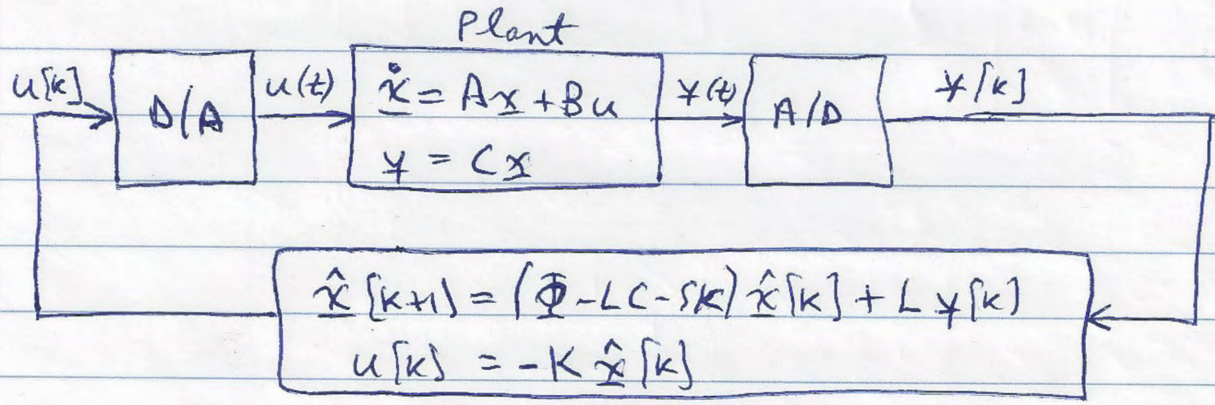
Observer-Based Regulators for Multiple-Output Plants

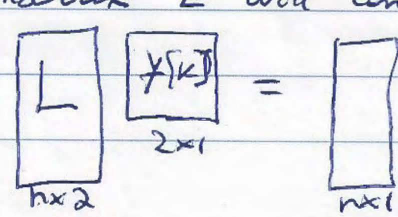
Recall the cart/pendulum system. It has a single input (u = voltage applied to the motor power amplifier) and two measured output signals: pendulum angular position and motor angular position. The linear state-space model is

$$\dot{\underline{x}} = A\underline{x} + Bu \qquad x_1 = \text{pend position}$$
$$\underline{y} = C\underline{x} \qquad x_2 = \text{pend velocity}$$
$$x_3 = \text{motor position}$$
$$\underline{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \text{pend pos} \\ \text{motor pos} \end{bmatrix} \qquad x_4 = \text{motor velocity}$$
$$= \begin{bmatrix} x_1 \\ x_3 \end{bmatrix}, \text{ thus } C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

An observer-based regulator will have the usual form:



Note: the observer gain matrix L will contain two columns:



Using the same derivation as before,
if $\underline{e}[k] = \underline{x}[k] - \underline{\hat{x}}[k]$ then $\underline{e}[k+1] = (\Phi - LC)\,\underline{e}[k]$

and $\underline{e}[k] \to \underline{0}$ if all the eigenvalues of $(\Phi - LC)$ are inside the unit circle

$$\underbrace{\overset{\text{known}}{\Phi}}_{n\times n} - \underbrace{L}_{n\times 2}\;\underbrace{C}_{2\times n}$$

The place command can be used to calculate L as before:

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Tso = Ts/5   (for a 5-times faster observer)
sopoles = S4/Tso
zopoles = exp(T* sopoles)
L = place(phi', C', zopoles)'
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So far everything is the same as the case of a single-output plant. What is different in the multiple-output case?

Answer: the stability margins may be <u>very</u> poor.

For example $\boxed{\text{dsm\_regob(phi, gamma, C, K, L)}}$ might give the following result:

Upper Gain Margin	0.04 dB
Lower Gain Margin	−0.1 dB
Phase Margin	1°

These margins are catastrophically bad!

What causes this problem?

Fact: For a multiple-output plant there are an infinite number of L matrices such that the eigenvalues of $(\Phi - LC)$ = zopoles

L = place(phi', C', zopoles)' computes one of these L matrices but the one it computes may result in poor stability margins.

The stability margins depend on Φ, Γ, C, K, L (see dsm_regob) but the place algorithm does not know about Γ or K.

A solution to this problem has been developed at URI, a new way to calculate observer gains for an observer-based regulator with a multiple-output plant:

sampling interval

[L, delta1, delta2] = obg_reg(phi, gamma, C, K, zopoles, T)

Searches for the L that gives the largest possible δ_1, δ_2 (modern stability robustness bounds) and thus gives good classical stability margins.