II) $Q' \Rightarrow wp(DO, R)$

Instead of proving this, we prove 5 other conditions.

1. $Q' \Rightarrow Inv$

$(\underset{\sim}{Q} \wedge d = \min(x,y)) \Rightarrow (\underset{\sim}{Q} \wedge \forall k \in [d+1 .. \min(x,y)] : \neg(k|x \wedge k|y))$

Now $d = \min(x,y)$, then the interval $[\min(x,y)+1 .. \min(x,y)]$ is empty. $\underbrace{\forall k \in \emptyset : \sim\sim}_{TRUE}$

2. $Inv \wedge \neg \pi \Rightarrow R$

In our case: $(Q \wedge \forall k \in [d+1 .. \min(x,y) : \neg(k|x \wedge k|y)) \wedge \underbrace{\neg\neg(d|x \wedge d|y)}_{d|x \wedge d|y}$

equals to R.

$d|x \wedge d|y$; the negation of the loop condition

$Inv \wedge \neg\pi \Rightarrow R$ ✓

3. $Inv \Rightarrow \pi \vee \neg\pi$

$Inv \Rightarrow \underbrace{\neg(d|x \wedge d|y) \vee (d|x \wedge d|y)}$

this is not TRUE, but cannot be undefined as $d \neq 0$ because $d : \mathbb{N}^+$

4. $Inv \wedge \pi \Rightarrow t > 0$

$Inv \wedge \pi \Rightarrow d > 0$ ✓, as $d : \mathbb{N}^+$

5. $Inv \wedge \pi \wedge t = t_0 \Rightarrow wp(S_0, Inv \wedge t < t_0)$ for any $t_0 \in \mathbb{Z}$

$Inv \wedge \neg(d|x \wedge d|y) \wedge d = t_0 \Rightarrow \underbrace{wp(d := d-1, Inv \wedge d < t_0)}_{(Inv \wedge d < t_0)^{d \leftarrow d-1} \wedge d-1 \in \mathbb{N}^+}$

RHS formula:

$(\underset{\sim}{Q} \wedge \underbrace{\forall k \in [d .. \min(x,y)] : \neg(k|x \wedge k|y)}_{\text{break the formula into two parts}} \wedge d-1 < t_0 \wedge d-1 \in \mathbb{N}^+)$

↑ included in Inv

$\forall k \in [d+1 .. \min(x,y)] : \neg(k|x \wedge k|y)$ ✓ since Inv

$\neg(d|x \wedge d|y)$ ✓ since π

$\underbrace{d-1 < t_0}_{\text{✓ as } d = t_0} \wedge \underbrace{d-1 \in \mathbb{N}^+}_{\text{✓ as } d : \mathbb{N}^+ \text{ and } d \neq 1 \ (\pi)}$