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Part 1: short questions.

1. Three properties of the operation of set union.
For any sets A and B.
$A \cup \emptyset = A$
$A \cup (B \cup C) = (A \cup B) \cup C$ (associativity)
$A \cup B = B \cup A$ (commutativity)
$A \cup A = A$ (idempotence)
$A \subseteq B \iff A \cup B = B$

2. Define what is called the domain of a binary relation $R \subseteq A \times B$.
~~$dmn(R) = \{x \in X \mid$~~
$dmn(R) = \{a \in A \mid \exists b \in B : (a, b) \in R\}$,

3. Define $R \subseteq A \times A$ transitive.
R transitive ~~if $\forall x, y, z$~~
if $\forall a, b, c \in R$:
$(aRb \wedge bRc) \Rightarrow aRc$.

4. Define what is equivalence relation.
Let X be a set. A binary relation R on X is called an equivalence relation.
If it is, $\begin{cases} \text{reflexive,} \\ \text{symmetric,} \\ \text{transitive.} \end{cases}$

5. $f : X \to Y$ injective.
It means: $\forall x_1, x_2 \in X$: $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$.

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6. $3i$

real part: ~~3~~. 0.

imaginary part: 3.

7. Theorem: A permutation with repetition is a sequence of m different kinds of elements containing k_1 number of elements of the first kind, k_2 number of elements of the second kind, $\ldots$, k_m number of elements of the m^{th} kind.

$$\frac{n!}{k_1! \cdot k_2! \cdot \cdots \cdot k_m!}$$

where $n = k_1 + k_2 + \cdots + k_m$.

8. A binary relation on a set X is called a partial order if:
① relexive ② transitive ③ anti-symmetric.

9. $z = a+bi$, $|z| = \sqrt{a^2+b^2}$: taking the value in range $[-\pi, \pi]$ or $[0, \pi]$. the argument of a nonzero $z \in \mathbb{C}$ is the angle $\varphi = \arg(z) \in [0, 2\pi)$ such that $z = r(\cos\varphi + i\sin\varphi)$ where $r = |z|$.

10. Binomial theorem:
For any $x, y \in \mathbb{R}$ and $n \in \mathbb{N}$ we have.

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}.$$

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11. Variations with repetition:
n^k, a sequence of length k chosen from n different elements (order matters, an element can occur more than once).

12. A triple $G=(\varphi, E, V)$ is called graph, if E and V are sets such that $V \neq \phi$, $V \cap E = \phi$ and $\varphi: E \to \{\{v,v'\} \mid v, v' \in V\}$.
E: sets of edges.
V: sets of vertices (nodes)
φ: incidence function.
The map φ assigns to each element of E an unordered pair of elements in V.

Part 2:

P_1: $x \in (A \cup (B \cup C)) \Leftrightarrow (x \in A) \vee (x \in (B \cup C)) \Leftrightarrow$
$\Leftrightarrow (x \in A) \vee ((x \in B) \vee (x \in C)) \Leftrightarrow$
$\Leftrightarrow ((x \in A) \vee (x \in B)) \vee (x \in C) \Leftrightarrow$
$\Leftrightarrow (x \in (A \cup B)) \vee (x \in C) \Leftrightarrow$
$\Leftrightarrow x \in ((A \cup B) \cup C)$.

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P2. $(R\circ S)^{-1} = S^{-1} \circ R^{-1}$.

$(z,x) \in (R\circ S)^{-1} \Leftrightarrow (x,z) \in R\circ S \Leftrightarrow$

$\Leftrightarrow \exists y : (x,y) \in S \wedge (y,z) \in R \Leftrightarrow$

$\Leftrightarrow \exists y : (y,x) \in S^{-1} \wedge (z,y) \in R^{-1}$ #

$= (z,x) \in S^{-1} \circ R^{-1}$.

P3. De Moivre's formula.

Let $z, w \in \mathbb{C}$ be nonzero complex numbers.

$z = |z|(\cos\varphi + i\sin\varphi)$, $w = |w|(\cos\psi + i\sin\psi)$,

and let $n \in \mathbb{N}^+$.

$zw = |z|(\cos\varphi + i\sin\varphi) \cdot |w|(\cos\psi + i\sin\psi) =$

$= |z||w|(\cos\varphi\cos\psi - \sin\varphi\sin\psi + i(\cos\varphi\sin\psi + \sin\varphi\cos\psi))$

$= |z||w|(\cos(\varphi+\psi) + i\sin(\varphi+\psi))$.