5) $f(x) = \frac{1}{\sqrt{(1+2x)^3}}$, $(x > -\frac{1}{2})$ $I = (0, \frac{1}{4})$

$T_2(x) = f(a) + f'(x)(x-a) + \frac{f''(x)}{2!}(x-a)^2$

f centered at 0, so $a=0$

$T_2(x) = f(0) + f'(0).x + \frac{f''(0)}{2}.x^2$

$\Rightarrow f(0) = \frac{1}{1} = 1$ $f(x) = (1+2x)^{-\frac{3}{2}} \Rightarrow f'(x) = (1+2x)^{-\frac{5}{2}}.-\frac{3}{2}.2$

$= -3(1+2x)^{-\frac{5}{2}} \Rightarrow f'(0) = -3$

$f''(x) = -3(1+2x)^{-\frac{7}{2}}.-\frac{5}{2}.2 = 15(1+2x)^{-\frac{7}{2}} \Rightarrow f''(0) = 15$

So $T_2(x) = 1 + (-3)x + \frac{15}{2}x^2 = \underline{\frac{15x^2}{2} - 3x + 1}$ $(x \in \mathbb{R})$

Error of Estimation

$F(x) \approx T_2(x)$, $x \in (0, \frac{1}{4})$

$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3$

The error $= |f(x) - T_2(x)| = \left|\frac{f'''(c)x^3}{3!}\right| = \frac{1}{6}(-105).(1+2c)^{-\frac{9}{2}}.x^3$

with c between $-\frac{1}{2}$ and x

$f'''(x) = 15.(1+2x)^{-\frac{9}{2}}.-\frac{7}{2}.2 = -105(1+2x)^{-\frac{9}{2}}$