Analysis II., Sample Test 2

1. Find the integrals:

a) $\int \frac{5}{3x^2+4}\,dx \quad (x \in \mathbb{R})$ b) $\int \frac{dx}{1+\cos x} \quad (x \in (-\pi, \pi))$

c) $\int \cos^5 x\,dx \quad (x \in \mathbb{R})$ d) $\int \frac{\ln x}{x \cdot (1+\ln^2 x)}\,dx \quad (x > 0)$

2. Find the integrals:

a) $\int \frac{x^2}{x^2-1}\,dx \quad (x \in (-1,1))$ b) $\int \frac{dx}{x^3-4x^2+4x} \quad (x > 2)$

c) $\int x^2 \cdot \cos x\,dx \quad (x \in \mathbb{R})$ d) $\int \arcsin x\,dx \quad (x \in (-1,1))$

3. Find the improper integrals:

a) $\int_0^{+\infty} (x-1) \cdot e^{-x}\,dx$ b) $\int_0^1 \frac{dx}{x(1+\ln^2 x)}$

4. Determine the area bounded by the following curves:

$$y = x^2,\ y = \tfrac{x^2}{2},\ y = 2x$$

5. Determine the arc length of the graph of the following function:

$$f(x) = x^{3/2} \quad (0 \le x \le 4)$$

6. Revolve the following curve around the x-axis and determine the volume of the resulting solid:

$$y = \sin x \quad (0 \le x \le \pi)$$

7. Calculate the following multiple integrals:

a) $\iint_{[1,4]\times[1,2]} e^{x+y}\,dx\,dy$ b) $\iiint_{[1,2]\times[0,1]\times[0,2]} xy^2z^3\,dx\,dy\,dz$

8. Integrate the function $f(x,y) = x^3 + 4y$ over the region bounded by the curves $y = x^2$ and $y = 2x$.

9. Determine the local extreme values of the following function:

$$f(x,y) = x^2 - 4xy + y^3 + 4y \qquad \left((x,y) \in \mathbb{R}^2\right)$$

10. Determine the global extreme values of the following function on the set H:

$$f(x,y) = x^3 + y^3 - x^2 - 2xy - y^2, \qquad H = \left\{(x,y) \in \mathbb{R}^2 \ :\ 0 \le x \le 2,\ 0 \le y \le 2\right\}$$