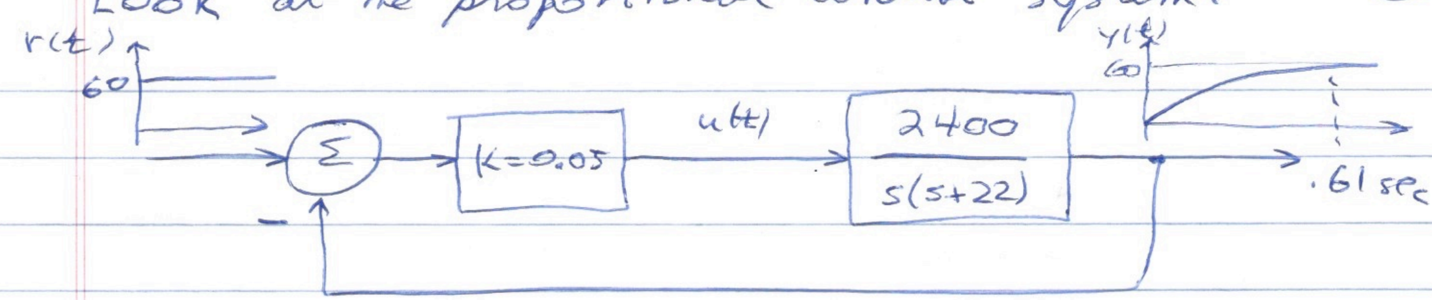
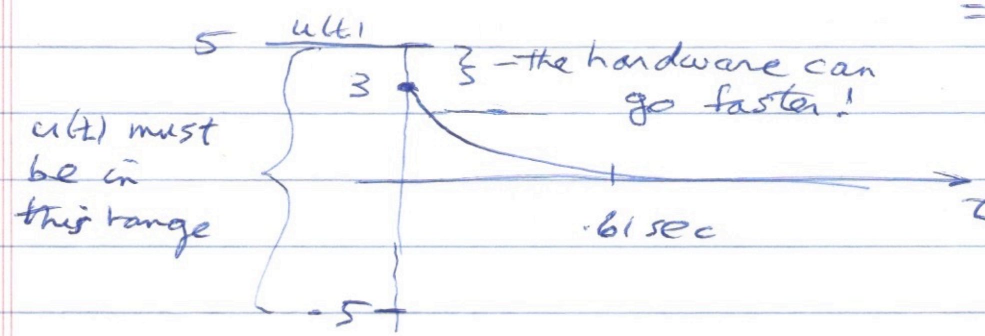
Look at the proportional control system:



What does u(t) look like? $u(0) = .05(r(0) - y(0))$
$= .05 \times 60 = 3$

Note: $u(0) = 60 \cdot K$

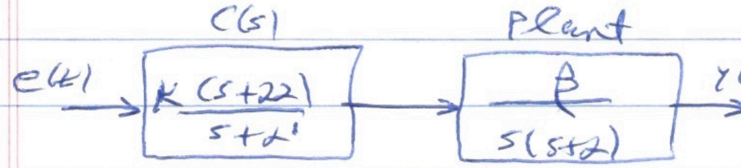


A Second Control System (Phase Lead Compensator)

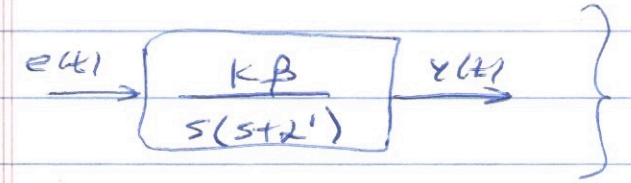
Replace the gain block with a compensator

$$C(s) = \frac{K(s+\alpha)}{(s+\alpha')} = \frac{K(s+22)}{(s+\alpha')}$$

The compensator zero at -22 cancels the plant pole at -22 and adds a pole at $-\alpha'$ (choose α' to be bigger than 22 to get a faster system). The forward path is then



which is equivalent to



Exactly like the forward path of the proportional control system with α replaced by α'.

For any choice of α', $K = \dfrac{\alpha'^2}{4\beta}$

will give a critically damped closed-loop system. Recall $u(0) = 60 \cdot K$. Set this equal to 5, the max possible $u(t)$ value:

$$\frac{60 \cdot \alpha'^2}{4\beta} = 5 \qquad \text{with } \beta = 2{,}400.$$

$$\alpha' = \sqrt{\frac{20 \cdot \beta}{60}} = \sqrt{\frac{2400}{3}} = 28.28$$

The double pole is now at $-\dfrac{\alpha'}{2} = -14.14$,

the gain is $K = \dfrac{\alpha'^2}{4\beta} = \dfrac{(28.28)^2}{4 \times 2400} = 0.083$

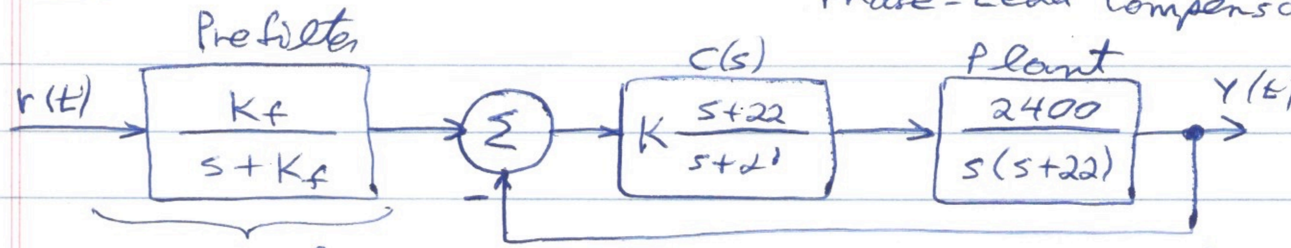
and the settling time is $T_s = \dfrac{4.62}{14.14} \times 1.45 = \boxed{0.475\text{ec}}$

faster than proportional control!

Can we make the system go faster without violating the constraints?

Yes, if we add a prefilter.

HW 1: Third Control System - Prefilter and Phase-Lead Compensator



one real pole at $-K_f$

if $K = \frac{(\alpha')^2}{4 \times 2400}$ there will be a double pole at $\frac{-\alpha'}{2}$.

Fact: a 3rd-order system can have a fast step response without overshoot if it has complex-conjugate poles.

"fudge factor"

Choose $K = \frac{f \cdot (\alpha')^2}{4 \times 2400}$ where $\boxed{1.25 \le f \le 1.5}$

bigger than gain for critical damping, ∴ complex conjugate poles at $-\frac{\alpha'}{2} \pm j\,x$

choose $K_f = \frac{\alpha'}{2}$ so prefilter pole is neither faster nor slower than the complex poles

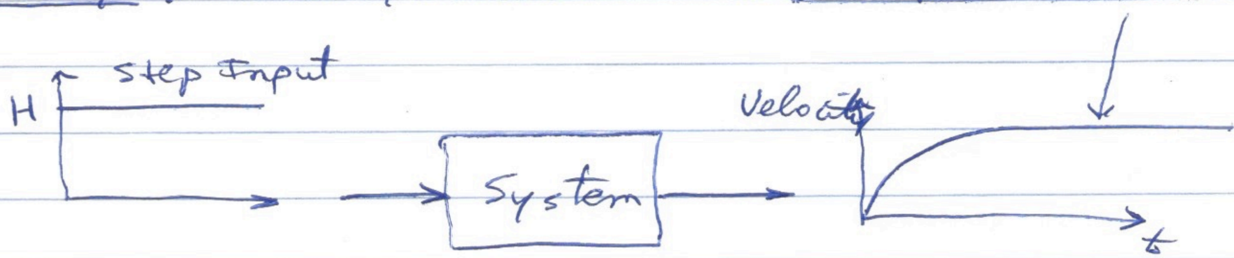
<u>Advice</u>

- Pick a value of f
- Pick a value of α'
- Let $K = \frac{f \cdot (\alpha')^2}{4 \times 2400}$
- Let $K_f = \frac{\alpha'}{2}$
- Simulate control system and check that $|u(t)| \le 5$ volts
- Adjust α'

trial and error over α'

Simple Model for a Positioning System.

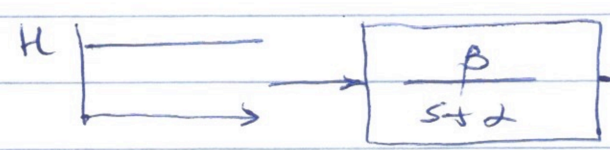
Consider a motor-driven positioning system. For a step input, what is the resulting velocity? Velocity increases to a steady-state value.



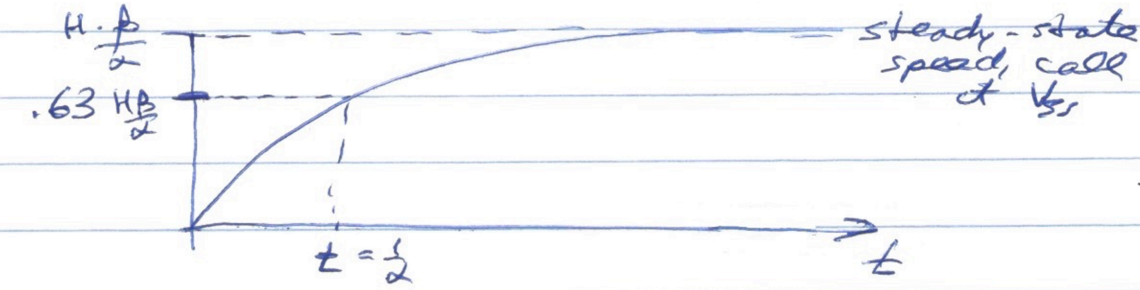
What is the corresponding position of the system? $\text{position} = \int_0^t \text{velocity}$

What transfer function represents this type of behavior?

A step response that rises to a steady-state value corresponds to the transfer function



$$v(t) = H \cdot \frac{\beta}{\alpha}\left(1 - e^{-\alpha t}\right)$$



$V_{ss} = \frac{H \cdot \beta}{\alpha}$. When $t = 1/\alpha$, the velocity is

$$v(1/\alpha) = \frac{H\beta}{\alpha}\underbrace{\left(1 - e^{-\alpha/\alpha}\right)}_{1-e^{-1} = .63}$$

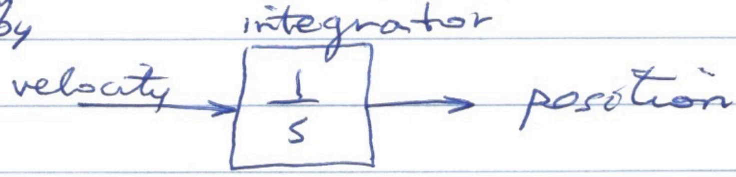
So α is the reciprocal of the time at which the step response reaches 63% of its steady-state value.

Recall Laplace Transform formulas:

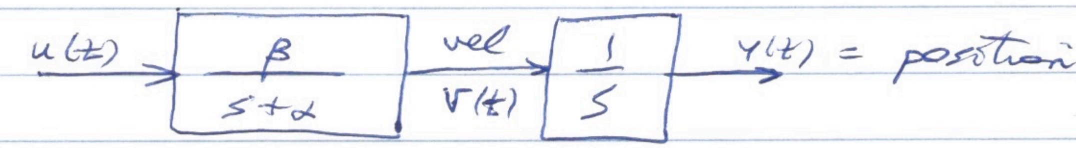
The "dot" means $\frac{d}{dt}$

function	Laplace Transform
$f(t)$	$F(s)$
$\dot{f}(t)$	$sF(s)$ (if $f(0)=0$)
$\int_0^t f(\tau)\,d\tau$	$\frac{1}{s}F(s)$

Thus, integrating velocity to get position is represented by



Combining the integrator with the previous velocity transfer function yields



or $\dfrac{\beta}{s(s+\alpha)}$ for the complete positioning system.

State-Space Models (See Section 3.3 in book)

Look at first transfer function (velocity):

$$\frac{V(s)}{U(s)} = \frac{\beta}{s+\alpha} \Longrightarrow (s+\alpha)V(s) = \beta\, U(s)$$

$$sV(s) + \alpha V(s) = \beta\, U(s)$$

take inverse Laplace transforms

$$\boxed{\dot{V}(t) + \alpha V(t) = \beta\, u(t)} \quad (1)$$