

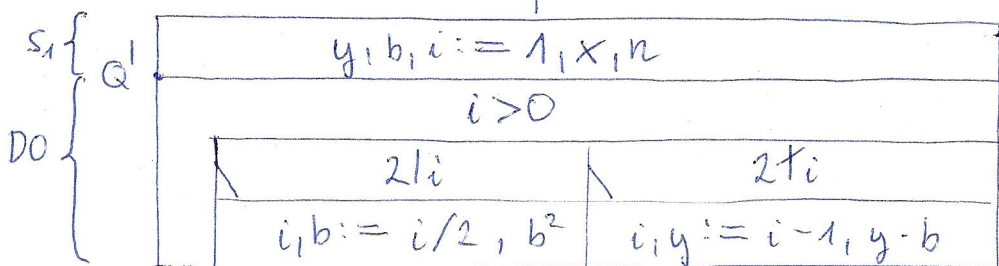
$$A = (x: \mathbb{N}^+, n: \mathbb{N}, y: \mathbb{N})$$

$$B = (x': \mathbb{N}^+, n': \mathbb{N})$$

$$Q = (x = x' \wedge n = n')$$

$$R = (Q \wedge y = x^n)$$

$i: \mathbb{N}$ and $b: \mathbb{N}^+$ are auxiliary variables



$$Q' = (Q \wedge y = 1 \wedge b = x \wedge i = n)$$

$$Iuv = (Q \wedge y \cdot b^i = x^n)$$

$$t: i$$

We want to prove the total correctness formula in order to prove that S solves the problem:

$$Q \Rightarrow wp(S, R)$$

Now, S is a sequence. Let us apply the verification rule of sequence. Instead of proving $Q \Rightarrow wp(S, R)$, it is enough to prove two others:

$$I) Q \Rightarrow wp(S_1, Q')$$

$$II) Q' \Rightarrow wp(DO, R)$$

$S = (S_1; DO)$ is a sequence,
 Q' is the intermediate condition

$$I) Q \Rightarrow wp(\underbrace{y, b, i := 1, x, n}_{\text{program}}, \underbrace{Q'}_{\text{logical function}})$$

$$wp(y, b, i := 1, x, n, Q') = Q' \wedge y \leftarrow 1, b \leftarrow x, i \leftarrow n =$$

$$(Q \wedge y = 1 \wedge b = x \wedge i = n) \wedge y \leftarrow 1, b \leftarrow x, i \leftarrow n =$$

$$(Q \wedge 1 = 1 \wedge x = x \wedge n = n) = Q$$

$$Q \Rightarrow Q \checkmark \text{ this is true}$$

$$II) Q' \Rightarrow wp(DO, R)$$

We prove this by using the verification rule of loop, and proving 5 other conditions.

$$1. Q' \Rightarrow Iuv$$

We want to prove that DO loop takes us from Q' to R . (Not from Q to R .)

$Q' \Rightarrow Iuv$ means: for any state where Q' is true, Iuv is also true. We do not need to prove that Iuv is always true. In fact, Iuv is not always true; but in case Q' is true then Iuv is also true.