Analysis 2, Test 1, 14th of May 2020

Part 1

1. (10 points) Study the continuity and differentiability of the following function (a, b are real parameters) :

$$f(x) = \begin{cases} 1 + x - a \cdot \sin x, \text{ if } x \in (-\infty; 0) \setminus \{-\pi\}; \\ 1, \text{ if } x = -\pi; \\ b \cdot e^x + x, \text{ if } x \in [0; +\infty). \end{cases}$$

2. ($4 + 6$ points) i) Determine the tangent line to the graph of the following function at point $a = 0$:

$$f(x) := \sqrt{1 + \tan x} \ \ (x \in (-\pi/4; \pi/4)).$$

ii) Evaluate the following limit :

$$\lim_{x \to 1} \frac{x^x - x}{\ln x - x + 1}.$$

3. (10 points) Discuss the following function and sketch its graph :

$$f(x) := \left(\frac{1+x}{1-x}\right)^2 \ \ (x \in \mathbb{R} \setminus \{1\}).$$

Part 2

1. ($6 + 6$ points) Give the following integrals :

i) $\displaystyle\int \frac{\sin^3 x + \sqrt{\tan x}}{\cos^2 x}\, dx \quad (x \in I := (0; \pi/2)).$

ii) $\displaystyle\int \frac{3x - 1}{(x+3) \cdot (x^2 - 4x + 4)}\, dx \quad (x \in I := (-3; 2)).$

2. (8 points) Rotate the graph of the function

$$f(x) = x + \sin x \ \ (x \in [0; \pi])$$

around axis x and find the volume of the solid you get this way.

3. (10 points) Give the local extremum places and values for the function :

$$f(x, y) := x^4 + y^3 - 2x^2 - 3y^2 \ \ ((x, y) \in \mathbb{R}^2).$$