5) $f(x,y) = (x^2+y^2)\,e^{-y} \qquad ((x,y)\in \mathbb{R}^2)$

$\partial_1 f(x,y) = 2x \cdot e^{-y} = 0 \Rightarrow x = 0$

$\partial_2 f(x,y) = -e^{-y}(x^2+y^2) + 2y\,e^{-y} = -e^{-y}(x^2+y^2-2y) = 0$

$\Rightarrow y^2 - 2y = 0 \Rightarrow y = 0,\ y = 2$

possible local extrema in $(0,0)$ and $(0,2)$

$$f''(x,y) = \begin{pmatrix} 2e^{-y} & -2xe^{-y} \\ -e^{-y}\cdot 2x & -e^{-y}(2y-2+x^2+y^2-2y) \end{pmatrix}$$

$$f''(0,0) = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \Rightarrow \det = 4 > 0 \quad \overset{f_{11}=2>0}{\text{local min in } (0,0)}$$

$$f''(0,2) = \begin{pmatrix} 2e^{-2} & 0 \\ 0 & -2\cdot e^{-2} \end{pmatrix} \Rightarrow \det = -4e^{-4} < 0 \text{ and } f_{11} = 2e^{-2} > 0$$

no local ext. in $(0,2)$