basis of vector space

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A set of vectors $\{\bar{v}_1, \bar{v}_2, \dots, \bar{v}_n\}$ is said to form a basis for a vector space. IF:

① The vectors $\bar{v}_1, \bar{v}_2, \dots \bar{v}_n$ span the vector space.

② The vectors $\bar{v}_1, \bar{v}_2, \dots \bar{v}_n$ L.Ind.

For example: $\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}$ are a basis of $\mathbb{R}^3$.

Do the vectors: $\begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}1\\2\\3\end{bmatrix}$,

also form a basis for $\mathbb{R}^3$?

Independ?

$$\left[\begin{array}{ccc|c}1&0&1&0\\1&1&2&0\\1&0&3&0\end{array}\right] \sim \left[\begin{array}{ccc|c}1&0&1&0\\0&1&1&0\\0&0&2&0\end{array}\right] \sim \left[\begin{array}{ccc|c}1&0&1&0\\0&1&1&0\\0&0&1&0\end{array}\right]$$

$-1R_1 + R_2 \to R_2$
$-1R_1 + R_3 \to R_3$

$\frac{1}{2}R_3 \to R_3$

$-1R_3 + R_1 \to R_1$
$-1R_3 + R_2 \to R_2$

$$\sim \left[\begin{array}{ccc|c}1&0&0&0\\0&1&0&0\\0&0&1&0\end{array}\right]$$

$$\begin{array}{ccc} a_1 & a_2 & a_3 \end{array}$$
$$\left[\begin{array}{ccc|c}1&0&0&0\\0&1&0&0\\0&0&1&0\end{array}\right]$$

$\therefore\ a_1 = 0$
$a_2 = 0$ $\Rightarrow$ Trival solution. $\therefore$ L.Ind.
$a_3 = 0$.

Span.

$$a_1\begin{bmatrix}1\\1\\1\end{bmatrix} + a_2\begin{bmatrix}0\\1\\0\end{bmatrix} + a_3\begin{bmatrix}1\\2\\3\end{bmatrix} = \begin{bmatrix}x\\y\\z\end{bmatrix}$$

$$1a_1 + 1a_3 = x$$
$$1a_1 + 1a_2 + 2a_3 = y$$
$$1a_1 + 3a_3 = z.$$

Solve a_1, a_2, a_3.

Use sbstitution.

$$a_1 + a_3 = x \rightarrow a_1 = x - a_3$$
$$a_1 + 3a_3 = z.$$
$$x - a_3 + 3a_3 = z.$$
$$x + 2a_3 = z.$$
$$a_3 = \frac{z-x}{2}$$

$$a_1 = x - \frac{z-x}{2}$$
$$a_1 = \frac{3}{2}x - \frac{z}{2}.$$

$$a_1 + a_2 + 2a_3 = y$$
$$\frac{3}{2}x - \frac{1}{2}z + a_2 + 2\cdot\frac{z-x}{2} = y.$$
$$\frac{3}{2}x - \frac{1}{2}z + z - x + a_2 = y.$$
$$\frac{1}{2}x + \frac{1}{2}z + a_2 = y.$$
$$a_2 = y - \frac{1}{2}x - \frac{1}{2}z$$

$2x + 2z = 4a_2 - y.$

$$a_2 = y - \frac{1}{2}x - \frac{1}{2}z.$$

$$\cancel{a_2 = -\frac{1}{2}x + y - \frac{1}{2}z.}$$

So they ~~are~~ form a spanning set.