Data Mining & Machine Learning

CS37300
Purdue University

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Your First Classifier!

- Let's consider one of the simplest classifiers out there.
- Assume we have a training set $(x_1,y_1)\ldots(x_n,y_n)$
- Now we get a new instance x_{new} , how can we classify it?
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$$d(x_1, x_2) = 1 - \frac{x_1 \bigcap x_2}{x_1 \bigcup x_2} \qquad d(x_1, x_2) = \sqrt[2]{(x_1 - x_2)^2}$$

K Nearest Neighbors

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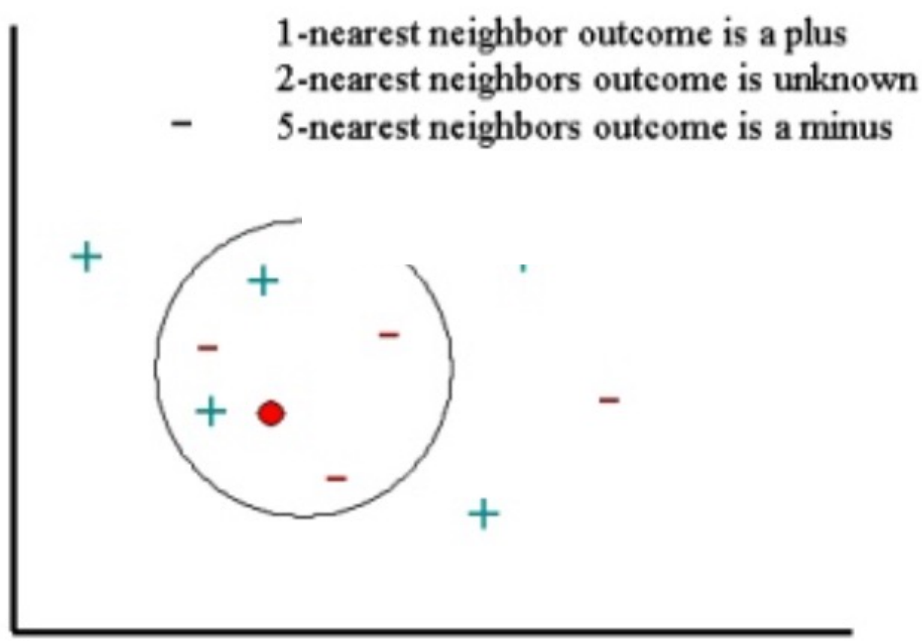

K Nearest Neighbors

- **Learning**: just storing the training examples
- **Prediction**:
  - Find the K training example closest to **x**
- **Predict a label:**
  - Classification: majority vote
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- KNN is a type of *instance based learning*
- This is called *lazy* learning, since most of the computation is done at prediction time

Let's analyze KNN

- ***What are the advantages and disadvantages of KNN?***
  - *What should we care about when answering this question?*
- ***Complexity***
  - ***Space** (how memory efficient is the algorithm?)*
    - *Why should we care?*
  - ***Time** (computational complexity)*
    - *Both at training time and at test (prediction) time*
- ***Expressivity***
  - *What kind of functions can we learn?*

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KNN needs to maintain all training examples!
-Datasets can be HUGE

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Analyzing K Nearest Neighbors

- We discussed the importance of finding a good model space
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  - Constrained (we can search effectively, using the data we have)
- Let's try to characterize the model space, by looking at the **decision boundary**

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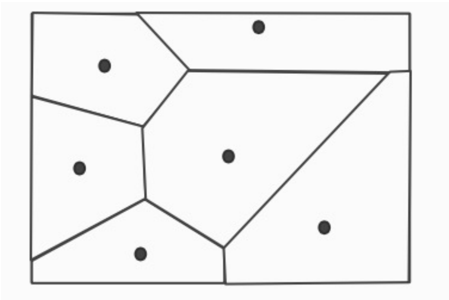

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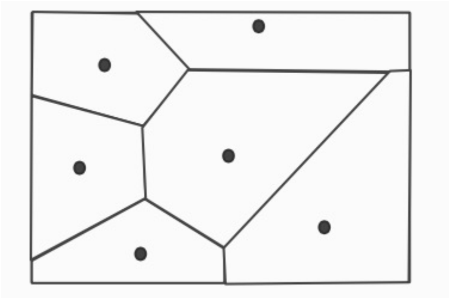
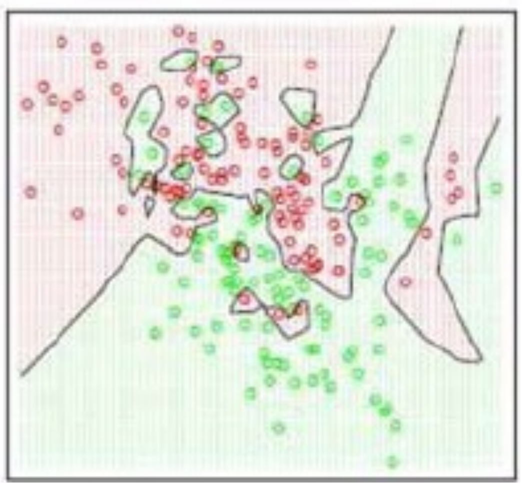
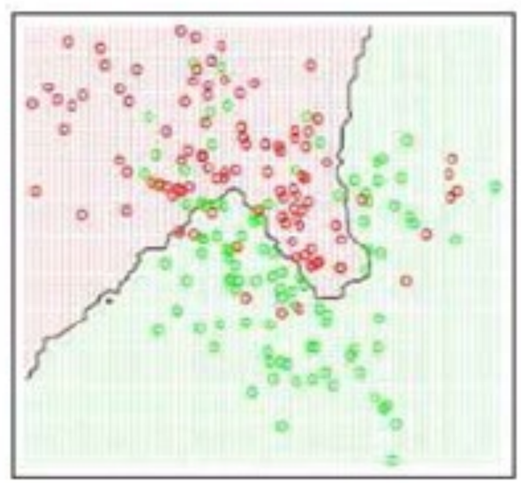


If we define the model space to be our choice of K
Does the complexity of the model space increase of decrease with K?

Analyzing K Nearest Neighbors

- Which model has a higher K value?





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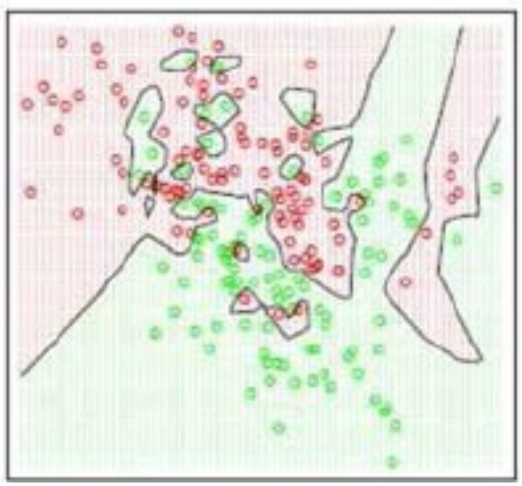
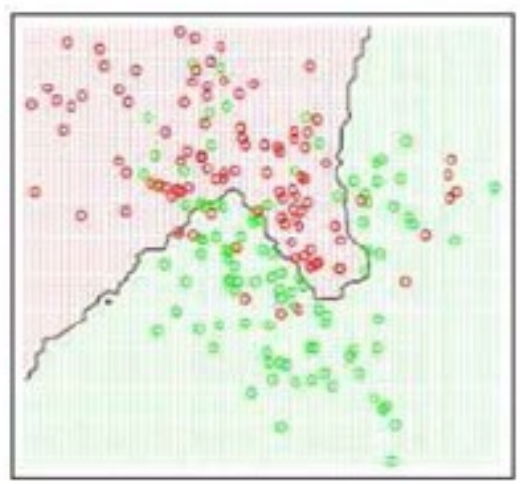
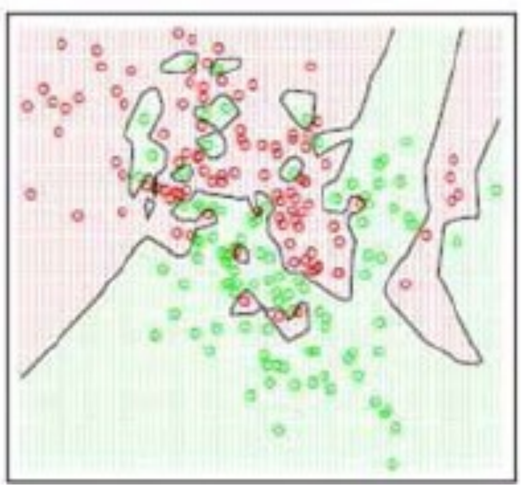
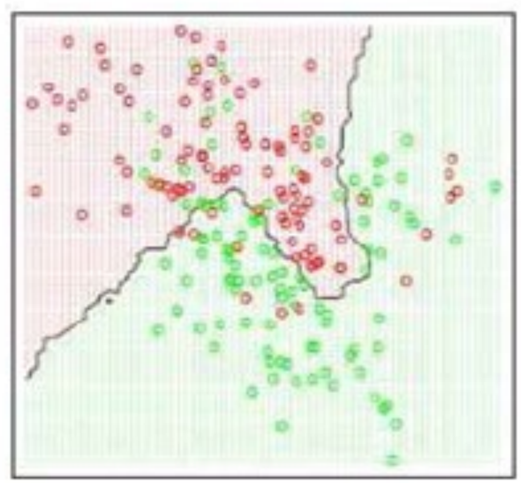




Analyzing K Nearest Neighbors

- Which model has a higher K value?
- Which model is more complex?
- Which model is more sensitive to noise?





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What will happen if we keep increasing K, up to the point that K=n ?

n = is the number of examples we have

How should we determine the value of K?

- Higher K values result in less complex functions (less expressive)
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- Option 1: Find the K that minimizes the training error.
  - <u>Training error</u>: after learning the classifier, what is the number of errors we get on the training data.
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- Option 2: Find K that minimizes the **validation error**.
  - Validation error: set aside some of the data (validation set). what is the number of errors we get on the validation data, after training the classifier.

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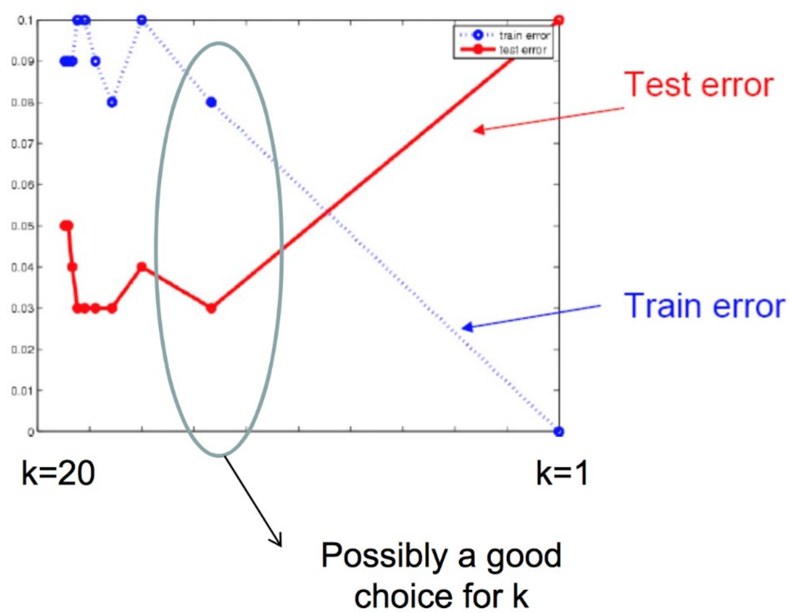

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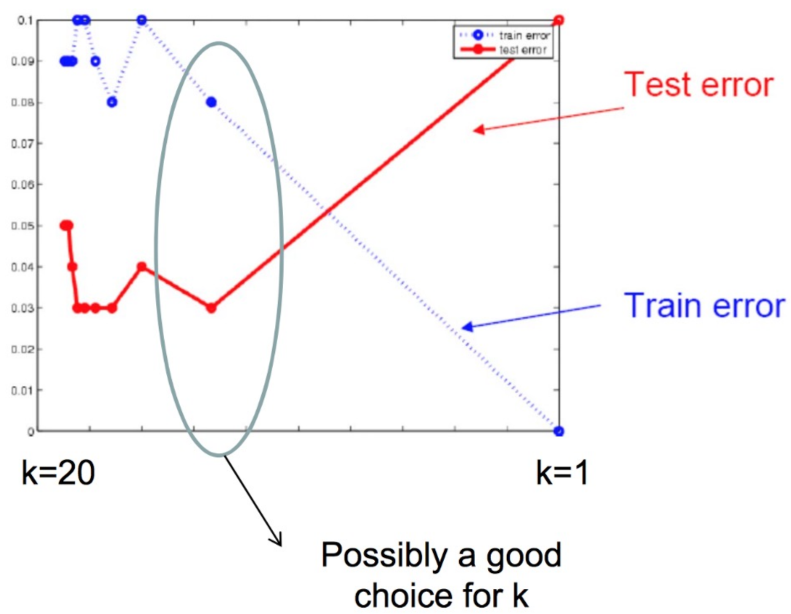

In general – using the training error to tune parameters will always result in a more complex hypothesis! **(why?)**

KNN Practical Consideration

- Finding the right representation is key
  - KNN is very sensitive to irrelevant attributes
- Choosing the right distance metric is important
  - Many options!
  - Popular choices:

– Euclidean distance

$$||\mathbf{x}_1 - \mathbf{x}_2||_2 = \sqrt{\sum_{i=1}^{n} (\mathbf{x}_{1,i} - \mathbf{x}_{2,i})^2}$$

– Manhattan distance

$$||\mathbf{x}_1 - \mathbf{x}_2||_1 = \sum_{i=1}^{n} |\mathbf{x}_{1,i} - \mathbf{x}_{2,i}|$$

– L_p-norm

- Euclidean = L_2
- Manhattan = L_1

$$||\mathbf{x}_1 - \mathbf{x}_2||_p = \left(\sum_{i=1}^{n} |\mathbf{x}_{1,i} - \mathbf{x}_{2,i}|^p\right)^{\frac{1}{p}}$$

Representation matters!

Img: Tensorflow W2Vec

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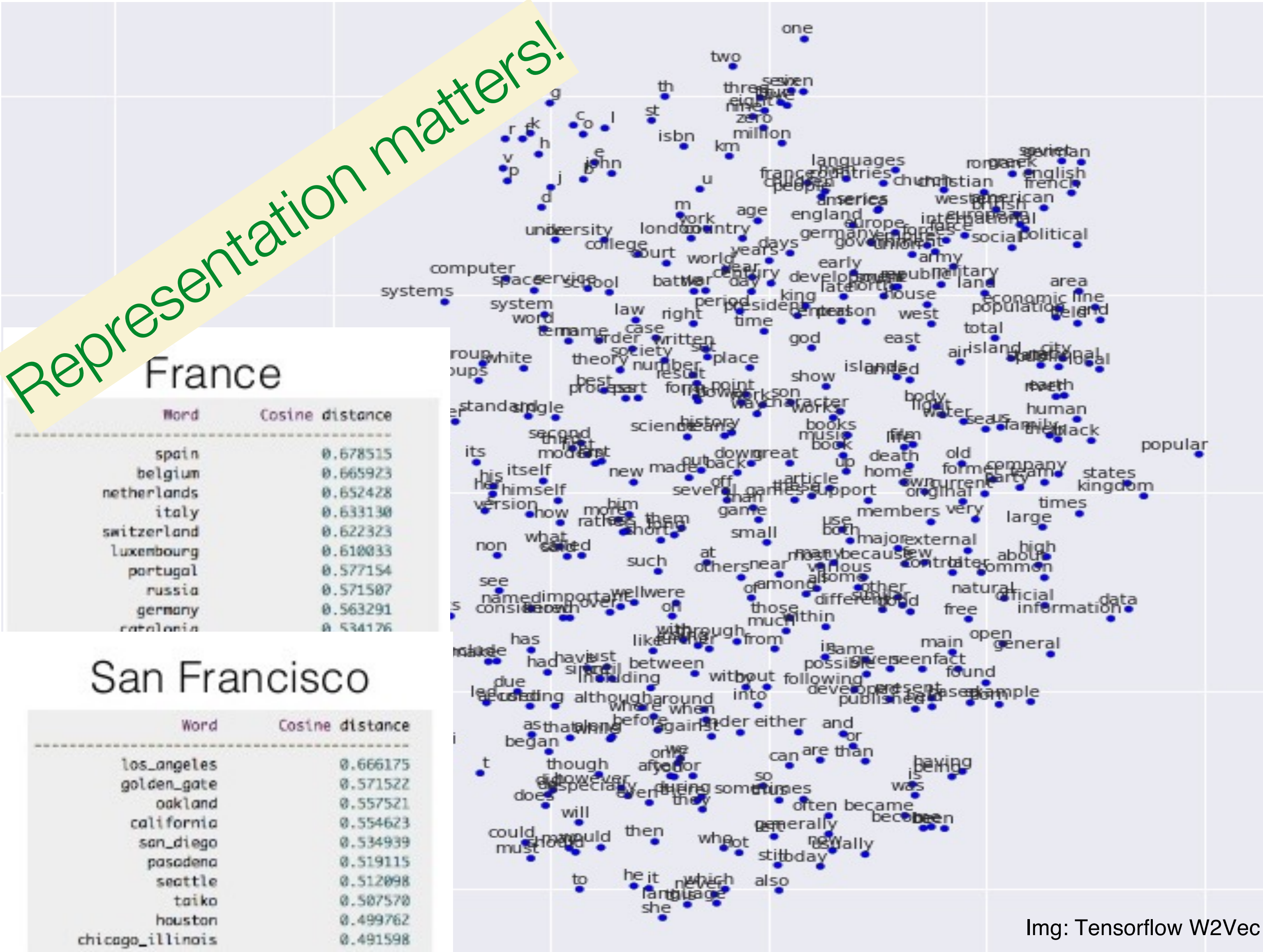

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France

Word	Cosine distance
spain	0.678515
belgium	0.665923
netherlands	0.652428
italy	0.633130
switzerland	0.622323
luxembourg	0.610033
portugal	0.577154
russia	0.571507
germany	0.563291
catalonia	0.534176

San Francisco

Word	Cosine distance
los_angeles	0.666175
golden_gate	0.571522
oakland	0.557521
california	0.554623
san_diego	0.534939
pasadena	0.519115
seattle	0.512098
taiko	0.507570
houston	0.499762
chicago_illinois	0.491598

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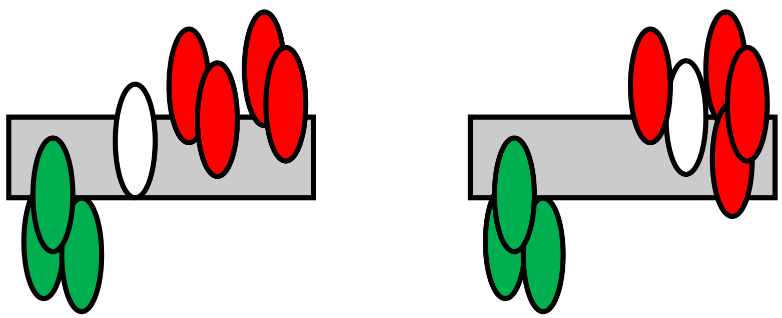

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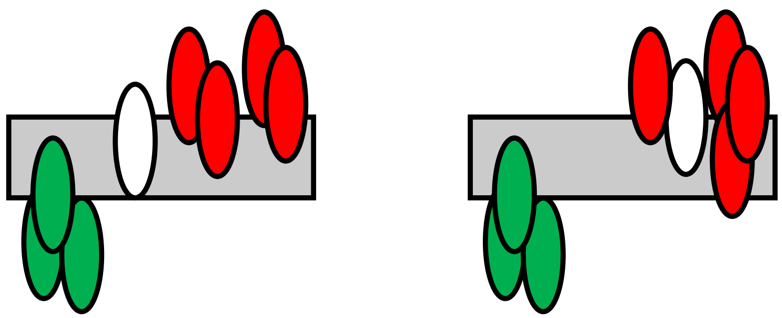

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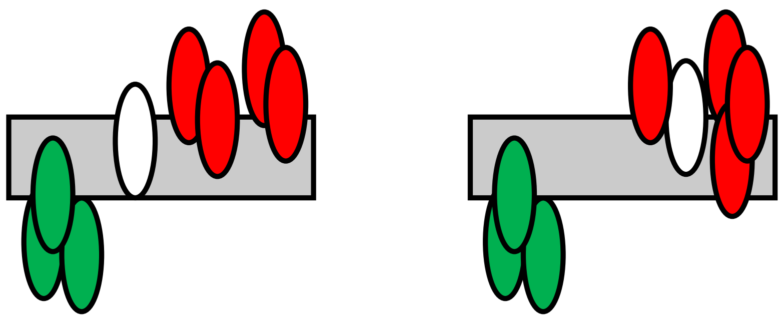

- What is the difference between the two scenarios?
- How can we reason about it?

Nearest neighbor

- Strengths:
  - Simple model, easy to implement
- Weaknesses:
  - Inefficient inference: time and space O(n)
    - (Inference time improvable with approximations, appropriate data structures)
  - Curse of dimensionality:
    - As number of features increase, you need an exponential increase in the size of the data to ensure that you have ”usable” nearby examples for any given data point

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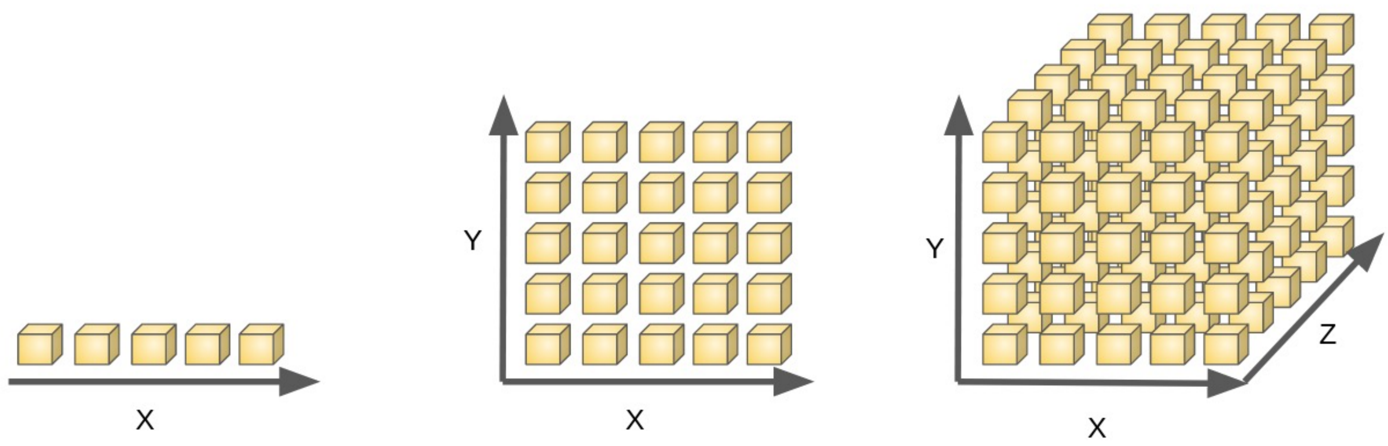

kNN Can Learn ANY Function (with enough data)

- Flexibility: Nearest Neighbor rules can learn **any concept** (with enough data)

- If n training examples are sampled independently from a distribution,
- if we choose $k_n \to \infty$ as $n \to \infty$, but not too fast so $\frac{k_n}{n} \to 0$ as $n \to \infty$, then
- kNN's classifier will converge to an optimal predictor

- **This is called "universal consistency"**