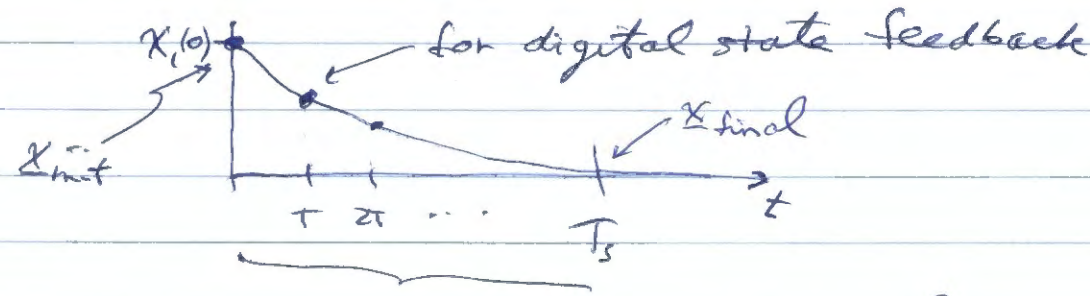
Choosing the sampling interval, T, for a digital state-feedback regulator. (Section 6.4.4 in book).

It is known that to drive a controllable n^{th} order discrete-time system from an arbitrary initial state, $\underline{x}_{init}$, to an arbitrary final state, $\underline{x}_{final}$, requires at least $\underline{n}$ time steps.

For a regulator, $\underline{x}_{final} = \underline{0}$ (drive all state variables to zero). We want to achieve a settling time of T_s seconds. So a plot of the first state variable of the regulated plant might look as follows:



Q: how many samples in T_s sec?

A: about $\frac{T_s}{T}$ samples.

So $\frac{T_s}{T}$ should be $\geq n$ (for an nth order plant)

rewrite as

minimum # of samples needed $\rightarrow n \leq \frac{T_s}{T}$

insert factor of 2 for safety

$$2n \leq \frac{T_s}{T} \leq 20n$$

multiply by 10 to get an upper bound

Take reciprocals of every term (have to change the sense of the equalities):

$$\frac{1}{2n} \geq \frac{T}{T_s} \geq \frac{1}{20n}$$

multiply by T_s

$$\frac{T_s}{2n} \geq T \geq \frac{T_s}{20n}$$

rewrite as: slow sampling

(fast sampling) $\frac{T_s}{20n} \leq T \leq \frac{T_s}{2n}$

Recommended value for T: $\frac{T_s}{20n}$ (could go as large as $\frac{T_s}{2n}$)

In order to guarantee controllability of (Φ, Γ) assuming (A, B) is controllable, we also need $T < \frac{\pi}{2\beta_{max}}$. For safety, increase denominator: $T < \frac{\pi}{5\beta_{max}}$ where β_{max} is the maximum imaginary part of any plant pole ($\text{eig}(A)$).

The guideline for selecting the sampling interval is to choose the <u>smaller</u> of these numbers:

$$T = \min\left(\frac{T_s}{20n}, \frac{\pi}{5\beta_{max}}\right)$$

How to choose desired CL pole locations in the s-plane (spoles)

There are <u>three</u> ideas that should be used.

(1) <u>Use normalized Bessel poles</u>, <u>scaled</u> to have a <u>desired settling time</u>, T_s.

Table 6.2 in the book shows these poles for systems of order 1, 2, ..., 10. For any order system, these poles provide less than 1% overshoot and 1-second settling time. From Table 6.2 in book

System order	Poles	Matlab variable name
1	-4.62	s1
2	$-4+j2.34,\ -4-j2.34$	s2
3	[* * *]	s3
⋮		⋮
10		s10

in sroots.mat on course web site. In m-file type "load sroots"

To get a desired settling time divide the Bessel poles by T_s.

Example: to scale the three normalized Bessel poles use $\dfrac{s3}{T_s}$