Theory of Programming
exam - sample

1. (20 points)

Given an array x of n integer numbers. Calculate the sum of the elements in such a way that if index i is an odd number, then the additive inverse of $x[i]$ has to be considered in the sum.
Examples:

(a) If the input array is x=[1,2,3,4] then the result should be -1+2-3+4=2.
(b) If the input array is x=[-1,-2,-3,4] then the result should be 1+(-2)+3+4=6.
(c) If the input array is x=[1] then the result should be -1.
(d) If the size of the input array is 0 then the result should be 0.

The specifications of the problem is given:
$A = (x : \mathbb{Z}^n, s : \mathbb{Z})$
$Pre = (x = x')$
$Post = (Pre \wedge s = \sum_{i=1}^{n}(-1)^i \cdot x[i])$

2. (20 points)

$w, r := 0, 0$
$\{w = 0 \wedge r = 0\}$
parbegin $W_1\| \dots \|W_n\|R_1\| \dots \|R_m$ **parend**
$\{FALSE\}$

R_j:

```
while TRUE do
{I ∧ r_j = 0}
        await w=0 then
                r := r + 1; r_j := 1
        ta;
        {I ∧ r_j = 1}
        read;
        {I ∧ r_j = 1}
        [r := r − 1; r_j := 0]
        work;
        {I ∧ r_j = 0}
od
{FALSE}
```

W_i:

```
while TRUE do
{I ∧ w_i = 0}
        work;
        await w = 0 ∧ r = 0 then
                w := 1; w_i := 1
        ta;
        {I ∧ w_i = 1}
        write;
        {I ∧ w_i = 1}
        [w := 0; w_i := 0]
        {I ∧ w_i = 0}
od
{FALSE}
```

$I = ((r = 0 \lor w = 0) \land w = \sum_{i=1}^{n} w_i \land r = \sum_{j=1}^{m} r_j)$ where
w_i: $\{0, 1\}$. Its value is 1 if the ith writer process is writing.
r_j: $\{0, 1\}$. Its value is 1 if the jth reader process is reading.

Prove that the readers/writers program (let us denote it by S) is free from deadlock.

3. (25 points)

Question 1: Write down the conditions that are needed to prove in order to show the interference freedom of the above proof outlines.
Question 2: Prove these conditions!
$A = (x : \mathbb{N})$

$Pre = (x = x' \land even(x) \land x > 0)$

$Post = (x = 1)$

$\{x = x' \land even(x) \land x > 0\}$
parbegin $S_1 \| S_2$ **parend**
$\{x = 1\}$

$\{Pre(S_1) = Inv\}$
S_1:

$\{Inv\}$
while $x > 2$ do
$\{Inv \land x > 2\}$
    x:= x−2
    $\{Inv\}$
od
$\{Post(S_1) = (Inv \land x \leq 2)\}$

S_2:

$\{Pre(S_2) = (even(x) \land x > 0)\}$
x:= x−1
$\{Post(S_2) = odd(x)\}$

Variant function: x
$Inv = (x > 0)$

4. (35 points)

$A = (x : \mathbb{N}, n : \mathbb{N}_0, z : \mathbb{N})$

$Pre = (x = x' \land n = n')$

$Post = (z = x'^{n'})$

Let us assume we already proved the deadlock freedom and interference freedom, you can ignore proving them. Prove the total correctness of the following program with respect to the given problem.

$\{x = x' \wedge n = n'\}$
$z := 1;$
$\{Inv\}$
parbegin $S_1 \| S_2$ **parend**
$\{z = x'^{n'}\}$

$\{Inv\}$
 S_1:
 	
 $\{Inv\}$
 while $n \neq 0$ do
 $\{Inv \wedge n \neq 0\}$
         n,z := n−1,z·x
 od
 $\{z = x'^{n'} \wedge n = 0\}$
	$\{Inv\}$
	 S_2:
	
	 $\{Inv\}$
	 while $n \neq 0$ do
	 $\{Inv\}$
	         await even(n) then
	                 x,n := x·x,n/2
	         ta
	 od
	 $\{z = x'^{n'} \wedge n = 0\}$

Inv denotes the invariant of the loops and is given as follows:
$Inv = (z \cdot x^n = x'^{n'})$
Variant function of the loops: t: n