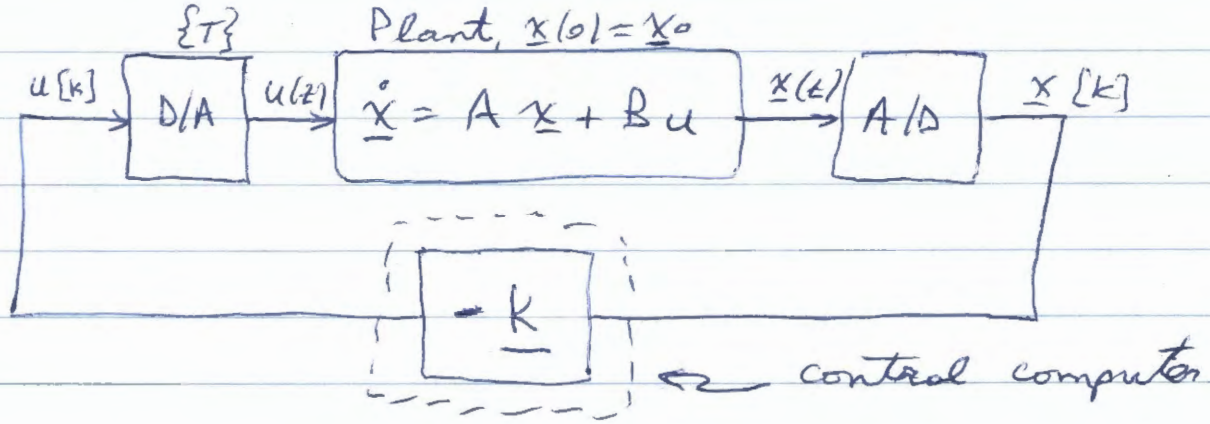
Digital State Feedback Regulators

A *regulator* is a control system that drives all plant state variables to zero.

Examples: balance an inverted pendulum on a cart; aircraft autopilot - level flight)

The architecture of a digital state-feedback regulator is as follows:

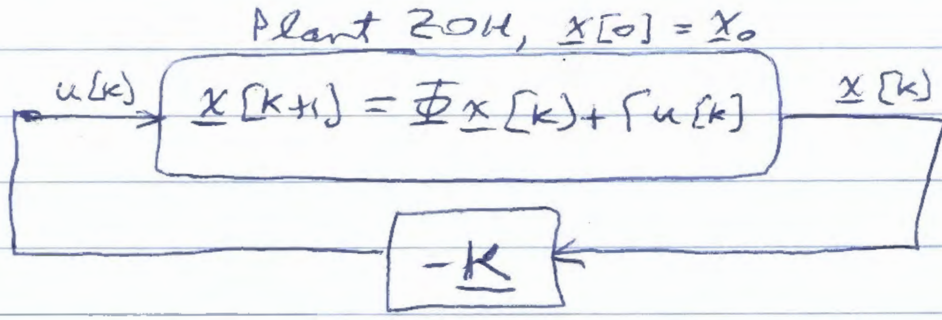


where $\underset{1\times n}{\underline{K}} = [k_1 \; k_2 \cdots k_n]$ is the *feedback gain vector*.

We will show how to calculate $\underline{K}$ so that $\underline{x}(t) \to \underline{0}$ in a specified amount of time; namely, in T_s seconds (settling time).

In order to calculate $\underline{K}$ we will replace the D/A - plant - A/D blocks with the ZOH plant equivalent model to get a discrete-time design model.

Design model for digital state-feedback regulator:



Note that $u[k] = -\underline{K}\,\underline{x}[k] = -[k_1 \cdots k_n] \begin{bmatrix} x_1[k] \\ \vdots \\ x_n[k] \end{bmatrix}$

$$= -k_1 x_1[k] \cdots - k_n x_n[k]$$

A model for the closed-loop regulated system is obtained by substituting the feedback connection equation $u[k] = -\underline{K}\,\underline{x}[k]$ into the plant equation:

$$\begin{aligned} \underline{x}[k+1] &= \Phi \underline{x}[k] + \underline{\Gamma}(-\underline{K}\,\underline{x}[k]) \\ &= \Phi \underline{x}[k] - \underline{\Gamma}\underline{K}\,\underline{x}[k] \\ &= (\Phi - \Gamma \underline{K})\,\underline{x}[k] \end{aligned}$$

Note: $\underset{n\times 1}{\Gamma}\ \underset{1\times n}{\underline{K}} = \underset{n\times n}{\boxed{}}$

$\underline{x}[k+1] = \underbrace{(\Phi - \Gamma \underline{K})}_{n\times n}\,\underline{x}[k]$ closed-loop system

is of the form $\underline{x}[k+1] = \text{"}A\text{"}\,\underline{x}[k]$

whose solution is $A^k \underline{x}_0$

(from Chapter 3)

Thus, the state of the regulated system is given by:

$$\underline{x}[k] = (\Phi - \Gamma K)^k \underline{x}_0$$

and $\underline{x}[k] \to \underline{0}$ if eigenvalues of $(\Phi - \Gamma \underline{K})$ are all inside the unit circle.

Given (Φ, Γ) we want to calculate a feedback gain vector so that $\text{eig}(\Phi - \Gamma K)$ are inside unit circle.

Procedure to design a digital state-feedback regulator:

Given a plant $\dot{\underline{x}} = A\underline{x} + Bu$ to regulate:

$\boxed{\text{Step 1}}$: Specify the desired settling time T_s seconds (will usually be given)

$\boxed{\text{Step 2}}$*: Choose the sampling interval, T. *

$\boxed{\text{Step 3}}$: >> [phi, gamma] = c2d(A,B,T)

(calculate ZOH model)

$\boxed{\text{Step 4}}$*: spoles = [* · · · *] pick n stable analog pole locations with T_s sec settling time

$\boxed{\text{Step 5}}$: >> zpoles = exp(T*spoles)

(use ZOH pole mapping formula)

$\boxed{\text{Step 6}}$: >> K = place(phi, gamma, zpoles)

The result will be that $\text{eig}(\Phi - \Gamma K) = \text{zpoles}$

* we will explain how to do this later.