Summary of Tracking System Design for SISO Plants with Reference Step Commands

Given an nth-order plant (A, B, C) and a desired settling time T_s.

Choose T to satisfy $\frac{T_s}{20(n+1)} \le T \le \frac{T_s}{2(n+1)}$ and $T \le \frac{\pi}{5\beta_{max}}$

Guideline: choose $T = \min\left(\frac{T_s}{20(n+1)}, \frac{\pi}{5\beta_{max}}\right)$

```
>> [phi, gamma] = c2d(A,B,T)
>> spoles = [...]        choose n+1 CL poles using
                         rules for choosing regulator
                         poles
```

Φ_a `>> phia = 1`

Γ_a `>> gammaa = 1`

```
>> zpoles = exp(T*spoles)
```

- Form design model (Φ_d, Γ_d)
- Calculate K_d and partition into K_1 and K_2
- Calculate stability robustness δ_1, δ_2

The last three steps are done by a new function:

```
>> [K1, k2, delta1, delta2] =
        tsd(phi, gamma, C, phia, gammaa,
            zpoles, T, 'place')
```

tsd: tracking system design

'place': function to compute Kd

Example: Consider the plant

$$\left.\begin{aligned}\dot{\underline{x}} &= \begin{bmatrix}0 & 1\\ 0 & -22\end{bmatrix}\underline{x} + \begin{bmatrix}0\\ 2400\end{bmatrix}u\\ y &= \begin{bmatrix}1 & 0\end{bmatrix}\underline{x}\end{aligned}\right\}(A,B,C)$$

This is $\frac{2400}{s(s+22)}$, the plant from HW 1!

Suppose $T_s = 0.35$ sec

Plant poles = eig(A) = 0, −22

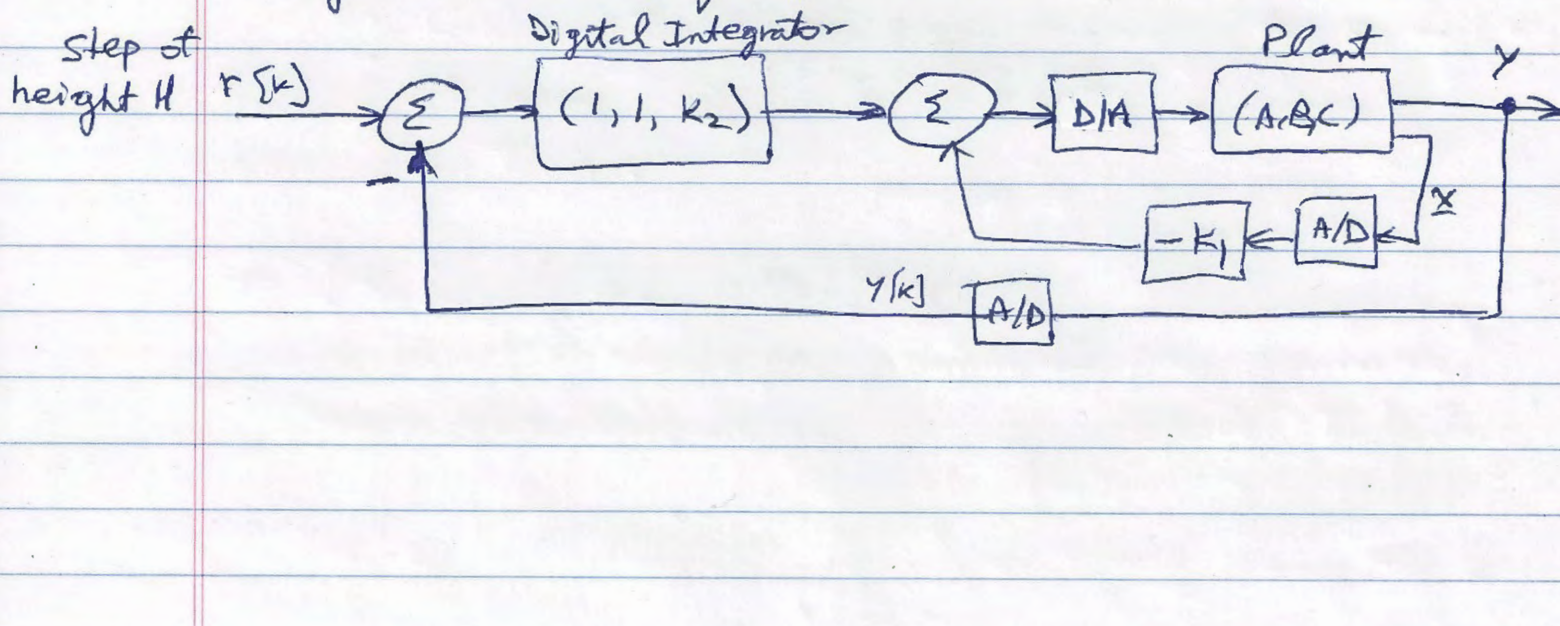
$s1/T_s = -13.2$ so −22 is a SDPP

Choose spoles = [−22 $s2/T_s$]

(need a total of 2+1=3 CL poles) — add two scaled Bessel poles

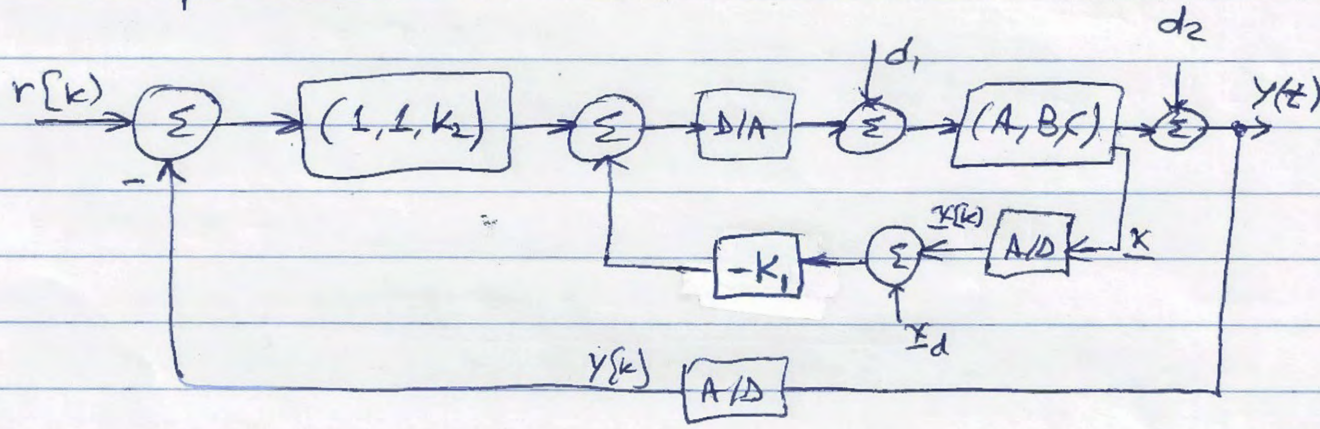
use Matlab code on previous page.

Digital Tracking System:



Another benefit of tracking systems designed to track step (constant) commands will reject additive constant disturbances anywhere in the loop

Example: d_1, d_2, $\underline{x}_d$ constant disturbances



The result is still that $y[k] \to r[k]$ (zero steady state error)

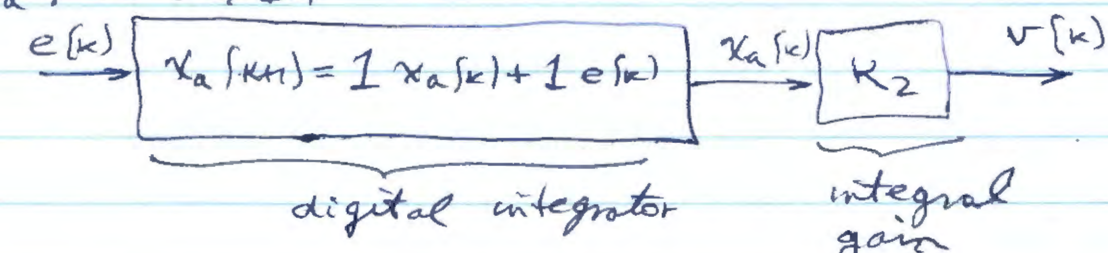
We need only one eigenvalue equal to 1 so choose $q = 1$ (one state variable for the additional dynamics)

and choose $\Phi_a = 1$ (then the eigenvalue of the 1×1 matrix Φ_a is 1).

Only requirement on Γ_a is that (Φ_a, Γ_a) must be controllable. Thus Γ_a can be any nonzero number. The simplest choice is $\Gamma_a = 1$.

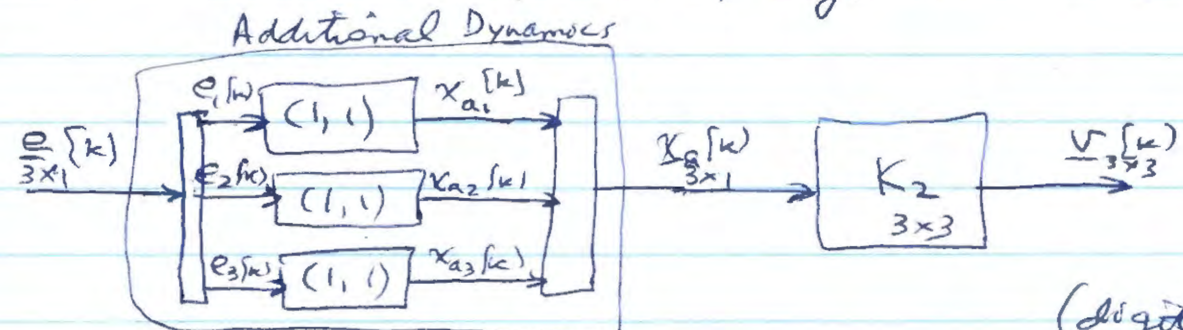
Additional Dynamics for Step Tracking (SISO)

$(\Phi_a, \Gamma_a) = (1, 1)$



Consider tracking for a MIMO plant (m-input, m-output). Then $\underline{r}[k] = \begin{bmatrix} c_1 \\ \vdots \\ c_m \end{bmatrix}$, $\underset{m\times1}{\underline{e}[k]}$, $\underset{m\times1}{\underline{v}[k]}$

The correct thing to do is to "replicate" the SISO additional dynamics, e.g. for m=3



The additional dynamics consists of 3 parallel (digital) integrators

$$\underline{x}_a[k+1] = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\Phi_a} \underline{x}_a[k] + \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\Gamma_a} \underline{e}[k]$$