Programmming theory
2. midterm-exam

1. $A = (w{:}\{0,1\}, r{:}\{0,1\})$ (20 points)

   So we have two variables, w and r, and they can store the values 0 and 1 only.

   – How many sequences are assigned to the state $\{w{:}1, r{:}1\}$ by the program given below?
   – Write down briefly how the program works when starting from the state $\{w{:}1, r{:}1\}$, also provide a sequence assigned to that state.
   – Prove that the following program is free from deadlock. Detailed explanation is required.

   $w, r := 0, 0;$
   $\{w = 0 \wedge r = 0\}$
   parbegin $S_1 \| S_2$ **parend**
   $\{FALSE\}$

S_1:	S_2:
while TRUE do	while TRUE do
$\{Inv \wedge r = 0\}$	$\{Inv \wedge w = 0\}$
await w=0 then	await $w = 0 \wedge r = 0$ then
$r := 1$	$w := 1$
ta;	ta;
$\{Inv \wedge r = 1\}$	$\{Inv \wedge w = 1\}$
$r := 0$	$w := 0$
$\{Inv \wedge r = 0\}$	$\{Inv \wedge w = 0\}$
od	od
$\{FALSE\}$	$\{FALSE\}$

   $Inv = (r = 0 \vee w = 0).$

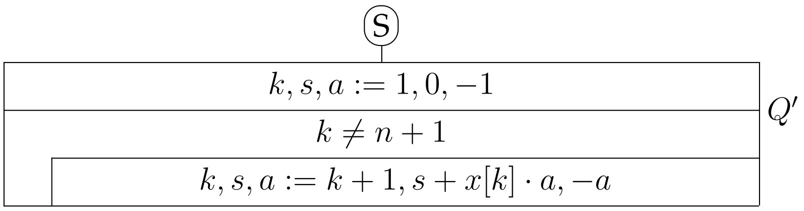
2. A problem is given by its specification:
   $A = (x{:}\mathbb{Z}^n, s{:}\mathbb{N})$ (25 points)

   $Q = (x = x')$

   $R = (Q \wedge s = \sum\limits_{i=1}^{n} (-1)^i \cdot x[i])$

   So, x is an array containing integer numbers, where the first index is 1 and the last index is n (the length of the array).
   Of course, $x[i]$ denotes the element of the array at the index i ($1 \leqslant i \leqslant n$).

   Let S denote the following program:



$Inv = (Q \wedge s = \sum_{i=1}^{k-1} (-1)^i \cdot x[i] \wedge k \in [1..n+1] \wedge a = (-1)^k)$ is the invariant of the loop, the variant function is $n + 1 - k$

$Q' = (Q \wedge s = 0 \wedge k = 1 \wedge a = -1)$ is the interediate condition of the sequence program S.
a:$\mathbb{Z}$ is an auxiliary variable of the program.

Prove that S is totally correct with respect to the given problem. We know that in case $Q \implies wp(S, R)$ holds, then S solves the given problem.
Detailed explanation is required.

3. $A = (x{:}\mathbb{N}^+)$ (15 points)
$Q = (x = x' \wedge even(x))$
$R = (x = 1)$

We have started to prove the total correctness of the following program, that is a parallel block. Let us assume that we have proven deadlock freedom and interference freedom. Write down all the other conditions that are sufficient to prove in order to show the total correctness of the program. Prove all of them.
You have to prove that the following hold:

(a) both components are totally correct
(b) the so-called entry condition holds
(c) the so called exit condition holds

$even(x)$ denotes that x is an even number (can be divided by 2), whereas $odd(x)$ means that $x \bmod 2 = 1$.

$\{x = x' \wedge even(x)\}$
parbegin $S_1 \| S_2$ **parend**
$\{x = 1\}$

S_1:

$\{Inv\}$
while $x > 2$ do
$\{Inv \wedge x > 2\}$
  x:= x−2
$\{Inv\}$
od
$\{Inv \wedge x \leq 2\}$

S_2:

$\{even(x)\}$
x:= x−1
$\{odd(x)\}$

Variant function (we usually denote that by t) is x
Loop invariant is $Inv = (x > 0)$