5 into 3

$$\dot{x}_3 = -35x_3 - 39(r - x_1)$$
$$= 39x_1 - 35x_3 - 39r \qquad (6)$$

5 into 4

$$u = x_3 + 3(r - x_1)$$
$$= -3x_1 + x_3 + 3r \qquad (7)$$

7 into 1 $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -22 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 2400 \end{bmatrix}(-3x_1 + x_3 + 3r)$

or

$$\dot{x}_1 = -22x_1 + x_2$$
$$\dot{x}_2 = -7200x_1 + 2400x_3 + 7200r$$

Combine with 6 to get

System from r to y

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \underbrace{\begin{bmatrix} -22 & 1 & 0 \\ -7200 & 0 & 2400 \\ 39 & 0 & -35 \end{bmatrix}}_{A}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 7200 \\ -39 \end{bmatrix}}_{B} r$$

From 2

$$y = \underbrace{\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}}_{C}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{0}_{D} \cdot r$$

If we wanted the system from r to u, for example, it would have the same A and B as the system from r to y but the output equation (see 7) would be

$$u = \underbrace{\begin{bmatrix} -3 & 0 & 1 \end{bmatrix}}_{C}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{3}_{D} \cdot r$$

Obtaining State-Space Models from differential equations with state variables defined.

Procedure: write first derivatives of each state variable in terms of state variables and input. Then combine the equations.

Example:

$$(1)\quad \ddot{y}_1 + 2\dot{y}_1 - 3y_1 = u$$
$$(2)\quad \dot{y}_1 + \dot{y}_2 + 4y_2 = 0$$

differential equations

$$(3)\quad y = y_1 + y_2$$

output

$$(4)\quad x_1 = y_1$$
$$(5)\quad x_2 = \dot{y}_1$$
$$(6)\quad x_3 = y_2$$

state variables defined [In this course I will always define the state variables for you.]

Start procedure:

$$\dot{x}_1 = \dot{y}_1 = x_2 \quad \text{or} \quad \dot{x}_1 = x_2$$

from (4) from (5)

$$\dot{x}_2 = \ddot{y}_1 = -2\dot{y}_1 + 3y_1 + u = -2x_2 + 3x_1 + u$$

from (5) from (2) from (4) and (5)

$$\dot{x}_3 = \dot{y}_2 = -\dot{y}_1 - 4y_2 = -x_2 - 4x_3$$

from (6) from (3) from (5) and (6)

Combine all the "x-dot" equations:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 3 & -2 & 0 \\ 0 & -1 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u$$

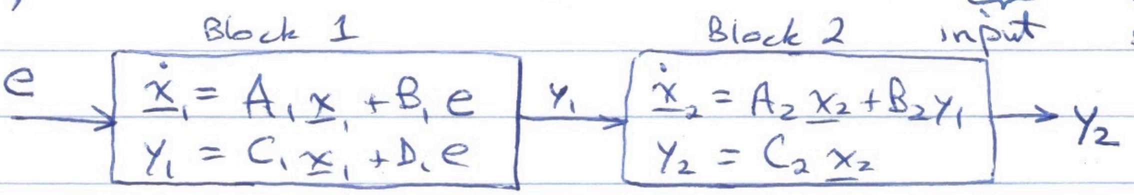
Suppose the output of the system in this example is $y = y_1 + y_2$. (See ③ on previous page)

Then $y = x_1 + x_3$ or $y = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

Obtaining state-Space Models from an interconnection of State-Space Models.

Procedure: Write the "$\underline{x}$-dot" equations for each block. Then combine and write output equation. The overall state vector combines the state vectors from each block

Example: find the state-space model from $\underbrace{e}_{\text{input}}$ to $\underbrace{y_2}_{\text{output}}$



Block 1: $\dot{\underline{x}}_1 = A_1 \underline{x}_1 + B_1 \overset{\text{input to overall system}}{\overset{\downarrow}{e}}$

Block 2:
$$\begin{aligned} \dot{\underline{x}}_2 &= A_2 \underline{x}_2 + B_2 y_1 \\ &= A_2 \underline{x}_2 + B_2 (C_1 \underline{x}_1 + D_1 e) \\ &= B_2 C_1 \underline{x}_1 + A_2 \underline{x}_2 + B_2 D_1 e \end{aligned}$$

Overall Model:

$$\left.\begin{aligned} \begin{bmatrix} \dot{\underline{x}}_1 \\ \dot{\underline{x}}_2 \end{bmatrix} &= \begin{bmatrix} A_1 & 0 \\ B_2 C_1 & A_2 \end{bmatrix} \begin{bmatrix} \underline{x}_1 \\ \underline{x}_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 D_1 \end{bmatrix} e \\ y_2 &= \begin{bmatrix} 0 & C_2 \end{bmatrix} \begin{bmatrix} \underline{x}_1 \\ \underline{x}_2 \end{bmatrix} \end{aligned}\right\} \text{final ("A", "B", "C")}$$

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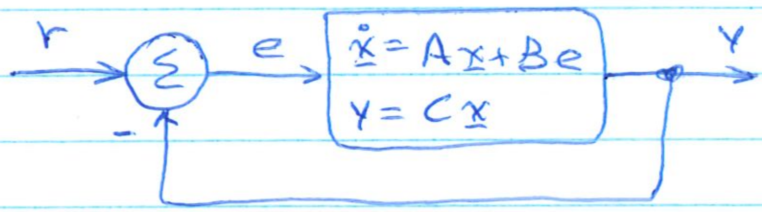
Another example: find the state-space model from r to y.



write x-dot equation for all blocks (only one)

$$\dot{\underline{x}} = A\underline{x} + Be$$ - substitute in the connection equation(s)

$$= A\underline{x} + B(r-y)$$

$$= A\underline{x} + Br - By$$ - substitute $y = C\underline{x}$

$$= A\underline{x} + Br - BC\underline{x}$$

$$\dot{\underline{x}} = (A-BC)\underline{x} + Br$$

Answer:

$$\dot{\underline{x}} = (A-BC)\underline{x} + Br$$

$$y = C\underline{x}$$