2) $\int_1^2 x \cdot \ln(x-1)\, dx$

$(f\cdot g)' = f'g \cdot g'\cdot f \Rightarrow \int f'g = fg - \int fg'$

$f'(x) = x \quad , \quad g(x) = \ln(x-1)$

$f(x) = \frac{x^2}{2} \quad , \quad g'(x) = \frac{1}{x-1}$

$\Rightarrow \int_1^2 x \ln(x-1)\, dx = \left[\frac{x^2}{2} \cdot \ln(x-1)\right]_1^2 - \int_1^2 \frac{x^2}{2} \cdot \frac{1}{x-1}$

$\Rightarrow \frac{1}{2}\int_1^2 \frac{x^2}{x-1} = \frac{1}{2}\int_1^2 \frac{x^2-x+x}{x-1} = \frac{1}{2}\left(\int_1^2 x + \int_1^2 \frac{x}{x-1}\right)$

$= \frac{1}{2}\left(\int_1^2 x + \int_1^2 1 + \int_1^2 \frac{1}{x-1}\right) = \frac{1}{2}\left[\frac{x^2}{2}\right]_1^2 + \frac{1}{2}[x]_1^2 + \frac{1}{2}\left[\ln|x-1|\right]_1^2$

$= \frac{1}{2}\cdot\left(\underbrace{2 - \frac{1}{2}}_{3/2} + \underbrace{4 - 1}_{3} + \underbrace{\ln 1}_{0} - \lim_{a\to 1^+}(\ln|a-1|)\right) = \frac{9}{4} - \frac{1}{2}\lim_{a\to 1^+}(\ln|a-1|)$

$\Rightarrow \left[\frac{x^2}{2} \cdot \ln(x-1)\right]_1^2 = \left(2\cdot 0 - \frac{1}{2}\lim_{a\to 1^+} \ln(a-1)\right)$

All is $\Rightarrow -\frac{9}{4} + \frac{1}{2}\lim_{a\to 1^+} \ln|a-1| - \frac{1}{2}\lim_{a\to 1^+} \ln|a-1| = \underline{\underline{\frac{-9}{4}}}$