Problem set 5.:
Complex numbers – algebraic and polar forms of complex numbers

Question 1.
Calculate the following, giving your answers in algebraic form.

$\sqrt{-16}$ $\sqrt{-25}$ $(2i)^2$ $2i+5i$ $\dfrac{4i}{2i}$

Question 2.
Let $z \in \mathbb{C}, z = -2+7i$. Determine the following:

$\operatorname{Re} z$ $\operatorname{Im} z$ $-z$ $\overline{z}$ $|z|$

Question 3.
Calculate the value $\dfrac{4+3i}{(2-i)^2}$ giving your answer in algebraic form.

Question 4.
Solve the following equation on the set of complex numbers: $\dfrac{x+i-3i\overline{x}}{x-4} = i-1$.

Question 5.
Find the complex number(s) $z \in \mathbb{C}$ satisfying the conditions:

$$\left|\frac{z-3}{2-\overline{z}}\right| = 1 \wedge \operatorname{Re}\left(\frac{z}{2+i}\right) = 2$$

Question 6.
Let $z \in \mathbb{C}, z = 2+5i$. Find the absolute value and the argument of z. Represent z on the complex plane (also called Gaussian plane).

Question 7.
Write the following complex numbers in polar form:

(a) $1+i$
(b) $-\sqrt{3}+i$
(c) $\dfrac{9}{2} - \dfrac{9\sqrt{3}}{2}i$
(d) $-\dfrac{\sqrt{14}}{2} - \dfrac{\sqrt{14}}{2}i$
(e) $4i$
(f) i
(g) 10

Question 8.
Calculate the following, using the polar form of complex numbers:

(a) $\left(\frac{9}{2} - \frac{9\sqrt{3}}{2}i\right)\left(-\frac{\sqrt{14}}{2} - \frac{\sqrt{14}}{2}i\right)$

(b) $\left(-\frac{3\sqrt{3}}{2} - \frac{3}{2}i\right)\left(\frac{\sqrt{3}}{3} + \frac{1}{3}i\right)$

(c) $\dfrac{-\frac{3\sqrt{3}}{2} - \frac{3}{2}i}{\frac{\sqrt{3}}{3} + \frac{1}{3}i}$

(d) $\left(\frac{5\sqrt{3}}{12} - \frac{5}{12}i\right)^{10}$

(e) $\left(-\frac{\sqrt{10}}{2} - \frac{\sqrt{10}}{2}i\right)^{15}$

(f) $\left(\frac{5}{2} - \frac{5\sqrt{3}}{2}i\right)^{23}$

(g) $(1+i)^8 \cdot \left(5\sqrt{3} - 5i\right)^3$

(h) $\left(\dfrac{\frac{3}{2} + \frac{3\sqrt{3}}{2}i}{-\frac{5\sqrt{3}}{2} + \frac{5}{2}i}\right)^{12}$

(i) $\left(1 - \dfrac{\sqrt{3} - i}{2}\right)^{24}$

Question 9.
Determine the complex roots below:
(a) 2^{nd} roots of -60;
(b) 3^{rd} roots of -60;
(c) 6^{th} roots of $1 - \sqrt{3}i$;
(d) 5^{th} roots of $-7\sqrt{3} + 7i$;
(e) 8^{th} roots of $-\frac{7}{2} + \frac{7}{2}i$;
(f) 2^{nd} roots of $-6\sqrt{3} + 6i$;
(g) 7^{th} roots of $\dfrac{\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^8}{(1+i)^5}$;

Question 10.

Using the polar form of complex numbers, calculate the value of $z = \dfrac{\left(2 + 2\sqrt{3}i\right)^{10}}{(-1+i)^{83}}$, giving your answer both in algebraic and in polar forms. Find all complex numbers w such that $w^3 = z$, giving your answers in polar form.

Question 11.

Express $z = \dfrac{(1+i)^8}{(1 - \sqrt{3}i)^6}$ in algebraic form.