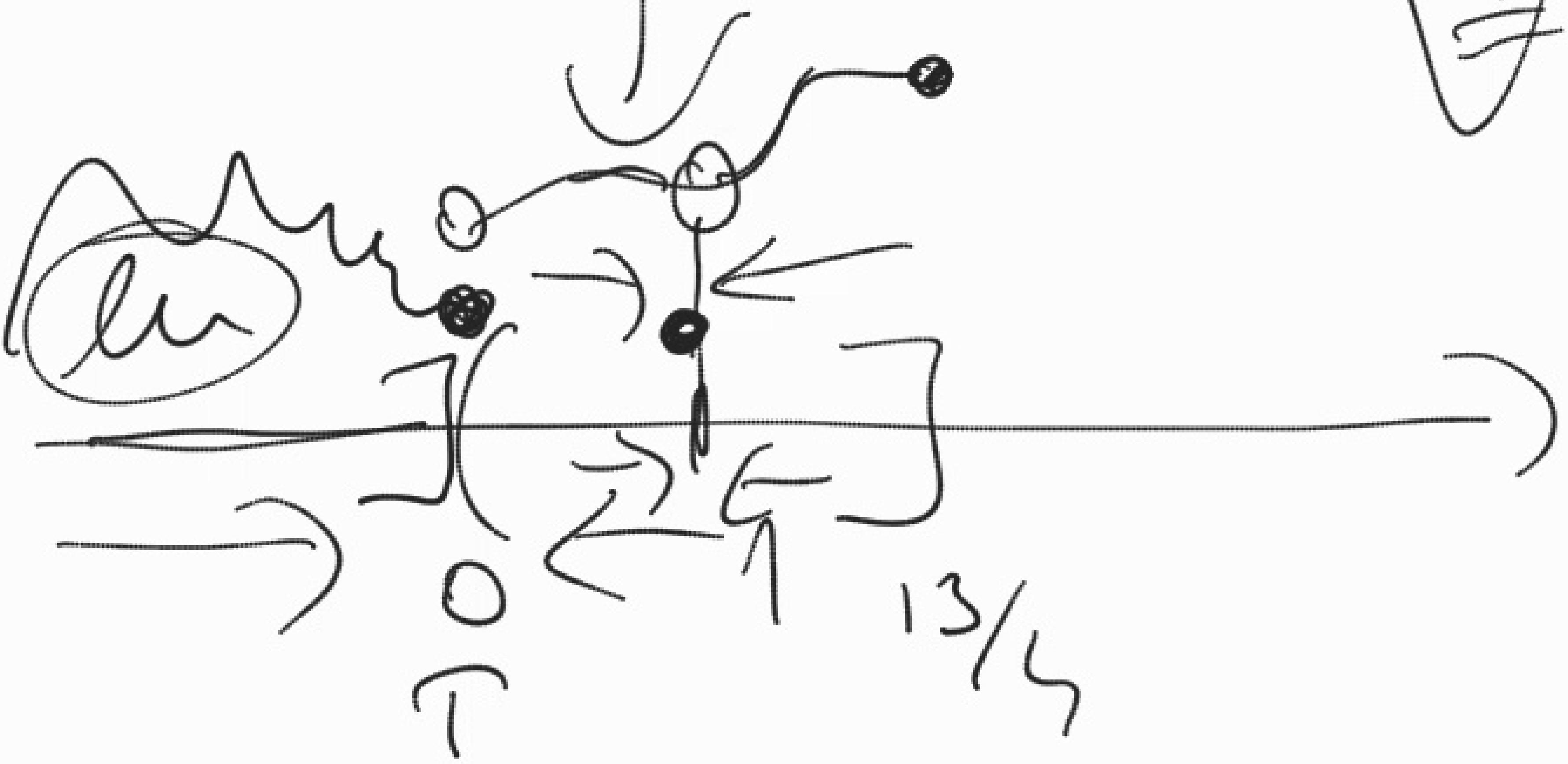
1) $f(x) = \begin{cases} \ln(1-x), & x \leq 0 \\ \dfrac{\sqrt{13-4x}-3}{1-\sqrt{x}}, & x \in (0; \frac{13}{4}] \setminus \{1\} \\ \dfrac{12}{13-4x}, & x > \frac{13}{4} \\ & x = 1 \end{cases}$

$\lim\limits_{x \to 1}$

1 if $x \in \mathbb{R} \setminus \{0; \frac{13}{4}\}$



② $x=0$ $f(0)=\ln 1=0$

$\lim\limits_{x\to 0-0} \ln(1-x)=\ln 1=0$

$\lim\limits_{x\to 0+0} \dfrac{\sqrt{13-4x}-3}{1-\sqrt{x}}=\dfrac{\sqrt{13}-3}{1}$

$=\sqrt{13}-3$; $\neq$

$\Rightarrow f\notin C\{0\}$, 0 is a jump. / type 1

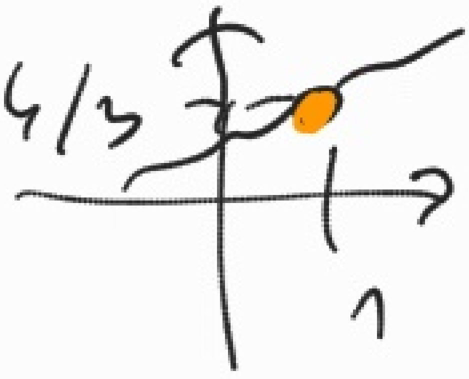
$$\lim_{x \to \frac{13}{4}-0} \frac{\sqrt{13-4x}-3}{1-\sqrt{x}} = \frac{-3}{1-\frac{\sqrt{13}}{2}}$$

$$\lim_{x \to \frac{13}{4}+0} \left(\frac{12}{13-4x}\right) = \frac{12}{13-4\left(\frac{13}{4}+0\right)}$$

$$= \frac{12}{\cancel{13}-\cancel{13}-0} = \boxed{-\infty} = \infty \quad f \notin C\left\{\frac{13}{4}\right\}$$

second kind

③ $x = 1$



$$f(1) = \frac{12}{13-4\cdot 1} = \frac{12}{9} = \frac{4\cdot \cancel{2}}{\cancel{6}_3} = \frac{4}{3} = \frac{4}{3}$$

so $f \in C\{1\}$

$$\lim_{x\to 1} \frac{\sqrt{13-4x}-3}{(1-\sqrt{x})} = \text{"}\frac{0}{0}\text{"} = \lim_{x\to 1} \frac{(13-4x-9)\cdot(1+\sqrt{x})}{(\sqrt{13-4x}+3)(1-x)}$$

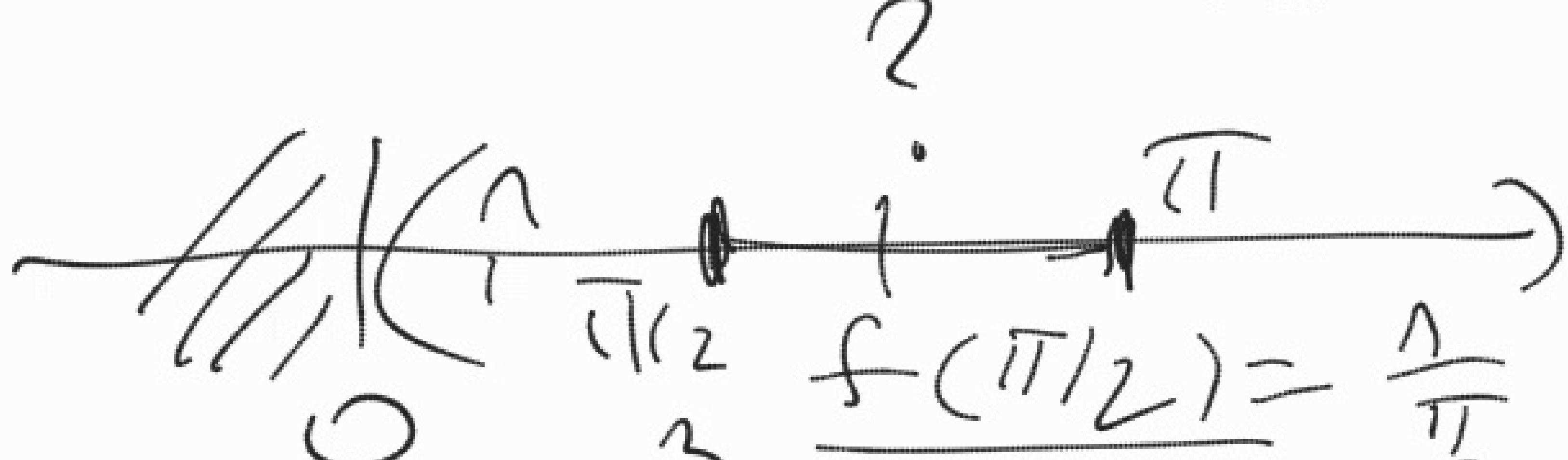
$$= \lim_{x\to 1} \frac{4\cancel{(1-x)}(1+\sqrt{x})}{(\sqrt{}+3)\cancel{(1-x)}}$$

a) $\cos^3 x + \frac{1}{x} = 0 \quad (x > 0)$

b) tangent line in $x = \pi$

c) Bolzano theorem?

$$f(x) := \cos^3 x + \frac{1}{x}$$



$$f(\pi/2) = \frac{1}{\frac{\pi}{2}} = \frac{2}{\pi} > 0$$

$$f(1) = \cos^3 1 + 1 > 0$$

$$f(\pi) = \cos^3 \pi + \frac{1}{\pi} = -1 + \frac{1}{\pi} < 0$$

$\exists\ \frac{\pi}{2} < c < \pi$ so that

$\Longrightarrow f(c) = 0$ ✓

$f \in C\left[\frac{\pi}{2}, \pi\right]$ and

$\left[\frac{\pi}{2}, \pi\right]$ is compact

b) $\underline{\pi} \Longrightarrow$

tan line here:

$$\boxed{y = f(\pi) + f'(\pi)\cdot(x - \pi)}$$

$$y = -1 + \frac{1}{\pi} + \left(-\frac{1}{\pi^2}\right)(x - \pi)$$

$$y = -\frac{1}{\pi^2} \cdot x - 1 + \frac{1}{\pi} + \frac{1}{\pi}$$

$$\boxed{y = -\frac{1}{\pi^2} x + \frac{2}{\pi} - 1}$$

$$f'(\pi) = 3\cos^2\pi\left(-\overset{0}{\overset{\shortparallel}{\sin\pi}}\right) - \frac{1}{\pi^2} = \underline{\underline{-\frac{1}{\pi^2}}}$$

$$① \lim_{x \to 0} (\cos 2x)^{\frac{1}{\tan x}} =$$

$$\boxed{f(x)^{g(x)} = e^{g(x) \ln f(x)} = e^{g(x) \cdot \ln f(x)}}$$

$$= \lim_{x \to 0} e^{\frac{1}{\tan x} \cdot \ln(\cos 2x)} =$$

STEP 2

$$= e^{\lim\limits_{x \to 0} \frac{\ln(\cos 2x)}{\tan x}} = e^{l}$$

$\underline{\exp \in C}$

$\boxed{\text{where}}$ $\frac{0}{0}, \frac{\infty}{\infty}$ $L'H$

$$l = \lim_{x \to 0} \frac{\ln(\cos 2x)}{\tan x} =$$

$$= \lim_{x \to 0} \frac{(\ln(\cos 2x))'}{(\tan x)'} \qquad \frac{0}{0}$$

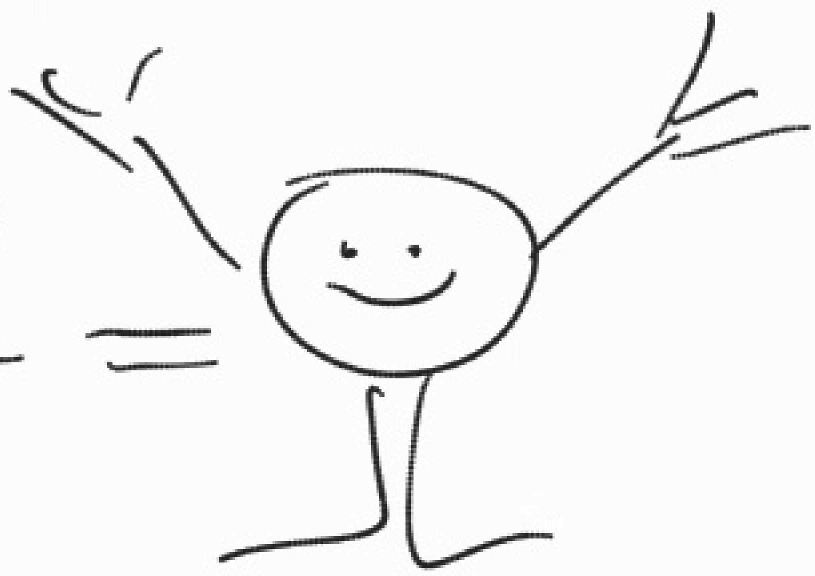
STEP 3

$=$

$$= \lim_{x \to 0} \frac{\overset{1 \leftarrow}{\dfrac{1}{\cos 2x}} \cdot \overset{2}{2} \cdot \overset{0 \leftarrow}{(-\sin 2x)}}{\dfrac{1}{\cos^2 x} \rightarrow 1} =$$

$$\left(\ln(\cos 2x)\right)'$$

$$= \frac{1 \cdot 2 \cdot 0}{1} =$$



<u>So</u>

$$e^{l} = e^{0} = \boxed{\text{ONE}}$$

$$f(x)=\sqrt{2x+1} \quad \left(x>-\frac{1}{2}\right)$$

$$T_2(x)=\,? \qquad a=0$$

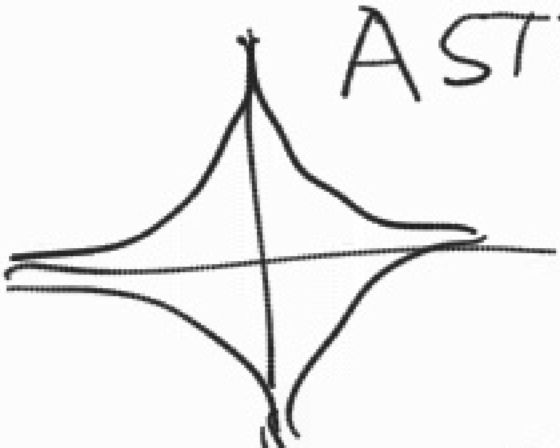
Error on $\left(-\frac{1}{4}, \frac{1}{4}\right)$

f''' 0000

STEP 1

$$T_2(x)=f(0)+f'(0)\cdot x+$$

ASTEROID



$$+\frac{f''(0)}{2!}\cdot x^2$$

$$f(x)=(2x+1)^{\frac{1}{2}}$$

$$f'(x)=\frac{1}{\cancel{2}}\cdot(2x+1)^{-\frac{1}{2}}\cdot\cancel{2}=(2x+1)^{-\frac{1}{2}}$$

$$f''(x) = -\frac{1}{\cancel{2}} \cdot (2x+1)^{-3/2} \cdot \cancel{2} =$$

$$= -(2x+1)^{-3/2}$$

$$f'''(x) = \frac{3}{2} \cdot (2x+1)^{-5/2} \cdot 2 =$$

$$= 3(2x+1)^{-5/2}$$

So
$$\begin{cases} f(0) = \sqrt{2\cdot 0+1} = 1 \\ f'(0) = (2\cdot 0+1)^{-\frac{1}{2}} = 1 \\ f''(0) = -1 \end{cases}$$

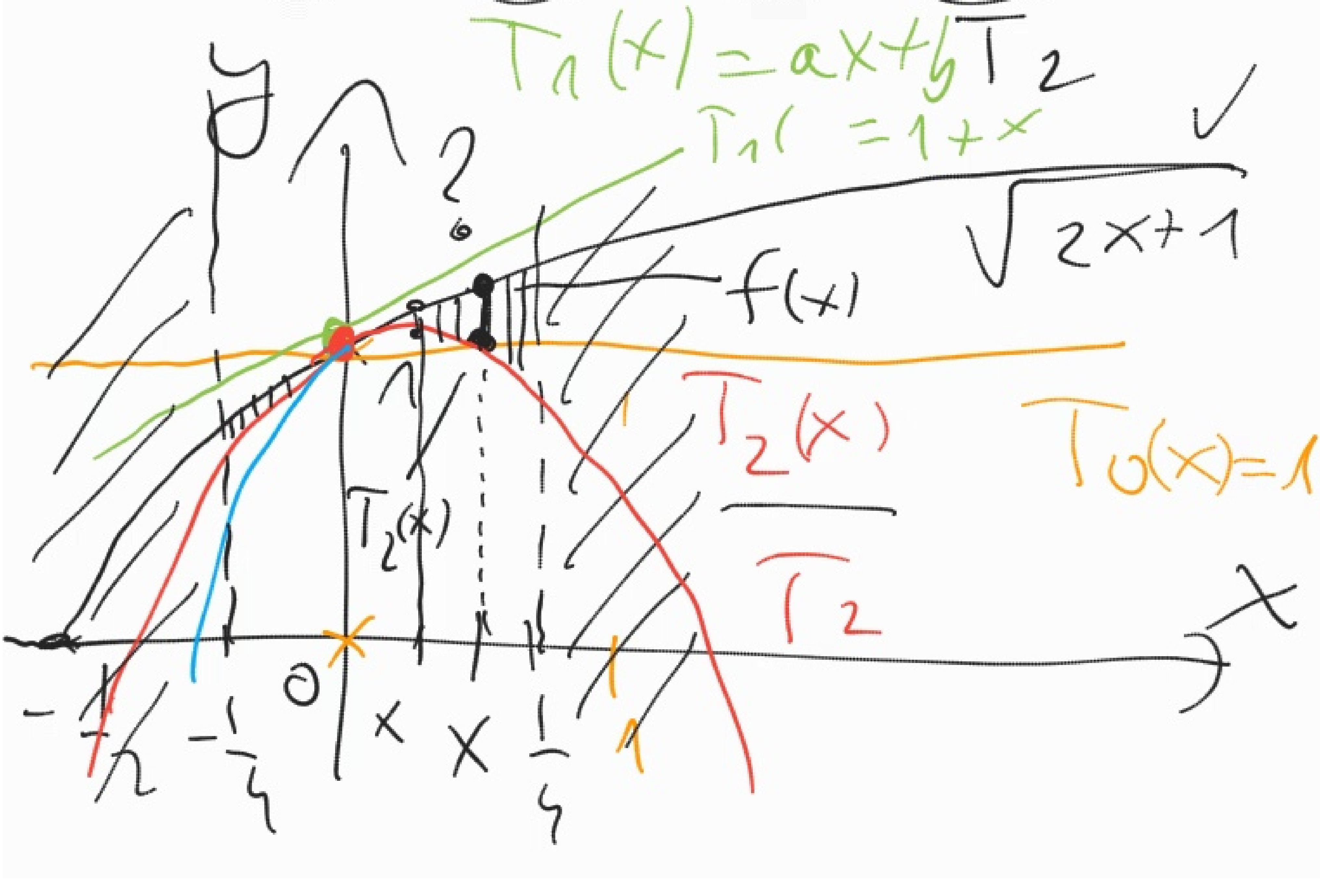
$c \in$
0 and
x

$$f'''(c) = \frac{3}{\sqrt{(2c+1)^5}}$$

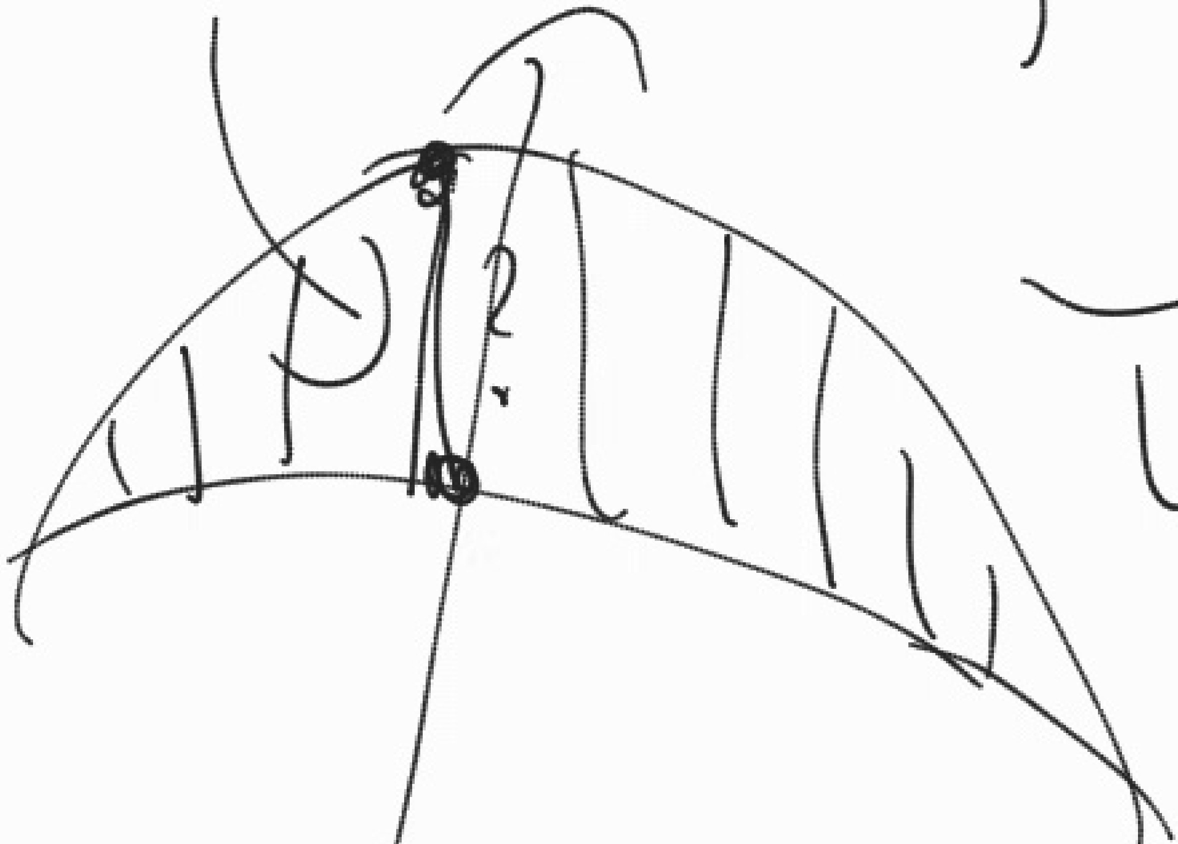
So $f(0)+f'(0)\cdot x+\dfrac{f''(0)}{2!}x^2$

$$T_2(x) = \underbrace{1 + x}_{T_1} + \frac{-1}{2}\cdot x^2$$

$$= \underbrace{\underbrace{\underline{1}}_{T_0} + x - \frac{1}{2}x^2}_{} \approx$$



$$|f(x) - T_2(x)| = \left| \frac{f'''(c)}{3!} \cdot x^3 \right|$$

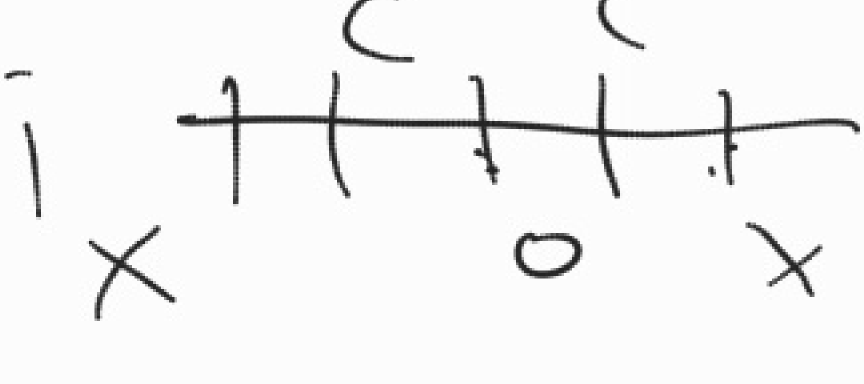


Lagrange error

Taylor formula

$$f(x) = T_2(x) + \frac{f'''(c)}{3!} x^3$$

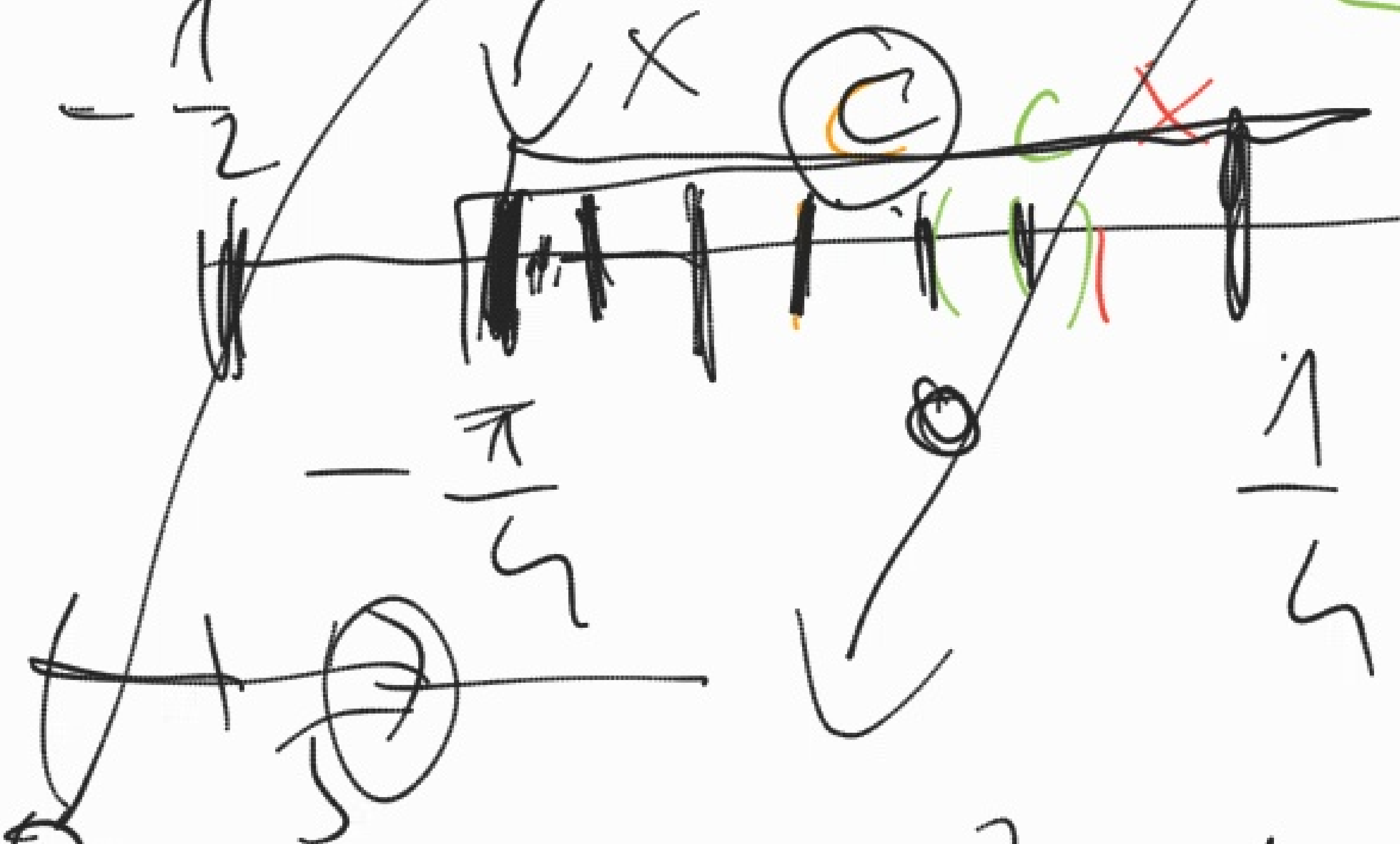
$(a=0)$;

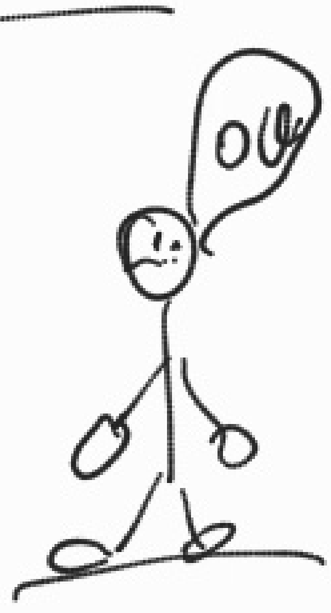


$$= \frac{1}{3!} \left| \frac{3c}{\sqrt{(2c+1)^5}} \right| \cdot |x|^3$$

$c < 1/4$

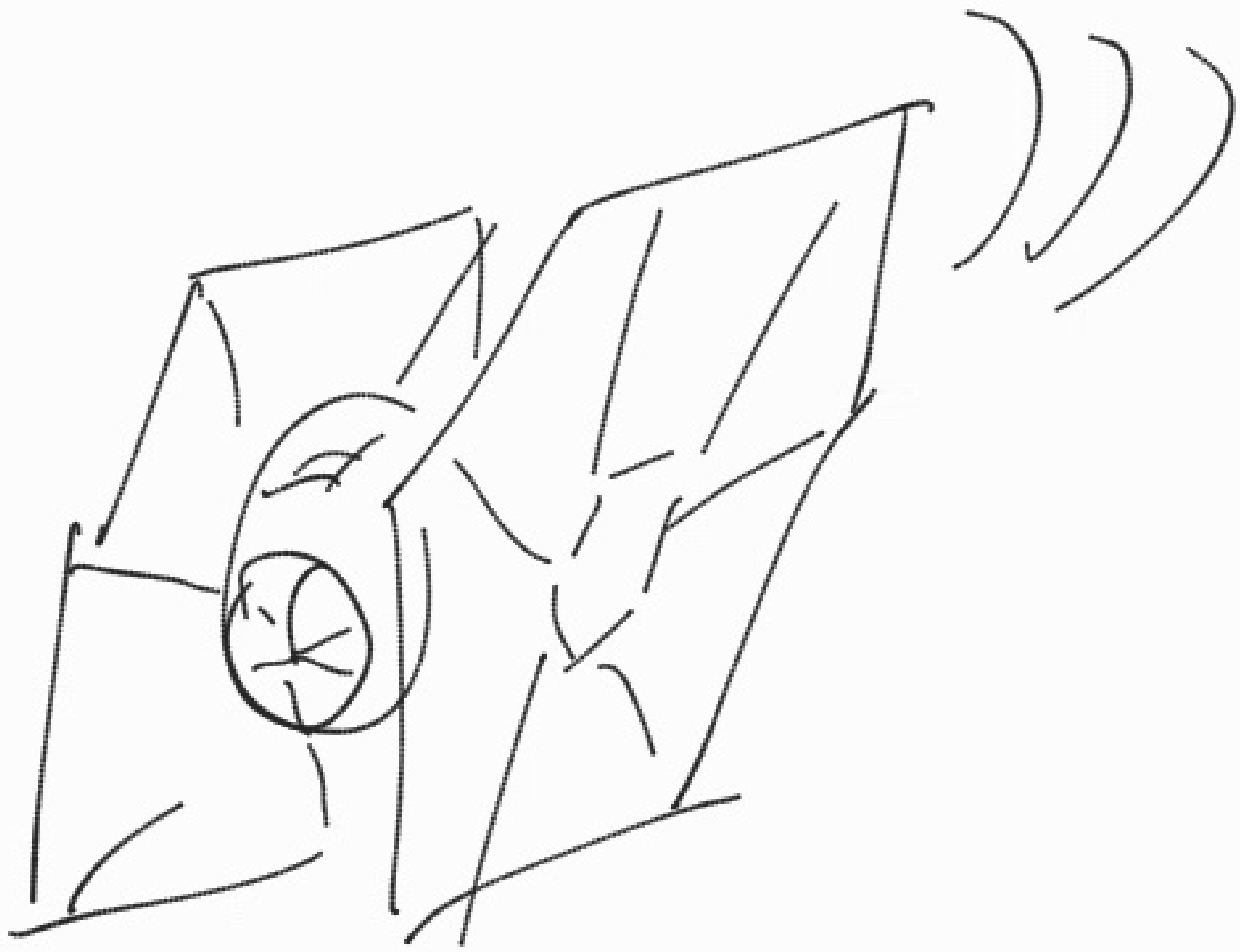
$$\leq \ldots < \left(\frac{1}{4}\right)^3$$





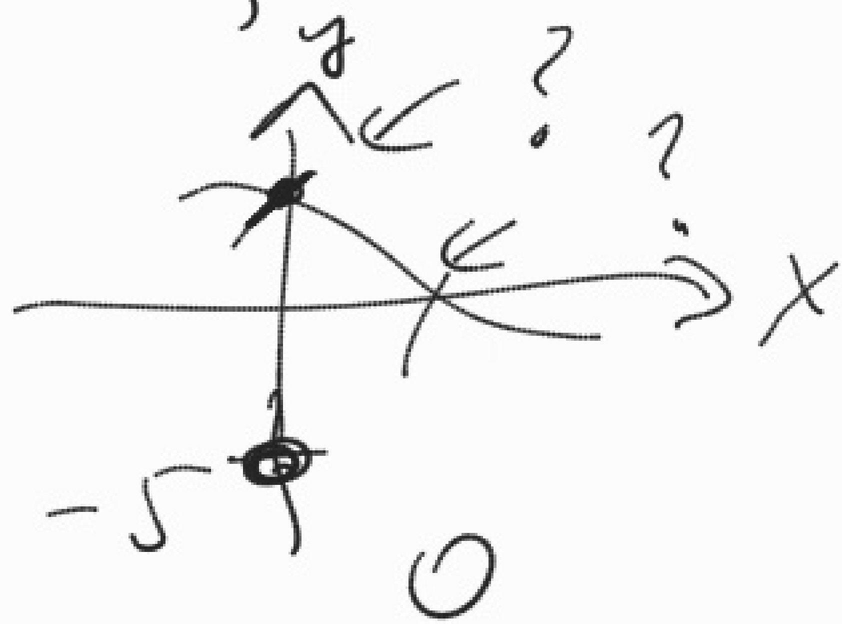
$$< \frac{1}{6} \cdot 3 \cdot \frac{1}{4}^3 \frac{1}{\sqrt{\left(2\left(-\frac{1}{4}\right)+1\right)^5}}$$

$$= \frac{1}{2} \cdot \frac{1}{64} \sqrt{2^5} = \frac{\sqrt{2} \cdot \cancel{4}}{2 \cdot \cancel{64}\,16} = \boxed{\frac{\sqrt{2}}{32}}$$



① $(x-5)e^x = f(x)$

$(x \in \mathbb{R})$

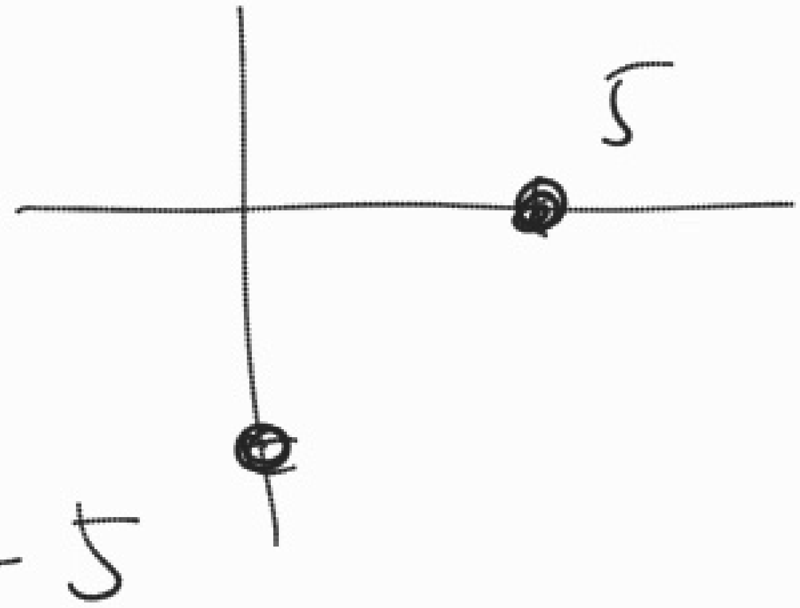


1°. STEP 1 :

$x=0 \Rightarrow f(0) = -5 \cdot e = \underline{\underline{-5}}$

$(x-5)\underbrace{e^x}_{\neq} = 0 \Leftrightarrow$

$\Leftrightarrow x-5=0 \Leftrightarrow x=5$



STEP 2

$$f'(x) = \left((x-5)\cdot e^{x}\right)' =$$

$$= (x-5)e^{x} + e^{x} =$$

$$= e^{x}(x-5+1) =$$

$$= e^{x}\cdot(x-4) \qquad (x\in\mathbb{R})$$

$$e^x(x-4) \begin{cases} =0 \iff x=4 \\ >0 \iff x>4 \\ <0 \iff x<4 \end{cases}$$

e^x: $+$ >0 $\forall x \in \mathbb{R}$

$\downarrow$ Table

STEP 3: f'' and sign

$$f''(x) = \left(e^x \cdot (x-4)\right)' =$$

$$= e^x \cdot (x-4) + e^x \cdot 1$$

$$= e^x(x-3) \quad (x \in \mathbb{R})$$

$$e^x(x-3) \begin{cases} = 0 \iff \underline{x = 3} \\ > 0 \iff \underline{x > 3} \\ < 0 \iff \underline{x < 3} \end{cases}$$

e^x ⊕ $\forall x \in \mathbb{R}$

↓ Table.

STEP 4 $e^{+\infty} \cdot +\infty$

$$\lim_{x \to +\infty} e^x(x-5) = +\infty$$

$$\lim_{x \to -\infty} e^x(x-5) = 0 \cdot (-\infty)$$

WHAT NEXT ⇒

$$\lim_{x \to -\infty} \frac{x-5}{e^{-x}} = "\frac{-\infty}{+\infty}" =$$

$$= \text{l'H} = \lim_{x \to -\infty} \frac{1}{-e^{-x}} =$$

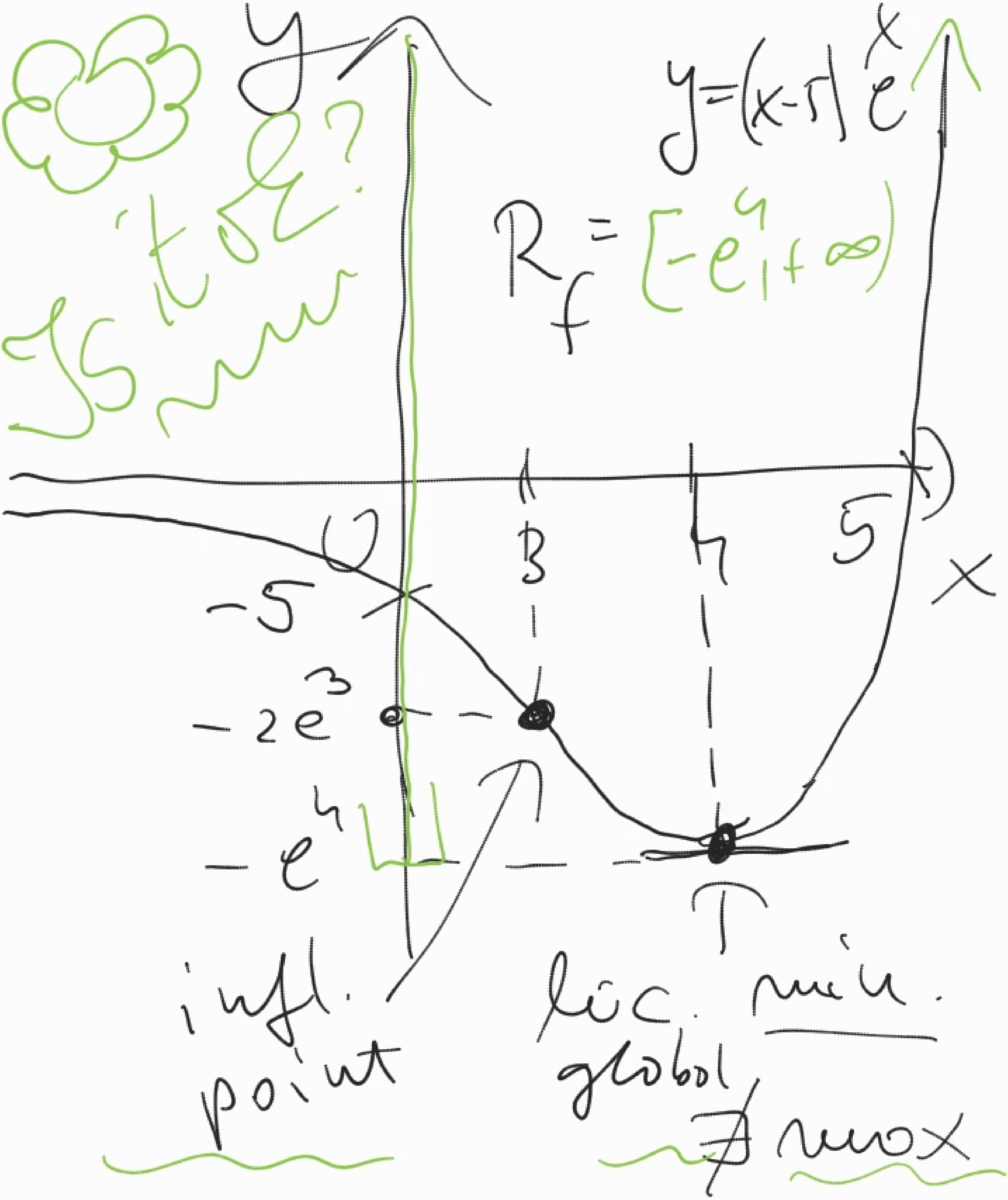
$$= \frac{1}{-\underbrace{e^{+\infty}}_{+\infty}} = \frac{1}{-\infty} = \boxed{-0}$$

STEP 5 Table

x	$-\infty$		3		4		$+\infty$
$f'(x)$		$-$		$-$	0	$+$ $+$	
$f(x)$	-0	$\searrow$		$\searrow$	$-e^4$	$\nearrow$	$+\infty$
$f''(x)$		$-$ $-$	0	$+$	$+$	$+$ $+$	
Convexity							

$$f(4) = e^4(4-5) = -e^4$$

STEP 6 GRAPH.



Is it ok?
y
$y=(x-5)e^x$
$R_f = [-e^4, +\infty)$
0
3
4
5
x
-5
$-2e^3$
$-e^4$
infl. point
loc. min. global
∄ max