

**Analysis–2 , Test 3 from Part 1 , 19. December 2019.**

1. (8 points) Study the continuity of the following function :

$$f(x) = \begin{cases} \ln(1-x), & \text{if } x \in (-\infty; 0]; \\ \frac{\sqrt{13-4x}-3}{1-\sqrt{x}}, & \text{if } x \in (0; 13/4] \setminus \{1\}; \\ \frac{12}{13-4x}, & \text{if } x \in (13/4; +\infty) \cup \{1\}. \end{cases}$$

2. (3+3 points) Consider the function :

$$f(x) := \cos^3(x) + \frac{1}{x} \quad (x > 0).$$

a) Prove that the equation  $f(x) = 0$  has at least one solution on the given domain of definition.

b) Give the equation of the tangent line to the graph of  $f$  at the point  $a = \pi$ .

3. (6 points) Evaluate the following limit :

$$\lim_{x \rightarrow 0} \left( \cos(2x) \right)^{1/\tan(x)}.$$

4. (8 points) Evaluate the second order Taylor polynomial at  $a = 0$  for the function :

$$f(x) := \sqrt{2x+1} \quad (x > -1/2),$$

and estimate the error of this approximation on the interval  $(-1/4; +1/4)$ .

5. (12 points) Discuss the following function and sketch its graph :

$$f(x) := (x-5) \cdot e^x \quad (x \in \mathbb{R}).$$

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