

Q1: We make the observer 3 to 5 times faster. Why not make the observer as fast as possible (e.g. 10 or 100-times faster)?

A1: A fast observer is more sensitive to sensor noise. A 3-5 times faster observer often helps filter out sensor noise but a faster observer will not filter out noise.

Q2: How do you find the classical stability margins for an observer-based regulated system?

A2: Recall that the stability margins for a state-feedback regulator are computed by  $dsm(\phi, \gamma, K)$ . The stability margins for any single-input control system can be computed by  $dsm(\phi_L, \gamma_L, C_L)$ , where  $(\Phi_L, \Gamma_L, C_L)$  is a state-space model for the "loop gain" of the control system.

Q3: How do you find a state-space model for the "loop gain" of any control system?

A3: Use the procedure given on the next page.

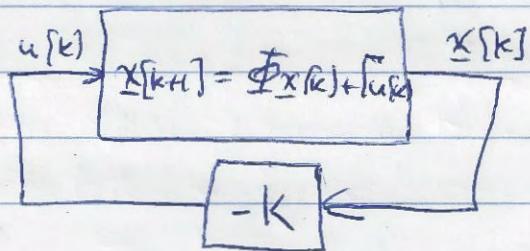
## Procedure to Calculate Loop Gain

Steps:

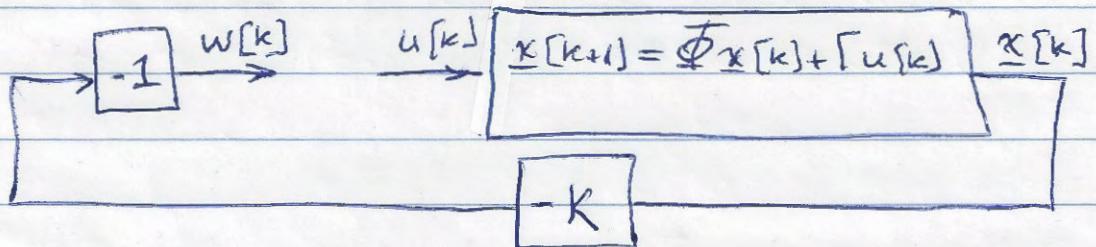
- (1) Break the loop open at the plant input  $u[k]$ .
- (2) Insert a minus sign on the "out" arrow and call the resulting output  $w[k]$ .
- (3) The state-space model from  $u[k]$  to  $w[k]$ , call it  $(\Phi_L, \Gamma_L, C_L)$ , is the loop gain.

The stability margins are calculated by  $dsm(\Phi_L, \Gamma_L, C_L)$

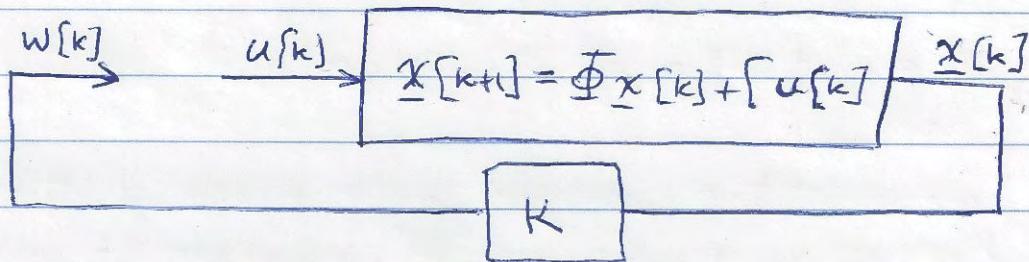
Example: State Feedback Regulator



Steps 1 and 2 :



Step 3: Simplify the diagram



The system from  $u[k]$  to  $w[k]$  is

$$\underline{x}[k+1] = (\underline{\Phi})\underline{x}[k] + (\underline{B}u[k])$$

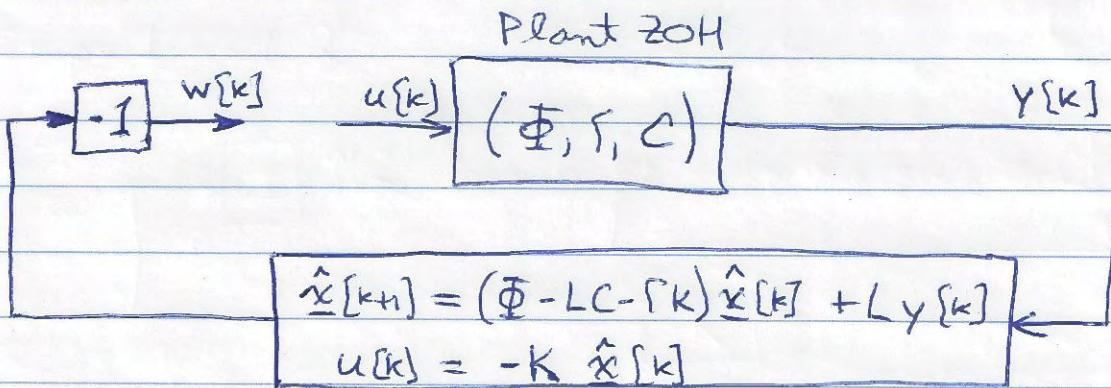
$$w[k] = (K)\underline{x}[k]$$

$$\text{so } \underline{\Phi}_L = \underline{\Phi}, \underline{B}_L = \underline{B}, \underline{C}_L = K$$

dsm (phi, gamma, K) we have already used this

Now use the procedure to find the loop gain of an observer-based regulated system (see bottom of pg. 57)

Steps 1 and 2



Step 3 Combine  $-1$  and  $-K$  to just  $K$

Step 3 The system from  $u[k]$  to  $w[k]$  uses the state vector

$$\begin{bmatrix} \underline{x}[k] \\ \hat{x}[k] \end{bmatrix}$$

From the block diagram  
on the previous page,

$$\begin{aligned} \underline{x}[k+1] &= \Phi \underline{x}[k] + \Gamma u[k] & y[k] \\ \hat{x}[k+1] &= (\Phi - LC - \Gamma k) \hat{x}[k] + LC \underline{x}[k] \end{aligned}$$

Combine into final model for loop gain

$$\begin{bmatrix} \underline{x}[k+1] \\ \hat{x}[k+1] \end{bmatrix} = \underbrace{\begin{bmatrix} \Phi & 0 \\ LC & (\Phi - LC - \Gamma k) \end{bmatrix}}_{\Phi_L} \begin{bmatrix} \underline{x}[k] \\ \hat{x}[k] \end{bmatrix} + \underbrace{\begin{bmatrix} \Gamma \\ 0 \end{bmatrix}}_{L} u[k]$$

$$w[k] = \underbrace{\begin{bmatrix} 0 & K \end{bmatrix}}_{C_L} \begin{bmatrix} \underline{x}[k] \\ \hat{x}[k] \end{bmatrix}$$

dsm( $\phi_{-L}, \gamma_{-L}, C_{-L}$ ) gives  
stability margins

The following function computes  $\Phi_L, \Gamma_L, C_L$   
for an observer-based regulator and then calls  
dsm.

dsm-regob( $\phi, \gamma, C, K, L$ )

gives the stability margins for an  
observer-based regulator