

Q1: We make the observer 3 to 5 times faster. Why not make the observer as fast as possible (e.g. 10 or 100-times faster)?

A1: A fast observer is more sensitive to sensor noise. A 3-5 times faster observer often helps filter out sensor noise but a faster observer will not filter out noise.

Q2: How do you find the classical stability margins for an observer-based regulated system?

A2: Recall that the stability margins for a state-feedback regulator are computed by $\text{dsm}(\phi, \gamma, K)$.

The stability margins for any single-input control system can be computed by $\text{dsm}(\phi_L, \gamma_L, C_L)$, where (Φ_L, Γ_L, C_L) is a state-space model for the "loop gain" of the control system.

Q3: How do you find a state-space model for the "loop gain" of any control system?

A3: Use the procedure given on the next page.

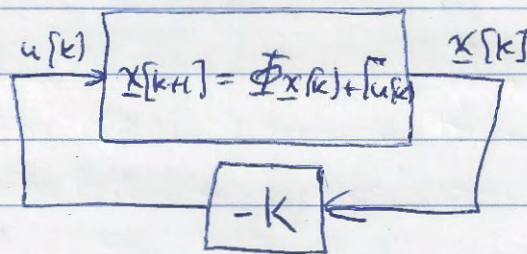
Procedure to Calculate Loop Gain

Steps:

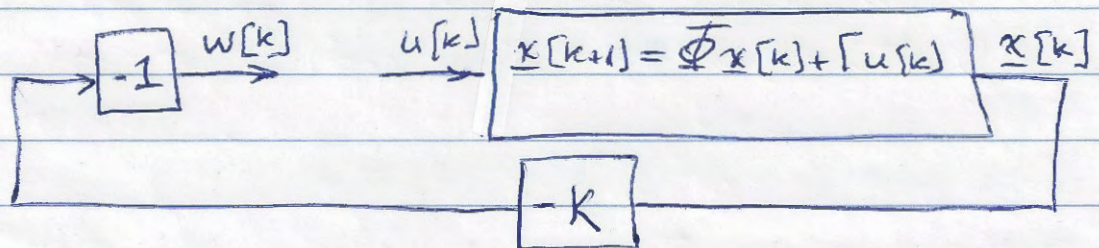
- (1) Break the loop open at the plant input $u[k]$.
- (2) Insert a minus sign on the "out" arrow and call the resulting output $w[k]$.
- (3) The state-space model from $u[k]$ to $w[k]$, call it (Φ_L, Γ_L, C_L) , is the loop gain.

The stability margins are calculated by $dsm(\Phi_L, \Gamma_L, C_L)$

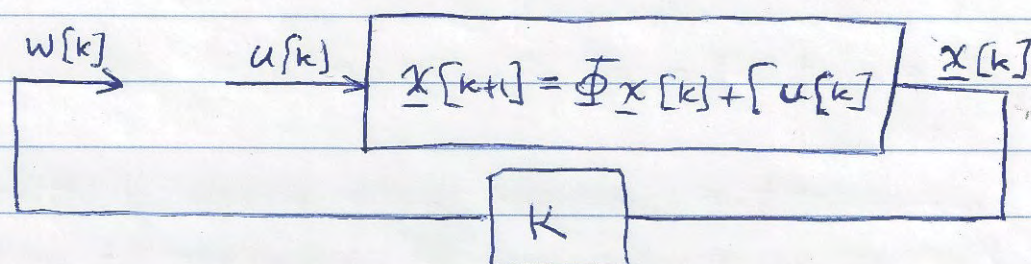
Example: State Feedback Regulator



Steps 1 and 2:



Step 3: Simplify the diagram



The system from $u[k]$ to $w[k]$ is

$$\underline{x}[k+1] = \underline{\Phi} \underline{x}[k] + \underline{\Gamma} u[k]$$

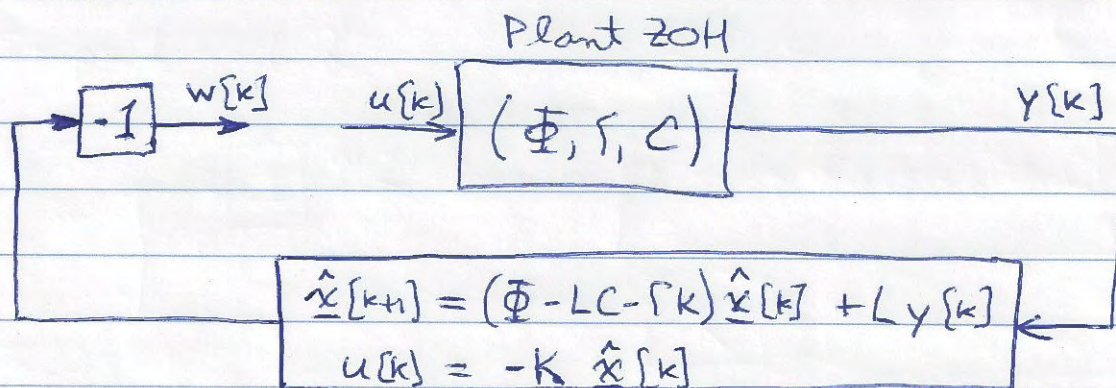
$$w[k] = \underline{K} \underline{x}[k]$$

so $\underline{\Phi}_L = \underline{\Phi}$, $\underline{\Gamma}_L = \underline{\Gamma}$, $\underline{C}_L = \underline{K}$

$\text{dsm}(\underline{\Phi}, \underline{\Gamma}, \underline{K})$ we have already used this

Now use the procedure to find the loop gain of an observer-based regulated system (see bottom of pg. (57))

Steps 1 and 2



Step 3 Combine -1 and $-K$ to just K

Step 3 The system from $u[k]$ to $w[k]$ uses the state vector $\begin{bmatrix} \underline{x}[k] \\ \hat{\underline{x}}[k] \end{bmatrix}$

From the block diagram on the previous page,

$$\begin{aligned} \underline{x}[k+1] &= \Phi \underline{x}[k] + \Gamma u[k] & \underbrace{y[k]} \\ \hat{\underline{x}}[k+1] &= (\Phi - LC - \Gamma K) \hat{\underline{x}}[k] + LC \underline{x}[k] \end{aligned}$$

Combine into final model for loop gain

$$\begin{bmatrix} \underline{x}[k+1] \\ \hat{\underline{x}}[k+1] \end{bmatrix} = \underbrace{\begin{bmatrix} \Phi & 0 \\ LC & (\Phi - LC - \Gamma K) \end{bmatrix}}_{\Phi_L} \underbrace{\begin{bmatrix} \underline{x}[k] \\ \hat{\underline{x}}[k] \end{bmatrix}}_{\hat{\underline{x}}[k]} + \underbrace{\begin{bmatrix} \Gamma \\ 0 \end{bmatrix}}_{\Gamma_L} u[k]$$

$$w[k] = \underbrace{\begin{bmatrix} 0 & K \end{bmatrix}}_{C_L} \begin{bmatrix} \underline{x}[k] \\ \hat{\underline{x}}[k] \end{bmatrix}$$

$dsm(\phi_L, \gamma_L, C_L)$ gives stability margins

The following function computes Φ_L, Γ_L, C_L for an observer-based regulator and then calls dsm .

$$dsm_regob(\phi, \gamma, C, K, L)$$

gives the stability margins for an observer-based regulator