

$$Q \Rightarrow I_w$$

$$(Q \wedge y=1 \wedge b=x \wedge i=n) \Rightarrow (Q \wedge y \cdot b^i = x^n)$$

We want to prove: in case LHS formula is true, then RHS formula is also true.

$$Q \wedge y \cdot b^i = x^n$$

$$1 \cdot x^n = x^n \quad \checkmark \quad \text{this is true}$$

$$2. I_w \wedge \neg \pi \Rightarrow R$$

$$(Q \wedge y \cdot b^i = x^n) \wedge i \leq 0 \Rightarrow (Q \wedge y = x^n)$$

As $i \leq 0$ and $i: \mathbb{N}$, then $i=0$.

$$\left. \begin{array}{l} y \cdot b^i = x^n \\ i = 0 \end{array} \right\}$$

$$y \cdot b^0 = x^n$$

$$y \cdot 1 = x^n$$

$$3. I_w \Rightarrow \pi \vee \neg \pi$$

$$I_w \Rightarrow i > 0 \vee i \leq 0$$

TRUE, as $i: \mathbb{N}$

$$4. I_w \wedge \pi \Rightarrow t > 0$$

$$(Q \wedge y \cdot b^i = x^n \wedge i > 0) \Rightarrow i > 0$$

In case the LHS formula is true, then $i > 0$ holds.

$$5. I_w \wedge \pi \wedge t = t_0 \Rightarrow \text{wp}(S_0, I_w \wedge t < t_0) \quad \text{for any } t_0 \in \mathbb{Z}$$

Now S_0 is a selection - IF statement - with two branches.

It is enough to prove 4 other conditions:

$$\alpha) I_w \wedge \pi \wedge t = t_0 \Rightarrow (2|i \vee 2 \nmid i) \wedge (2 \nmid i \vee 2|i) \quad \checkmark$$

TRUE, as $i: \mathbb{N}$, every natural number is either odd or even

$$\beta) I_w \wedge \pi \wedge t = t_0 \Rightarrow 2|i \vee 2 \nmid i \quad \checkmark$$

$$\gamma) I_w \wedge \pi \wedge t = t_0 \wedge 2|i \Rightarrow \text{wp}(i, b := i/2, b^2, I_w \wedge t < t_0)$$

$$(Q \wedge y \cdot b^i = x^n \wedge i > 0 \wedge i = t_0 \wedge 2|i) \Rightarrow (I_w \wedge t < t_0) \wedge 2|i$$

I_w

$$(Q \wedge y \cdot (b^2)^{i/2} = x^n \wedge i/2 < t_0 \wedge 2|i) =$$

$$(Q \wedge y \cdot b^i = x^n \wedge i/2 < t_0 \wedge 2|i)$$

I_w

$\left. \begin{array}{l} i: \mathbb{N} \\ i > 0 \\ i = t_0 \end{array} \right\} \checkmark$

$$\delta) I_w \wedge \pi \wedge t_0 = t \wedge 2 \nmid i \Rightarrow \text{wp}(i, y := i-1, y \cdot b, I_w \wedge t < t_0)$$