

Analysis II., Sample Test 1

1. Discuss the continuity of the following function:

$$f(x) = \begin{cases} \frac{x^2 + 2x - 8}{x^2 - x - 2} & \text{if } x \in \mathbb{R} \setminus \{-1, 2\} \\ 2 & \text{if } x \in \{-1, 2\} \end{cases}$$

2. Prove that the following equation has at least one solution in \mathbb{R} :

$$\cos x = x$$

3. Determine $f'(a)$ by the definition:

$$f(x) = \frac{1}{2x - 1}, \quad a = 3$$

4. Discuss the differentiability of the following function (a is a real parameter):

$$f(x) = \begin{cases} 1 - ax & \text{if } x < 0 \\ e^{-x^2} & \text{if } x \geq 0 \end{cases}$$

5. Determine the equation of the tangent line to the graph of f at the point $a = 0$:

$$f(x) = (2 + \sin x)^{\cos x}$$

6. Discuss the monotonicity, and the local and global extreme values of the following function:

$$f(x) = \frac{x}{x^2 + x + 1} \quad (x \in \mathbb{R})$$

What are the global extreme values if the function is only defined on $[-2, 0]$?

7. The hypotenuse of a right triangle is 1 unit. The legs of the triangle are denoted by x and y . When will $x + 2y$ be the largest?

8. Determine the following limits using L'Hospital's Rule:

$$\text{a) } \lim_{x \rightarrow 0+0} (\sin x \cdot \ln x) \quad \text{b) } \lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{1}{\sin x} \right) \quad \text{c) } \lim_{x \rightarrow 0+0} (\cos x)^{1/x^2}$$

9. Consider the function

$$f(x) = \sqrt{1 + 3x} \quad (x > -1/3).$$

Determine the second degree Taylor polynomial of f centered at 0, and estimate the error of approximation on the interval $(-\frac{1}{4}, \frac{1}{2})$.

10. Discuss the following function and sketch the graph:

$$f(x) = \frac{x^2}{(x + 1)^2} \quad (-1 \neq x \in \mathbb{R})$$