Analysis II, Sample Test 2

1. Determine the global extreme values of the following functions:

a) $f(x) = 1 + e^{2x^2 - x^4}$ $(x \in \mathbb{R})$ b) $f(x) = \ln(e^x - x)$ $(x \in \mathbb{R})$ c) $f(x) = \sin(\pi \cdot \sin x)$ $\left(x \in \left[-\dfrac{\pi}{2}, \pi\right]\right)$

2. a) We have a square and a disc on the plane so that the sum of their area is 1 m². When will the sum of their perimeter be the smallest?

2. b) We have a square and a disc on the plane so that the sum of their perimeter is 1 m. When will the sum of their area be the largest?

3. Determine the following limits:

a) $\lim\limits_{x \to 0} \dfrac{x - \sin x}{4x^4 + x^3}$ b) $\lim\limits_{x \to 0}(\cos x)^{\frac{1}{x^2}}$ c) $\lim\limits_{x \to 0+} x^{\arctan x}$ d) $\lim\limits_{x \to 1}\left(\dfrac{1}{\ln x} - \dfrac{1}{x - 1}\right)$

4. Let a) $f(x) = \sqrt[3]{1 + x}$ $(x \in \mathbb{R})$ b) $f(x) = \arctan x$ $(x \in \mathbb{R})$.
   i) Find the third Taylor-polynomial $T_3(x)$ centered at 0 and at 1.
   ii) Estimate the error of approximation $f(x) \approx T_3(x)$ for $x \in \left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$.
   a-iii) Approximate $\sqrt[3]{2}$ by means of $T_3(1)$, $\sqrt[3]{3}$ by means of $T_3(2)$, and estimate the errors.
   b-iii) Approximate $\dfrac{\pi}{4} = \arctan 1$ by means of $T_3(1)$, and estimate the error.

5. Discuss and sketch the graph of the functions:

a) $f(x) = \ln(x^2 + 2x + 2)$ $(x \in \mathbb{R})$ b) $f(x) = \dfrac{x^3 + x}{x^2 - 1}$ $(x \in \mathbb{R} \setminus \{-1, 1\})$

6. Find the integrals:

a) $\displaystyle\int e^{8x+5} - 6(x-1)^{100} + \dfrac{x + 9}{1 + 9x^2}\, dx$ b) $\displaystyle\int \tan x \cdot \sqrt{1 + \ln(\cos x)}\, dx$

c) $\displaystyle\int \dfrac{1 - x^4 + (1 + \arctan x)^{100}}{1 + x^2}\, dx$ d) $\displaystyle\int \sin^5 x \, dx$

e) $\displaystyle\int \sin^3 x \cdot \cos^{100} x\, dx$ f) $\displaystyle\int \sin^4 x \cdot \cos^6 x\, dx$