

$$S) \text{Inv} \wedge \Pi \wedge t = t_0 \wedge 2Ti \Rightarrow \underbrace{\text{wp}(i, y := i-1, y \cdot b, \text{Inv} \wedge t < t_0)}$$

we calculate this up

$$(\text{Inv} \wedge t < t_0) \wedge i \leq i-1, y \leq y \cdot b \wedge i-1 \in \mathbb{N}$$

The value of  $i-1$  has to be a natural number, as we have to guarantee that executing  $i, y := i-1, y \cdot b$  will terminate without error. Recall: the type of  $i$  is  $\mathbb{N}$ .

$$\underbrace{(\text{Q} \wedge y \cdot b^i = x^u \wedge i > 0)}_{\text{Inv}} \wedge i = t_0 \wedge 2Ti \Rightarrow ((\text{Q} \wedge y \cdot b^i = x^u \wedge i < t_0) \wedge i-1 \in \mathbb{N})$$

$$(\text{Q} \wedge \underbrace{y \cdot b \cdot b^{i-1} = x^u}_{y \cdot b^i = x^u} \wedge \underbrace{i-1 < t_0}_{i=t_0} \wedge i-1 \in \mathbb{N})$$

We know:  
 $\left. \begin{matrix} i: \mathbb{N} \\ 2Ti \end{matrix} \right\} i: \mathbb{N}^+$

$$A = (x: \mathbb{N}^+, y: \mathbb{N}^+, d: \mathbb{N}^+)$$

$$B = (x': \mathbb{N}^+, y': \mathbb{N}^+)$$

$$Q = (x = x' \wedge y = y')$$

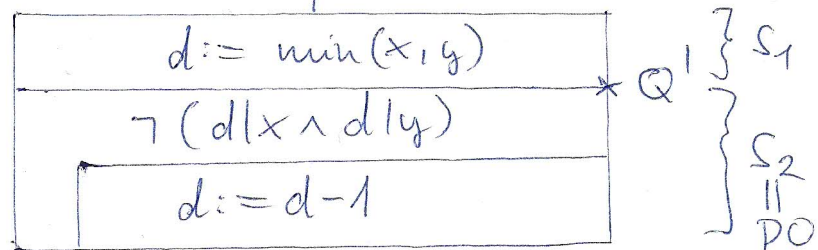
$$R = (Q \wedge d|x \wedge d|y \wedge \forall k \in [d+1.. \min(x, y)]: \neg (k|x \wedge k|y))$$

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$$Q' = (Q \wedge d = \min(x, y))$$

$$\text{Inv} = (Q \wedge \forall k \in [d+1.. \min(x, y)]: \neg (k|x \wedge k|y))$$

$$t := d$$



We want to prove:  $Q \Rightarrow \text{wp}(S, R)$ .  $S$  is a sequence.

$$\text{I) } Q \Rightarrow \text{wp}(S_1, Q')$$

$$\text{II) } Q \Rightarrow \text{wp}(\text{DO}, R)$$

$$\text{I) } Q \Rightarrow \text{wp}(d := \min(x, y), Q')$$

$$Q' \wedge d \leq \min(x, y)$$

$$(Q \wedge d = \min(x, y)) \wedge d \leq \min(x, y)$$

$$(Q \wedge \min(x, y) = \min(x, y))$$

$$Q \Rightarrow Q' \checkmark$$