Solutions 4

1.)

a.) $A = \{1,2,3,4,5\}$; $B = \{10,11,12,13,14\}$

$f \subseteq A \times B$; $f = \{(1,11); (2,11); (4,12); (5,10)\}$

(rem: $\forall x, y, y' : (x,y) \in f \wedge (x,y') \in f \Rightarrow y = y'$; $f \subseteq X \times Y$ is a function)

f: ✓

i: ✗ ; $(1,11) \in f \wedge (2,11) \in f$

s: ✗ ; $13 \notin \mathrm{Rng}(f) \wedge 14 \notin \mathrm{rng}(f)$

$\Rightarrow$ b: ✗

$\mathrm{rng}(f) = \{10,11,12\}$

$\mathrm{dmn}(f) = \{1,2,4,5\}$

b.) $A = \{1,2,3,4\}$; $B = \{a,b,c,d,e,f\}$

$f \subseteq A \times B$, $f = \{(1,a); (2,c), (3,e); (3,f), (4,a)\}$

f: ✗ ; $(3,e) \in f \wedge (3,f) \in f$

$\Rightarrow$ i; s, b : ✗

c.) $A = \{1,2,3,4,5\}$; $B = \{a,b,c,d,e,f\}$

$f \subseteq A \times B$; $f = \{(1,a); (4,e); (5,d)\}$

f: ✓

i: ✓ ; since $\forall x, x', y : ((x,y) \in f \wedge (x',y) \in f) \Rightarrow x = x'$

s: ✗ ; $\{b, c, f\} \not\subseteq \mathrm{rng}(f)$

~~dmn(f)~~ b: ✗

$\mathrm{dmn}(f) = \{1,4,5\}$

$\mathrm{rng}(f) = \{a,d,e\}$

d.) $A=\{1,2,3\}$; $B=\{1,3,5\}$; $f\subseteq A\times B$; $f=\{(1,1);(2,5);(3,5)\}$

$dmn(f)=\{1,2,3\}$

$rng(f)=\{1,5\}$

f: ✓ ; (def ✓)

i: ✗ ; $(2,5)\in f \wedge (3,5)\in f$

s: ✗ ; $3\notin rng(f)$

$\Rightarrow b$: ✗

4.) a.) $f\subseteq \mathbb{N}\times\mathbb{N}$; $f=\{(x,y)\subseteq \mathbb{N}\times\mathbb{N} \mid x|y\} = f=\{(x,y)\subseteq \mathbb{N}\times\mathbb{N} \mid y=k\cdot x;\ k\in\mathbb{Z}\}$

$\boxed{f: ✗}$; pl. $(2,4)\in f \wedge (2,6)\in f \wedge 4\neq 6$

b.) $f=\{(2,2);(3,3);(5,5);\ldots\}$ $\Rightarrow$ $\boxed{f: ✓}$ (def ✓) (bij ✓)

c.) $f\subseteq\{0,3,5\}\times\{1,2,5\}$; $x f y \Leftrightarrow xy=0$

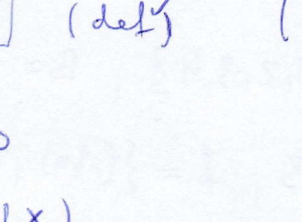
$f=\{(0,1);(0,2);(0,5)\}$ $\Rightarrow$ $\boxed{f: ✗}$ (def ✗)

$[(0,1)\in f;\ (0,2)\in f;\ 1\neq 2]$

d.) $f\subseteq\{1,2,5\}\times\{0,3,5\}$; $x f y \Leftrightarrow xy=0$

$f=\{(1,0);(3,0);(5,0)\}$ $\Rightarrow$ $\boxed{f: ✓}$

e.) $\boxed{f: ✗}$; $(112, 112)\in f \wedge (112, 121)\in f \wedge (112\neq 121)$



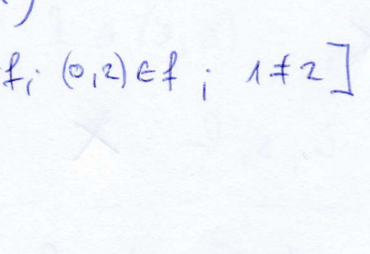
f.) $f=\{(x,y)\subseteq \mathbb{N}\times\mathbb{N} \mid 2x=y\}$

f: ✓ (def ✓)

g.)

$f=\{(x,y)\subseteq\mathbb{Z}\times\mathbb{Z} \mid x^2=y^2\}$

f: ✗ ; pl. $(1,1)\in f \wedge (1,-1)\in f \wedge 1\neq -1$



2.)

$R = \{(a,b) \subseteq A \times B \mid T_a = b\}$

$A = \{$ egy. Δ, ahol ~~[illegible]~~ a magasság $= m$ rögzített $\}$

$B = \{$ poz. valós számok

$$T_a = \frac{alap \cdot m}{2} = b \in B$$

függvény? -> igen ✓, ugyanis $\forall a \in A$-t meghatározza ~~[illegible]~~ az alapjának a nagysága (mivel m rögzített)

-> ha egy háromszög megfeleltethető az alapjával, akkor a területével is -> különböző alapokhoz különböző területek tartoznak => R fv. ✓

injektív? -> igen ✓, mivel a magasság rögzített, így minden alaphoz különböző terület tartozik

szürjektív? -> igen ✓; mivel ~~[illegible]~~ $rng(R) = B = \{$pozitív valós számok$\}$

(legyen $b \in B$. ekkor $\exists a$, hogy $T_a = b$, mert

$\hookrightarrow \frac{alap \cdot m}{2} = b$

$alap = \frac{2 \cdot b}{m}$ <- az a Δ, aminek az alapja ekkora)

bijektív? -> sz + i = ✓ => ✓

5.)

a.) $f_1 = \{ (x,y) \in \mathbb{R} \times \mathbb{R} \mid 7x = y^2 \} \subseteq \mathbb{R} \times \mathbb{R}$

f: ✗ ; $(1, \sqrt{7}) \in f_1 \wedge (1, -\sqrt{7}) \in f_1 \wedge \sqrt{7} \neq -\sqrt{7}$

b.) HW

c.) HW

d.) $f_4 = \{(x,y) \in \mathbb{R} \times \mathbb{R}_0^+ \mid y = |x|\} \subseteq \mathbb{R} \times \mathbb{R}_0^+$

f: ✓ $\forall x, y, y' : |x| = y \wedge |x| = y' \Rightarrow y = y'$

$$|x| = \begin{cases} x ; & \text{if } x \geq 0 \\ -x ; & x < 0 \end{cases}$$

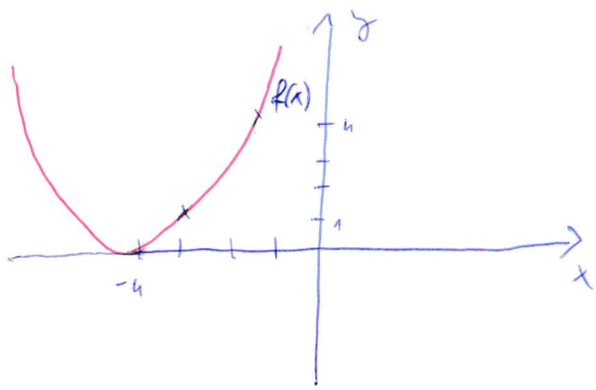
i: ✗ ($(1,1) \wedge (-1,1) \in f_4$)

s: ✓

↳ b: ✗

e.) $f_5 = \{(x,y) \in \mathbb{R} \times \mathbb{R} \mid y = (x+4)^2\} \subseteq \mathbb{R} \times \mathbb{R}$

f: ✓ (def)



i: ✗ ($(-3,1) \wedge (-5,1) \in f_5$)

s: ✗ ($\mathrm{dmn}(f_5) \neq \mathbb{R}$)

↳ b: ✗

f.) HW

g.) HW

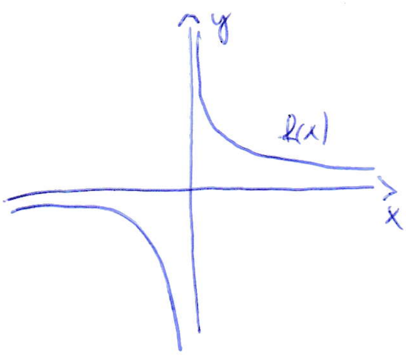
h.) $f_8 = \{ (x,y) \in (\mathbb{R} \setminus \{0\}) \times (\mathbb{R} \setminus \{0\}) \mid xy = 1 \} \subseteq (\mathbb{R} \setminus \{0\}) \times (\mathbb{R} \setminus \{0\})$

f: ✓

i: ✓

s: ✓

b: ✓



↳ $y = \frac{1}{x}$

i.) HW !

j.) $f_{10} = \{ (x,y) \in \mathbb{Z} \times \mathbb{Z} \mid |x-y| \leq 3 \}$

f: ✗ ; $(0,0) \in f_{10} \wedge (0,1) \in f_{10} \wedge 1 \neq 0$

6.) $A = \{2,3,6,8,9,12,18\} \subseteq \mathbb{N}^+$; $R \subseteq A \times A$; $aRb \Leftrightarrow a|b$

a.) r: ✓ ; $\{(2,2) ; (3,3) ; \dots , (18,18)\} \subseteq R$

t: ✓ ; $a|b \wedge b|c$

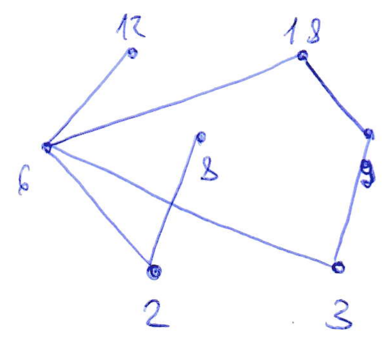
$$\underbrace{b = k \cdot a ;\ k \in \mathbb{Z} \qquad c = n \cdot b ;\ n \in \mathbb{Z}}$$

$c = n \cdot (k \cdot a) = (n \cdot k) \cdot a$; $n \cdot k \in \mathbb{Z} \Rightarrow a|c$

a: ✓ ; $[(a|b \wedge b|a) \Rightarrow a = b]$

$\forall a,b \in \mathbb{Z}$

b.)



8.)

a.) r: ✓ (why?).

t: ✓ (why?)

a: X ; eg: $x+1 \le x+2 \quad \wedge \quad x+2 < x+1 \quad \wedge \quad x+1 \neq x+2$

$$\begin{pmatrix} \deg(x+1) = 1 \\ \deg(x+2) = 1 \\ 1 \le 1 \end{pmatrix} \qquad \begin{pmatrix} - \shortparallel - \end{pmatrix}$$

$\Rightarrow$ X

b.) $R \subseteq \mathbb{Z} \times \mathbb{Z}$, $aRb \Leftrightarrow |a| \le |b|$

r: ✓

t: ✓

a: X , e.g. $\left[\underset{\substack{\Updownarrow \\ (1,1) \in R}}{(|1| \le |-1|)} \wedge \underset{\substack{\Updownarrow \\ (-1,1) \in R}}{(|-1| \le |1|)} \right] \wedge (1 \neq -1)$