Programming theory
1. midterm-exam
group B

You are allowed to use the short form $(a_1, \dots a_n)$ in order to denote the state $\{v_1{:}a_1, \dots v_n{:}a_n\}$.

1. Let $A = [1..5]$ be a statespace and $S \subseteq A \times (\bar{A} \cup \{fail\})^{**}$ be the following program:

$$S = \begin{Bmatrix} 1 \to <1,2,1> & 1 \to <1,4,5,1> & \\ 2 \to <2,4,5,2> & 3 \to <3, fail> & 3 \to <3,2,1> \\ 4 \to <4,1,5,4,2> & 4 \to <4,3,1,2,5> & \\ 5 \to <5,2,1> & 5 \to <5,3> & 5 \to <5> \end{Bmatrix}$$

   Let $F \subseteq A \times A$ be the following problem: $F = \{\, (1,1), (1,3), (2,2), (5,1), (5,5) \,\}$

   - Determine the following four sets: $S(1)$, $D_{p(S)}$, $p(S)(5)$, $p(S)$
   - Does S solve F? Detailed explanation is required.
   - Determine the weak program function of S, provide its domain as well.

   (12 points)

2. Consider the statespace A, program S and problem F that were given in task 1.

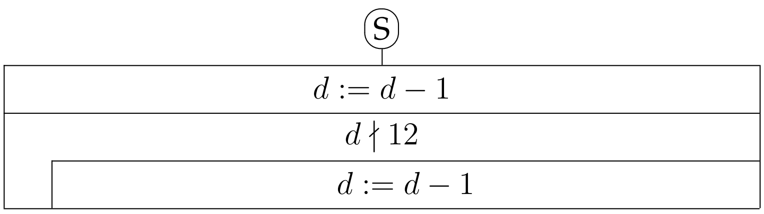
   (a) Let $Q, R : A \to \mathbb{L}$ be logical functions given such that $\lceil R \rceil = \{1, 2, 5\}$ and $\lceil Q \rceil = \{5\}$.

   - Determine the truth-set $\lceil wp(S, R) \rceil$.
   - Decide whether 5 is an element of $\lceil wp(S, Q) \rceil$.

   (6 points)

   (b) Provide a program $S_2 \subseteq A \times (\bar{A} \cup \{fail\})^{**}$, such that S_2 is a function (deterministic relation) and S_2 solves problem F. Detailed explanation is required. (6 points)

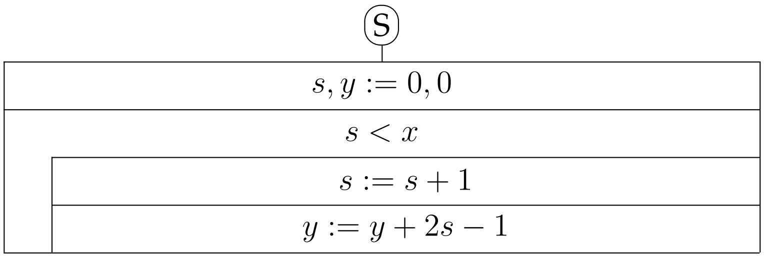
3. (a) $A = (d{:}\mathbb{N})$



   - Write down the sequences assigned to the states $\{d{:}15\}$, $\{d{:}8\}$ and $\{d{:}1\}$ by the program S.
   - What does the program function of S assign to the state $\{d{:}10\}$?

   (6 points)

(b) $A = (x:\mathbb{N}, y:\mathbb{N})$



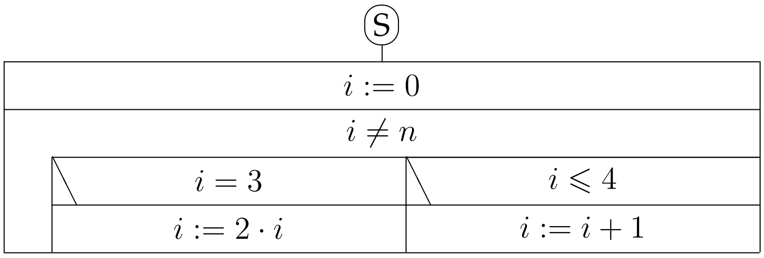
A is the base-statespace and $s:\mathbb{N}$ is an auxiliary (temporary) variable of the program S.
Write down the sequences assigned to the states $\{x:3, y:5\}$ and $\{x:4, y:5\}$ by the program S. (6 points)

4. (a) $A = (i : \mathbb{N}, n : \mathbb{N})$

Write down the sequences that are assigned to the states

- $\{i:4, n:6\}$ and
- $\{i:5, n:7\}$

by the following S program.



(5 points)

(b) Let $A = [1..4]$ be a statespace and let $S_1, S_2 \subseteq A \times (\bar{A} \cup \{fail\})^{**}$ be programs over A.

$$S_1 = \begin{Bmatrix} 1 \to < 1, \dots > & 2 \to < 2, 1 > & 2 \to < 2, 4 > \\ 3 \to < 3, 1, 4 > & 3 \to < 3, 2, fail > & 4 \to < 4, 1, 3 > \end{Bmatrix}$$

$$S_2 = \begin{Bmatrix} 1 \to < 1, 3, 2 > & 1 \to < 1, 2, 4 > & 2 \to < 2, \dots > \\ 3 \to < 3 > & 4 \to < 4, fail > & 4 \to < 4, 2, 1 > \end{Bmatrix}$$

- Determine the sequence $(S_1; S_2)$ of the two given programs.
- Let $\pi_1, \pi_2 \in A \to \mathbb{L}$ logical functions, such that $\pi_1 = \{(1, false), (2, true), (3, true)\}$ and $\pi_2 = \{(1, false), (2, true)\}$. Determine the selection (branches, IF statement) $(\pi_1:S_1, \pi_2:S_2)$.

(7 points)

5. (a) Let $S_1, S_2 \subseteq A \times (\bar{A} \cup \{fail\})^{**}$ be any arbitrary programs over the same base-statespace A.
Decide whether $S_1 \setminus S_2$ is a program over A as well. (3 points)

(b) Let $A = (x{:}\mathbb{N}, h{:}\mathbb{N})$ be a statespace and $Q, R \in A \to \mathbb{L}$ be logical functions such that $Q = (d = 1 \wedge h = 10)$ end $R = (h = 10^d \wedge 10^{d-1} \leqslant x)$. Decide whether $Q \implies R$ is true or not. (4 points)

(c) Let $A = [1..5]$, $S_0 \subseteq A \times (\bar{A} \cup \{fail\})^{**}$ be program, and $\pi\colon A \to \mathbb{L}$ such that $\lceil \pi \rceil = \{2, 3, 4, 5\}$.

$$S_0 = \begin{Bmatrix} 1 \to < 1,2,4 > & 2 \to < 2 > & 3 \to < 3,4,2 > \\ 3 \to < 3,5 > & 3 \to < 3,3,3,\ldots > & 4 \to < 4,5,3,4 > \\ 4 \to < 4,1,3 > & 5 \to < 5,5,\ldots > & \end{Bmatrix}$$

Determine the loop (π, S_0), we can denote by DO as well. (5 points)