2) Find the local extrema on the whole domain and the global extrema on the set $[-\frac{1}{2}, 1]$

$$f(x) = x^2 \cdot \sqrt{x+1} \qquad (x > -1)$$

$$f(-\tfrac{1}{2}) = (-\tfrac{1}{2})^2 \sqrt{-\tfrac{1}{2}+1} = \frac{1}{4} \cdot \sqrt{\tfrac{1}{2}} = \frac{1}{4\sqrt{2}}$$

$$f(1) = 1^2 \cdot \sqrt{1+1} = \sqrt{2}$$

$$f'(x) = 2x \cdot \sqrt{x+1} + \frac{x^2}{2\sqrt{x+1}} = x\left(2\sqrt{x+1} + \frac{x}{2\sqrt{x+1}}\right) = 0$$

$$\Rightarrow x_1 = 0 \quad , \quad \frac{4(x+1)+x}{2\sqrt{x+1}} = 0 = \frac{5x+4}{2\sqrt{x+1}} = 0 \Rightarrow x_2 = -\frac{4}{5}$$

$$f'(-\tfrac{1}{2}) = -1 \cdot \sqrt{\tfrac{1}{2}} + \frac{\frac{1}{4}}{2\sqrt{\frac{1}{2}}} = -\frac{1}{\sqrt{2}} + \frac{\sqrt{2}}{8} = \frac{-1}{\sqrt{2}} + \frac{1}{4\sqrt{2}} = -\frac{3\sqrt{2}}{8}$$

$$f(-\tfrac{4}{5}) = \frac{16}{25} \cdot \sqrt{\tfrac{1}{5}} = \frac{16}{25\sqrt{5}} \Rightarrow \text{local max}$$

$$f'(1) = \frac{5.1+4}{2\sqrt{1+1}} = \frac{9}{2\sqrt{2}}$$

$f'(x) = 0 \Leftrightarrow x_1 = 0$

$f'(x) > 0 \Leftrightarrow x_1 > 0$

$f'(x) < 0 \Leftrightarrow x_1 < 0$

$f'(x) = 0 \Leftrightarrow x_2 = -\frac{4}{5}$

$f'(x) > 0 \Leftrightarrow x_2 > -\frac{4}{5}$

$f'(x) < 0 \Leftrightarrow x_2 < -\frac{4}{5}$

x	-1	$-\frac{4}{5}$	$-\frac{1}{2}$	0	1	$+\infty$
$f'(x)$	$-$	$-\ -\ 0$	$-\ -\ -\ -\ -$	0	$+\ +\ +$	$+\ +$
$f(x)$	0 ↘	$\frac{16}{25\sqrt{5}}$	→	0	↗	

$$\lim_{x\to+\infty} f = \lim_{x\to+\infty} x^2\sqrt{x+1} = +\infty \Rightarrow \text{no global max.}$$

On the set $[-\frac{1}{2}, 1]$, f is $\downarrow$ on $[-1, 0)$,

f is $\uparrow$ on $(0, 1]$ so global max is $f(1) = \sqrt{2}$

global min is $f(0) = \underline{0}$