Programming theory

1st midterm exam - sample

You are allowed to use the short form $(a_1, \dots a_n)$ in order to denote the state $\{v_1{:}a_1, \dots v_n{:}a_n\}$.

1. Let $A = [1..5]$ be a statespace, $S \subseteq A \times (A \cup \{fail\})^{**}$ a program over the statespace A.

$$S = \left\{ \begin{array}{lll} 1 \to < 1,2,5,1 > & 1 \to < 1,4,3,5,2 > & 1 \to < 1,3,2,3,\dots > \\ 2 \to < 2,1 > & 2 \to < 2,4 > & 3 \to < 3,3,3,\dots > \\ 4 \to < 4,1,5,4,2 > & 4 \to < 4,3,1,2,5,1 > & 5 \to < 5,2,3,4 > \\ 5 \to < 5,2,fail > & 5 \to < 5,3,4 > & \end{array} \right\}$$

   Let $F \subseteq A \times A$ denote the following problem: $F = \{ (2,1),(2,4),(4,1),(4,2),(4,5) \}$

   (a) Determine the program function of S and its domain.

   (b) Determine the following two sets: $S(1)$ and $p(S)(2)$.

   (c) Decide whether S is totally correct with respect to the given problem F.

(12 points)

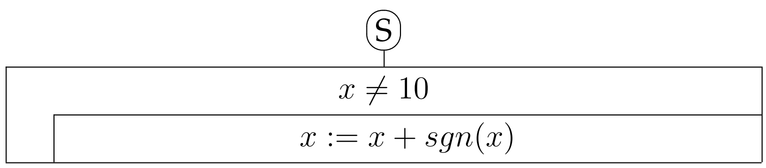
2. Consider the statespace A, program S and problem F that were given in task 1.

   (a) Let $Q, R : A \to \mathbb{L}$ be logical functions given such that $\lceil R \rceil = \{1,5\}$ and $\lceil Q \rceil = \{5\}$.
      - Determine the truth-set $\lceil wp(S,R) \rceil$.
      - Decide whether 4 is an element of $\lceil wp(S,Q) \rceil$.

(6 points)

   (b) Provide a program $S_2 \subseteq A \times (\bar{A} \cup \{fail\})^{**}$, such that S_2 solves problem F. Detailed explanation is required. (6 points)

3. (a) Let $H = \{a \in \mathbb{Z} \mid a \geqslant -5\}$
   $A = (x \colon H)$



   Write down the sequences assigned to the states 4, 13, -2, 0 and 10 by the program S. (6 points)

(b) $A = (x{:}\mathbb{Z}, y{:}\mathbb{Z})$.

S

$z := x$
$x := y$
$y := z$

A is the base-statespace and $z{:}\mathbb{Z}$ is an auxiliary (temporary) variable of the program S.

- Write down the sequences assigned to the state $\{x{:}3, y{:}8\}$.
- What does the programfunction of S assign to the state $\{x{:}3, y{:}5\}$?

(6 points)

4. (a) Let $S_1, S_2 \subseteq A \times A^{**}$ be any arbitrary programs, such that $S_1 \subseteq S_2$ holds. Decide whether $D_{p(S_1)} \subseteq D_{p(S_1)}$ holds or not. (6 points)

   (b) Let $A = [1..6]$ be the common base-statespace of the following programs $S_1, S_2 \subseteq A \times (\bar{A} \cup \{fail\})^{**}$:

$$S_1 = \begin{Bmatrix} 1 \rightarrow < 1,4,3 > & 1 \rightarrow < 1,2,4 > & 2 \rightarrow < 2,2,\ldots > \\ 2 \rightarrow < 2,1,4,6 > & 3 \rightarrow < 3,5,1 > & 4 \rightarrow < 4,5,3 > \\ 5 \rightarrow < 5,1,fail > & 6 \rightarrow < 6,3,1,5 > & \end{Bmatrix}$$

$$S_2 = \begin{Bmatrix} 1 \rightarrow < 1,3,2 > & 1 \rightarrow < 1,2,4 > & 2 \rightarrow < 2,6 > \\ 3 \rightarrow < 3,4 > & 4 \rightarrow < 4,fail > & 4 \rightarrow < 4,5,1 > \\ 5 \rightarrow < 5 > & 6 \rightarrow < 6,4,3,2 > & \end{Bmatrix}$$

   Let $\pi_1, \pi_2 \in A \rightarrow \mathbb{L}$ be logical functions such that
   $\pi_1 = \{(1, true), (2, true), (4, true), (5, false), (6, false)\}$ and
   $\pi_2 = \{(1, true), (2, false), (3, true), (4, true), (5, false)\}$.
   Determine the IF statement, the selection $(\pi_1{:}S_1, \pi_2{:}S_2)$.
   (6 points)

5. (a) Let $A = (a{:}\mathbb{N}, h{:}\mathbb{N})$ be a statespace and $Q, R \in A \rightarrow \mathbb{L}$ be logical functions such that $Q = (a = 10)$ end $R = (h = a^3)$. Decide whether $Q \implies R$ is true or not. (4 points)

   (b) Let $A = [1..5]$, $S_0 \subseteq A \times (\bar{A} \cup \{fail\})^{**}$ be programs, and $\pi\colon A \rightarrow \mathbb{L}$ such that $\lceil \pi \rceil = \{1, 2, 3, 4\}$.

$$S_0 = \begin{Bmatrix} 1 \rightarrow < 1,2,4 > & 2 \rightarrow < 2 > & 3 \rightarrow < 3,4,2 > \\ 3 \rightarrow < 3,5 > & 3 \rightarrow < 3,3,3,\ldots > & 4 \rightarrow < 4,5,3,4 > \\ 4 \rightarrow < 4,1,3 > & 5 \rightarrow < 5,5,\ldots > & \end{Bmatrix}$$

   Determine the loop (π, S_0). (8 points)