To make the observer problem look <u>exactly</u> like the regulator problem, use the following facts from linear algebra

<u>Fact 1:</u> A matrix M and its transpose M^T have the same eigenvalues.

<u>Fact 2:</u> $(M_1 + M_2)^T = M_1^T + M_2^T$

<u>Fact 3:</u> $(M_1 \cdot M_2)^T = M_2^T \cdot M_1^T$

We are interested in the eigenvalues of $\Phi - LC$. By Fact 1 we can work with the transpose of this matrix and use Facts 2 & 3.

$$(\Phi - LC)^T = \Phi^T - (LC)^T = \underbrace{\overset{\text{known}}{\boxed{\Phi^T}}}_{n \times n} - \underbrace{\overset{\text{known}}{\boxed{C^T}}}_{n \times 1} \underbrace{\overset{\text{unknown}}{\boxed{L^T}}}_{1 \times n}$$

same form as regulator: $\boxed{\Phi} - \boxed{\Gamma}\,\boxed{K}$

Regulator		Observer
Φ	$\rightarrow$	Φ^T
Γ	$\rightarrow$	C^T
K	$\rightarrow$	L^T (calculate L^T, then <u>transpose</u> result to get L.)

$\boxed{\text{Consider the example}}$ of a 4^{th}-order, single-input, single-output plant, (A, B, C). ZOH plant equivalent is (Φ, Γ, C) where [phi, gamma] = c2d(A,B,T)

(Example for (A, B, C) 4th-order plant.)

Calculate $\underline{K}$ (regulator gains)

T_s = regulator settling time

spoles = S4/Ts (all Bessel poles, for example)

zpoles = exp(T*spoles)

K = place(phi, gamma, zpoles)

Calculate $\underline{L}$ (observer gains)

T_{so} = observer settling time (guideline given below)

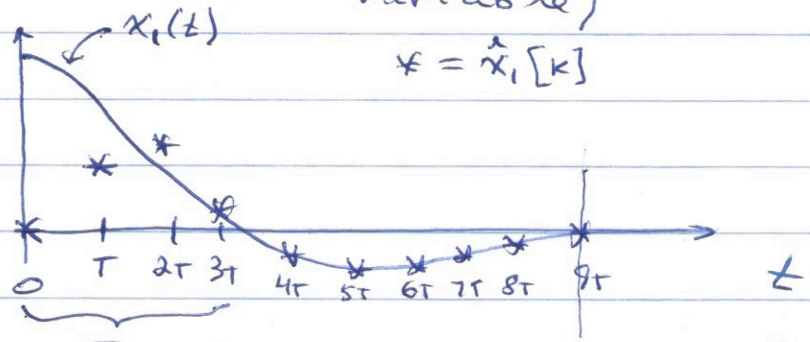
sopoles = S4/Tso

zopoles = exp(T* sopoles)

L = $\underbrace{\text{place}}_{\text{computes } L^T}$(phi', C', zopoles)'

need to transpose the result

Typical Picture (first state variable)



$T_{so} = 3T$, time it takes for $\hat{x}[k] \to x(kT)$

$T_s = 9T$ (time it takes for $\underline{x}(t) \to \underline{0}$)

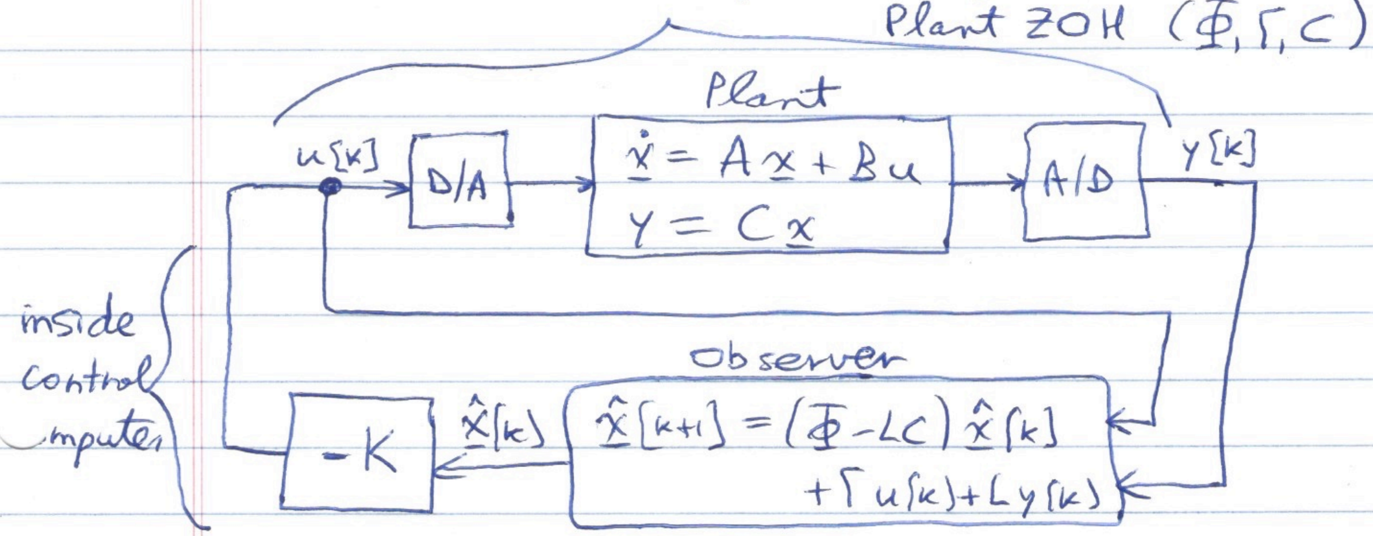
In order to use observer estimate $\hat{\underline{x}}[k]$ to perform state feedback regulation we need $T_{so} < T_s$. Guideline:

$$\boxed{\text{Choose } T_{so} = \frac{T_s}{f}, \text{ where } 3 \le f \le 5}$$

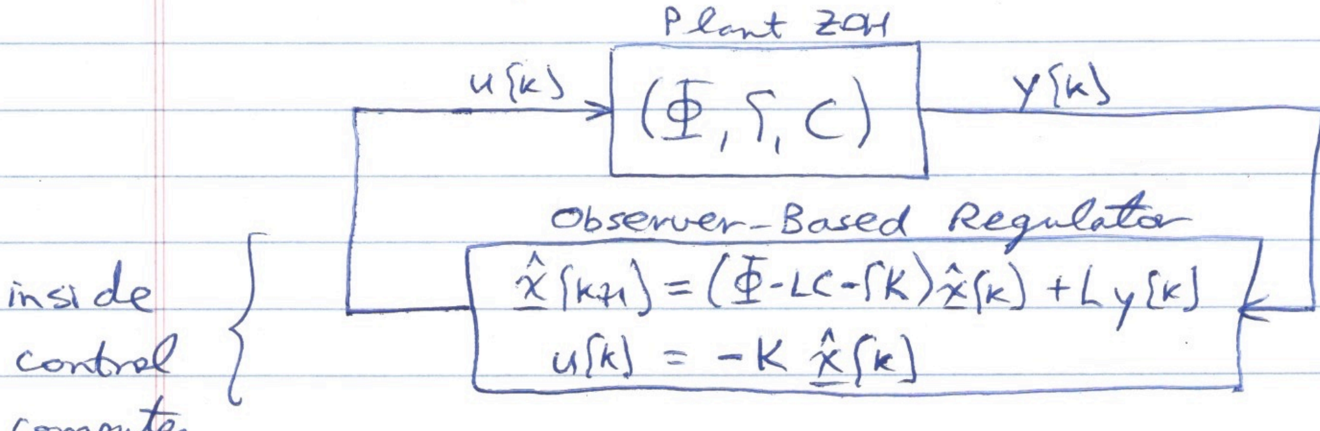
The observer is said to be "f times faster"

Idea of an observer-based regulator:

- Send the plant input and output ($u[k]$ and $y[k]$) to an observer. The observer calculates $\underline{\hat{x}}[k]$.
- Send the estimated state vector $\underline{\hat{x}}[k]$ through the state feedback gain vector K to create the plant input.



Because $u[k] = -K\underline{\hat{x}}[k]$, this diagram can be simplified as follows:



Note: this closed-loop system has order $2n$, (n^{th} order plant, nth order regulator).

$\boxed{\text{Where are the } 2n \text{ poles located?}}$

It can be shown that the $2n$ poles of the observer-based regulated system are in the following locations:

n of the poles are $\mathrm{eig}(\Phi - \Gamma K)$ = zpoles

and n of the poles are $\mathrm{eig}(\Phi - LC)$ = zopoles.

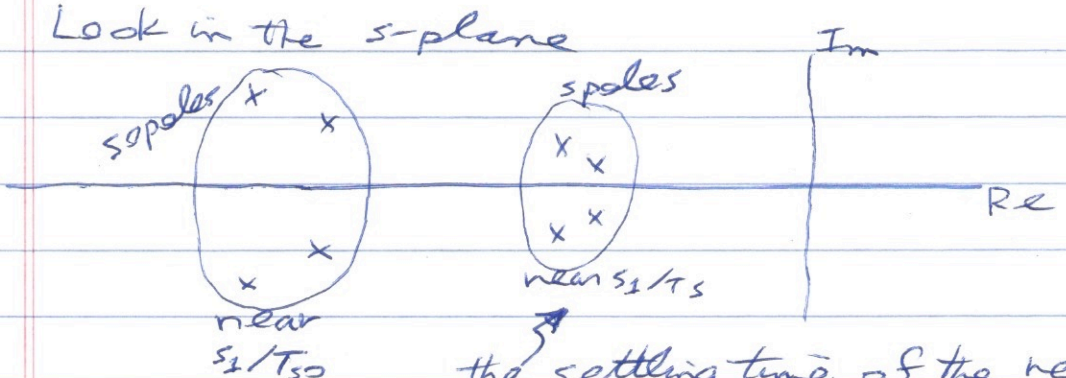
zpoles are the closed-loop poles we get with full state feedback $u[k] = -K\,\underline{x}[k]$.

zopoles are the poles of the observer.

Recall that zpoles = exp(T * spoles)

and zopoles = exp(T * sopoles), where spoles have a settling time of T_s seconds and sopoles have a settling time of T_{so} seconds, where $T_{so} = \dfrac{T_s}{f}$, $3 \le f \le 5$.

Look in the s-plane



the settling time of the regulated system is dominated by these poles.

Q1: We make the observer 3 to 5 times faster. Why not make the observer as fast as possible (e.g. 10 or 100-times faster)?

A1: A fast observer is more sensitive to sensor noise. A 3-5 times faster observer often helps filter out sensor noise but a faster observer will not filter out noise.

Q2: How do you find the classical stability margins for an observer-based regulated system?

A2: Recall that the stability margins for a <u>state-feedback regulator</u> are computed by dsm (phi, gamma, K).
The stability margins for <u>any</u> single-input control system can be computed by dsm (phi_L, gamma_L, C_L), where (Φ_L, Γ_L, C_L) is a state-space model for the "loop gain" of the control system.

Q3: How do you find a state-space model for the "loop gain" of any control system?
A3: Use the procedure given on the next page.