Stability Robustness for Multi-Input Systems

(Read sections 5.4, 5.5 and 5.6)

In Lab 3 we work with the 2 DOF AERO plant, which is a 2-input system. There are two motor-driven propellers that influence the yaw and pitch motions.

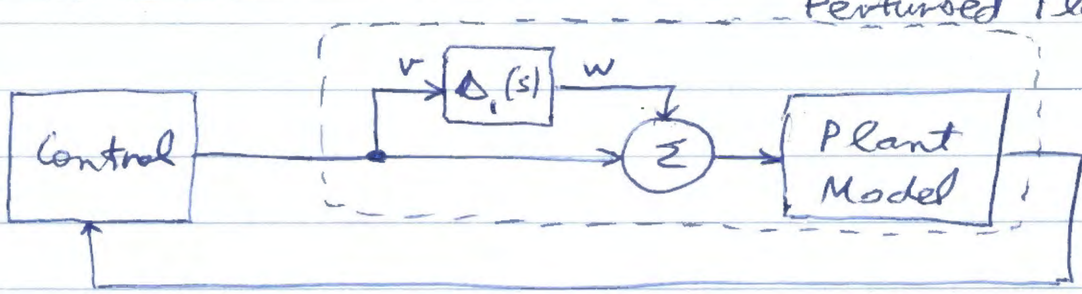
How do we compute "stability margins" for a multi-input system? One thought would be to insert individual perturbations (eg. g_1, g_2, ...) on each of the plant inputs, and let each of the g_is vary in gain or phase <u>one at a time</u>. The drawback to this approach is that a control system can have low sensitivity to perturbations on individual plant inputs but be <u>very sensitive</u> to small perturbations that occur simultaneously on multiple inputs.

Modern Stability Robustness

The modern approach works for both multi-input systems as well as single-input systems (which are a special case).

This approach considers two different ways of perturbing a plant model, as shown on the next page.

First Perturbation Model



The plant is a p-input system.
$\Delta_1(s)$ represents a stable, p-input, p-output unknown system. If $\Delta_1(s) = 0$ the closed-loop system is stable by design. The closed-loop system remains stable provided that

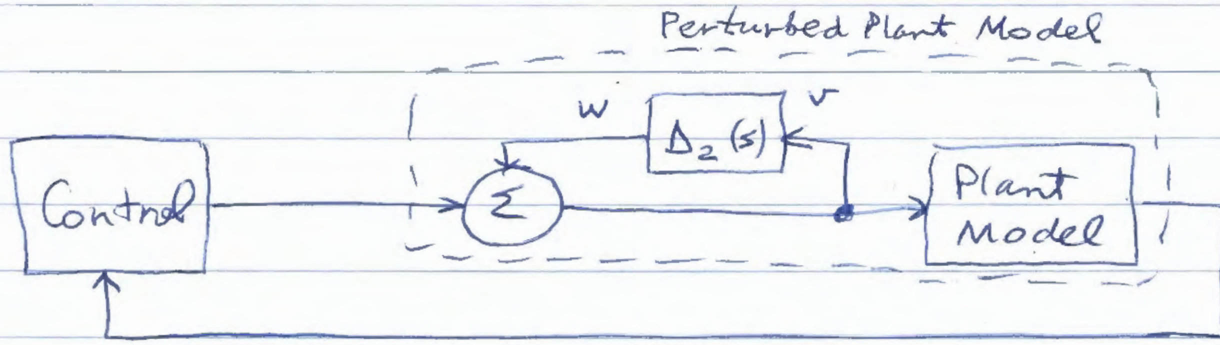
$$\underbrace{\|\Delta_1(s)\|_\infty}_{} < \delta_1 .$$

$\|\Delta_1(s)\|_\infty$, called the "system infinity norm", is a measure of the "size" of the perturbation. Includes simultaneous gain and phase perturbations on all plant inputs.

δ_1 is a number, called a "stability robustness bound" that can be computed for any control system

We want δ_1 to be as large as possible. A specific guideline will be given on the next page, after introducing the second perturbation model.

Second Perturbation Model



This closed-loop system will remain stable if $\|\Delta_2(s)\|_\infty < \delta_2$.

δ_2 is the second stability robustness bound

For control systems in which the plant and/or controller has a pole at $s=0$, or for any unstable plant, δ_1 and δ_2 cannot be larger than 1. } most useful control systems.

$$\boxed{\text{Guideline: We would like } \min(\delta_1, \delta_2) \geq 0.5}$$

Modern stability robustness bounds provide bounds on classical stability margins (ie the stability margins cannot be too small):

$$\hat{g}_{max} = \max\left(1+\delta_1, \frac{1}{1-\delta_2}\right)$$

$$\hat{g}_{min} = \min\left(1-\delta_1, \frac{1}{1+\delta_2}\right)$$

If the plant has gain perturbations $g_1, \cdots, g_p$ on the plant inputs, the control system remains stable if $\hat{g}_{min} < g_i < \hat{g}_{max}$, $i=1,\cdots,p$

Also, $\hat{\phi}_{max} = 2 \sin^{-1}\left(\dfrac{\max(\delta_1, \delta_2)}{2}\right)$

If the plant has phase perturbations $e^{-j\phi_1}, \ldots, e^{-j\phi_p}$ on the plant inputs, the control system remains stable if

$$|\phi_i| < \hat{\phi}_{max}, \quad i = 1, \ldots, p$$

Note that, for a single-input control system,

$$\underbrace{g_{max}}_{\text{classical upper gain margin}} > \hat{g}_{max}$$

$$\underbrace{g_{min}}_{\text{classical lower gain margin}} < \hat{g}_{min}$$

$$\underbrace{\phi_{max}}_{\text{classical phase margin}} > \hat{\phi}_{max}$$

A Note on Classical Gain Margins

Recall that upper and lower gain margins are reported in dB:

$$UGM = 20 \log_{10}(g_{max})$$

$$LGM = 20 \log_{10}(g_{min})$$

These expressions can be inverted by dividing both sides by 20, then raising both sides as exponents of 10:

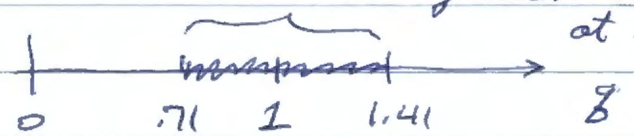
$$g_{max} = 10^{\frac{UGM}{20}} \qquad \left(\text{recall } 10^{\log_{10}(x)} = x \right)$$

$$g_{min} = 10^{\frac{LGM}{20}}$$

So $UGM = 3\text{ dB} \Rightarrow g_{max} = 10^{3/20} = 1.41$

$LGM = -3\text{dB} \Rightarrow g_{min} = 10^{-3/20} = .71$

we want to guarantee stability for at least this range of g.



What kind of classical gain margin estimates do we get from a modern stability robustness analysis when δ_1 and δ_2 both equal the smallest desired value of 0.5? From page 47

$$\hat{g}_{max} = \max\left(1+0.5, \frac{1}{1-0.5}\right) = 2$$

$$\hat{g}_{min} = \min\left(1-0.5, \frac{1}{1+0.5}\right) = 0.5$$

These estimates are <u>better</u> than ± 3 dB and they are valid for all plant inputs <u>simultaneously</u>.

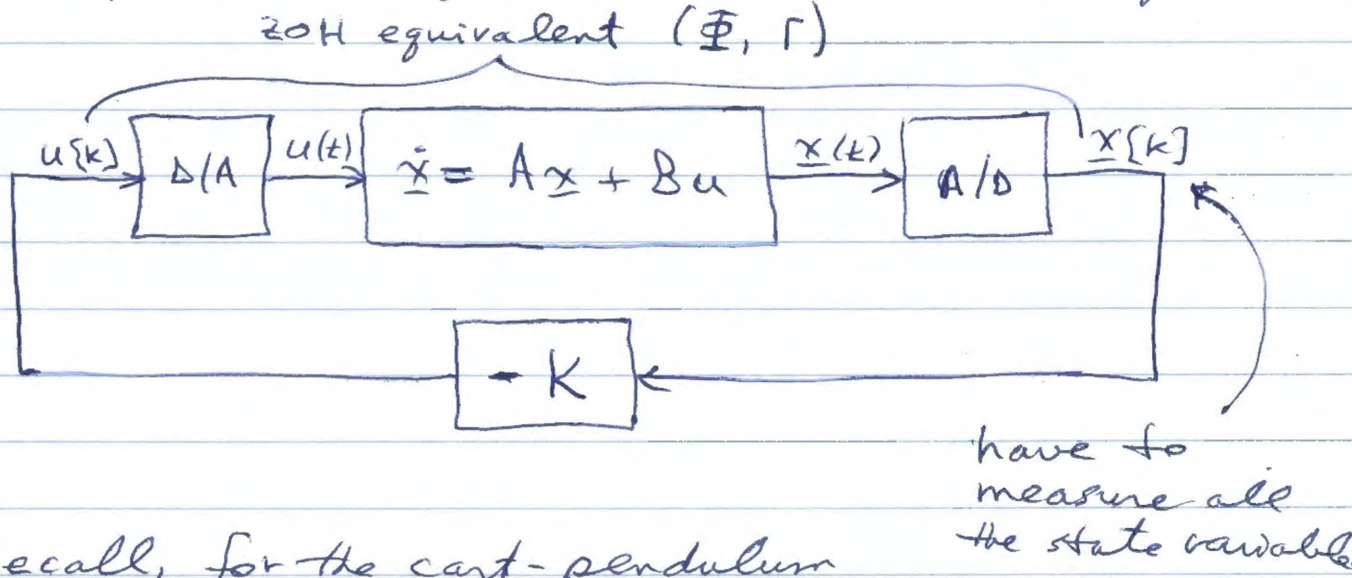
How about the phase margin estimate?

$$\hat{\phi}_{max} = 2 \sin^{-1}\left(\frac{.5}{2}\right) = 29^\circ$$

Very close to the classical guideline and it is valid for all plant inputs simultaneously.

Observers (Chapter 7)

Motivation: if we want to control a plant by state feedback, we need to measure all of the state variables. For example, a digital state-feedback regulator:



Recall, for the cart-pendulum system the state variables are:

x_1 - pendulum angular position - measured w/ encoder

x_2 - " " velocity - not measured!

x_3 - motor angular position - measured w/ encoder

x_4 - " " velocity - not measured!