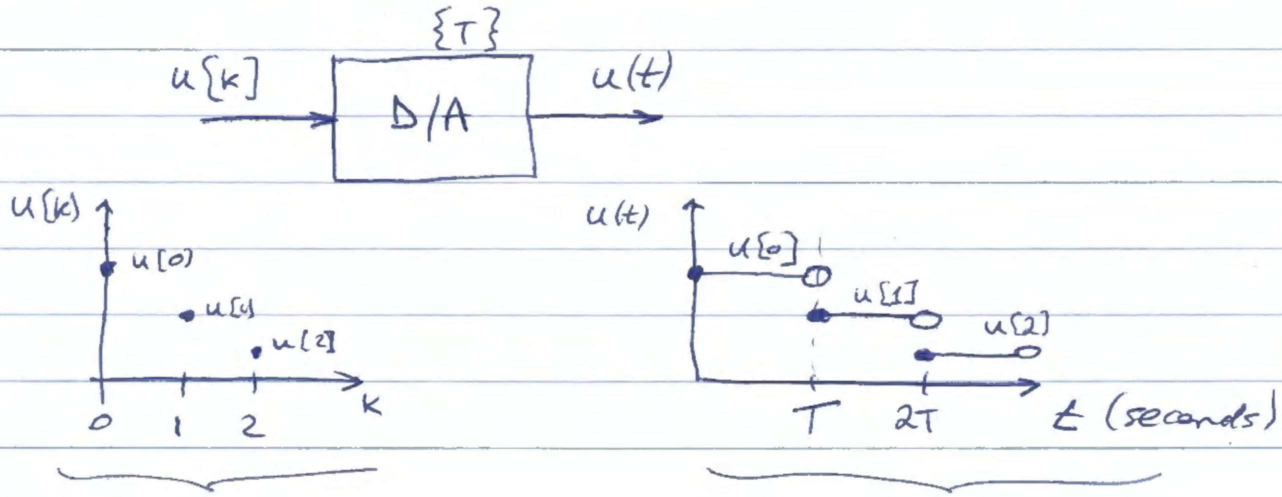
To do digital control we will need digital to analog (D/A) converters, and analog to digital (A/D) converters.

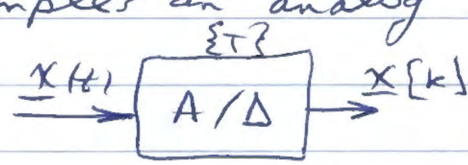
Consider a $\boxed{\text{D/A converter}}$ with sampling interval T seconds (Read section 4.1)



sequence of numbers — piecewise constant analog signal

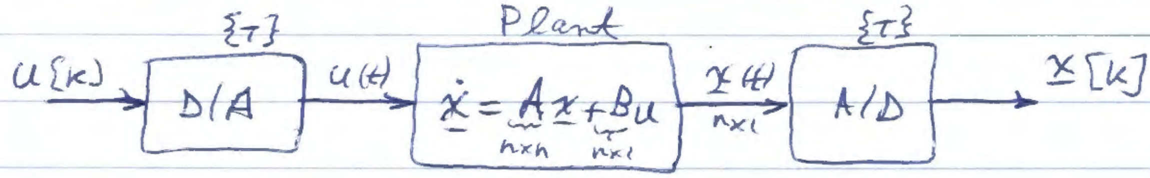
At each sampling instant $t = k \cdot T$ seconds, the D/A converter outputs <u>u[k] volts</u> and <u>holds that voltage constant</u> for T seconds, until $t = kT + T$, when the next sample is sent to the D/A. A constant value may be thought of as a polynomial of <u>degree zero</u>, and this type of D/A converter is referred to as a <u>zero-order hold</u> or ZOH.

An $\boxed{\text{A/D converter}}$ samples an analog signal every T seconds.



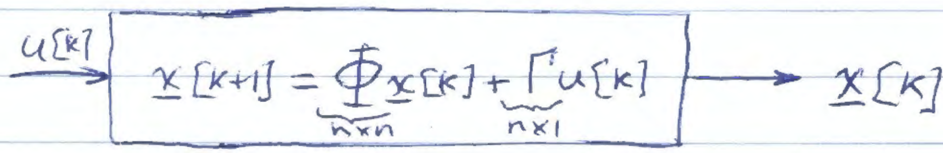
where $x[k] = x(t)\big|_{t=kT}$

Digital Control with all Plant State Variables Measured



plant poles: $s_i = \text{eig}(A)$, $i = 1, \cdots, n$

The cascade connection of D/A, plant, A/D is exactly represented by the following (zero-order hold) ZOH plant equivalent:



poles $z_i = \text{eig}(\Phi)$, $i = 1, \cdots, n$

How are the poles of the plant (eig(A)) related to the poles of the ZOH plant equivalent (eig (Φ)) ?

Recall the eigenvalue decomposition of A:

diagonal matrix

$$\underset{n\times n}{A} = \underset{n\times n}{U}\,\underset{n\times n}{\Lambda}\,\underset{n\times n}{U^{-1}}$$

the columns of U are the eigenvectors of A and the diagonal elements of Λ are the eigenvalues of A. Previously we have called these numbers λ_i. On this page we have called those numbers s_i.

$\boxed{\text{Note}}$: any time you decompose a matrix into the product of three matrices where the third is the inverse of the first and the middle matrix is diagonal, that is an eigenvalue decomp

It can be shown (Chapter 4) that the plant ZOH matrices are obtained from the following formulas:

$$\Phi = e^{A \cdot T}, \quad \Gamma = \int_0^T e^{A \cdot \tau} B \, d\tau$$

where (A, B) is the state-space model of the plant and T is the sampling interval for the D/A and A/D converters.

Recall that the matrix exponential can be written in terms of the eigenvalue decomposition of A:

$$\Lambda = \begin{bmatrix} s_1 & & \\ & \ddots & \\ & & s_n \end{bmatrix} \qquad \Phi = e^{AT} = U e^{\Lambda T} U^{-1}, \quad \Lambda = \mathrm{diag}(s_i)$$

$$A \cdot T = U(\Lambda \cdot T)U^{-1} \qquad = \underbrace{U \begin{bmatrix} e^{s_1 T} & & 0 \\ & e^{s_2 T} & \\ 0 & & \ddots \; e^{s_n T} \end{bmatrix} U^{-1}}_{\text{this is an eigenvalue decomposition of } \Phi}$$

scalar sampling interval

The eigenvalues of Φ are ($z_i = \mathrm{eig}(\Phi)$)

$$\boxed{z_i = e^{s_i T}, \quad i = 1, \ldots, n}$$

This is called the ZOH pole-mapping formula, and it maps the plant poles to the corresponding poles of the ZOH plant equivalent.