Practice 2.

1.) $A = \{1, 2, 3, 4\}$

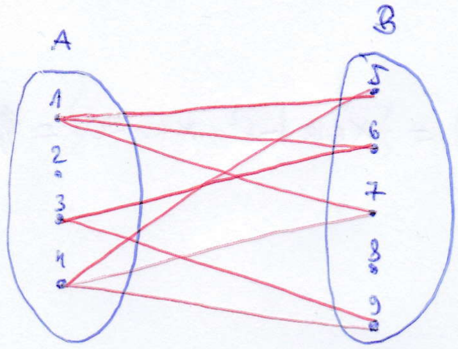
$B = \{5, 6, 7, 8, 9\}$

$\rho \subseteq A \times B$; $\rho = \{(1,5), (1,6); (1,7); (3,6); (3,9), (4,5); (4,7); (4,9)\}$

a.) $dmn(\rho) = \{1, 3, 4\}$

$rng(\rho) = \{5, 6, 7, 9\}$

b.)



c.) $H_1 = \{1, 2, 3\}$; $H_2 = \{4\}$

$\rho|_{H_1} = \{(1,5); (1,6); (1,7); (3,6); (3,9)\}$

$\rho|_{H_2} = \{(4,5); (4,7); (4,9)\}$

d.) ρ_1 : nope

ρ_2 : nope

ρ_3 : ✓

ρ_4 : nope

e.) $\rho^{-1} = \{(5,1); (6,1); (7,1); (6,3); (9,3); (5,4); (7,4); (9,4)\}$

$\rho(\{1,2\}) = \{5, 6, 7\}$

$\rho^{-1}(\{5,6\}) = \{1, 3, 4\}$

2.)
$\rho \subseteq \mathbb{Z} \times \mathbb{Z}$; $\rho = \{(a,b) \in \mathbb{Z} \times \mathbb{Z} \mid a = 2b\}$

$dmn(\rho) = \{$ páros számok $\}$

$rng(\rho) = \mathbb{Z}$

$\rho^{-1} = \{(a,b) \in \mathbb{Z} \times \mathbb{Z} \mid 2a = b\}$

$\rho(\{3,4,\dots,10\}) = \{2,3,4,5\}$

$\rho|_{\{1,2,\dots,6\}} = \{(2,1);(4,2),(6,3)\}$

3.) $R = \{(x,y) \in \mathbb{R} \times \mathbb{R} \mid y^2 = 2 - x - x^2\}$

$R(\{0\}) = \cancel{\{\pm\sqrt{2}\}} = \{-\sqrt{2}, \sqrt{2}\}$ $\qquad R^{-1}(\{0\}) = \{x \in \mathbb{R} \mid -x^2 - x + 2 = 0\} = \{2, +1\}$

$R^{-1} = \{(x,y) \in \mathbb{R} \times \mathbb{R} \mid x^2 = 2 - y - y^2\}$

$R(A) = \{y\}$; $y \in \mathbb{R}$; $A = ?$

$= \{y \in \mathbb{R} \mid \exists x \in A : (x,y) \in R\} = \{y \in \mathbb{R} \mid \exists x \in A : y^2 = 2 - x - x^2\} \Rightarrow y = 0$

$\Rightarrow \boxed{A_1 = \{-2, 1\} ; \quad A_2 = \{1\} ; \quad A_3 = \{-2\}}$

$R^{-1}(A) = \{x\}$; $x \in \mathbb{R}$; $A = ?$

$= \{x \in \mathbb{R} \mid \exists y \in A : (x;y) \in R\} \Rightarrow \exists! x : -x^2 - x + 2 = y^2$

$\Rightarrow -(x^2 + 2 \cdot \frac{1}{2} \cdot x + \frac{1}{4}) + \frac{9}{4} = y^2$

$\Rightarrow -(x + \frac{1}{2})^2 + \frac{9}{4} = y^2 \Rightarrow x = -\frac{1}{2}$

$\Rightarrow \frac{9}{4} = y^2 \Rightarrow y = \pm\frac{3}{2}$

$\boxed{\Rightarrow A_1 = \{-\frac{3}{2} ; \frac{3}{2}\} ; A_2 = \{-\frac{3}{2}\} ; A_3 = \{\frac{3}{2}\}}$

4.)

$A = \{1,2,3\}$

$B = \{a,b,c,d,e,f\}$

$C = \{2,4,6,8\}$

$R \subseteq A \times B$; $R = \{(1,a),(1,b),(2,c);(2,f);(3,d);(3,e);(3,f)\}$

$S \subseteq B \times C$; $S = \{(a,2);(a,4);(c,6);(c,8);(d,2),(d,4);(d,6);(f,8)\}$

$S \circ R = \{(x,z) \mid \exists y : (x,y) \in R \,,\, (y,z) \in S\}$

$S \circ R = \{(1,2);(1,4);(2,6);(2,8);(3,2);(3,4);(3,6);(3,8)\}$

6.) a.) $R = \{(x,y) \in \mathbb{R} \times \mathbb{R} \mid 4x = y^2 + 6\}$

$S = \{(y,z) \in \mathbb{R} \times \mathbb{R} \mid y - 1 = z\} = \{(y,z) \in \mathbb{R} \times \mathbb{R} \mid y = z + 1\}$

$S \circ R = \{(x,z) \in \mathbb{R} \times \mathbb{R} \mid 4x = y^2 + 6 \wedge y = z + 1\} = \{(x,z) \in \mathbb{R} \times \mathbb{R} \mid 4x = (z+1)^2 + 6\}$

$S = \{(x,y) \in \mathbb{R} \times \mathbb{R} \mid x - 1 = y\}$

$R = \{(y,z) \in \mathbb{R} \times \mathbb{R} \mid 4y = z^2 + 6\}$

$R \circ S = \{(x,z) \in \mathbb{R} \times \mathbb{R} \mid \underline{x - 1 = y \wedge 4y = z^2 + 6}\} = \underline{\{(x,z) \in \mathbb{R} \times \mathbb{R} \mid 4(x-1) = z^2 + 6\}}$

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b.) $R = \{(x,y) \in \mathbb{R} \times \mathbb{R} \mid x = 2y\}$

$S = \{(y,z) \in \mathbb{R} \times \mathbb{R} \mid z = y^3\}$

$S \circ R = \{(x,z) \in \mathbb{R} \times \mathbb{R} \mid x = 2y \wedge y^3 = z\} = \{(x,z) \in \mathbb{R} \times \mathbb{R} \mid \left(\frac{x}{2}\right)^3 = z\}$

$\Downarrow$

$y = \frac{x}{2}$

$S = \{(x,y) \in \mathbb{R} \times \mathbb{R} \mid x^3 = y\}$

$R = \{(y,z) \in \mathbb{R} \times \mathbb{R} \mid y = 2z\}$

$R \circ S = \{(x,z) \in \mathbb{R} \times \mathbb{R} \mid x^3 = y \wedge y = 2z\} = \{(x,z) \in \mathbb{R} \times \mathbb{R} \mid x^3 = 2z\}$