Basic notations

1 Sets

Informally, a set is a collection of objects, which are called the elements of the set. The elements of a set do not certainly have a common property, except that they are elements of the same set. For example, here is a set:

$$\{Budapest, \alpha, \emptyset, \sqrt[3]{2}\}$$

One way to define a set is by listing the elements of the set inside curly-brackets (for example, the set of logical values:)

$$\mathbb{L} ::= \{igaz, hamis\}$$

or by providing a condition (for example, the set of integer numbers that can be divided by 5 and are not greater than 100:)

$$\{\, x \in \mathbb{Z} \mid x \leqslant 100 \wedge 5|x \,\}$$

Definition: *The set $[a..b] ::= \{\, x \in \mathbb{Z} \mid a \leqslant x \wedge x \leqslant b \,\}$ is called interval (where a and b are integer numbers). It is empty, if $b < a$.*

Notation: *The cardinality of a set H is denoted by $|H|$. The fact, that set H is finite, can be expressed in this way: $|H| < \infty$.*

Special sets:

$\mathbb{N}$	– the set of all natural numbers (including 0)
$\mathbb{N}^+$	– the set of all positive integers
$\mathbb{Z}$	– the set of all integers
$\mathbb{L}$	– the set of logical values
$\emptyset$	– the empty set

2 Sequences

Notation: *Let H^{**} denote the set of all finite and infinite sequences of the elements of set H. H^∞ includes the infinite sequences; H^* contains the finite ones. So, $H^{**} = H^* \cup H^\infty$ and $H^* \cap H^\infty = \emptyset$. The length of the sequence $\alpha \in H^{**}$ is $|\alpha|$, in case of infinite sequence this value is denoted by ∞.*

3 Relations

Definition: *Let A and B be arbitrary not empty sets. The Cartesian product $A \times B$ is the set of all ordered pairs (a, b) where $a \in A$ and $b \in B$:*

$$A \times B ::= \{ (a, b) \mid a \in A \wedge b \in B \}$$

Definition: *Let A and B be arbitrary not empty sets. Any subset R (including the empty set) of $A \times B$ is called a relation.*

A relation can be considered as a mapping from the set A to B. In case $(x, y) \in R$, then we say that R assigns (or associates) y to x.

Definition: *Let A and B be arbitrary not empty sets and let $R \subseteq A \times B$ be any relation. The domain of relation R:*

$$\mathcal{D}_R ::= \{ a \in A | \exists b \in B : (a, b) \in R \}$$

the range of relation R:

$$\mathcal{R}_R ::= \{ b \in B | \exists a \in A : (a, b) \in R \}$$

the image of a (where $a \in A$) by R:

$$R(a) ::= \{ b \in B | (a, b) \in R \}$$

Definition: *Let A and B arbitrary not empty sets and let $R \subseteq A \times B$ be any relation. We say that R is deterministic, if*

$$\forall a \in A : |R(a)| \leqslant 1$$

Deterministic relations are called functions. The function $R \subseteq A \times B$, as a special relation, has a particular notation: $R \in A \to B$.

Notation: *In case for the function $f \in A \to B$ it also holds, that its domain equals to set A, then the following notation is used: $f : A \to B$. Notice that in this case the following holds:*

$$\forall a \in A : |f(a)| = 1$$

Remark: *Such $f \in A \to B$ functions, which does not associate exactly one single element to every element of A (in other words, their domain is not equal to A), are called partial functions.*

Remark: *Let $f : A \to B$ be an arbitrary function (now we know that f is a special relation that assigns a single element to every element of set A. Let $a \in A$ and $f(a) = \{b\}$ where $b \in B$ denotes the only element assigned to a. In this case, for the sake of simplicity, the image $f(a)$ can be written as the value b instead of the set $\{b\}$.*

4 Statespace

Problems are about data, programs also deal with data. The typevalue-set of a data is a set containing all possible values of the given data.

Definition: *Let $A_1, \ldots, A_n$ (where $n \in \mathbb{N}^+$) be typevalue-sets and let $v_1, \ldots, v_n$ be unique labels (more precisely: variables) identifying the typevalue-sets. The set $\{v_1 : a_1, \ldots, v_n : a_n\}$ (where $\forall i \in [1..n] : a_i \in A_i$) constructed from the variables and sets mentioned beforehand, is called* state.

In case a statespace contains only one component, the notation a_1 can be used instead of $\{v_1 : a_1\}$ to denote a state.

Definition: *Let $A_1, \ldots, A_n$ (where $n \in \mathbb{N}^+$) be typevalue-sets and let $v_1, \ldots, v_n$ be unique labels (more precisely: variables) identifying the typevalue-sets. The set of all possible $\{v_1 : a_1, \ldots, v_n : a_n\}$ variables (where $\forall i \in [1..n] : a_i \in A_i$) constructed from the variables and sets mentioned beforehand, is called statespace and denoted by $(v_1 : A_1, \ldots, v_n : A_n)$.*

$$(v_1 : A_1, \ldots, v_n : A_n) ::= \Big\{ \{v_1 : a_1, \ldots, v_n : a_n\} \,|\forall i \in [1..n] : a_i \in A_i \Big\}$$

Definition: *The labels (variables) of the statespace $A = (v_1 : A_1, \ldots, v_n : A_n)$ are considered as $v_i \colon A \to A_i$ functions, where $v_i(a) = a_i$ for any $a = \{v_1 : a_1, \ldots, v_n : a_n\}$ state.*

Definition: *Let $A = (v_1 : A_1, \ldots, v_n : A_n)$ and $B = (u_1 : B_1, \ldots, u_n : B_m)$ any statespaces ($n, m \in \mathbb{N}^+$ and $m \leqslant n$). We say that statespace B is a subspace of statespace A ($B \leq A$), if there exist an injective function $\varphi\colon [1..m] \to [1..n]$, where $\forall i \in [1..m] : B_i = A_{\varphi(i)}$.*

5 Problem

Definition: *Let A by an arbitrary statespace. Any $F \subseteq A \times A$ relation is called problem.*

The definition of problem is based on our approach, where we consider the problem as a mapping from the statespace to the same statespace. In this relation, for every element of the statespace we determine the goal states we intend to get starting from the given element of the statespace.

6 Program

Definition: *Let A be the so-called base state space and $\bar{A}$ be the set of all states which belong to the state spaces B whose subspace is A, i.e. $\bar{A} = \bigcup\limits_{A \leq B} B$. $\bar{A}$ does not contain the state **fail**. The relation $S \subseteq A \times (\bar{A} \cup \{\textbf{fail}\})^{**}$ is called **program** over A, if*

1. *$\mathcal{D}_S = A$*
2. *$\forall a \in A$: $\forall \alpha \in S(a) : |\alpha| \geq 1$ and $\alpha_1 = a$*
3. *$\forall \alpha \in \mathcal{R}_S : (\forall i \in \mathbb{N}^+ : i < |\alpha| \to \alpha_i \neq fail)$*
4. *$\forall \alpha \in \mathcal{R}_S : (|\alpha| < \infty \to \alpha_{|\alpha|} \in A \cup \{fail\})$*