Download the following files from the course web site, (`rjvaccaro.github.io/ele458`): `dsm.m` and `sroots.mat`.

Each of the following problems asks you to compute stability margins. You may use Matlab to compute the stability margins of a state feedback regulator as follows: `>> dsm(phi,gamma,K)`. You want the upper gain margin to be as large as possible (at least greater than 3 db), the lower gain margin to be as negative as possible (at least less than -3 db), and the phase margin to be as large as possible (at least greater than 30 degrees).

1. *Hydraulic Positioning System*

   The purpose of this problem is to design a digital regulator for a hydraulic positioning system. The hardware is represented by the following state-space model:

$$\dot{\mathbf{x}} = \begin{bmatrix} -30 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1225 & -1225 & -21 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 30 \\ 0 \\ 0 \\ 0 \end{bmatrix} u$$

   where some of the state variables are defined as follows

$$\begin{aligned} x_1 &= \text{valve position} \\ x_2 &= \text{normalized flow rate} \\ x_4 &= \text{position of piston.} \end{aligned}$$

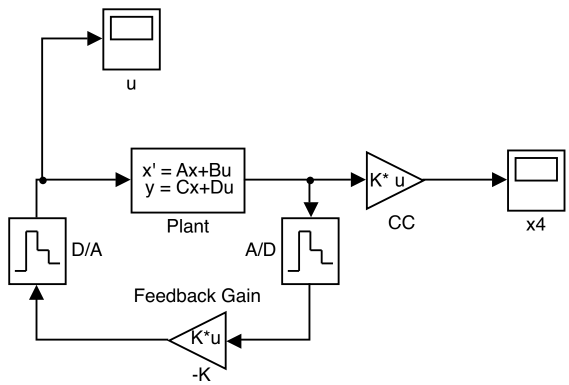
   The system has two real poles as well as a pair of complex-conjugate poles (type `eig(A)` to see the eigenvalues of $\mathbf{A}$, which are the poles of the system). The lightly damped complex poles model a resonance caused by the compressibility of the hydraulic fluid.

   Note the following regarding Simulink implementation of a digital state-feedback regulator (the Simulink model is shown in page 2):

   - The feedback gain matrix, $\mathbf{K}$, is implemented with a gain block. **You must double-click on the gain block and set the following field: *Multiplication* should be set to** `Matrix(K*u)`.
   - The feedback gain vector is a discrete-time gain while the plant is a continuous-time system. **Use zero-order hold (ZOH) blocks (found under *Discrete*) to model both D/A and A/D converters.** Set the sample time of these blocks to `T`.

   In the Simulink block diagram, the plant is a *Continuous-time state-space model.* Use `C=eye(4)` and `D` = `zeros(4,1)` to make the block output the entire state vector. The piston position signal x_4 is then obtained by sending the state vector through a gain block containing `CC=[0 0 0 1]`, with *Multiplication* set to `Matrix(K*u)`. The Simulink block diagram is shown on the next page.

   The following two problems show the importance of using all the rules for choosing `spoles`. In both problems, multiple rules apply. However, if `spoles` are chosen using only scaled Bessel poles, the resulting design will have poor stability margins. If all applicable rules are followed the design will have good stability margins.



Use the following initial state vector: `x0=[0;0;0;-1]` (put in m-file, then type `x0` in the Initial Conditions tab of the plant state-space block). This indicates that the piston starts at a position of -1 and the regulator drives piston position (and all state variables) to 0. Look at the x_4 and u scopes.

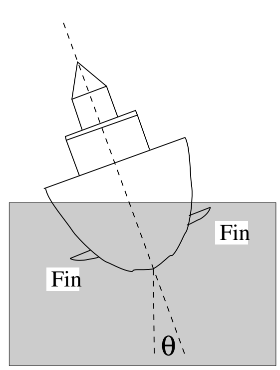
(a) The desired settling time for the system is `Ts = 0.35` seconds. Use a sampling interval of $T = 0.01$. Explain how this choice relates to the rule-of-thumb for choosing T. Choose the closed-loop s-plane pole locations using scaled Bessel poles; that is, the `s4` pole vector scaled to achieve the desired settling time (`s4/Ts`).

Calculate feedback-gain vector for *digital* a regulator. Don't forget to map `spoles` into the z plane using the ZOH pole-mapping formula! Find the stability margins for this regulator using `dsm`.

(b) Design a second regulator by picking a different set of closed-loop poles. Use added damping and sufficiently damped plant poles as appropriate. To get started, type the following two commands at the Matlab prompt: `eig(A)` followed by `s1/Ts`. Put the following numbers into `spoles`: any sufficiently damped plant poles, any plant poles that need added damping, and scaled Bessel poles as needed to get `spoles` containing n pole locations. Calculate the feedback gain vector. Compute the stability margins and compare with those from the all-Bessel regulator.

2. *Ship Stabilization System*

The roll of a ship can be regulated using fins to generate a stabilizing torque. The following figure shows a picture of a ship at roll angle θ.



The angle-of-attack of the fins is controlled by motors (fin actuators), and we assume that the torque generated by the fins is proportional to their relative angle-of-attack, called fin position. A state-space model of the system is

$$\begin{aligned}\dot{\mathbf{x}} &= \begin{bmatrix} 0 & 1 & 0 \\ -4 & -.4 & 40 \\ 0 & 0 & -4 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} u \\ y &= [1 \quad 0 \quad 0]\,\mathbf{x}.\end{aligned} \tag{1}$$

where the state variables are

$$\begin{aligned} x_1 &= \theta \text{ (radians)} \\ x_2 &= \dot{\theta} \text{ (radians/second)} \\ x_3 &= \text{fin angle (rad).} \end{aligned}$$

Note that the matrix $[1 \quad 0 \quad 0]$ in equation (1) above is NOT used in the Simulink state-space block for the plant. The matrix `CC`$=[1 \quad 0 \quad 0]$ is used in a gain block to extract the first state variable (x_1) and feed it to the scope.

(a) Set the simulation time to 20 seconds. Plot the roll angle x_1 for the **open-loop response** of the ship (i.e. with no feedback; set `K=[0 0 0]` in response to the following initial state

$$\mathbf{x}(0) = \begin{bmatrix} .17 \\ 0 \\ 0 \end{bmatrix}$$

which corresponds to the ship at a $10°$ roll angle. You should see a roll period of about 3 seconds. **Note that stability margins are meaningless here** as there is no feedback control when the feedback gains are all zero.

(b) Set the simulation time to 10 seconds. Design a digital state feedback regulator that results in a settling time of `Ts=4.5` seconds. Use the rule-of-thumb to choose the sampling interval. Use added-damping and sufficiently damped poles as appropriate. Plot the roll angle of the closed-loop system in response to the initial state used in (a). Find the stability margins of this regulator.

(c) Calculate the feedback gain vector for an all-Bessel regulator and compute the stability margins. Simulate the response and see good results. Calculate the stability margins and see unacceptable results! In other words, **simulation with the nominal plant model does *not* show whether the stability margins are good or bad.** That is why the stability margins should always be calculated.