3) $f(x) = x \cdot e^{-x} \quad (x \in \mathbb{R})$

1) Intersection with axes

$x = 0 \Rightarrow f(0) = 0$

$y = 0 \Rightarrow f(x) = 0 \Rightarrow x = 0$

2) f' and its sign

$f'(x) = e^{-x} - e^{-x} \cdot x = e^{-x}(1 - x)$

$f'(x) = 0 \Leftrightarrow (1 - x) = 0, \; x = 1$

$f'(x) > 0 \Leftrightarrow x < 1$

$f'(x) < 0 \Leftrightarrow x > 1$

3) f'' and its sign (convexity)

$f''(x) = (2 - x) \cdot e^{-x}$

$f''(x) = 0 \Leftrightarrow x = 2$

$f''(x) > 0 \Leftrightarrow x < 2$

$f''(x) < 0 \Leftrightarrow x > 2$

4) Limits and asymptotes

$$\lim_{x \to \pm\infty} x \cdot e^{-x} = \lim_{x \to \pm\infty} \frac{x}{e^x} = \frac{\infty}{\infty} \overset{LH}{=} \lim_{x \to \pm\infty} \frac{1}{e^x} = \frac{\pm 0}{=}$$