

Analysis 2, Test 2 Sample 1

1. Give the following integrals :

i) $\int \frac{\sin x \cdot \cos x}{\cos^4 x - \sin^4 x} dx \quad (x \in I := (0; \pi/4)).$

ii) $\int \frac{(x + \ln x)^2}{x} dx \quad (x \in I := (0; +\infty)).$

2. Find the area of the region between the graphs of the functions $f(x) = x - 1$ ($x \in \mathbb{R}$) and $g(x) = 1 - x^2$ ($x \in \mathbb{R}$).

3. Rotate the graph of the function

$$f(x) = \cos x + \sqrt{2 \cdot \sin x} \quad (x \in [0; \pi/2])$$

around the axis x and find the volume of the solid you get this way.

4. Give the local extremum places and values for the function :

$$f(x, y) := x^3 - 3xy + y^2 \quad ((x, y) \in \mathbb{R}^2).$$

5. Evaluate the double integral of the function $f(x, y) := x \cdot e^y$ ($(x, y) \in \mathbb{R}^2$) on the bounded and closed region enclosed by the following curves :

$$y = x^2, y = 2 - x, y = 0.$$

Analysis 2, Test 2 Sample 2

1. Give the following integrals :

i) $\int_4^5 \frac{2x^2 - 12x + 14}{(1 - x) \cdot (x - 3)^2} dx.$

ii) $\int_2^3 \frac{x}{\sqrt{x^2 - 4}} dx.$

2. Evaluate the following improper integral :

$$\int_0^{+\infty} (x^2 + 1) \cdot e^{-2x} dx.$$

3. Find the arclength of the graph of the function $f(x) = \frac{x^3}{6} + \frac{1}{2x}$ ($x \in [1/2; 1]$).

4. Integrate the function $f(x) = xy + \frac{z}{y}$ ($x \in [0; \pi/2]$) on the 3-dimensional interval :

$$I := [1, 2] \times [1, 4] \times [-1; 0].$$

5. Find the absolute extremum places and values for the function :

$$f(x, y) := y \cdot (2x - 3) \quad ((x, y) \in A),$$

where A is the closed and bounded region enclosed by the curves :

$$y = x^2, y = 0, x = 2.$$