specific levels (e.g., intermediate level of integration) might be thought of as a limiting factor for non-risk related gains from exposures. Proposition 3 provides the implications of different levels of integration on welfare-losses. If the welfare-enhancing level of integration in normal times is less (or more) than the welfare-losses-minimizing level of integration, then the government can set a target in between these two levels while considering both risk-related and non-risk related gains.

14 Conclusion

Bank rescues have always played a key role in maintaining stability in the financial system. Recently, 2007-2008 global financial crisis has brought bank resolution to the forefront of the discussions. In this study, I develop a coalition formation framework to analyze financial contagion and stability while considering both the government's and banks' incentives to prevent costly bank failures.

The results alter fundamental insights into financial stability and government bailouts and challenge the well-known concepts of too-big-to-fail or too-connected-to-fail. Moreover, the rescue formation framework presented in this paper provides various insights into different behaviors of agents against each other's fragility in interconnected systems and how a social planner can solve these problems via implementing ex-ante regulatory policies on the network structure and ex-post intervention via resource transfers to agents.

Lastly, this study focuses on the role of pre-existing counterparty risks while abstracting away from endogenous portfolio selection of banks. The next step of research is investigating the endogenous formation of the interbank network and banks' asset portfolio selection in the pre-crisis period. This would help us better understand whether the endogenous portfolio selection of banks can work as a commitment mechanism for rescues in crisis times.

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15 Appendix

1. Contagion Algorithm

By incorporating the bankruptcy condition (with no rescue attempts), the total assets of the bank hit by the shock is given by:

$$V_r = (\sum_{j \in N} C_{rj} V_j) + (p_r - s - b_r), \tag{3}$$

which is equal to the sum of its interbank assets and the recovery value of its primary asset, where $b_r = \beta_r$ if bank r defaults and 0 otherwise. Similarly, the total assets of bank $i \neq r$ when the payments are realized is given by:

$$V_i = (\sum_{j \in N} C_{ij} V_j) + (p_i - b_i), \tag{4}$$

where $b_i = \beta_i$ if bank i defaults and 0 otherwise.

In matrix notation:

$$\mathbf{V} = (I - C)^{-1}(\mathbf{p} - \mathbf{b}), \tag{5}$$

where $\mathbf{p} = [p_1, ..., p_r - s, ..., p_n]'$ is the vector of primary assets after the shock, and $\mathbf{b}$ is the vector of realized liquidation costs such that $b_i = \beta_i$ if bank i defaults and 0 otherwise.

The total liabilities of bank i is equal to the sum of its interbank and external liabilities, which is given by:

$$L_i = (\sum_{j \in N} C_{ji} V_i) + l_i. \tag{6}$$

Lastly, by incorporating the bankruptcy condition, shareholders' equity (or net worth) of bank i is equal to:

$$e_i = max\{0, V_i - L_i\}. \tag{7}$$

Following equations (4) and (5), shareholders' equity can be rewritten as:

$$e_i = max\{0, v_i - l_i\}, \tag{8}$$

where $v_i = V_i - (\sum_{j\in N} C_{ji}V_i)$ is the total assets of bank i minus its liabilities to other banks. For $\hat{C}_{ii} := 1 - \sum_{j\in N} C_{ji} > 0$, it holds that $v_i = \hat{C}_{ii}V_i$.

As a result, $\hat{C}_{ii}$[24] is the portion of the total assets of bank i which is not claimed by other banks in the network. The vector $\mathbf{v}$ can be rewritten in matrix notation as $\mathbf{v} = \hat{C}(I - C)^{-1}(\mathbf{p} - \mathbf{b})$.

For, $A = \hat{C}(I - C)^{-1}$, we have:

$$\mathbf{v} = A(\mathbf{p} - \mathbf{b}). \tag{9}$$

Consequently, the shareholders' equity is given by:

$$\mathbf{e} = max\{0, A(\mathbf{p} - \mathbf{b}) - \mathbf{l}\}. \tag{10}$$

$A = \hat{C}(I - C)^{-1}$ matrix is a column stochastic matrix, called the *dependency matrix*. The shareholders' equity and the payments to internal and external creditors of each bank are determined simultaneously via the payment solution satisfying Equations (3) to (8). There always exists a payment solution and there can be multiple solutions.[25] As in Elliott, Golub, Jackson [35], the contagion algorithm is based on the best-case solution in which as few banks as possible fail. The contagion algorithm works as follows:

At step t of the algorithm, let $\aleph_t$ be the set of failed banks. Initialize $\aleph_0 = \emptyset$.[26] At step $t \geq 1$:

(i) Let b_{t-1} be a vector with element $b_i = \beta_i$ if $i \in \aleph_{t-1}$ and $b_i = 0$ otherwise.

(ii) Let $\aleph_t$ be the set of all j such that entry j of the following vector is negative:

$$A[\mathbf{p} - \mathbf{b}_{t-1}] - l$$

(iii) Terminate if $\aleph_t = \aleph_{t-1}$. Otherwise return to step 1. The algorithm provides us the domino failures in the network, where various banks default at each step,

[24] $\hat{C}_{ii}$ is assumed to be strictly positive. In matrix notation, $\hat{C}$ is an $n \times n$ diagonal matrix such that $\hat{C}_{ii} > 0$ $\forall i$ and $\hat{C}_{ij} = 0$ $\forall i \neq j$. By this assumption, the inverse $(I - C)^{-1}$ is well defined and non-negative.

[25] See Elliott, Golub, Jackson [35] for a detailed discussion on the existence and the multiplicity of payment solution. There exists two main sources of the multiplicity of the payment solution. One source is based on the story of self-fulfilling failures as in the Diamond and Dybvig [27] model, and the other source of the multiple solution is based on the interdependencies in the financial network.

[26] For $s = 0$, it is considered that a given financial network (C, F) is such that all banks are always able to pay back their external and interbank liabilities in full and the net worth of each bank is non-negative.

which are triggered by the defaults in the previous steps. By using this algorithm one can find the set of failures for a given financial network (C, F). This hierarchical default structure works for any given financial network. Lastly, Lemma 2 in the Supplementary Appendix provides the restructuring of the dependencies after the formation of rescue coalitions.

Proof of Theorem 1:

Step 1) I show that in any given network (C, F), there exists no equilibrium outcome where there are liquidation-driven failures.

Step 2) I show that for any given network (C, F), there is a distress-cost-minimizing government transfer vector $\mathbf{t}^*$ such that $(C, F, \mathbf{t}^*)^{M^*}$ is an equilibrium outcome where all liquidation-driven failures are always prevented under certain conditions, and all failures are always prevented under certain conditions. The second step finalizes the proof.

Proof of Step 1) Consider any financial network (C, F) and an outcome $(C, F, \mathbf{t})^M$ where there exists some liquidation-driven failures. Next, I show that there exists $(C, F, \mathbf{t}')^{M'}$ such that a distress-cost-minimizing government always prefers $\mathbf{t}'$ to $\mathbf{t}$, where following the coalition formation there exists no liquidation-driven-failures in $(C, F, \mathbf{t}')^{M'}$ and, hence, $(C, F, \mathbf{t})^M$ cannot be an equilibrium outcome.

Call the set of failures in $(C, F, \mathbf{t})^M$ by $(\aleph_M^T)$. Based on the contagion algorithm, call the non-rescued second set of failures in $(\aleph_M^T)$ by $(\aleph_M^2)$. The set $(\aleph_M^2)$ is the first set of liquidation-driven-failures after the coalitions are formed, so it is the set of non-rescued first set of liquidation-driven failures. Such non-rescued first set of liquidation-driven-failures can lead to other liquidation-driven-failures as well. If all banks in the set $(\aleph_M^2)$ is rescued, then there exists no liquidation driven failures.

I will show that it is always preferred by the government to deviate from $\mathbf{t}$ and make further transfers to $(\aleph_M^2)$ to prevent their failure. By considering the selective rescues, one can prove it by the iteration technique, but since the claim is for any set of non-rescued $(\aleph_M^2)$, it is equivalent to the iteration technique.

Preventing the set of failures $(\aleph_M^2)$ immediately implies that contagion algorithm stops after the shock-driven failures, and this will end the proof of Step 1.

To prevent the failure of $(\aleph_M^2)$, the amount of government transfer required if there are no transfers by banks would be as follows:

Consider the partition $(C, F, \mathbf{t})^M$, and consider that in addition to the coalitions in M, the coalition of banks in the set $\aleph_M^2$. Denote the new partition $(C, F, \mathbf{t}')^{M'}$. We know that when banks in $\aleph_M^2$ is rescued, all the remaining liquidation-driven-falures are also prevetented. Then, the required level of government transfer to coalition $\aleph_M^2$

in $(C, F, \mathbf{t}^{'})^{M^{'}}$ in the absence of any contribution from other banks to this particular coalition is: $\sum_{j\in\aleph^1_{M'}}\sum_{i\in\aleph^2_M}\beta_j A_{ij} + \sum_{i\in\aleph^2_M} sA_{ir} - \sum_{i\in\aleph^2_M}(v_i - l_i)$.

Following the minimum level of government transfer to coalition $\aleph^2_M$, the government wants the benefits being larger than transfers. The benefits from rescuing these banks is $B \geq \sum_{i\in\aleph^2_M}\beta_i$, and $B = \sum_{i\in\aleph^2_M}\beta_i$ if the set $(\aleph^T_M) \setminus (\aleph^1_M) \cup (\aleph^2_M)$ is empty (if the failure of banks in $\aleph^2_M$ would not cause any further liquidation-driven failures).

So, the condition below must hold:

$$\sum_{i\in\aleph^2_M}\beta_i - \left[\sum_{j\in\aleph^1_{M'}}\sum_{i\in\aleph^2_M}\beta_j A_{ij} + \sum_{i\in\aleph^2_M} sA_{ir} - \sum_{i\in\aleph^2_M}(v_i - l_i)\right] \geq 0$$

which can be rewritten as follows:

$$\sum_{i\in\aleph^2_M}\zeta\left[p_i - \sum_{j\in\aleph^1_M} A_{ij}p_j\right] + \sum_{i\in\aleph^2_M}(v_i - l_i - sA_{ir}) \geq 0$$

(Here, note that for bank r, p_r also takes the shock into account, so it is $p_r - s$.)

For any bank in $i \in \aleph^2_M$, $v_i - l_i - sA_{ir} \geq 0$ always holds. Then, for $\sum_{i\in\aleph^2_M}\zeta\left[p_i - \sum_{j\in\aleph^1_M} A_{ij}p_j\right] \geq 0$, such an additional transfer by the government always decreases the welfare-losses and therefore the outcome where there exists a liquidation-driven-failure cannot be an equilibrium outcome. As long as for any $i \in N$, $p_i \geq \sum_{j\neq i} A_{ij}p_j$ holds, the condition above always holds in any given financial network. Then, the government prefers $\mathbf{t}^{'}$ to $\mathbf{t}$.

Lastly, I show that if there exists any equilibrium outcome $(C, F, \mathbf{t})^M$ under $\mathbf{t}$, then the equilibrium outcome would be $(C, F, \mathbf{t}^{'})^{M^{'}}$ under $\mathbf{t}^{'}$ where only the additional coalition of $\aleph^2_M$ is formed different from the outcome M.

If the government makes such required transfers to set $\aleph^2_M$, then the non-rescued first set of failures remains same $\aleph^1_{M'} = \aleph^1_M$, because including any distressed bank $i \in \aleph^1_{M'}$ to the coalition $\aleph^2_M$ is costly and making the minimum required transfers that would make the net worth of coalition $\aleph^2_M = 0$ imply that including any distressed bank $i \in \aleph^1_{M'}$ to the coalition $\aleph^2_M$ would result in the failure of the larger coalition. Lastly, if the formation of $\aleph^2_M$ prevents the failure of further liquidation driven failures which were in some coalitions in $(C, F, \mathbf{t})^M$, whenever the $\aleph^2_M$ is formed, then such already existing rescue coalitions will now only involve either healthy banks, or some healthy banks and some banks from the set of $(\aleph^1)^R \cup (\aleph^2)^R$. By Lemma 2, a coalition of healthy banks can be either formed or not formed at an equilibrium without changing their net worths (so one can consider such banks do not form such coalitions after the transfers to $\aleph^2_M$).

Next, denote the set of prevented first set of liquidation-driven failures by $(\aleph^2)^R$, where $(\aleph^2)^R \subseteq \aleph^2$, and denote the set of prevented shock-driven failures by $(\aleph^1)^R$, where $(\aleph^1)^R \subseteq \aleph^1$. Lemma 2 in the supplementary appendix implies that the formation of the coalition $\aleph^2_M$ does not effect the decision of the rest of the system to rescue any bank in the set $(\aleph^1)^R$ and the set $(\aleph^2)^R$. Eventually, a distress-costs-minimizing government can prefer to make a higher transfer or make transfers to the set $(\aleph^1_M)$ eventually, but in this step of the proof, I specifically show that even the formation of the additional coalition of $\aleph^2_M$ is preferred by the government, so that $(C, F, \mathbf{t})^M$ cannot be an equilibrium outcome.

Lastly, the proof shows that the government would prefer to make the additional transfers to $\aleph^2_M$ even under zero additional contribution to $\aleph^2_M$ by other banks and even the rescue of $\aleph^2_M$ does not prevent any additional failures. Therefore, Step 1 holds regardless of banks' contributions, and hence the network structure. This is the end of the proof of Step 1.

Proof of Step 2)

Take the set $N \setminus \aleph^1$. Following the Step 1, there exists no equilibrium in which a failure outside the set of $N \setminus \aleph^1$ occurs. The same rescue principle in Theorem 1 applies again. Consider an equilibrium outcome (existence of equilibrium is showed at the end of the proof) under $\mathbf{t}'$, where all liquidation-driven failures are prevented but a subset $J' \subseteq \aleph^1$ is not rescued. For any such subset $J' \subseteq \aleph^1$, the minimum amount of government transfer required to rescue J' is:

$$\sum_{j \in J'} sA_{jr} + \sum_{k \in \aleph^1 \setminus J'} A_{jk}\beta_k - \sum_{j \in J'} (v_j - l_j).$$

Next, I find the set $J' \subseteq \aleph^1$ that is not rescued, and show under which conditions that set is empty.

If the subset $J' \subseteq \aleph^1$ is not rescued, then for any subset J'' of the subset J', the following condition must hold:

$$\sum_{j \in J'' \subseteq J'} \zeta \left[p_j - \sum_{k \in J' \setminus J''} A_{jk} p_k \right] + \left(\sum_{j \in J'' \subseteq J'} (v_j - l_j) - \sum_{j \in J'' \subseteq J'} sA_{jr} \right) < 0$$

If there exists such a subset $J' \subseteq \aleph^1$, then the set J' is not rescued.

The condition can be rewritten as follows:

$$\frac{\sum_{j \in J'' \subseteq J'} \zeta \left[p_j - \sum_{k \in J' \setminus J''} A_{jk} p_k \right] + \sum_{j \in J'' \subseteq J'} (v_j - l_j)}{\left(\sum_{j \in J'' \subseteq J'} A_{jr} \right)} < s$$

Then, there exists a threshold level of $s_{J''}$ for each subset $J'' \subseteq J'$ such that for $s \leq s^* = min\{s_{J''}\}$, all banks are rescued at equilibrium. The similar arguments about formation of coalitions in Step 1 will follow here as well, and whenever the condition above is satisfied, all banks in the set $\aleph^1$ will be always rescued. Then, as long as for any $i \in N$, $p_i \geq \sum_{j \neq i} A_{ij} p_j$ holds, then s^* is increasing in ζ. This finalizes the Step 2. Again the government transfer here assumes no additional transfers from other banks, however, banks can also make transfers depending on their incentives, similar to the argument in Step 1.

At an equilibrium where the government makes less transfers and banks make some positive transfers via forming coalitions, the realized failures in this case cannot be a superset of the realized failures when only the government makes the transfers. In that case if banks would form a coalition that excludes some $\aleph^1$, then a distress-cost-minimizer government always prefers to make some transfer and prevent all failures as explained above.

Lastly, there always exists an equilibrium. Consider any network (C, F) and a shock s hitting bank r. Suppose that the equilibrium outcome when the government transfer $\mathbf{t} = \mathbf{0}$ is $(C, F, 0)^{M_0}$. If there exists no equilibrium outcome, the government can increase the vector $\mathbf{t}$ until it reaches an equilibrium transfers and outcome that minimizes the distress costs. It is always guaranteed that there exists a $\bar{\mathbf{t}}$ where there exists an equilibrium. In the most extreme case, the government can always makes transfers to each bank in a way that banks do not need to form any coalition, so that each bank staying single becomes the equilibrium outcome. This completes the proof.

Proof of Theorem 7:

The dependency matrix and vector $\mathbf{v}$ is given by Equation 10.

If a network satisfies the following properties, then it is a welfare-minimizing network:

i) for $s = s_S$, there exists a strong Nash equilibrium in which all potential failures (if exist) are prevented with no government assistance. (whenever the banks in the system have the capability to prevent all failures, then they always prevent all failures)

ii) for $s = s_L$, all failures but the failure of the shocked bank can be prevented in full (if exist) with no government assistance. (whenever the banks in the system do not have the capability to prevent all failures, then they can always prevent all failures but the failure of the shocked bank)

Any network satisfying the properties above weakly dominates any other network because the welfare-losses are minimized both for $s = s_S$ and for $s = s_L$ (for $s = s_L$ government can assist in the rescue of the shocked bank depending on the shock level, but the government assistance is minimized in such a case). So, it is the minimum possible value for $H(C, F, \mathbf{t})^M$.

Step 1) First, the Assumption of $\sum_{i=1}^{n}(p_i - l_i) < min\{p_1, ..., p_n\}$ implies that the shock to any bank can be large enough that banks are not capable of preventing all failures. In such a case, the government assistance is required to prevent all failures. Whenever the government decides to assists, the level of assistance does not depend on the level of connectivity but only depends on $\sum_{i=1}^{n}(p_i - l_i)$, which is exogenously given. Whenever, the government decides not to rescue the shocked bank, then the rest of banks can prevent all potential contagious failures (all potential failures except the shocked bank) if $1 - A_{kk} \leq \sum_{j \in N \setminus k}(v_j - l_j)$. Therefore, for each bank k, the following property is the first desired property.

$$(1 - A_{kk})\, p_k = \sum_{j \in N \setminus k}(v_j - l_j)$$

where $(1 - A_{kk})$ measures the total dependency of other banks to bank k. In this case, failure of a bank causes a loss to the rest of the system equal to the total net worth of remaining banks.

By using $\mathbf{v} = A\mathbf{p}$, the expression can be rewritten as:

$$\sum_{i \in N \setminus k}\sum_{j \in N \setminus k} A_{ij} p_j = \sum_{j \in N \setminus k} l_j \text{ for all } k \in N \text{ } (k \text{ is a singleton})$$

This equation implies that the dependency of other banks to bank k, $(1 - A_{kk})$, is at intermediate levels since it is a function of $\mathbf{p}$ and $\mathbf{l}$, and it is always greater than zero.

Step 2) For $(1 - A_{kk})\, p_k = \sum_{j \in N \setminus k}(v_j - l_j)$ for all $k \in N$ (k is a singleton), it holds that whenever the shocked bank (a randomly chosen bank) is distressed either

i) there exists at least one extra bank that is distressed (there exists a bank $j \in N \setminus r$ s.t. $(1 - A_{jr})\, p_r > v_j - l_j$), or

ii) $(1 - A_{jr})\, p_r = (v_j - l_j)$ holds for each $j \in N \setminus r$ (net worth of each bank in the set of $\{N \setminus r\}$ drops to zero, as in the complete network case).

If the second condition holds (such as in complete network), then we are done. If the first condition holds, move to step 3.

Step 3) Next, I show that whenever the condition below holds and the banks do not have capability to rescue the shocked bank, then there exists no group of banks that can deviate from grand coalition and have a shareholders' value greater than zero without government support.

$$\sum_{i \in K}\sum_{j \in K} A_{ij} p_j \leq \sum_{i \in K} l_i \text{ for all } K \subset N$$

Any $K \subset N$ defaults. For any $K \subset N \setminus r$, the condition above implies that either

i) $\sum_{j \in N \setminus k} A_{kr} p_r > \sum_{j \in K} (v_j - l_j)$, otherwise

ii) there exists a bank in $j \in N \setminus \{K \cup r\}$ such that $A_{jr} p_r > v_j - l_j$. Then, for any consequent steps the similar scenario holds, and eventually the loss emanating from failing banks outside of group K exceeds $\sum_{j \in K} (v_j - l_j)$. So, any coalition that does not involve bank r also defaults, so deviation is not profitable.

Next, consider the small shocks where banks can coalitionally prevent all failures. In such a case, the sharing rule guarantees that there exists no profitable deviation by a coalition smaller than the grand coalition that involves bank r. The reason is that for a deviation to be profitable it should be that any remaining bank must be solvent, otherwise as shown above the total loss exceed $\sum_{j \in K} (v_j - l_j)$ and the coalition defaults. Consider a partition $\{K, N \setminus K\}$ such that coalition K involves bank r and can absorb the shock, and the remaining banks are solvent. Then, given the sharing rule the grand coalition gives at least as much as payoff as each bank in coalition K receives in partition $\{K, N \setminus K\}$. Thus, the grand coalition becomes an equilibrium outcome. Then, depending on the shock level, the government chooses the transfers $\mathbf{t}$ and decide whether to assist in the rescue of the shocked bank or not (which can be done by choosing $t_r \geq 0$, while setting transfers to other banks to zero).

Any financial network (C, F) that does not satisfy these two properties is not a social-welfare maximizing network. First, whenever part (i) does not hold and the integration level for any bank is greater than that level, for sufficiently large shocks, the government has to step in to maintain the stability in the rest of the system that weakly reduces the social welfare. On the other hand, if there exists a bank that has a lower integration level than thar given level, then the contributions of other banks in the rescue of that bank would be lower, which again weakly reduces social welfare. Secondly, whenever part (ii) does not hold, then as discussed above there always exists a coalition that can deviate from the grand coalition and end up with a value greater than zero, which implies that there exist cases where the welfare is less than the first-best case. This finalizes the proof.

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16 Supplementary Appendix

LEMMA **1** *i)* $A_{ij} = \sum_{k \in N} A_{ik} C_{kj}$ *for all* $i \neq j$.

ii) $A_{ii} = \hat{C}_{ii} + \sum_{k \in N} A_{ik} C_{ki}$ *for all* $i \in N$.

Proof of Lemma 1:

First, I show that $A = AC + \hat{C}$ holds for any given C matrix such that $\hat{C}$ is a diagonal matrix with entries $\hat{C}_{ii} = (1 - \sum_{j \in N} C_{ji}) > 0$ for all i and $\hat{C}_{ij} = 0$ for all $i \neq j$, where $A = \widehat{C}(I - C)^{-1}$.

Suppose that $A = AC + \hat{C}$ holds. Then,

$A = AC + \hat{C}$ implies

$\hat{C}(I - C)^{-1} = \hat{C}(I - C)^{-1}C + \hat{C}$, which can be rewritten as:

$\hat{C}(I - C)^{-1} = \hat{C}[(I - C)^{-1}C + I]$.

Since $\hat{C}$ is a diagonal matrix with non-zero diagonal elements,

$(I - C)^{-1} = [(I - C)^{-1}C + I]$ holds, which can be rewritten as:

$(I - C)^{-1}(I - C) = I$, and hence $I = I$ holds, which completes the proof.

$A = AC + \hat{C}$ implies that $A_{ii} = \hat{C}_{ii} + \sum_{k \in N} A_{ik} C_{ki}$ holds for all $i \in N$, and $A_{ij} = \sum_{k \in N} A_{ik} C_{kj}$ holds for all $i \neq j$.

end of proof.

LEMMA **2** *Consider a financial network* (C, F) *in which a set of banks* $\phi \subseteq N$ *form the set of coalitions* $M = \{m_1, ..., m_n\}$, *which satisfies the properties below:*

$$N^M = (N \setminus \phi) \cup M$$

$$\begin{array}{llll} p_j^M = p_j & l_j^M = l_j & \beta_j^M = \beta_j & \forall j \in N \setminus \phi \\ p_{m_k}^M = \sum_{k \in m_k} p_k & l_{m_k}^M = \sum_{k \in m_k} l_k & \beta_{m_k}^M = \sum_{k \in m_k} \beta_k & \forall m_k \in M \end{array}$$

Then, for any given set M, *there exist unique structures for* C^M *and* A^M *that satisfy*

$$\begin{array}{lll} V_j^M = V_j & L_j^M = L_j & \forall j \in N \setminus \phi \\ V_{m_k}^M = \sum_{k \in m_k} V_k & L_{m_k}^M = \sum_{k \in m_k} L_k & \forall m_k \in M \end{array}$$

for every asset value vector $p \in \mathbb{R}_+^N$. *The unique structures have the following properties:*

$$C^M_{ij} = C_{ij} \;\; \forall i,j \in N \setminus \phi \qquad C^M_{m_kj} = \textstyle\sum_{k\in m_k} C_{kj} \;\; \forall j \in N \setminus \phi$$
$$C^M_{m_km_k} = \left(\frac{\sum_{k\in m_k}\sum_{l\in m_k}(C_{lk}V_k)}{\sum_{k\in m_k} V_k}\right) \;\forall m_k \in M \qquad C^M_{jm_k} = \left(\frac{\sum_{k\in m_k}(C_{jk}V_k)}{\sum_{k\in m_k} V_k}\right) \;\forall j \in N \setminus \phi$$
$$C^M_{m_km_l} = \left(\frac{\sum_{l\in m_l}\sum_{k\in m_k}(C_{kl}V_l)}{\sum_{l\in m_l} V_l}\right) \;\forall m_k, m_l \in M$$

$$A^M_{ij} = A_{ij} \;\; \forall i,j \in N \setminus \phi \qquad A^M_{m_kj} = \textstyle\sum_{k\in m_k} A_{kj} \;\; \forall j \in N \setminus \phi$$
$$A^M_{m_km_k} = \left(\frac{\sum_{k\in m_k}\sum_{l\in m_k}(A_{lk}p_k)}{\sum_{k\in m_k} p_k}\right) \;\forall m_k \in M \qquad A^M_{jm_k} = \left(\frac{\sum_{k\in m_k}(A_{jk}p_k)}{\sum_{k\in m_k} p_k}\right) \;\forall j \in N \setminus \phi$$
$$A^M_{m_km_l} = \left(\frac{\sum_{l\in m_l}\sum_{k\in m_k}(A_{kl}p_l)}{\sum_{l\in m_l} p_l}\right) \;\forall m_k, m_l \in M$$

First, Lemma 2 above abstracts away from government transfers, but it directly applies under government transfers as well. To include government transfers, each p_j should be replaced by p'_j where $p'_j = p_j + t_j$.

The restructuring of the claims can be summarized as follows:

i) the claims among the banks that are not involved in any coalition (singleton banks) remain same,

ii) the claims that a given coalition holds in a singleton bank is equal to the sum of the claims that each bank in that coalition holds in that singleton bank,

iii) the claims of a singleton bank in a coalition is equal to the weighted sum of its claims in each bank in that coalition,

iv) the claims of a given coalition in another coalition is a weighted sum of the claims of each bank in the banks in the other coalition.

Proof of Lemma 2:

The equations below are satisfied before the coalitions:

$$V_j = (\sum_{k\in\phi} C_{jk}V_k) + (\sum_{i\in N\setminus\phi} C_{ji}V_i) + p_j$$
$$\sum_{k\in m_k} V_k = (\sum_{k\in m_k}\sum_{j\in N} C_{kj}V_j) + \sum_{k\in m_k} p_k$$

Call the claims matrix given in Lemma 2 C^*. The equations below are satisfied after the coalitions:

$$V^M_j = (\sum_{i\in N\setminus\phi} C^*_{ji}V^M_i) + \sum_{m_k\in M}(C^*_{jm_k}V^M_{m_k}) + p^M_j \;\; \forall j \in N \setminus \phi$$
$$V^M_{m_k} = (\sum_{j\in N\setminus\phi} C^*_{m_kj}V^M_j) + C^*_{m_km_k}V^M_{m_k} + (\sum_{m_l\neq m_k} C^*_{m_km_l}V^M_{m_l}) + p^M_{m_k} \;\; \forall m_k \in M$$

I claim that C^* satisfies the given properties in Lemma 2. In order to show this, I rewrite the equations after the coalitions by using the asset values and the new claims C^* after the coalitions, which are given in Definition 2:

$$V^M_j = (\sum_{i\in N\setminus\phi} C_{ji}V^M_i) + \sum_{m_k\in M}\left(\frac{\sum\limits_{k\in m_k}(C_{jk}V_k)}{\sum\limits_{k\in m_k} V_k} V^M_{m_k}\right) + p_j \;\; \forall j \in N \setminus \phi$$

$V^M_{m_k} = (\sum_{j \in N \setminus \phi} \sum_{k \in m_k} C_{kj} V^M_j) + \left(\frac{\sum_{k \in m_k} \sum_{l \in m_k} (C_{lk} V_k)}{\sum_{k \in m_k} V_k} \right) V^M_{m_k} + \sum_{m_l \neq m_k} \left(\frac{\sum_{l \in m_l} \sum_{k \in m_k} (C_{kl} V_l)}{\sum_{l \in m_l} V_l} \right) V^M_{m_l} + \sum_{k \in m_k} p_k$

We know that the financial system before the coalitions has a unique solution. We also know that the new system has a unique solution. Then, if we plug $V_j = V^M_j$ $\forall j \in N \setminus \phi$ and $V^M_{m_k} = \sum_{k \in m_k} V_k$ $\forall m_k \in M$ into the equations for V^M_j and $V^M_{m_k}$, we get:

$V_j = (\sum_{i \in N \setminus \phi} C_{ji} V_i) + \sum_{m_k \in M} (\sum_{k \in m_k} C_{jk} V_k) + p_j = (\sum_{k \in \phi} C_{jk} V_k) + (\sum_{i \in N \setminus \phi} C_{ji} V_i) + p_j$, and

$\sum_{k \in m_k} V_k = (\sum_{k \in m_k} \sum_{j \in N} C_{kj} V_j) + \sum_{k \in m_k} p_k$

These are the equations that we had before the coalitions. Thus, from the uniqueness property, $V_j = V^M_j$ $\forall j \in N \setminus \phi$ and $V^M_{m_k} = \sum_{k \in m_k} V_k$ $\forall m_k \in M$ is also the unique solution for the system after the coalitions. So, C^* satisfies the properties given in Lemma 2.

Next, I show that A^* also satisfies the properties given in Lemma 2. The properties of healthy and distressed banks implies that $e_j = e^M_j$ $\forall j \in N \setminus \phi$ and $e^M_m = \sum_{k \in m_k} e_k$ $\forall m_k \in M$ holds for the shareholders' equity. Equation (7) implies $v = Ap$ holds before the shock, and hence $e = v - l = Ap - l$ holds before the shock. Since l is given exogenously, $v = v^M$ must hold, and hence $A^M p^M = Ap$ must hold.

For any given (C, F), A is the unique Leontief inverse matrix derived from the given C matrix**.** By the definition of healthy and distressed banks, for a given set of coalitions, one can find p^M and v^M. Next, I show that $v^M = A^* p^M$ holds for every $p \in \mathbb{R}^N_+$, which means that A^* satisfies the given properties in Lemma 2.

$v^M_j = \sum_{m_k \in M} (A^*_{j m_k} p^M_{m_k}) + \sum_{i \in N \setminus \phi} A^*_{ji} p^M_i = \sum_{m_k \in M} \left(\frac{\sum_{k \in m_k} (A_{jk} p_k)}{\sum_{k \in m_k} p_k} \right) \sum_{k \in m_k} p_k + \sum_{i \in N \setminus \phi} (A_{ji} p_i) = \sum_{i \in N} A_{ji} p_i = v_j$

holds for every $p \in \mathbb{R}^N_+$. Similarly,

$v^M_{m_k} = A^*_{m_k m_k} p^M_{m_k} + \sum_{m_l \neq m_k} (A^*_{m_k m_l} p^M_{m_l}) + \sum_{j \in N \setminus \phi} A^*_{m_k j} p^M_j = \left(\frac{\sum_{k \in m_k} \sum_{l \in m_k} (A_{lk} p_k)}{\sum_{k \in m_k} p_k} \right) \sum_{k \in m_k} p_k + \sum_{m_l \neq m_k} \left(\frac{\sum_{l \in m_l} \sum_{k \in m_k} (A_{kl} p_l)}{\sum_{l \in m_l} p_l} \right) \sum_{l \in m_l} p_l + \sum_{j \in N \setminus \phi} \sum_{k \in m_k} A_{kj} p_j = \sum_{k \in m_k} \sum_{j \in N} A_{kj} p_j = \sum_{k \in m_k} v_k$

holds for every $p \in \mathbb{R}^N_+$. Thus, $v^M = A^* p^M$ holds for every $p \in \mathbb{R}^N_+$.

This completes the first part of the proof that C^M and A^M satisfy the given properties of coalitions for every $p \in \mathbb{R}^N_+$.

Next, I show that there exist unique structures for C^M and A^M satisfying the desired properties for every asset values vector $p \in \mathbb{R}^N_+$.

First, I show that A^* is the unique structure satisfying the properties in Lemma

2 for every $p \in \mathbb{R}^N_+$.

I already showed that $v^M = A^* p^M$ holds for every $p \in \mathbb{R}^N_+$ for any given M. Fix any given M. Suppose that there exist another matrix A^{**} satisfying the desired properties on total assets and liabilities for every $p \in \mathbb{R}^N_+$ for M. Then, $v^M = A^* p^M$ and $v^M = A^{**} p^M$ hold for every $p \in \mathbb{R}^N_+$, which implies $A^{**} p^M = A^* p^M$ holds for every $p \in \mathbb{R}^N_+$. Then, $A^{**}\epsilon_j = A^*\epsilon_j \; \forall \, e_j$ holds where ϵ_j is the vector such that j^{th} element of ϵ_j is equal to 1 and all other elements of ϵ_j are equal to ϵ.

$A^{**}\epsilon_j = A^*\epsilon_j \; \forall \, \epsilon_j$ implies $A^*_{kj} = A^{**}_{kj} \; \forall k, j$ as $\epsilon \to 0$. Then, this implies $A^* = A^{**}$. Thus, A^* is unique.

Next, I show the uniqueness of C^*. I already showed that $V^M = (I - C^*)^{-1} p^M$ holds for every $p \in \mathbb{R}^N_+$ for any given M. Suppose that there is another C^{**} satisfying $V^M = (I - C^{**})^{-1} p^M$ for every $p \in \mathbb{R}^N_+$ for M. Then, similarly, $(I - C^*)^{-1} = (I - C^{**})^{-1}$ must hold.

$(I - C^*)^{-1} = (I - C^{**})^{-1} \Longrightarrow ((I - C^*)^{-1})^{-1} = ((I - C^{**})^{-1})^{-1} \Longrightarrow I - C^* = I - C^{**}$ holds from the uniqueness of the inverse matrix, which also implies $C^* = C^{**}$. Thus, C^* is unique.

Lastly, $A^*$and C^* being the unique structures satisfying the desired properties for every $p \in \mathbb{R}^N_+$ for any given M implies that $A^* = \hat{C}^*(I - C^*)^{-1}$, where $\hat{C}^*$ is a diagonal matrix derived from C^*.

Thus, A^M and C^M are the unique structures satisfying the desired properties for every asset values vector for any given set of coalitions, which completes the proof.

Lastly, following the formation of coalitions, the contagion analysis builds on the dependencies given by $A^M = \hat{C}^M (I - C^M)^{-1}$.

Proof of Proposition 1

Consider a clustered economy where there are k banks in each cluster and $\frac{N}{k}$ disconnected clusters.

By some further algebra, Lemma 2 implies that in a clustered network of consisting of $\frac{N}{k}$ disconnected clusters:

$A_{ij} = \frac{c(1-c)}{k-1}$ for any pair (i, j) in the same cluster and $i \neq j$, and $A_{ii} = 1 - c(1 - c)$, and $A_{ij} = 0$ for any pair (i, j) in different clusters.

Step 1) First, at equilibrium, any bank in the set of $N \setminus J_r$ is not involved in any rescue coalition. The banks in the set $N \setminus J_r$, without being involved in any coalition, has zero losses. Therefore, these banks would only be involved in a rescue coalition if the government makes transfers such that these banks receive weakly higher payoffs then staying isolated. This implies that in addition to the rescue costs, the government should make additional transfer to make any bank in the set of

$N \setminus J_r$ that are involved in a rescue coalition better off. A welfare-losses-minimizing government never makes such an extra transfer. On the other hand, in the absence of such government assistance, these banks are never involved in a rescue coalition. The proof of this claim is below.

By Lemma 1, $A_{ki} = 0$ for any $k \in N \setminus J_r$, $i \in J_r$ where J_r is the set of banks located in the connected component that includes bank r. This implies that any $k \notin J_r$ is a healthy bank and has a net worth of $e_j = 1 - l - A_{ir} = 1 - l$.

Next, consider the set of coalitions M as an outcome of the game. Consider that there exists a coalition $m_k \in M$ which includes a healthy bank h. For m_k to be a rescue coalition, there must be a bank $i : i \in m_k \cap J_r$. Consider the set of coalitions $M' = M \setminus m_k$, an alternative outcome of the game. $e_h^{M'} = 1 - l$ holds for any healthy bank $h \in m_k$. Then, if $e_h^M < 1 - l$ holds for any healthy bank $h \in m_k$, then the strategy $s'_h = \{h\}$ is always a profitable deviation for h, and M cannot be an equilibrium set of coalitions in that case. So, for M to be an equilibrium outcome, $e_h^M \geq 1 - l$ must hold, and thus $e_{m_k}^M \geq 1 - l$ must also hold.

Suppose that $e_{m_k}^M \geq 1 - l$. Then, either there exists at least one more healthy bank in m_k, or there exists at least one distressed bank j in m_k such that j is a healthy bank in $(C, F)^{M \setminus m_k}$. Otherwise, if all banks in $m_k \setminus h$ are distressed banks in $(C, F)^{M \setminus m_k}$, then $v_j^{M \setminus m_k} < l_j$ holds for all $\{j : j \neq h, j \in m_k\}$, which implies that $e_{m_k}^M < 1 - l$. As a result, M cannot be an equilibrium outcome.

Consider that h is the only healthy bank in m_k, but there exists at least one distressed bank j such that j would not default in $(C, F)^{M \setminus m_k}$. Denote the set of such banks $\{j : j \in m_k \cap (N^{M \setminus m_k} \setminus \aleph^{M \setminus m_k})\}$ by D. Then, for any arbitrarily selected sharing rule, e_j^M holds if $e_j^M < e_j^{M \setminus m_k} = 1 - l$ holds for some $j \in D$. In this case, $s'_j = \{j\}$ is a profitable deviation for any $\{j : j \in m_k \cap (N^{M \setminus m_k} \setminus \aleph^{M \setminus m_k})\}$, and hence M cannot be an equilibrium outcome.

Next, consider that there exists more than one healthy bank in m_k. In this case, for $e_j^M \geq 1 - l$ to hold, either all banks are healthy banks and reallocate the net worth, which implies that m_k is never a rescue coalition, or if m_k is a rescue coalition the condition above holds for some $j \in D$ or there exists a healthy bank $i \in m_k$ s.t. $e_i^M < 1 - l$. Then, for any such bank i having $e_i^M < 1 - l$, $s'_i = \{i\}$ is a profitable deviation. As a result, M cannot be an equilibrium outcome in this case as well. This completes the proof by showing that there exists no equilibrium outcome in which any bank in $N \setminus J_r$ is involved in a rescue coalition.

Step 2) Consider the cluster J_r that bank r is involved.

For, $s > s^0 = \frac{1-l}{1-c(1-c)}$ the bank hit by the shock is always distressed. The level of s^0 is independent from the cluster size. In this case, either the counterparties are not distressed but they can contribute to the rescue of the shocked bank at some levels, or all counterparties are also distressed. In the latter case, Proposition 6 implies that the counterparties of the shocked bank can rescue each other (with no government assistance) only via rescuing the shocked bank. So, whenever all counterparties are also distressed but they are liquiditation-driven failures, then:

(i) if the government decides to assist in the rescue of the shocked bank, the required transfer is:

$max\{0, s - k(1 - l)\}$

(ii) if the government decides to assist in the rescue of the set $J_r \setminus r$, the required transfer is:

$c(1 - c)\,(s + \zeta(1 - s)) - (k - 1)(1 - l)$

Note that Proposition 2 implies if all banks in the set $J_r \setminus r$ are shock-driven failures, then the government and banks either rescue all banks in the set $J_r \setminus r$ or rescues non of them. The same principal in Proposition 2 on d-ring lattices applies exactly in the same way here because the complete network case in d-ring lattices represents an island in an island-isolated network.

Then, if the government find one of these two rescue plan (rescuing the shocked bank only or rescuing all banks in the set J_r) welfare improving, and whenever the additional transfer required to rescue the shocked bank (additional transfer required to rescue $J_r \setminus r$ instead of rescuing $J_r \setminus r$) is less than the additional benefit of rescuing the shocked bank, then the government always prevents the failure of the shocked-bank instead of rescuing the rest. So, if

$s - k(1-l) - c(1-c)\,(s + \zeta(1 - s)) + (k-1)(1-l) \leq \zeta\,(1 - s)$, then the government prevents all failures. It can be rewritten as:

$s - c(1 - c)s + \zeta s - c(1 - c)\zeta s - (1 - l) - c(1 - c)\zeta \leq \zeta$

$s \leq s^1 = \frac{\zeta(1+c(1-c))+(1-l)}{(1-c(1-c))(1+\zeta)}$

The condition above shows that the government's decision to rescue the shocked bank or not is independent from the cluster size.

If the government prefers one of these rescue plans to not to rescue any shock-driven failures, then for $s \leq \frac{\zeta(1+c(1-c))+(1-l)}{(1-c(1-c))(1+\zeta)}$, the government always prevents the first failure. The required transfer is $s - k(1 - l)$. As k rises, the required transfer decreases. On the other hand, for $s > \frac{\zeta(1+c(1-c))+(1-l)}{(1-c(1-c))(1+\zeta)}$, the government always prevent the failure of the rest, if it decides to engage in rescues. The transfer to the $J_r \setminus r$ is $c(1 - c)\,(s + \zeta(1 - s)) - (k - 1)(1 - l)$. As k rises, the required transfer decreases.

Theorem 1 shows that, if the counterparties are liquidation-driven-failures, then the government is always engaged in rescue plans via positive transfers. The government does not engage in any rescue, including the rescue of the $J_r \setminus r$ whenever the counterparties of the shocked bank are also potential shock-driven failures and when the benefits are less than the costs, which occurs when

$1 - l < \frac{c(1-c)s}{k-1}$ (counterparties are shock-driven-failures), and

$c(1-c)\left(s + \zeta(1-s)\right) - (k-1)(1-l) > \zeta(k-1)$, (rescue of $J_r \setminus r$ is costly), which boils down to a single equation as follows:

$s > s^* = \frac{\zeta(k-1)+(k-1)(1-l)-c(1-c)\zeta}{c(1-c)(1-\zeta)}$

Lastly, by some line of algebra $s_0 < s_1$ and $s_1 < s^*$ always hold. Consequently, there are four cases:

Case i) for $s > s^*$, all banks in the isolated cluster that bank r is involved fails.

Case ii) for $s^* \geq s > s^1$, all banks but the shocked bank are rescued (it might also hold that $s^* < s^1$, which is discussed below)

Case iii) for $s^0 < s \leq s^1$, all banks are rescued (either only the shocked bank is rescued if it is the only distressed bank or all banks in the set J_r are rescued). For $s^* < s^1$, s^1 is replaced with s^*.

Case iv) for $s < s^0$, there exists no potential failures and no rescue coalitions.

In case (i), s^* is increasing in k. So, the higher the k, the higher the threshold level of shock for case (i) occur.

In case (ii), s^1 is independent from the cluster size. For fixed c, the total losses of counterparties from the failure of the shocked bank is $min\{(k-1)(1-l), c(1-c)(s + \zeta(1-s))\}$, which is fixed. If banks in $J_r \setminus r$ are healthy banks after the failure of the shocked bank, then there are no rescues for this shock level. If banks in $J_r \setminus r$ are distressed, then, as k rises, banks' total contributions in the rescue of $J_r \setminus r$, which is equal to $(k-1)(1-l)$, rises.

The level of s^* is increasing in k, and in some cases $s^* < s^1$ can hold given that s^1 is independent from the cluster size. In such a case, there exists no case (ii) as above, and the level in case (iii) also changes (s^1 is replaced with s^*) . Then, for $s > s^*$, all banks in the isolated cluster that bank r is involved fails, and for $s^0 < s \leq s^*$, all banks are rescued.

In case (iii), whenever all banks in J_r are distressed and rescued, banks' total contributions in the rescue of r is equal to $k(1-l)$, which is increasing in k. Whenever, only the shocked bank is distressed and rescued and the banks in $J_r \setminus r$ are healthy banks (this happens for k greater than a threshold level), then banks' total contributions in the rescue of r is $c(1-c)\zeta(1-s)$, which is independent from the cluster size. For $s^* < s^1$, s^1 is replaced with s^* where s^* is increasing in k.

In case (iv), no rescues occur, and this is again independent from the cluster size.

$s^* = \frac{\zeta(k-1)+(k-1)(1-l)-c(1-c)\zeta}{c(1-c)(1-\zeta)}$ is increasing in ζ and k. This equation gives us the

$\zeta^* = \frac{sc(1-c)-(k-1)(1-l)}{(k-1)+sc(1-c)-c(1-c)}$, and $k^* = \frac{sc(1-c)(1-\zeta)+\zeta(1+c(1-c))+1-l}{\zeta+1-l}$.

This completes the proof of (i) and (ii).

Proofs of parts (iii) and (iv):

The equation above for s^* implies that there exists $0 \leq \zeta^{***} < 1$ such that for $\zeta \geq \zeta^{***}$, there exists no such $s^* \leq p_r$ as described in part (i).

The desired condition for ζ is that:

$\zeta \geq \frac{sc(1-c)-(k-1)(1-l)}{(k-1)+sc(1-c)-c(1-c)}$

Then, for $s \leq \frac{(k-1)(1-l)}{c(1-c)}$, it always holds. For $s > s^{***} = \frac{(k-1)(1-l)}{c(1-c)}$, there exists a ζ^{***} such that for $\zeta \geq \zeta^{***}$, all banks in the distressed-cluster are rescued, regardless of the cluster size.

The threshold level can be found by setting $k = 2$, because the conditions here are designed for $k \geq 2$. For $k = 1$ only the shocked bank's rescue condition matters, it is independent from the cluster size in any islands-isolated network. Then, for $k = 2$, and for the maximum possible shock, $s = 1$, the threshold level is $\zeta^{***} = \frac{c(1-c)-(1-l)}{1}$. For $\zeta \geq \zeta^{***}$, all banks in the distressed-cluster except the shocked bank are rescued, for any $k \geq 2$, regardless of the shock level. The rescue decision for the shocked bank is independent from the cluster size, as shown previously. This complete the proof for part (iv).

Lastly, we can find the other threshold level by setting $k = N$. Then, for $\zeta < \zeta^{**} = \frac{sc(1-c)-(N-1)(1-l)}{(N-1)+sc(1-c)-c(1-c)}$, and $s > s^{**} = \frac{(N-1)(1-l)}{c(1-c)}$, all counterparties of the shocked bank fail, regardless of the cluster size. Therefore, $k = 1$ minimizes the distressed-costs if $\zeta < \zeta^{**}$ and $s > s^{**} = \frac{(N-1)(1-l)}{a}$.

This completes the proof.

Proof of Proposition 2

In a d-ring lattice network, the shock-driven failures are either the shocked bank or the shocked bank and its all counterparties.

Next, I show that if there exists any potential liquidation failure, this implies that all banks are distressed. Consider the first set of liquidation-driven failures. These banks are the banks from $r + 1$ to $r + d + 1$ if shocked bank is the only potential shock-driven failure, or from $r + d + 1$ to $r + 2d + 1$ if both shocked bank and its all counterparties are potential shock-driven failures. Based on the contagion algorithm, for each bank in that set, $v_i - l_i - \sum_{k \in \aleph^1} A_{ik}\beta_k < 0$ holds, where $\aleph^1$ is the set of shock-driven failures. Then, whenever such failures occur, then for the counterparties of these banks, the same condition applies and $v_i - l_i - \sum_{k \in \aleph^2} A_{ik}\beta_k < 0$ holds. Therefore, if there exists any potential liquidation-driven failure, then all banks are distressed banks.

Next, I will show that with no government assistance and excluding any banks in the set $N \setminus \aleph^1$, there is an equilibrium that banks in $N \setminus \aleph^1$ form a coalition. This implies that we can start the analysis from that outcome and check if adding some more banks from the set $\aleph^1$ to that coalition (with government assistance if it is welfare-loss-reducing).

The total payoff of the banks in the coalition $N \setminus \aleph^1$ with no government intervention is $max\left\{0, T(1-l) - \sum_{i \in N \setminus \aleph^1} \sum_{j \in \aleph^1} A_{ij}\beta_j\right\}$, where $T = N - d - 1$ is the number of banks in the set $N \setminus \aleph^1$, Since the banks in the set $N \setminus \aleph^1$ have zero dependency to bank r, they have no losses from the shock.

If $(N-d-1)(1-l) < \sum_{i \in N \setminus \aleph^1} \sum_{j \in \aleph^1} A_{ij}\beta_j$, then any coalition in $N \setminus \aleph^1$ would fail if it does not include a bank from $N \setminus \aleph^1$. For, $(N-d-1)(1-l) \geq \sum_{i \in N \setminus \aleph^1} \sum_{j \in \aleph^1} A_{ij}\beta_j$, then there exists a coalition in $N \setminus \aleph^1$ that can prevent their own failures and stop the liquidation driven failures. In that case, consider the smalles set of banks $N \setminus \aleph^1$ ordered from $r+d+1$ to $r+d+k$ such that, the coalition of k banks is such that $k(1-l) \geq \sum_{i \in K} \sum_{j \in \aleph^1} A_{ij}\beta_j$, but excluding any set of banks from the coalition K would always lead to K' such that $k'(1-l) < \sum_{i \in K'} \sum_{j \in \aleph^1} A_{ij}\beta_j$.

Since the coalition K stops the contagious failures, the banks from $r+d+k+1$ to N have no incentives to deviate with some banks in $N \setminus \aleph^1$ since it would not provide any further gain to them. For the banks from $r+d+k+1$ to N, the only losses from failures are their losses from the initial failures of $\aleph^1$. However, their losses do not lead to their failures since K is defined as the smallest coalition that would prevent all liquidation-driven failures. If there is any such case, then those banks would be added to the coalition K, and this would always happen in an orderly way starting from from $r+d+k+1$. The banks in the set K have no incentives to deviate with some other banks in K, because it would lead to their failure , again since K is the smallest set preventing the failures of banks in K. Then, the only possible deviation is including the banks in the set $\aleph^1$. Lastly, for $(N-d-1)(1-l) < \sum_{i \in N \setminus \aleph^1} \sum_{j \in \aleph^1} A_{ij}\beta_j$, then $K = N \setminus \aleph^1$ would be the coalition where there is no deviation by a subset of it.

Consequently, for such set of K the total contributions of banks for forming the coalition of $K \cup D$ where $D \subset \aleph^1$ would be equal to their potential welfare losses from the failure of the banks in D, which is equal to $\sum_{i \in K} \sum_{j \in D} A_{ij}\beta_j$. For banks in $N \setminus (\aleph^1 \cup K)$, call it set K^{-}, their contribution would be equal to $\sum_{i \in K^{-}} \sum_{j \in D} A_{ij} \beta_j$.

Alternatively, instead of only the coalition of K, the coalition of $N \setminus \aleph^1$ can be formed with the payoff conditions as described above, and the government intervention would be designed to support this $N \setminus \aleph^1$.

Next, consider the government assistance and rescue formation.

Step 2) I will show that if both the shocked bank and its all counterparties are potential shock-driven failures, then either all of the counterparties of the shocked bank are rescued or non of the counterparties of the shocked bank are rescued. Moreover Theorem 1 implies that there exists no liquidation-driven failures. So, there are only three possible cases:

- case i) government and banks rescue all banks

- case ii) government and banks rescue all banks but the shocked bank

- case iii) government and banks do not prevent any shock-driven failures.

Proof of Step 2:

Suppose that the shocked bank and its counterparties are shock-driven failures. Consider the set $\aleph^1 \setminus r$, and denote the set of these banks by $S = \{r+1, ..., r+d\}$. Now, consider rescue of the banks in this set. I will show that if bank $r+k \in \aleph^1 \setminus r$ is rescued, then bank $r+k+1 \in \aleph^1 \setminus r$ is also rescued, and I will show that if bank $r+k \in \aleph^1 \setminus r$ is rescued, then $r+k-1 \in \aleph^1 \setminus r$ is also rescued. This implies that whenever any bank is rescued in the set $\aleph^1 \setminus r$, then all banks in this set are rescued. So, either all of these banks are rescued or non of these banks are rescued.

I start with the coalition $N \setminus \aleph^1$, and then I will add one-by-one the banks in the set $\aleph^1 \setminus r$. Take any of these banks, call it bank k.

Then, the government prefers adding only bank k to the coalition $N \setminus \aleph^1$ if

$\zeta + \sum_{i \in N \setminus \aleph^1} \zeta A_{ik} \geq sA_{kr} - (1-l)$ where $\sum_{k \in N \setminus \aleph^1} \zeta A_{ik}$ is the contributions of the banks in the set $N \setminus \aleph^1$ in the rescue of bank k. Next, take bank $k+1$ and add it to the coalition of $(N \setminus \aleph^1) \cup k$.

Then, the government prefers adding only bank $k+1$ to the coalition $(N \setminus \aleph^1) \cup k$ if

$\zeta + \sum_{i \in (N \setminus \aleph^1) \cup k} \zeta A_{i,k+1} \geq sA_{k+1,r} - (1-l)$ where $\sum_{i \in (N \setminus \aleph^1) \cup k} \zeta A_{i,k+1}$ is the contributions of the banks in the set $(N \setminus \aleph^1) \cup k$ in the rescue of bank $k+1$. First, $A_{k,k+1} = 0$. However, $\sum_{i \in N \setminus \aleph^1} \zeta A_{i,k+1} \geq \sum_{i \in N \setminus \aleph^1} \zeta A_{ik}$ always holds, again from the d-ring lattice property: The links emanating from $k+1$ and entering into $N \setminus \aleph^1$ is weakly larger than the links emanating from k and entering into $N \setminus \aleph^1$.

Lastly, given that $A_{k+1,r} = A_{kr}$, then if k is rescued, $k+1$ is also rescued.

Next, I show that if k is rescued then $k-1$ also rescued.

Then, the government prefers adding only bank $k-1$ to the coalition $(N \setminus \aleph^1) \cup k$ if

$\zeta + \sum_{i \in (N \setminus \aleph^1) \cup k} \zeta A_{i,k-1} \geq s A_{k-1,r} - (1-l)$ where $\sum_{i \in (N \setminus \aleph^1) \cup k} \zeta A_{i,k-1}$ is the contributions of the banks in the set $(N \setminus \aleph^1) \cup k$ in the rescue of bank $k-1$. Given that $A_{k-1,r} = A_{kr}$, then if k is rescued, $k-1$ is also rescued.

This completes the proof of Step 2.

Step 3) Next, I show that there exists a d^* such that for $d < d^*$ and for $\zeta \leq \zeta^*$, the shocked bank and its counterparties always fail, but for $d \geq d^*$, all counterparties of the shocked bank are always rescued whenever they are distressed.

From the proof of Theorem 1, case (i) above holds iff:
$\frac{\zeta(1-s)+\zeta d+\zeta T+(d+1)(1-l)}{1} \geq s$

where

$T = \frac{\bar{a}}{d} \sum_{k \in [0,d]} min\{1-l, min\{2d+1-N, d-k\}\}$. (how much of the liquidation costs due to the failure of banks in the set $\aleph^1$ are absorbed by the banks in the set $N \setminus \aleph^1$, which is equal to the the total contribution of these banks). The $min\{2d+1-N, d-k\}$ condition follows from the d-ring lattice property. The losses are absorbed by counterparties but if the network is sufficiently small or d is sufficiently large, then the losses from the failure of a bank in the set $\aleph^1$ are also partially absorbed by some banks in $\aleph^1$ that are preceding that bank in the d-ring lattice ordering.

This gives us the threshold level of shock for rescuing all banks, if the government decides to engage in rescue of either the shocked bank or its counterparties.

So, the threshold level for non-rescue of all shock-driven failures is:
$s^*(\bar{a}, l, \zeta, d) = \frac{\zeta(1-s)+\zeta d+\zeta T+(d+1)(1-l)}{1}$
where T is a function of $\bar{a}$.

Then, for $s > s^*(\bar{a}, l, \zeta, d)$ and $\zeta < \zeta^* = \frac{s-T-(d+1)(1-l)}{d+1-s}$, all counterparties of the shocked bank fail, and the shocked bank fail.

First, s^* is increasing in ζ. Secondly, T is always less than 1. An increase in d by one would increase the potential links from a bank to its preceding nodes by one, so the total increase in the number of links between the banks in $aleph^1$ is always less than or equal to d. So, the total decrease in T is always less than or equal to dA_ij, which is always less than one. Then s^* is increasing in d.

Proof of part (iii):

Since the gains from rescuing all of the counterparties of the shocked bank increase in the number of counterparties, d, the threshold level for part (iv) is calculated for the largest d, which is $d = N - 1$.

Then, for $d = N - 1$, the new condition for ζ, which I call ζ^{**}, is such that $\zeta^{**} = \frac{s-N(1-l)-min\{\overline{a},(N-1)(1-l)\}}{N-s}$. In the complete network case, $T = min\{\overline{a}, (N-1)(1-l)\}$.

For $s \leq N(1-l) - min\{\overline{a}, (N-1)(1-l)\}$, there exists no case as in part (iii) because in that case $\zeta^{**} \leq 0$, and so there is no $\zeta < \zeta^{**}$. For $s > s^{**} = N(1-l) - min\{\overline{a}, (N-1)(1-l)\}$ and $\zeta < \zeta^{**} = \frac{s-N(1-l)-min\{\overline{a},(N-1)(1-l)\}}{N-s}$, in any d-ring lattice network, all counterparties of the shocked bank fail.

Lastly, the rescue of the shocked bank is independent from the diversification level, because the additional benefit of rescuing it is independent from diversification, and the additional cost of rescuing it is equal to $sA_r r - (1-l)$, which is again independent from diversification.

Therefore, $d = 1$ becomes the welfare losses-minimizing network for for $s > s^{**} = N(1-l) - min\{\overline{a}, (N-1)(1-l)\}$ and $\zeta < \zeta^{**} = \frac{s-N(1-l)-min\{\overline{a},(N-1)(1-l)\}}{N-s}$.

Proof of part (iv):

Since s^* is increasing in d, the threshold level for part (iv) is calculated for the smallest d, which is $d = 1$. Then, for $d = 1$, the new condition for ζ, which I call ζ^{***}, is such that $\zeta^{***} = \frac{1-2(1-l)-min\{\overline{a},(1-l)\}}{1}$, where I also consider the highest possible shock, $s = 1$. In ring network, $T = min\{\overline{a}, (1-l)\}$.

The condition above is always less than 1.

Then, for $\zeta \geq \zeta^{***}$, either all banks or all banks but the shocked bank is rescued in any given d-ring lattice $\psi(\overline{a}, d, \overline{F})$.

As shown above, the rescue of the shocked bank is independent from the diversification level.

This completes the proof.

Proof of Proposition 3

c^* is such that $1 - A_{ii}(c^*) = (N-1)(1-l)$.

For $c \leq c^*$,

whenever $s \leq \frac{1+N(1-l)}{2}$, the government always prevents the shocked bank's failure.

whenever $s > \frac{1+N(1-l)}{2}$, the government only transfers to the rest if needed.

Case (i) For, $1 - l < \frac{c(1-c)}{d}$ and $s > \frac{1-l}{1-c(1-c)}$, then all banks are potentially failing.

(i) when the government intervenes for the rescue of the shocked bank, the required transfer is:

$max\{0, s - N(1-l)\}$

(ii) when the government intervenes for the rescue of the set $m_k \setminus r$, the required transfer is:

$c(1-c) - (N-1)(1-l)$

Then, whenever the additional transfer required to rescue the shocked bank is less than the additional benefit of rescuing the shocked bank, then the government always prevents the initial failure. So, if

$s - t(1-l) - (c(1-c) - (k-1)(1-l)) \leq 1 - s$

$s \leq \frac{2+c(1-c)-l}{2}$

For $s \leq \frac{2+c(1-c)-l}{2}$, the government always prevent the first failure. The required transfer is $s - N(1-l)$. The required transfer is independent from c.

For $s > \frac{2+c(1-c)-l}{2}$, the government always prevent the rest. The transfer to the rest is $max 0, c(1-c) - (N-1)(1-l)$. For $c(1-c) \leq (N-1)$, no need for government assistance. For $c(1-c) \geq (N-1)$. As c rises, the required transfer rises.

Case (ii) For, $1 - l \geq \frac{c(1-c)}{d}$ and $s > \frac{1-l}{1-c(1-c)}$, then only bank r is potentially failing. In this case, similarly, for $s \leq \frac{2+c(1-c)-l}{2}$, the government intervenes to prevent the first failure.

(i) when the government intervenes for the rescue of the shocked bank, the required transfer is:

$max\{0, N(1-l) - s - (N-1)(1-l) - c(1-c)\} = max\{0, (1-l) - s - c(1-c)\}$. As $c(1-c)$ rises, the required transfer decreases; however the total amount that banks would transfer is equal to $(N-1)(1-l) - c(1-c)$, which is equal to their sum of net worth if there was no such rescue. So, for $c(1-c) \leq (N-1)(1-l)$, the required transfer is decreasing in c.

(ii) no government assistance is required except for rescuing the shocked bank, since there are no other potential failures.

Case (iii)

For, $s \leq \frac{1-l}{1-c(1-c)}$, then no bank is potentially failing, and since the shock is irreversible and there are no liquidation costs, no transfer is made.

Lastly, as k rises, $\frac{c(1-c)}{d}$ decreases so the threshold level of "switching from case (i) to case (ii)" decreases, but the government transfer is independent from d both in case (i) and case (ii).

As a result, there exists a threshold level of c^* such that $1 - A_{ii}(c^*) = (N-1)(1-l)$, where $c = c^*$ minimizes the distress costs, and for $c \leq c^*$, the distress-costs are weakly incrasing in c, and for $c \geq c^*$, the distress-costs are weakly incrasing in c, where c^* is independent from d.

This completes the proof.

A Special Case: Star Network

The star structure can be described as follows. A single bank is located at the core and connected to all other banks and each other bank is connected to only the core bank. In addition, the core bank's primary asset is larger than each periphery bank's primary asset. Formally, a star network (C, F) that consists of a single core bank $c \in N$ and a set of periphery banks $\{N \setminus c\}$ (denoted by p) is such that the integration level of the core bank and any periphery bank is equal to $(N-1)\alpha_p$ and α_c, respectively, where $C_{cp} = \alpha_c$, $C_{pc} = \alpha_p$, $C_{pp'} = 0$, and $C_{cc} = C_{pp} = 0$ holds. I also normalize the primary asset value of any periphery bank to $p_p = 1$, and $p_c > p_p = 1$ holds for the primary assets of banks. I also consider that $\sum\limits_{i \in N} (p_i - l_i) \leq p_p = 1$, which ensures ensures that the capacity constraint hits for large enough shocks.

Proposition 4 *Consider an economy with four banks. There exists exogenously given (p_c, l_c, p_p, l_p) and interbank claims $\alpha_c^*(p_c, l_c, p_p, l_p)$ and $\alpha_p^*(p_c, l_c, p_p, l_p)$ such that a star network with claims $\alpha_c^*(p_c, l_c, p_p, l_p)$ and $\alpha_p^*(p_c, l_c, p_p, l_p)$ is a distress-costs-minimizing network.*

Proposition 4 captures the heterogeneity in both asset sizes and connectivity. The result implies that a star network can be a distress-costs-minimizing network for certain exogenously given primary assets and liabilities. For simplicity, I consider four banks in Proposition 4 that can be extended to n banks with the same insights. Next, I provide a numerical of a star network.

Example 1 *For exogenously given $p_p = 1$, $p_c = 5$, $l_p = \frac{42}{44} < 1$, and $l_c = \frac{1033}{220} < 5$ and $N = 4$, the star network with $\alpha_c^* = \frac{4}{10}$ and $\alpha_p^* = \frac{1}{10}$ is a distress-costs-minimizing network.*

As illustrated in Example 1 and Figure 4, the claims (in ratio) that the core bank holds in each periphery bank is higher than the claims that each periphery bank holds in the core bank (the flow (in ratio) from periphery to core is higher than the flow from core to a periphery). This guarantees that the connectedness of each periphery bank to the rest of the system is at sufficient levels and there exists no clustering in loss transmission.

In addition to the analysis on the star network, one might think about the core-periphery structure, which is another network structure that has been extensively studied in the literature. The core-periphery structure is in some sense an extended version of the star network, which includes some additional connectivity layers such

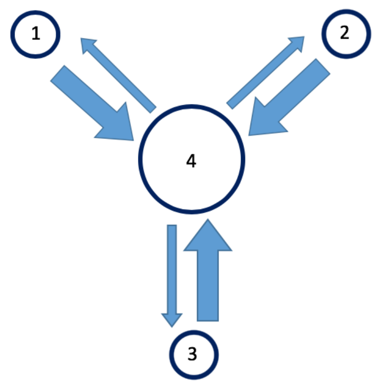

Figure 4: A distress-costs-minimizing network structure in Example 1: *Star Network*

as connectivity among core banks. Therefore, in addition to the insights driven from the star network, the distress-costs-minimizing connectivity among core banks play a key role in the effectiveness of rescues in a core-periphery structure. The earlier results on d-ring lattices provides us an idea about the connectivity of the core banks. Consequently, putting together the results on d-ring lattices (for core-core connectivity) and star network (for core-periphery connectivity) guide us to understand distress-costs-minimizing connectivity in a core-periphery network.

Proof of Proposition 4:

For given claims in a star network, the matrix $(I - C)$ can be represented as an "arrowhead matrix" such that:

$$I - C = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & -\alpha_p \\ 0 & 1 & 0 & \cdots & 0 & -\alpha_p \\ 0 & 0 & 1 & \cdots & 0 & -\alpha_p \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & -\alpha_p \\ -\alpha_c & -\alpha_c & -\alpha_c & \cdots & -\alpha_c & 1 \end{bmatrix}$$

where core node is represented as the n^{th} element of the matrix above.

Following Theorem 2.1 in Najafi et al. (2014), the modified Sherman-Morrison inverse of $(I - C)$ is as follows:

$$(I - C)^{-1} = (I - S_1)\left(I - \tfrac{1}{1+\omega}(S_2(I - S_1))\right)$$

where $1 + \omega \neq 0$, $\omega = -(n-1)\alpha_p\alpha_c$, and S_1 and S_2 are strictly lower and strictly upper triangular parts of $(I - C)$ such that $(I - C) = I + S_1 + S_2$.

Then,

$$
S_2(I-S_1)=\begin{bmatrix}0&0&0&\cdots&0&-\alpha_p\\0&0&0&\cdots&0&-\alpha_p\\0&0&0&\cdots&0&-\alpha_p\\\vdots&\vdots&\vdots&\ddots&\vdots&\vdots\\0&0&0&\cdots&0&-\alpha_p\\0&0&0&\cdots&0&0\end{bmatrix}\begin{bmatrix}1&0&0&\cdots&0&0\\0&1&0&\cdots&0&0\\0&0&1&\cdots&0&0\\\vdots&\vdots&\vdots&\ddots&\vdots&\vdots\\0&0&0&\cdots&1&0\\\alpha_c&\alpha_c&\alpha_c&\cdots&\alpha_c&1\end{bmatrix}
$$

$$
I-\tfrac{S_2(I-S_1)}{1+\omega}=\begin{bmatrix}\frac{1-(n-2)\alpha_p\alpha_c}{1-(n-1)\alpha_p\alpha_c}&\frac{\alpha_p\alpha_c}{1-(n-1)\alpha_p\alpha_c}&\cdots&\frac{\alpha_p\alpha_c}{1-(n-1)\alpha_p\alpha_c}&\frac{\alpha_p}{1-(n-1)\alpha_p\alpha_c}\\\frac{\alpha_p\alpha_c}{1-(n-1)\alpha_p\alpha_c}&\frac{1-(n-2)\alpha_p\alpha_c}{1-(n-1)\alpha_p\alpha_c}&\cdots&\frac{-\alpha_p\alpha_c}{1-(n-1)\alpha_p\alpha_c}&\frac{\alpha_p}{1-(n-1)\alpha_p\alpha_c}\\\vdots&\vdots&\vdots&\vdots&\vdots\\\frac{\alpha_p\alpha_c}{1-(n-1)\alpha_p\alpha_c}&\frac{\alpha_p\alpha_c}{1-(n-1)\alpha_p\alpha_c}&\cdots&\ddots&\frac{\alpha_p}{1-(n-1)\alpha_p\alpha_c}\\0&0&0&\cdots&1\end{bmatrix}
$$

$$
(I{-}C)^{-1}=\begin{bmatrix}1&0&0&\cdots&0&0\\0&1&0&\cdots&0&0\\0&0&1&\cdots&0&0\\\vdots&\vdots&\vdots&\ddots&\vdots&\vdots\\0&0&0&\cdots&1&0\\\alpha_c&\alpha_c&\alpha_c&\cdots&\alpha_c&1\end{bmatrix}\begin{bmatrix}\frac{1-(n-2)\alpha_p\alpha_c}{1-(n-1)\alpha_p\alpha_c}&\frac{\alpha_p\alpha_c}{1-(n-1)\alpha_p\alpha_c}&\cdots&\frac{\alpha_p\alpha_c}{1-(n-1)\alpha_p\alpha_c}&\frac{\alpha_p}{1-(n-1)\alpha_p\alpha_c}\\\frac{\alpha_p\alpha_c}{1-(n-1)\alpha_p\alpha_c}&\frac{1-(n-2)\alpha_p\alpha_c}{1-(n-1)\alpha_p\alpha_c}&\cdots&\frac{-\alpha_p\alpha_c}{1-(n-1)\alpha_p\alpha_c}&\frac{\alpha_p}{1-(n-1)\alpha_p\alpha_c}\\\vdots&\vdots&\vdots&\vdots&\vdots\\\frac{\alpha_p\alpha_c}{1-(n-1)\alpha_p\alpha_c}&\frac{\alpha_p\alpha_c}{1-(n-1)\alpha_p\alpha_c}&\cdots&\ddots&\frac{\alpha_p}{1-(n-1)\alpha_p\alpha_c}\\0&0&0&\cdots&1\end{bmatrix}
$$

$$
(I-C)^{-1}=\begin{bmatrix}\frac{1-(n-2)\alpha_p\alpha_c}{1-(n-1)\alpha_p\alpha_c}&\frac{\alpha_p\alpha_c}{1-(n-1)\alpha_p\alpha_c}&\cdots&\frac{\alpha_p\alpha_c}{1-(n-1)\alpha_p\alpha_c}&\frac{\alpha_p}{1-(n-1)\alpha_p\alpha_c}\\\frac{\alpha_p\alpha_c}{1-(n-1)\alpha_p\alpha_c}&\frac{1-(n-2)\alpha_p\alpha_c}{1-(n-1)\alpha_p\alpha_c}&\cdots&\frac{\alpha_p\alpha_c}{1-(n-1)\alpha_p\alpha_c}&\frac{\alpha_p}{1-(n-1)\alpha_p\alpha_c}\\\vdots&\vdots&\vdots&\vdots&\vdots\\\frac{\alpha_p\alpha_c}{1-(n-1)\alpha_p\alpha_c}&\frac{\alpha_p\alpha_c}{1-(n-1)\alpha_p\alpha_c}&\cdots&\ddots&\frac{\alpha_p}{1-(n-1)\alpha_p\alpha_c}\\\frac{\alpha_c}{1-(n-1)\alpha_p\alpha_c}&\frac{\alpha_c}{1-(n-1)\alpha_p\alpha_c}&\frac{\alpha_c}{1-(n-1)\alpha_p\alpha_c}&\cdots&\frac{1}{1-(n-1)\alpha_p\alpha_c}\end{bmatrix}
$$

$$
\hat{C}=\begin{bmatrix}1-\alpha_c&0&\cdots&0\\0&\ddots&0&\vdots\\\vdots&0&1-\alpha_c&0\\0&\cdots&0&1-(n-1)\alpha_p\end{bmatrix}
$$

The dependency matrix A is equal to:

$$
A=\hat{C}(I{-}C)^{-1}=\begin{bmatrix}\frac{(1-\alpha_c)(1-(n-2)\alpha_p\alpha_c)}{1-(n-1)\alpha_p\alpha_c}&\frac{(1-\alpha_c)\alpha_p\alpha_c}{1-(n-1)\alpha_p\alpha_c}&\cdots&\frac{(1-\alpha_c)\alpha_p\alpha_c}{1-(n-1)\alpha_p\alpha_c}&\frac{(1-\alpha_c)\alpha_p}{1-(n-1)\alpha_p\alpha_c}\\\frac{(1-\alpha_c)\alpha_p\alpha_c}{1-(n-1)\alpha_p\alpha_c}&\frac{(1-\alpha_c)(1-(n-2)\alpha_p\alpha_c)}{1-(n-1)\alpha_p\alpha_c}&\cdots&\frac{(1-\alpha_c)\alpha_p\alpha_c}{1-(n-1)\alpha_p\alpha_c}&\frac{(1-\alpha_c)\alpha_p}{1-(n-1)\alpha_p\alpha_c}\\\vdots&\vdots&\vdots&\vdots&\vdots\\\frac{(1-\alpha_c)\alpha_p\alpha_c}{1-(n-1)\alpha_p\alpha_c}&\frac{(1-\alpha_c)\alpha_p\alpha_c}{1-(n-1)\alpha_p\alpha_c}&\cdots&\ddots&\frac{(1-\alpha_c)\alpha_p}{1-(n-1)\alpha_p\alpha_c}\\\frac{\alpha_c(1-(n-1)\alpha_p)}{1-(n-1)\alpha_p\alpha_c}&\frac{\alpha_c(1-(n-1)\alpha_p)}{1-(n-1)\alpha_p\alpha_c}&\frac{\alpha_c(1-(n-1)\alpha_p)}{1-(n-1)\alpha_p\alpha_c}&\cdots&\frac{1-(n-1)\alpha_p}{1-(n-1)\alpha_p\alpha_c}\end{bmatrix}
$$

Then, for $p_p=1$ for all $p\in N\setminus c$, and for given p_c, before any shock hits, we have:

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} \frac{(1-\alpha_c)(1-(n-2)\alpha_p\alpha_c)}{1-(n-1)\alpha_p\alpha_c} & \frac{(1-\alpha_c)\alpha_p\alpha_c}{1-(n-1)\alpha_p\alpha_c} & \frac{(1-\alpha_c)\alpha_p\alpha_c}{1-(n-1)\alpha_p\alpha_c} & \frac{(1-\alpha_c)\alpha_p}{1-(n-1)\alpha_p\alpha_c} \\ \frac{(1-\alpha_c)\alpha_p\alpha_c}{1-(n-1)\alpha_p\alpha_c} & \frac{(1-\alpha_c)(1-(n-2)\alpha_p\alpha_c)}{1-(n-1)\alpha_p\alpha_c} & \frac{(1-\alpha_c)\alpha_p\alpha_c}{1-(n-1)\alpha_p\alpha_c} & \frac{(1-\alpha_c)\alpha_p}{1-(n-1)\alpha_p\alpha_c} \\ \frac{(1-\alpha_c)\alpha_p\alpha_c}{1-(n-1)\alpha_p\alpha_c} & \frac{(1-\alpha_c)\alpha_p\alpha_c}{1-(n-1)\alpha_p\alpha_c} & \frac{(1-\alpha_c)(1-(n-2)\alpha_p\alpha_c)}{1-(n-1)\alpha_p\alpha_c} & \frac{(1-\alpha_c)\alpha_p}{1-(n-1)\alpha_p\alpha_c} \\ \frac{\alpha_c(1-(n-1)\alpha_p)}{1-(n-1)\alpha_p\alpha_c} & \frac{\alpha_c(1-(n-1)\alpha_p)}{1-(n-1)\alpha_p\alpha_c} & \frac{\alpha_c(1-(n-1)\alpha_p)}{1-(n-1)\alpha_p\alpha_c} & \frac{1-(n-1)\alpha_p}{1-(n-1)\alpha_p\alpha_c} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ p_c \end{bmatrix}$$

where bank 4 is the core bank and other banks are the periphery banks. Then, by some lines of algebra,

$v_p = \frac{(1-\alpha_c)(p_p+\alpha_p p_c)}{1-(n-1)\alpha_p\alpha_c}$
$v_c = \frac{(1-(n-1)\alpha_p)(\alpha_c p_p(n-1)+p_c)}{1-(n-1)\alpha_p\alpha_c}$
For $n=4$ and $p_p=1$:
$v_p = \left[\frac{(1-\alpha_c)(1+\alpha_p p_c)}{1-3\alpha_p\alpha_c}\right]$ for all $p \in N \setminus c$, and $v_c = \frac{(3\alpha_c+p_c)(1-3\alpha_p)}{1-3\alpha_p\alpha_c}$.

Moreover, the assumption $\sum\limits_{i\in N}(p_i - l_i) \leq p_p$ guarantees that for sufficiently large shocks banks are not capable of preventing all failures. The conditions for the optimal network are as follows.

i) For the core bank, the optimality requires:
$\sum\limits_{i\in N\setminus c} A_{ic} = 1 - A_{cc} = \sum\limits_{i\in N\setminus c}(v_i - l_i)$, which can be rewritten as:
$(n-1)\frac{(1-\alpha_c)\alpha_p}{1-(n-1)\alpha_p\alpha_c} = (n-1)\left[\frac{(1-\alpha_c)(p_p+\alpha_p p_c)}{1-(n-1)\alpha_p\alpha_c} - l_p\right]$
By further simplification:

$$l_p = \left[\frac{(1-\alpha_c)(p_p+\alpha_p p_c-\alpha_p)}{1-(n-1)\alpha_p\alpha_c}\right]$$

$$\frac{\alpha_p}{\alpha_c} = \left[\frac{(l_p+p_p(\alpha_c-1))}{\alpha_c(p_c-1-\alpha_c(p_c-1-(n-1)l_p))}\right]$$

For $n=4$,

$$\frac{\alpha_p}{\alpha_c} = \frac{(l_p+\alpha_c-1)}{\alpha_c(p_c-1-\alpha_c(p_c-1-3l_p))}$$

This gives us the first condition.

ii) For each periphery bank $p \in N \setminus c$, the optimality requires:
$1 - A_{pp} = (v_c - l_c) + \sum\limits_{i\in N\setminus(p\cup c)}(v_i - l_i)$
$1 - A_{pp} = \left[\frac{(1-(n-1)\alpha_p)(\alpha_c p_p(n-1)+p_c)}{1-(n-1)\alpha_p\alpha_c} - l_c\right] + (n-2)\left[\frac{(1-\alpha_c)(p_p+\alpha_p p_c)}{1-(n-1)\alpha_p\alpha_c} - l_p\right]$
Then, I plug $l_p = \left[\frac{(1-\alpha_c)(p_p+\alpha_p p_c-\alpha_p)}{1-(n-1)\alpha_p\alpha_c}\right]$ into above and get:
$1 - A_{pp} = \left[\frac{(1-(n-1)\alpha_p)(\alpha_c p_p(n-1)+p_c)}{1-(n-1)\alpha_p\alpha_c} - l_c\right] + (n-2)\left[\frac{(1-\alpha_c)\alpha_p}{1-(n-1)\alpha_p\alpha_c}\right]$
$(n-2)\frac{(1-\alpha_c)\alpha_p\alpha_c}{1-(n-1)\alpha_p\alpha_c} + \frac{\alpha_c(1-(n-1)\alpha_p)}{1-(n-1)\alpha_p\alpha_c} = \left[\frac{(1-(n-1)\alpha_p)(\alpha_c p_p(n-1)+p_c)}{1-(n-1)\alpha_p\alpha_c} - l_c\right] + (n-2)\left[\frac{(1-\alpha_c)\alpha_p}{1-(n-1)\alpha_p\alpha_c}\right]$

Then, by rewriting the equation above:
$l_c = -(n-2)(\frac{(1-\alpha_c)\alpha_p\alpha_c}{1-(n-1)\alpha_p\alpha_c}) - \frac{\alpha_c(1-(n-1)\alpha_p)}{1-(n-1)\alpha_p\alpha_c} + \left[\frac{(1-(n-1)\alpha_p)(\alpha_c p_p(n-1)+p_c)}{1-(n-1)\alpha_p\alpha_c}\right] + (n-2)\left[\frac{(1-\alpha_c)\alpha_p}{1-(n-1)\alpha_p\alpha_c}\right]$

$$l_c = \frac{(n-2)(1-\alpha_c)^2\alpha_p + (\alpha_c(p_p(n-1)-1)+p_c)(1-(n-1)\alpha_p)}{1-(n-1)\alpha_p\alpha_c}$$

For $p_p = 1$ and $n = 4$, we get:
$l_c = \frac{2\alpha_p(1-\alpha_c)^2+(2\alpha_c+p_c)(1-3\alpha_p)}{1-3\alpha_p\alpha_c}$

$$\frac{\alpha_p}{\alpha_c} = \frac{(2\alpha_c+(p_c-l_c))}{\alpha_c[(10-3l_c)\alpha_c-2\alpha_c^2+3p_c-2]}$$

This gives us the second condition.
By combining the two equations for $\frac{\alpha_p}{\alpha_c}$, we get:

$$\alpha_p = \frac{2\alpha_c+(p_c-l_c)}{\alpha_c[(10-3l_c)\alpha_c-2\alpha_c^2+3p_c-2]} = \frac{(l_p+\alpha_c-1)}{\alpha_c(p_c-1-\alpha_c(p_c-1-3l_p))}$$

which gives us a solution for α_c as a function of p_c, l_c, p_p, and l_p. By plugging $\alpha_c^*(p_c, l_c, p_p, l_p)$ into conditions above, we get $\alpha_p^*(p_c, l_c, p_p, l_p)$.

Lastly, for each bank to be initially solvent before any shock hits, it should be that $v_c \geq l_c$ and $v_p \geq l_p$. First, as one can see from the equations above, $v_p \geq l_p$ always holds. Secondly, $v_c \geq l_c$ holds iff $\frac{\alpha_c(1-3\alpha_p)}{2\alpha_p(1-\alpha_c)^2} \geq 1$ holds, which is the third condition.

These two conditions (the combination of first two conditions, and the third condition) together characterize the solution for star network. Then there exists (p_c, l_c, p_p, l_p) such that a star network with claims $\alpha_c^*(p_c, l_c, p_p, l_p)$ and $\alpha_p^*(p_c, l_c, p_p, l_p)$ satisfies these initial conditions for the optimality. Example 1 is an example showing the existence of such star networks.

Next, I show that in such star network with $\alpha_c^*(p_c, l_c, p_p, l_p)$ and $\alpha_p^*(p_c, l_c, p_p, l_p)$, whenever there is a distressed bank, the grand coalition is formed at equilibrium (with or without government intervention, depending on the shock level).

Under the assumption that $\sum_{i \in N}(p_i - l_i) \leq p_p$, it is always true that whenever a periphery bank is hit by a shock and becomes distress, then the core bank and the other periphery banks also become distressed. Similarly, whenever the core bank is hit by the shock and becomes distressed, then under the assumption that $p_c > 1$, all other banks are distressed banks as well because $A_{ic} = v_i - l_i$ for all $i \in N \setminus c$ implying that $A_{ic}p_c > v_i - l_i$ for all $i \in N \setminus c$.

Next, in the given star network, $\sum_{i \in N}(p_i - l_i) \leq p_p$ implies that any single default would cause a total loss at least as much as the total available capital in the system. Moreover, whenever the shocked bank is in distress, any coalitional deviation that involves the shocked bank would default due to the shock. Any coalitional deviation that does not involve the shocked bank would default. Therefore, whenever any coalition that does not involve some banks and if those banks default, then any other coalition among the remaining banks would also default. Therefore, similar to the previous proofs, it follows form here that the grand coalition is equilibrium (with or without government assistance, depending on the shock level) whenever there exists any distressed bank following a given shock.

Example 1 on Star Network.

I set $p_p = 1$, $p_c = 5$, $l_p = \frac{42}{44} < 1$, $l_c = \frac{1033}{220} < 5$.

Then, $\alpha_p = 0.1$ and $\alpha_c = 0.4$ is an optimal network, where $v_c = \frac{1085}{220}$ for the core bank, and $v_p = \frac{45}{44}$ for each periphery bank, and the dependency matrix is:

$$A = \begin{bmatrix} \frac{69}{110} & \frac{3}{110} & \frac{3}{110} & \frac{3}{44} \\ \frac{3}{110} & \frac{69}{110} & \frac{3}{110} & \frac{3}{44} \\ \frac{3}{110} & \frac{3}{110} & \frac{69}{110} & \frac{3}{44} \\ \frac{14}{44} & \frac{14}{44} & \frac{14}{44} & \frac{35}{44} \end{bmatrix}.$$

Example 2 on Selective Rescues

EXAMPLE **2** *Consider a financial network (C, F) with the following interbank claims and bank characteristics:*

$$C = \begin{bmatrix} 0 & 0.1 & 0 & 0.1 \\ 0.05 & 0 & 0 & 0 \\ 0.05 & 0 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \end{bmatrix}$$

$$l_i = 0.93 \textit{ for all } i \in N$$
$$p_i = 1 \textit{ for all } i \in N$$
$$\sum_j C_{ji} = 0.1 \textit{ for all } i \in N$$
$$s = 0.25 \textit{ hits bank 1}$$

Additional Results

PROPOSITION **5** *Consider a financial network (C, F) such that the liquidation costs are equal to zero ($\zeta_i = 0$ for all $i \in N$), the shocked bank (bank r) has high external liabilities (there exists an l_r^* such that l_r is greater than the threshold level l_r^*), and there exists at least one distressed bank in (C, F). Then, there exists no strong Nash equilibrium in which any bank is involved in a rescue coalition, and there exists a strong Nash equilibrium in which no rescue coalition has been formed.*

Rogers and Veraart [67] show that the existence of liquidation costs is crucial for rescues. A similar result applies in this framework as well, which is given in Proposition 5. Proposition 5 shows that if liquidation costs are zero ($\zeta_i = 0, \forall i \in N$), then potential rescuer banks have no incentives to form rescue coalitions. Similarly, the government has no incentives for assisting in rescues. The reason is that any rescue that does not prevent any liquidation costs is unprofitable for rescuer banks as well as for society because rescue is a costly action since total assets of a distressed bank is lower than its total liabilities.

Proof of Proposition 5:

First, a distress-cost-minimizing government never contributes to rescues whenever liquidation costs are zero. So, in the remaining part of the proof, I set government bailout transfers to zero.

Secondly, $l_r > l_r^*$ is a condition that rules out formation of some possible non-rescue coalitions, where a set of banks affect the propagation of the initial shock into the system via formation of a coalition that includes bank r. I rule out such unintuitive cases.

Next, I provide the complementary result below. Proposition 6 is on the existence of rescue coalitions in financial networks. Definition below follows from the contagion algorithm.

PROPOSITION **6** *Consider a version of the rescue model without the government intervention (only banks can be involved in rescue actions). Then, consider a potentially contagious financial network (C, F) and the financial network $(C, F)^M$ that is formed by the set of coalitions M such that $\overline{dr}_i = \overline{dr}_j$ for all $(i, j) \in m_k$ for all $m_k \in M$, and $\nexists m_k \in M$ such that $r \in m_k$. Then, there exists no rescue coalition $m_k \in M$.*

Next, I provide Definition 4, which I use for the proof of 6.

DEFINITION **4** *In the plain vanilla contagion case, bank i's distress rank is equal to $\overline{dr}_i = t$ if bank i would default at step t of the contagion if there were no rescue attempt in financial network (C, F). For any healthy bank, that is a bank $i \in N \setminus \aleph_T$, bank i's distress rank is equal to $\overline{dr}_i = N$.* [27]

Proof of Proposition 6:

Index the coalitions in M based on the distress ranks of the banks involved, and denote a given coalition by m_k if $\overline{dr}_i = \overline{dr}_j = k \ \forall (i, j) \in m_k$, and denote the set of coalitions m_k by M_k. Consider that bank r is not involved in any coalition, $r \notin \phi$.

[27] The distress rank for a healthy bank can be at most N. In the extreme case, consider a network where only one bank defaults at each step of the algorithm and only one bank remains healthy ($N \setminus \aleph_T$ consists of a single bank). In that case, the distress rank is equal to $N - 1$ for the bank that fails lastly and, thus, the distress rank is equal to N for the remaining single healthy bank. This example shows that the distress rank is always less than or equal to N for a healthy bank in a given financial network, and that's why I fix it to N for generality.

Suppose that there exists a coalition $m_1 \in M$. We know that $v_i - l_i - A_{ir}s < 0$ holds for all $i \in m_1$. By Lemma 2, $A^M_{m_1 r} = \sum_{i \in m_1} A_{ir}$ holds since $r \notin \phi$. Then, $\sum_{i \in m_1} (v_i - l_i) - A^M_{m_1 r} = \sum_{i \in m_1} (v_i - l_i) - \sum_{i \in m_1} A_{ir} < 0$ also holds, which means that m_1 defaults at step 1 in $(C, F)^M$. This result holds for any $m_1 \in M_1$. On the other hand, by Lemma 2, $A_{jr} = A^M_{jr}\ \forall j \notin \phi$, which implies that $v_j - l_j - A_{jr} = v_j - l_j - A^M_{jr} < 0$ holds for all $\{j : j \notin \phi, \overline{dr}_j = 1\}$; thus any such bank j defaults at step 1 in $(C, F)^M$. Lastly, Lemma 2 implies that any $\{j : j \notin \phi, \overline{dr}_j > 1\}$ remains solvent at step 1 in $(C, F)^M$. Similar argument follows for each subsequent step of the algorithm. Formally, for $p_i = 1$ for all $i \in N$, and $\beta_i = \overline{\beta}$ for all $i \neq r$, we have:
$A^M_{m_k m_l} \sum_{l \in m_l} \beta_l = \left(\frac{\sum_{l \in m_l} \sum_{k \in m_k} (A_{kl} p_l)}{\sum_{l \in m_l} p_l} \right) \sum_{l \in m_l} \beta_l = (\sum_{l \in m_l} \sum_{k \in m_k} A_{kl}) \overline{\beta}$.

Therefore, for any m_k, its total loss due to the failures up to the k^{th} step of the contagion algorithm in $(C, F)^M$ is equal to $(\sum_{l : l < k, l \in M_l} \sum_{k \in m_k} A_{kl}) \overline{\beta} + \sum_{i \notin \phi, i \in \aleph_{k-1}} \sum_{k \in m_k} A_{ki}) \overline{\beta} + \sum_{k \in m_k} A_{kr}(s + \beta_r)$, which is equal to the sum of the individual losses of each bank in m_k due to the failures up to the k^{th} step of the contagion algorithm that would be realized if coalition m_k has not been formed, all else equal.

For any individual bank that has the distressed level of k, we know that $v_k - l_k - A_{kr}(s + \beta_r) - (\sum_{i \in \aleph_{k-1}} A_{ki}) \overline{\beta} < 0$ holds. Thus, $\sum_{k \in m_k} (v_k - l_k - A_{kr}(s + \beta_r)) - (\sum_{l : l < k, l \in M_l} \sum_{k \in m_k} A_{kl}) \overline{\beta} - (\sum_{i \notin \phi, i \in \aleph_{k-1}} \sum_{k \in m_k} A_{ki}) \overline{\beta} < 0$ also holds, and hence; any m_k defaults at step k of the contagion algorithm in the network $(C, F)^M$. This holds for any $m_k \in M_k$ and any k.Thus, there exists no rescue coalition in M, which completes the proof.

Proposition 6 highlights the fact that a coalition that only involves banks having same distress ranks does not prevent any failure unless there exists any other coalition which is formed by banks having different distress ranks.

- Proof of Proposition 5 continued:

For $\zeta = 0$, $\aleph_T = \aleph_1$ (the contagion algorithm stops after one step) where the only potential failures are the potential first step failures. $1 - l - A_{ir}s < 0$ holds for all $i \in \aleph_1$, and $1 - l - A_{ir}s \geq 0$ holds for all $i \in N \setminus \aleph_1$.

By Proposition 6, there exists no rescue coalition which is formed among banks in the set $\aleph_1 \setminus r$ or among banks in the set $N \setminus \aleph_1$.

Then, any rescue coalition must involve at least one bank from each set $N \setminus \aleph_1$ and $\aleph_1 \setminus r$.

Consider an outcome of the game which involves at least one such rescue coalition. Select any of these rescue coalitions arbitrarily, and denote that coalition by m_k. For any such outcome, the net worth of m_k is equal to $e^M_{m_k} = \sum_{i \in m_k \setminus \aleph_1} (v_i - l_i - A_{ir}s) + \sum_{i \in \{\aleph_1 \cap m_k\}} (v_i - l_i - A_{ir}s)$, where there exist $K-h$ number of healthy banks and h number of distressed banks involved in m_k.

For any distressed bank $i \in \{\aleph_1 \cap m_k\}$, $v_i - l_i - A_{ir}s < 0$ holds, and therefore, there always exists a healthy bank $i \in m_k$ that has a payoff less than $v_i - l_i - A_{ir}s$. Thus, given the strategies of other banks, revealing the strategy $s_i = \{i\}$ is a profitable deviation for any such healthy bank $i \in m_k$.

This holds for any arbitrarily selected rescue coalition m_k for any set of strategies resulting in set of coalition M that involves at least one rescue coalition.

Next, I show that there exists a SNE in which no coalition has been formed. Consider the strategies such that $s_i = \{i\}$ for all $i \in N$, which results in an outcome $M = \emptyset$. First, there exists no coalitional deviation among the banks in the set $N \setminus \aleph_1$ (healthy banks), since for any coalitional deviation the payoffs are summed. Similarly, for any coalitional deviation among the banks in the set $\aleph_1 \setminus r$, the payoffs would be equal to zero. In addition, as I showed above, any coalitional deviation that involves at least one healthy bank and one distressed bank from the set of $\aleph_1 \setminus r$ is a non-profitable deviation for any healthy bank involved in such a coalitional deviation. Lastly, for $l_r > l^*_r$, there exists no profitable coalitional deviation that involves bank r. Therefore, $s_i = \{i\}$ for all i is an equilibrium set of strategies.

A Discussion on Cooperative and Non-Cooperative Game Settings

The coalition formation framework built on a contagion model has two specific features. First, bank rescues are similar to public goods, where all banks weakly gain some benefits from any rescue that prevent a costly bankruptcy, whereas rescuer banks pay some costs. The (potential) costs for rescuers might exceed the (potential) benefits in some cases. Secondly, rescue coalitions are complements to each other and, hence, the superset of a given set of rescue coalitions is also a rescue coalition, for everything else constant. These two specifications together imply that both a cooperative game setting and a non-cooperative game setting would work similarly and provide the same insights and results about welfare analysis. Similarly, among non-cooperative game settings, solution concepts of strong Nash equilibrium and coalition-proof Nash equilibrium provide the same results. In the paper, I define a non-cooperative game for rescue formation and use strong Nash equilibrium as a solution concept. Alternatively, a cooperative (or coalitional) game setting could be used under proper *core* definitions that captures the externalities[28] and the possibility

[28]Partition Function Games (PFGs) are the cooperative games that capture the externalities in a given economy, different than the Characteristics Function Games. In a PFG, payoffs of agents in a

of formation of coalitions smaller than the grand coalition.

A Brief History of Rescues in the Financial System

Throughout the history, bank rescues played a key role in maintaining stability in the financial system and the real economy. The historical and recent rescue practices in the financial sector have had three main properties: *(i)* government assistance (rescues might occur with or without government assistance), *(ii)* selectivity (rescues might exclude some of the distressed banks and include some others), and *(iii)* coalition formation (rescues occur in the form of (multiple) coalitions by (multiple) banks). The model proposed in this paper is the first model that captures these three main properties of rescue formation in distressed times.

Bank of England's coordination of Barings' rescue with no government bailouts in 1890 is an example of a successful case. Similarly, the resolution of the Alsatian crisis in 1828 and the Hamburg crisis of 1857 illustrate the role of the private sector in handling distress and restoring confidence in the economy.[29] In the second half of the 19^{th} century and early 20^{th} century, New York Clearing House and other clearinghouse associations, which are private institutional frameworks, were resolving distress and restoring confidence in the US financial system, before the foundation of the Federal Reserve System in 1913.[30]

More recently, the resolution of Long-Term Capital Management (LTCM) in 1998 is an example of a multi-bank rescue consortium. The 14-member consortium injected around 3.6 billion dollars to save LTCM, with no government-assistance.[31] On the

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coalition depends on the partition of the economy, so depends on what other agents do.

[29] See Kindleberger and Aliber [58] for further discussion on these two cases as well as the history of financial crises.

[30] In a recent work, Anderson, Erol, and Ordoñez [6] study the implications of the founding of the Federal Reserve System. Besides, James [57] explain the effectiveness of rescue mergers in the early 19^{th} as follows:

> "In some communities financial reconstruction was attempted by arrangements for a strong bank to merge with a weakened bank or, if several weakened banks were involved, by establishing a new institution with additional capital to take over the liabilities of the failing banks, the stockholders of which took a loss." (James, 1938)

[31] In the Report of the Presidents' Working Group on Financial Markets [69], it is explained as follows:

> "The firms in the consortium saw that their losses could be serious, with potential losses to some firms amounting to $300 million to $500 million each...The self-interest of these firms was to find an alternative resolution that cost less than they could expect to lose in the event of default." (Rubin et al., 1999)

other hand, the story in the 2007-2008 financial crisis was quite different than the LTCM case. Geithner [46] explains the coalitional rescue attempts in the Lehman Brothers case as follows:

> "We told the bankers from the night before to divide themselves into three groups: one to analyze Lehman's toxic assets to help facilitate a potential *merger*, one to investigate an LTCM-style *consortium* that could take over the firm and gradually wind down its positions, and one to explore ways to prepare for a *bankruptcy* and limit the attendant damage." (Geithner, 2015)

Consequently, the rescue did not happen, and Lehman Brothers filed for bankruptcy. Still, the other largest financial institutions formed government-assisted or non-assisted rescue mergers, e.g., Bank of America-Merrill Lynch and JP Morgan Chase-Bear Stearns.[32] The rescue coalitions protected the financial system against unintended consequences of counterparty risks[33] in the 2007-2008 period.

Lastly, in line with its practical importance, there is a historical debate on rescue of distressed banks and financial stability, going back to Thornton (1802) and Bagehot (1873). In his famous book "Lombard Street: A Description of the Money Market," Bagehot explains the importance of timely rescue actions and sharing the costs of a distress. Similar to what Bagehot argues, this paper shows that timely rescues are essential for minimizing welfare losses. Furthermore, reserves must be lent to banks "whenever the security is good" in Bagehot's words, which corresponds to sufficiently small financial shocks in this framework. This paper contributes to this debate by showing how the connectedness between banks affect bailout costs and welfare losses, and provides network-neutrality results on bailout decisions of the government.

[32] The mega-mergers in that period are as follows (the acquiring institution(s) - the acquired institution (the date)): JP Morgan Chase - Bear Stearns (March 2008); Banco Santander - Alliance&Leicester (July 2008); Bank of America - Merrill Lynch (September 2008); Lloyds - HBOS (September 2008 to January 2009), Wells Fargo - Wachovia (October 2008), BNP Paribas - Fortis (May 2009).

[33] Segoviano and Singh [71] quantify counterparty risks in a sample of financial institutions including JP Morgan Chase, Citibank, Bank of America, Goldman Sachs, Merrill Lynch, Lehman, Morgan Stanley, Credit Suisse, Bear Stearns, Wachovia, and Wells Fargo. Counterparty liabilities of broker-dealers in OTC derivatives market as of March 2008 were as follows (in billion dollars): JP Morgan Chase Bank: 68, Citibank: 126, Bank of America: 29, Goldman Sachs: 104, Merrill Lynch: 59, Lehman Brothers: 36, Morgan Stanley: 69, Credit Suisse: 70.

See ECB Report [38] for counterparty risks associated with credit default swaps (CDSs).

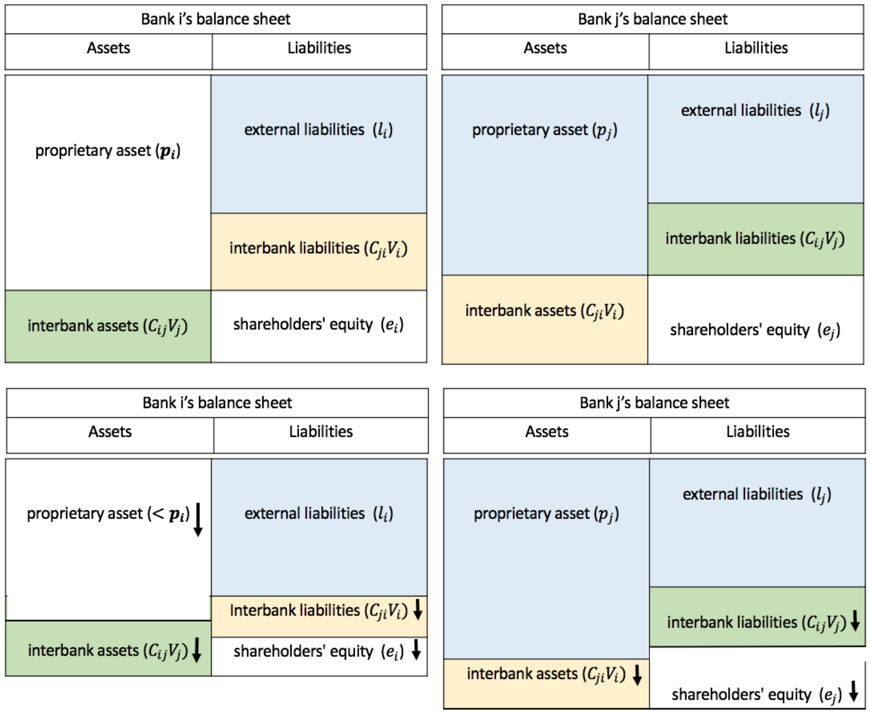

Figure 5: Interdependencies in balance sheets

The Linearities in Interbank Contracts—Figure 5 illustrates how a decline in the primary asset value of bank i affects the interbank liabilities and the net worth of bank i and j.

Figure 5 shows the linearities in interbank assets and liabilities. Such linearities might be thought of as the voluntary "write-downs" of interbank liabilities in distressed times. For instance, if any bank faces a sharp reduction in mortgage loan repayments by households, then its assets decrease and the claim structure implies that its interbank liabilities also drop at some rate. Given that the external liabilities always remain fixed, the drop in its total liabilities is always smaller than the drop in its total assets. This feature makes the model with bankruptcies robust to the feature of linearities. Equation (3) shows that when the total assets of bank i (V_i) decreases by one unit, its interbank liabilities decrease in amount of $\sum_{j \in N} C_{ji} < 1$ units.