# Problem set 8.: Further enumeration problems

**Question 1.**
Consider all 5-digit numbers consisting of the digits 2, 3, 4, 5 and 7 containing each of these digits exactly once.
(a) How many such numbers exist?
(b) How many of the above numbers are even?
(c) How many of the above numbers are divisible by four?

**Question 2.**
How many different six-digit numbers can be formed using each of the digits 0, 1, 2, 3, 4, 5 exactly once which are divisible by 5?

**Question 3.**
A group of ten people organize a prize draw: 4 different books can be won, and each person can win at most one book. How many different outcomes are possible?

**Question 4.**
In a factory, 4% of the 500 locks manufactured during a shift are faulty. In how many different ways can we choose 10 locks out of the 500 locks such that there are

(a) exactly 5 faulty locks
(b) at least 2 faulty locks

among the ones chosen?

**Question 5.**
A class of 32 students are electing a student committee from among its members. The committee consists of 1 secretary and 4 other committee members. Paul Smith is a student in the class. In how many ways can the committee be selected in such a way that

(a) Paul Smith is the secretary of the committee;
(b) Paul Smith is a non-secretary member of the committee;
(c) Paul Smith is a member of the committee?

**Question 6.**
In how many different ways can 9 people get on a tram of 3 carriages in such a way that exactly 3 people get into each carriage?

**Question 7.**
A deck of Hungarian playing cards consists of 32 cards, which are divided in four suits: acorns, hearts, leaves and bells with 8 cards in each suit. Each suit contains one ace. We draw 5 cards from the 32-card deck. How many different outcomes are possible such that
(a) all cards drawn are hearts;
(b) there is exactly one hearts among the cards drawn;
(c) there is at least one card which is hearts among the ones drawn;
(d) there are 2 hearts and 3 leaves among the cards drawn;
(e) the 5 cards drawn contain at least one card from each suit;

(f) the 5 cards drawn contain exactly 1 ace and 4 hearts;
(g) all cards drawn are ace or hearts?

**Question 8.**
A party was attended by 9 men and 12 women. During the party 7 different pairs had a dance. In how many different ways could these 7 pairs be formed from the people at the party if each pair consists of a man and a woman and any person can be a member of more than one pairs? The order in which the pairs danced does not matter.

**Question 9.**
In how many different ways can we place 24 identical balls into 8 different boxes, if
(a) we can leave any of the boxes empty;
(b) we need to put at least 1 ball into each box;
(c) we need to put at least 2 balls into each box?

**Question 10.**
A flea is jumping on the number line: at each jump it moves 1 unit in the positive or 1 unit in the negative direction. In how many different ways can the flea get from 0 to 10 with exactly 18 jumps?

**Question 11.**
A bus ticket has a grid of 9 cells printed on it numbered from 1 to 9. When validating the ticket, the ticket machine punches the ticket through 3 or 4 of these 9 cells. How many different combinations of punch holes can be created this way?

**Question 12.**
There are two different straight lines given in the plane: $e$ and $f$. We are given 5 different points on line $e$ and 7 different points on line $f$. (All points given are pairwise different, hence we have 12 distinct points in total). How many triangles exist with the property that all vertices of the triangle are among these 12 points?

**Question 13.**
(a) Find the sum of all the different 3-digit numbers which contain each of the digits $1, 2, 3$ exactly once.
(b) Find the sum of all the different 6-digit numbers which contain each of the digits $1, 2, 3, 4, 5, 6$ exactly once.
(c) What is the sum of all the different 6-digit numbers which can be formed from the digits $1, 2, 3, 4, 5, 6$ in such a way that not every digit has to be used and any digit can appear more than once?

**Question 14.**
When rolling a die three times, how many possible sequences of results exist which contain 6 at least once?

**Question 15.**
A standard 52-card deck of French playing cards contains 4 aces and 4 kings. In how many ways can the 52 cards be handed out to 4 players in such a way that each player gets exactly 13 cards

containing 1 ace and 1 king?

**Question 16.**
In how many different ways can we distribute 30 balls in 100 compartments if each compartment has to contain exactly 6 or 0 balls and
(a) all the balls are different;
(b) all the balls are different and we take into account the order of the balls inside each compartment;
(c) all the balls are different and we do not take into account the order of the balls inside the compartments?

**Question 17.**
Solve the equation $0,7 \cdot \binom{25}{x} = \binom{23}{x}$ on the set $\{0, 1, 2, ..., 23\}$ (where $\binom{25}{x}$ and $\binom{23}{x}$ denote binomial coefficients).

**Question 18.**
Let $n, k \in \mathbb{N}$, $k \leq n$. Prove the following identities:
(a) $\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}$
(b) $k \cdot \binom{n}{k} = n \cdot \binom{n-1}{k-1}$