Solutions PS_05

Q1:

$\sqrt{-16} = 4i$

$\sqrt{-25} = 5i$

$(2i)^2 = 4i^2 = -4$

$2i + 5i = 7i$

$\frac{4i}{2i} = 2$

Q2. $z = -2 + 7i$

$Re(z) = -2$; $Im(z) = 7$; $-z = 2 - 7i$; $\bar{z} = -2 - 7i$; $|z| = \sqrt{(-2)^2 + 7^2} = \sqrt{53}$

Q3.

multiply by the conjugate of the denom

$$\frac{4+3i}{(2-i)^2} = \frac{4+3i}{(2-i)(2-i)} = \frac{4+3i}{4-2i-2i+i^2} = \frac{4+3i}{3-4i} = \frac{4+3i}{3-4i} \cdot \frac{3+4i}{3+4i} =$$

$$= \frac{(4+3i)(3+4i)}{3^2+4^2} = \frac{12+16i+9i+12i^2}{25} = \frac{25i}{25} = i$$

Q7

a) $z = 1+i$; $r = |z| = \sqrt{1^2+1^2} = \sqrt{2}$

Firstly, you need to find r and φ

$$\left.\begin{array}{l} \cos\varphi = \frac{a}{r} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \\ \sin\varphi = \frac{b}{r} = \frac{\sqrt{2}}{2} \end{array}\right\} \varphi = 45° = \frac{\pi}{4}$$

$$\Rightarrow z = |z| \cdot (\cos\varphi + i \cdot \sin\varphi) = \boxed{\sqrt{2} \cdot \left(\cos\frac{\pi}{4} + i \cdot \sin\frac{\pi}{4}\right)}$$

b.) $z = -\sqrt{3} + i$; $r = |z| = \sqrt{(-\sqrt{3})^2 + 1^2} = \sqrt{4} = 2$

$$\left.\begin{array}{l} \cos\varphi = \frac{a}{r} = -\frac{\sqrt{3}}{2} \\ \sin\varphi = \frac{b}{r} = \frac{1}{2} \end{array}\right\}$$

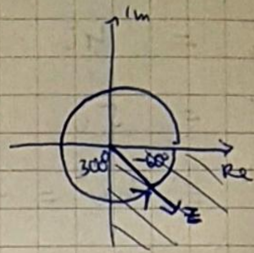
re(z) is negative; Im(z) is positive $\Rightarrow$

$\Rightarrow \varphi = 150° = \frac{5\pi}{6}$

$$\Rightarrow \boxed{z = 2 \cdot \left(\cos\frac{5\pi}{6} + i \cdot \sin\frac{5\pi}{6}\right)}$$

c.) $z = \frac{9}{2} - \frac{9\sqrt{3}}{2} i$; $|z| = r = \sqrt{\left(\frac{9}{2}\right)^2 + \left(-\frac{9\sqrt{3}}{2}\right)^2} = \sqrt{\frac{81}{4} + 3 \cdot \frac{81}{4}} = 9$

$$\left.\begin{aligned} \cos\varphi &= \frac{a}{r} = \frac{\frac{9}{2}}{9} = \frac{1}{2} \\ \sin\varphi &= \frac{b}{r} = \frac{-\frac{9\sqrt{3}}{2}}{9} = -\frac{\sqrt{3}}{2} \end{aligned}\right\} \begin{aligned} &Re(z) > 0 ; \\ &Im(z) < 0 ; \end{aligned} \Rightarrow$$



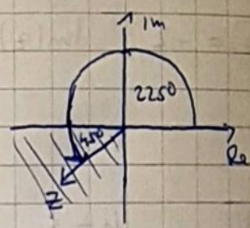
$\Rightarrow \varphi = 300° = \frac{5\pi}{3}$

$\Rightarrow \boxed{z = 9\left(\cos\left(\frac{5\pi}{3}\right) + i \cdot \sin\left(\frac{5\pi}{3}\right)\right)}$

d.) $z = -\frac{\sqrt{14}}{2} - \frac{\sqrt{14}}{2} i$; $|z| = \sqrt{\left(-\frac{\sqrt{14}}{2}\right)^2 + \left(-\frac{\sqrt{14}}{2}\right)^2} = \sqrt{\frac{14}{4} + \frac{14}{4}} = \sqrt{7}$

$\cos\varphi = \frac{a}{r} = \frac{-\frac{\sqrt{14}}{2}}{\sqrt{7}} = \frac{-\frac{1}{2} \cdot \sqrt{2} \cdot \cancel{\sqrt{7}}}{\cancel{\sqrt{7}}} = -\frac{\sqrt{2}}{2}$

$\sin\varphi = \frac{b}{r} =$ — " — $= -\frac{\sqrt{2}}{2}$



$\Rightarrow \varphi = 225° = \frac{5\pi}{4}$

$\Rightarrow z = \sqrt{7} \cdot \left(\cos\left(\frac{5\pi}{4}\right) + i \cdot \sin\left(\frac{5\pi}{4}\right)\right)$

Q8) a.) $\overset{z}{\overset{\shortparallel}{\left(\frac{9}{2} - \frac{9\sqrt{3}}{2} i\right)}} \overset{w}{\overset{\shortparallel}{\left(-\frac{\sqrt{14}}{2} - \frac{\sqrt{14}}{2} i\right)}}$

from Q7/c this is $9\left(\cos\left(\frac{5\pi}{3}\right) + i \cdot \sin\frac{5\pi}{3}\right)$

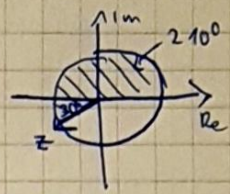
from Q7/d this is $\sqrt{7}\left(\cos\left(\frac{5\pi}{4}\right) + i \cdot \sin\left(\frac{5\pi}{4}\right)\right)$

$\Rightarrow z \cdot w = 9\sqrt{7}\left(\cos\left(\frac{5\pi}{3} + \frac{5\pi}{4}\right) + i \cdot \sin\left(\frac{5\pi}{3} + \frac{5\pi}{4}\right)\right) =$

$= 9\sqrt{7}\left(\cos\left(\frac{35}{12}\pi\right) + i \cdot \sin\left(\frac{35}{12}\pi\right)\right) = \boxed{9\sqrt{7}\left(\cos\left(\frac{11}{12}\pi\right) + i \cdot \sin\left(\frac{11}{12}\pi\right)\right)}$

c.) $\dfrac{-\frac{3\sqrt{3}}{2} - \frac{3}{2}i}{\frac{\sqrt{3}}{3} + \frac{1}{3}i} = \dfrac{z}{w}$

- $z = -\dfrac{3\sqrt{3}}{2} - \dfrac{3}{2}i \Rightarrow |z| = \sqrt{\left(-\dfrac{3\sqrt{3}}{2}\right)^2 + \left(-\dfrac{3}{2}\right)^2} = \sqrt{3 \cdot \dfrac{9}{4} + \dfrac{9}{4}} = 3$



$\cos\alpha = \dfrac{Re(z)}{|z|} = \dfrac{-\frac{3\sqrt{3}}{2}}{3} = -\dfrac{\sqrt{3}}{2}$

$\sin\alpha = \dfrac{Im(z)}{|z|} = \dfrac{-\frac{3}{2}}{3} = -\dfrac{1}{2}$

$\varphi = 210° = \dfrac{7\pi}{6}$; $z = 3\left(\cos\left(\dfrac{7\pi}{6}\right) + i \cdot \sin\left(\dfrac{7\pi}{6}\right)\right)$

- $w = \dfrac{\sqrt{3}}{3} + \dfrac{1}{3}i$, $|w| = \sqrt{\left(\dfrac{\sqrt{3}}{3}\right)^2 + \left(\dfrac{1}{3}\right)^2} = \sqrt{\dfrac{4}{9}} = \dfrac{2}{3}$

$$\left.\begin{aligned} \cos(\beta) &= \frac{Re(w)}{|w|} = \frac{\frac{\sqrt{3}}{3}}{\frac{2}{3}} = \frac{\sqrt{3}}{2} \\ \sin(\beta) &= \frac{Im(w)}{|w|} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2} \end{aligned}\right\} \varphi = 30° = \frac{\pi}{6}$$

$w = \dfrac{2}{3} \cdot \left(\cos\left(\dfrac{\pi}{6}\right) + i \cdot \sin\left(\dfrac{\pi}{6}\right)\right)$

- $\dfrac{z}{w} = \dfrac{|z|}{|w|} \cdot \left(\cos(\alpha - \beta) + i \cdot \sin(\alpha - \beta)\right) = \dfrac{3}{\frac{2}{3}}\left(\cos\left(\dfrac{7\pi}{6} - \dfrac{\pi}{6}\right) + i \cdot \sin\left(\dfrac{7\pi}{6} - \dfrac{\pi}{6}\right)\right) =$

$\Rightarrow \boxed{\dfrac{z}{w} = \dfrac{9}{2} \cdot \left(\cos(\pi) + i \cdot \sin(\pi)\right)}$