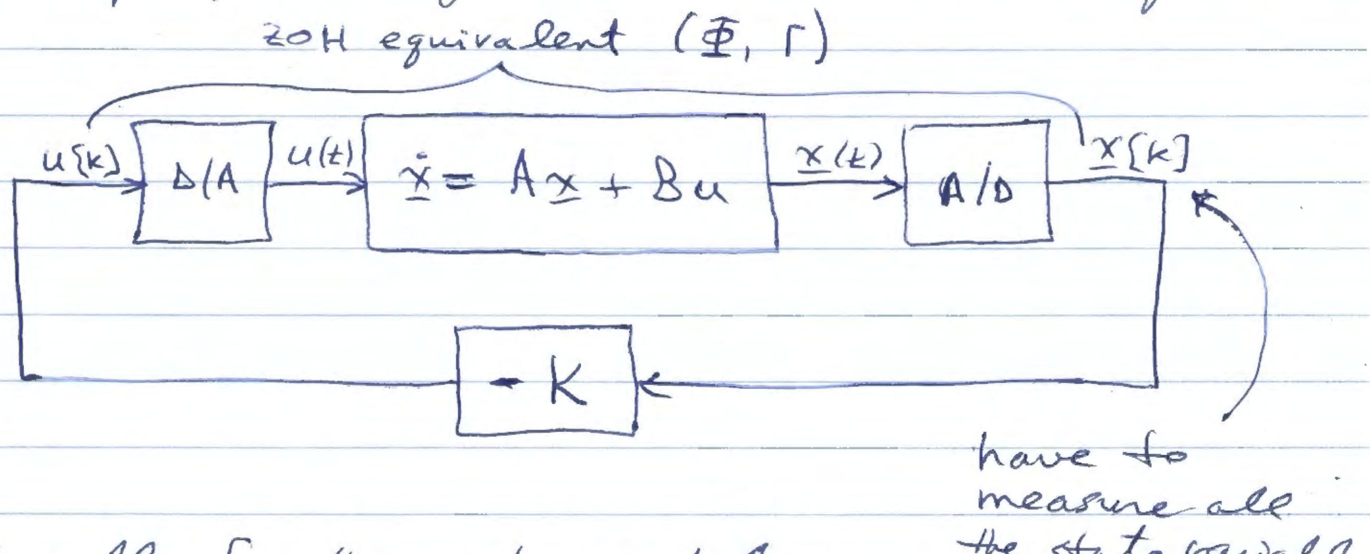
How about the phase margin estimate?

$$\hat{\phi}_{max} = 2 \sin^{-1}\left(\frac{.5}{2}\right) = 29^{\circ}$$

Very close to the classical guideline and it is valid for all plant inputs simultaneously.

Observers (Chapter 7)

Motivation: if we want to control a plant by state feedback, we need to measure all of the state variables. For example, a digital state-feedback regulator:

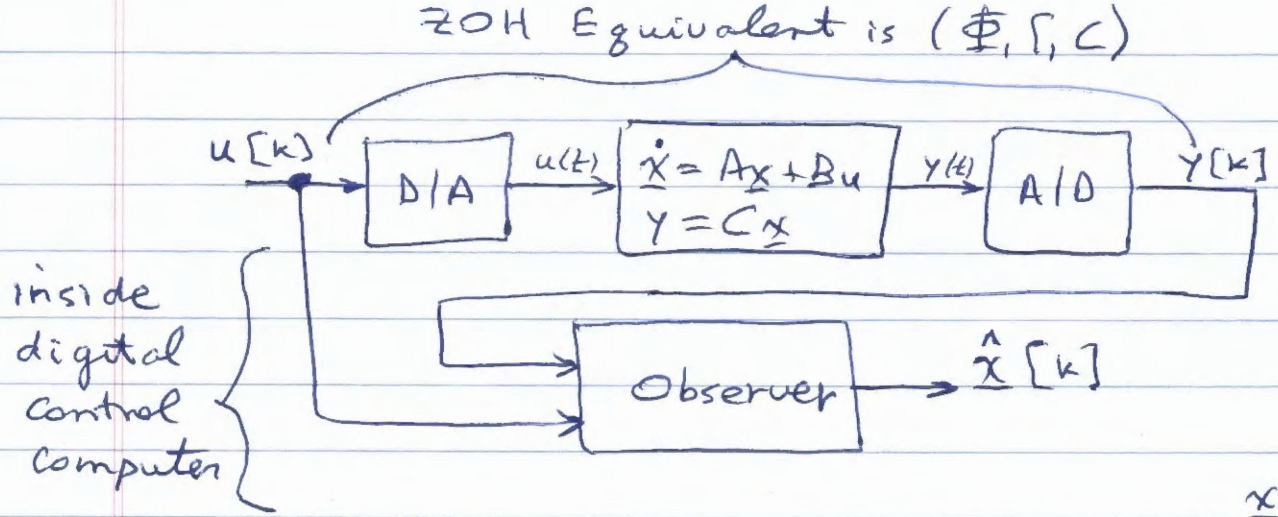


Recall, for the cart-pendulum system the state variables are:

x_1 - pendulum angular position - measured w/ encoder
x_2 - " " velocity - not measured!
x_3 - motor angular position - measured w/ encoder
x_4 - " " velocity - not measured!

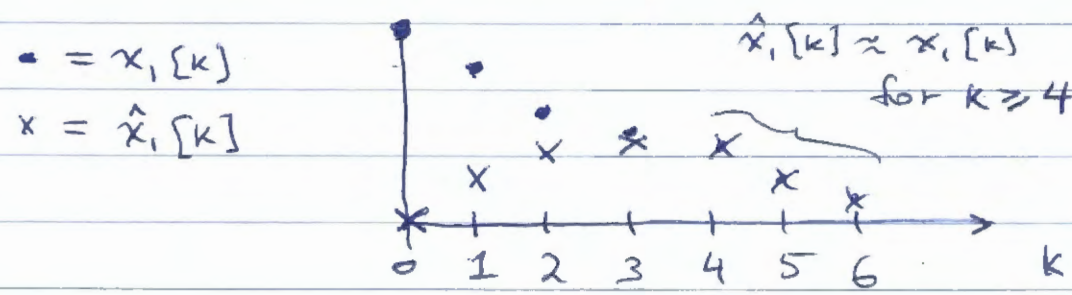
Suppose only the plant output $y = C\underline{x}$ is measured? (Assume y is one signal for now)

A digital observer is a system that takes $u[k]$ and $y[k]$ as inputs and produces an estimate $\hat{\underline{x}}[k]$ of the plant state vector $\underline{x}(t)\Big|_{t = k \cdot T}$



The desired behavior is $\hat{\underline{x}}[k] \rightarrow \underline{x}(kT) = \underline{x}[k]$ as k increases

Look at one state variable, say $x_1[k]$



In this example, the observer settling time is

$$T_{so} = 4 \cdot T \text{ seconds}$$

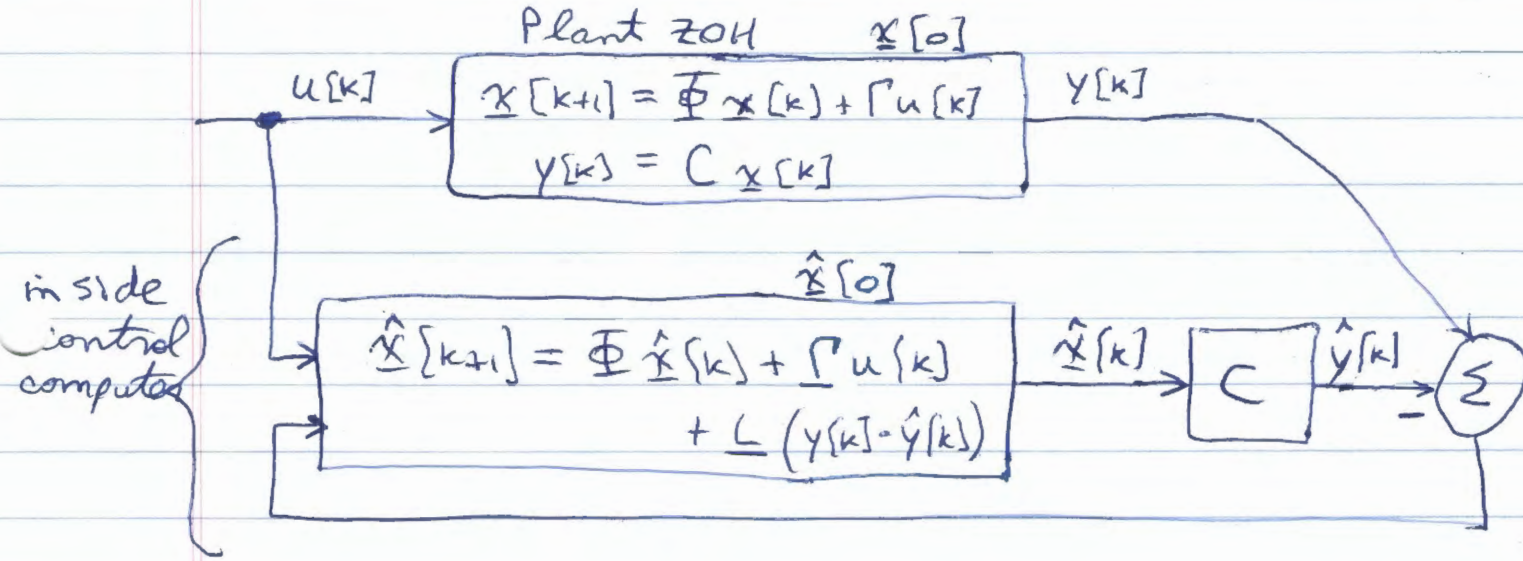
T_{so} is the amount of time it takes for $\hat{\underline{x}}$ to converge to $\underline{x}$.

Note that observer has to "hit" n moving targets (each of the n plant state variables).

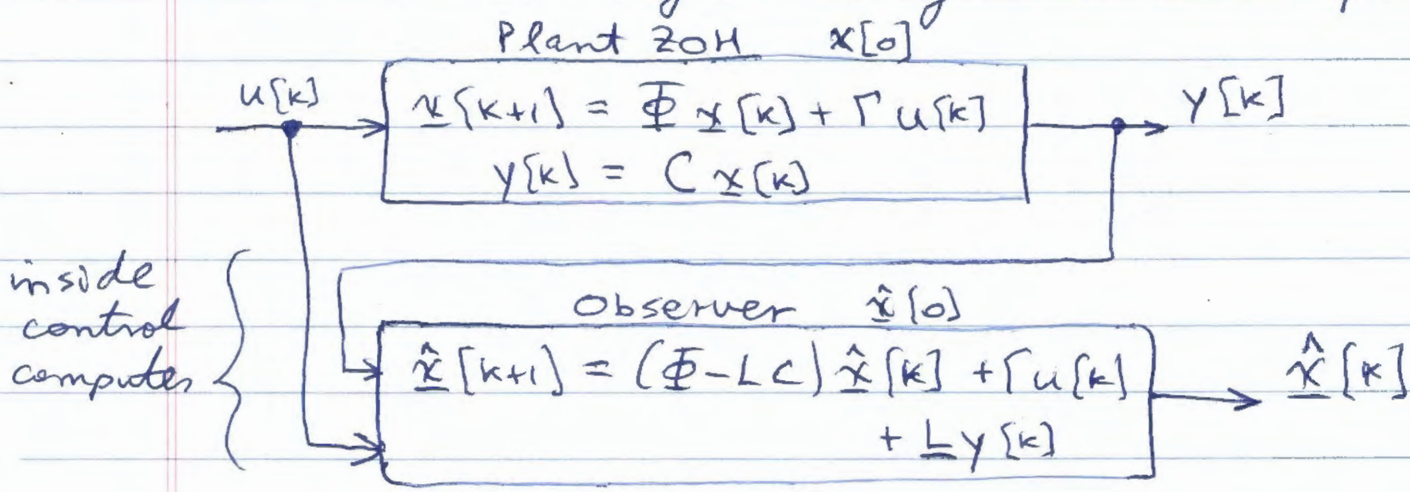
Derive observer equation as follows:

(1) Copy the plant ZOH equation

(2) Add a correction term through an observer gain vector $\underline{L} = \begin{bmatrix} l_1 \\ l_2 \\ \vdots \\ l_n \end{bmatrix}$ (for a single-output plant).



Re-draw this diagram using the fact that $\hat{y}[k] = C\hat{\underline{x}}[k]$



Calculate $\underline{L}$ so that $\hat{\underline{x}}[k] \to \underline{x}[k]$

Analyze observer performance:

Define an error vector: $\underline{e}[k] = \underline{x}[k] - \hat{\underline{x}}[k]$

true plant state vector

observer estimate

Assume $\underline{e}[0] \neq \underline{0}$

The goal is $\underline{e}[k] \to \underline{0}$. That is, the observer should drive the error vector to zero.
This is similar to a state-feedback regulator driving the plant state vector to zero, $\underline{x}[k] \to \underline{0}$.

From the block diagram at the bottom of page 52 we can write the following:

$$\underline{e}[k+1] = \underline{x}[k+1] - \hat{\underline{x}}[k+1] \quad \left(\begin{array}{l}\text{by the definition} \\ \text{of } \underline{e}[k]\end{array}\right)$$

$$= \left\{\Phi \underline{x}[k] + \cancel{\Gamma u[k]}\right\} - \left\{(\Phi - LC)\hat{\underline{x}}[k] + \cancel{\Gamma u[k]} + LC\,\underline{x}[k]\right\}$$

$$= \Phi \underline{x}[k] - (\Phi - LC)\hat{\underline{x}}[k] - LC\,\underline{x}[k]$$

$$= (\Phi - LC)\,\underline{x}[k] - (\Phi - LC)\,\hat{\underline{x}}[k]$$

$$= (\Phi - LC)(\underline{x}[k] - \hat{\underline{x}}[k])$$

$$\underbrace{\underline{e}[k+1] = (\Phi - LC)\,\underline{e}[k]}, \quad \underline{e}[\underline{0}] \neq 0$$

We have seen an equation of this form twice before, in Chap. 3 and in Chap. 6.

<u>Chapter 3</u>

$$\underline{x}[k+1] = A\,\underline{x}[k]$$

solution $\;\underline{x}[k] = A^k \underline{x}_0$

$\underline{x}[k] \to \underline{0}$ if all <u>eigenvalues</u> of A are <u>inside the unit circle</u>

<u>Chapter 6</u> State feedback regulator:

$$\underline{x}[k+1] = (\Phi - \Gamma K)\,\underline{x}[k]$$

$\underline{x}[k] \to \underline{0}$ if all <u>eigenvalues</u> of $\Phi - \Gamma K$ are <u>inside the unit circle</u>.

Calculate K to make this happen. (Choose spoles, map to zpoles = exp(T*spoles).

<u>Chapter 7</u> Observer Design (see bottom of pg 53)

$$\underline{e}[k+1] = (\Phi - LC)\,\underline{e}[k]$$

$\underline{e}[k] \to \underline{0}$ if all <u>eigenvalues</u> of $\Phi - LC$ are <u>inside the unit circle</u>.

Look more carefully at the two problems:

Regulator: $$\underset{n\times n}{\overset{\text{Known}}{\boxed{\Phi}}} - \underset{n\times 1}{\overset{\text{Known}}{\boxed{\Gamma}}}\;\underset{1\times n}{\overset{\text{unknown}}{\boxed{K}}}$$

Observer: $$\underset{n\times n}{\overset{\text{Known}}{\boxed{\Phi}}} - \underset{n\times 1}{\overset{\text{unknown}}{\boxed{L}}}\;\underset{1\times n}{\overset{\text{Known}}{\boxed{C}}}$$

not identical problems