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Efficient geometry of flexible solar panels optimized for the latitude of New York City

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ABSTRACT

The long-term goal of the project is to create and justify a reliable mathematical model that expresses the efficiency of geometrical shapes of non-tracking flexible solar panels. However, the amount of solar energy absorbed by a non-tracking flexible solar panel depends on many parameters: the direction of the sun beam, reflected light, and temperature, etc., which would make a complete model mathematically complicated. In the current model, we limit our consideration to the direction of the sunbeam. In order to simulate the exposure of the panel, we describe the trajectory of the Sun and base the model on the mathematical flux that uses the sun rays as the vector field. To be precise, the efficiency of a geometrical panel is defined as the flux density, which is the ratio of the mathematical flux and the surface area. Our current model was evaluated for the latitude of New York City and we determined the efficiency of the optimized flat panels, cylindrical panels, and conical panels. The analysis was largely done through geometrical studies and numerical integration with software programs Python, Maple, Mathematica, and MATLAB.

Keywords: Flexible Solar Panels, New York, cylindrical shape, simulation, optimization, Python, Mathematica

1. INTRODUCTION

The goal of this project is to provide a consistent analysis of geometrical surfaces that may serve as shapes of flexible solar panels. The flexible technology was introduced in 2011, as described by Barr *et al.*,[1] indicating that solar panels can be printed on medium as flexible and fragile as paper. Since shaped panels offer more options than flat panels, providing a complete mathematical model is necessary before experiments can take place. Thus the results of this project may in the future provide a starting point for the experiments with shaped panels at particular locations.

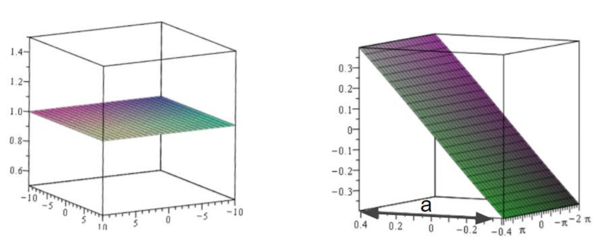

Figure 1. Flat Panel

Figure 2. Catenoid

Figure 3. Cylinder

This article focuses on New York City latitude. The main motivation of this choice is our own location and the availability for future experiments, where we can experimentally verify our results using geometrical panels.

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Table 1. Comparison of Illumination Times

Day	Model Illumination Time	Actual Illumination Time in 2018[4]
Winter Solstice $w = 0$	2.37 rad=9.07 hrs	9 hrs 15 min 18 sec
Spring Equinox $w = 91.25$	π rad=12 hrs	12 hrs
Summer Solstice $w = 182.5$	3.9 rad =14.92 hrs	15 hrs 5 min 36 sec
Fall Equinox $w = 273.75$	π rad=12 hrs	12 hrs

In the prequel, Marciniak *et al.*[2] and Pandya *et al.*[3] analyzed some geometrical shapes at the North Pole, the Equator, and the geostationary satellite above the Equator. All those locations, even if geographically distant, have the property that the length of the day remains (nearly) constant throughout the year. This is not the case in our current work, which makes the model more complicated and lengthens the time of calculations by computer software. We optimize the flat panel presented of Figure 1, a section of a catenoid presented on Figure 2, and a section of a cylinder presented on Figure 3.

2. THE SUN

The trajectory of the Sun on the Celestial Sphere has been studied for centuries. However, for the purpose of completeness of the project, we describe the position of the sun using a particular coordinate system that is convenient for our calculations. The z axis is chosen to be perpendicular to the Plane of the Sun, directing north. The y axis points towards the East, which is defined as the position of the "sunrise" on the day of the Spring Equinox. The x axis is perpendicular to both z and y such that the coordinate system x, y, z is positively oriented. The trajectory of the sun on day w after the Winter Solstice can be modeled according to the

$$\vec{F}(t) = \langle \cos\delta_w \sin t, \cos\delta_w \cos t, \sin\delta_w \rangle \,, \tag{1}$$

where $t \in [0, 2\pi]$ is the "time" of one revolution. Here δ_w is the declination angle approximated as follows:

$$\delta_w = 0.41 - \frac{0.41\,|w - 182.5|}{91.25}. \tag{2}$$

The value of 91.25 is the number of days in a quarter of a year: $\frac{365}{4} = 91.25$ and 182.5 is the number of days in a half year $\frac{365}{2} = 182.5$. For location with the latitude ϕ on the Northern Hemisphere, the range of the parameter t is determined by the inequality $x\cos\phi + z\sin\phi > 0$, which describes the position of the plane of the sun for a location with latitude ϕ (excluding $\phi = 90^0$ which is the North Pole), thus

$$\cos\delta_w \sin t \cos\phi + \sin\delta_w \sin\phi > 0, \tag{3}$$

which implies that

$$\sin t > -\tan\phi \tan\delta_w. \tag{4}$$

Thus

$$\arcsin(-\tan\phi\tan\delta_w) < t < \pi - \arcsin(-\tan\phi\tan\delta_w). \tag{5}$$

Substituting the latitude of NYC $\phi = 40.7128^0 = 0.71rad$, one obtains illumination times (including sunrise and sunset, later denoted as t_{min} and t_{max}, respectively) for each day. Table 1 compares the illumination times during the Solstices and Equinoxes from the model and the actual observations in 2018.[4] The difference in time (about 10 minutes per day) comes partially from the approximations in the model but mainly from the assumption that the sunrise and the sunset occur when the center of the sun passes the horizon. For the actual observations the sunrise and the sunset happen when the edge of the sun passes the horizon.

3. THE METHOD

Flux is a scalar quantity that measures the flow through a surface and is usually introduced among Multivariable Calculus topics as by Steward.[5] In the formula for the flux Φ, the surface of the panel is denoted by S and the vector field $\vec{F}$ is the radiation of the Sun parameterized by its trajectory on the Celestial Sphere as described by the equation:

$$\Phi = \iint_S \vec{F} \cdot d\vec{S} = \iint_D \vec{F} \cdot \vec{n}\, dA. \tag{6}$$

Here $\vec{n}$ denotes the normal vector to the surface S and D is the range of its parametrization. The daily flux $\mathcal{F}_w$ on day w can be written as:

$$\mathcal{F}_w = \int_{t_{min}}^{t_{max}} \Phi\, dt = \iiint_E \vec{F} \cdot \vec{n}\, dA\, dt. \tag{7}$$

The range for the parameter $t \in [t_{min}, t_{max}]$, is determined for latitude ϕ according to equation 5. Here $E = \{(t, u, v) \in T \times D : \vec{F}(t) \cdot \vec{n} > 0\}$ and the condition $\vec{F}(t) \cdot \vec{n} > 0$ is implied by the fact that the solar panel is not losing energy if the sun is shining from the opposite direction than the normal vector $\vec{n}$. Thus restricting the parameters to the region E limited by this condition prevents subtracting the "negative" part of the mathematical flux.

The daily efficiency on day w is defined to be the ratio of the daily flux $\mathcal{F}_w$ and the area of the panel, and is called the Daily Exposure $\mathcal{DE}_w$:

$$\mathcal{DE}_w = \frac{\mathcal{F}_w}{Area}. \tag{8}$$

The Annual Exposure $\mathcal{AE}$ is the sum of Daily Exposures over the entire year, the summation for 365 days is expressed as:

$$\mathcal{AE} = \sum_{w=0}^{364} \mathcal{DE}_w. \tag{9}$$

4. ANALYSIS OF THE PANELS

Table 2. Flat Tilted Panel in NYC latitude

Value of a	Annual Exposure
$a = -0.8$	541.125
$a = -0.4$	640.733
$a = 0$	694.009
$a = 0.4$	654.212
$a = 0.8$	563.796

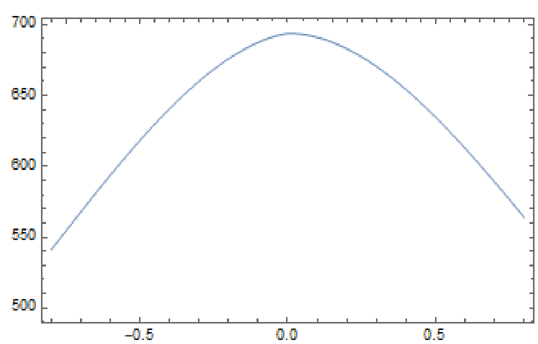

Figure 4. Annual Exposure of a flat panel in NYC

4.1 Optimization of a flat panel

The surface of a flat tilted panel is parameterized by the formula $\vec{r}(y, z) = \langle a - az, y, z \rangle$, where $0 \leq y \leq 1$ and $0 \leq z \leq 1$ and $a \in [0, \infty]$ is a tilting parameter.

The area of such a panel is $\sqrt{1+a^2}$. The normal vector $\vec{n}$ is evaluated as the cross product of the tangent vectors $\vec{r}_y = \langle 0,1,0\rangle$ and $\vec{r}_z = \langle -a,0,1\rangle$. Thus $\vec{n} = \langle 1,0,a\rangle$. The daily flux $\mathcal{F}_w$ of such a panel can be calculated from equation 7:

$$\mathcal{F}_w = \iiint_E \langle \cos\delta_w \sin t, \cos\delta_w \cos t, \sin\delta_w\rangle \cdot \langle 1,0,a\rangle \, dA\, dt \tag{10}$$

and simplifies to

$$\mathcal{F}_w = \iiint_E (\cos\delta_w \sin t + a\sin\delta_w) \, dA\, dt. \tag{11}$$

Since the integrand does not depend either on y or z, integrating with respect to $dA = dydz$ simplifies the integral into:

$$\mathcal{F}_w = \int_{t_{min}}^{t_{max}} \cos\delta_w(\sin t + a\tan\delta_w)dt. \tag{12}$$

The condition $\sin t + a\tan\delta_w > 0$ provides additional restrictions on the parameter t.

Further the Daily Exposure is calculated as:

$$\mathcal{DE}_w = \frac{\mathcal{F}_w}{Area} = \frac{\mathcal{F}_w}{\sqrt{1+a^2}} \tag{13}$$

Then the Annual Exposure is calculated as:

$$\mathcal{AE} = \sum_{w=0}^{364} \mathcal{DE}_w \tag{14}$$

Results of numerical integration for the values of the Annual Exposure for appropriate values of a are presented in Table 2 and on Figure 4.

4.2 Segment of a cylinder

A cylindrical panel, with radius 1 and height 1 is parameterized by the equations $\vec{r}(u,v) = \langle \sin u, \cos u, -v\rangle$. The range of the parameters is $u \in [\frac{-\pi}{2}+b, \frac{3\pi}{2}-b]$ and $v \in [-1,0]$. The area of the panel is equal to the circumference of the base $2\pi - 2b$ times the height, which is simply $2\pi - 2b$.

Table 3. Cylindrical Segment in NYC latitude

Value of b	Annual Exposure
$b = \frac{7\pi}{12}$	610.918
$b = \frac{8\pi}{12}$	641.829
$b = \frac{9\pi}{12}$	667.273
$b = \frac{10\pi}{12}$	686.224
$b = \frac{11\pi}{12}$	697.913

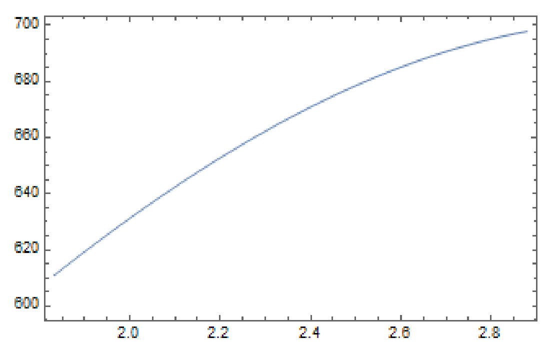

Figure 5. Annual Exposure of a cylindrical segment in NYC

4.2.1 Daily Flux

The daily flux will be computed as follows. The tangent vectors are: $\vec{r}_u = \langle \cos u, -\sin u, 0\rangle$ and $\vec{r}_v = \langle 0, 0, -1\rangle$ thus the normal vector is: $\vec{n} = \vec{r}_u \times \vec{r}_v = \langle \cos u, -\sin u, 0\rangle \times \langle 0, 0, -1\rangle = \langle \sin u, \cos u, 0\rangle$. The daily flux is given by the integral:

$$\mathcal{F}_w = \iiint_E \vec{F}(t) \cdot \vec{n} = \iiint_E \langle \cos\delta_w \sin t, \cos\delta_w \cos t, \sin\delta_w\rangle \cdot \langle \sin u, \cos u, 0\rangle \, dvdudt \tag{15}$$

which simplifies to

$$\mathcal{F}_w = \iiint_E \cos\delta_w \left(\sin t \sin u + \cos t \cos u\right) \, dvdudt \tag{16}$$

After applying the trigonometric identity $\sin t \sin u + \cos t \cos u = \cos(u - t)$ the integral becomes:

$$\mathcal{F}_w = \cos\delta_w \iiint_E \cos(u - t) \, dvdudt. \tag{17}$$

The region E is determined by the property $\cos(u - t) > 0$, and the boundaries for t determined by the location in NYC and determined by equation 5.

4.2.2 Exposure

The annual exposure is computed as the sum with respect to parameter w of daily exposures, i.e.,

$$\mathcal{AE} = \sum_{w=0}^{364} \mathcal{DE}_w \tag{18}$$

The results are obtained numerically from software and presented in Table 3 and Figure 5.

4.3 Optimization of the catenoid

The properties of the catenoid has been studied since the $XVIII^{th}$ century. The motivation of using this surface for the purpose of solar panels lies in the fact that catenoid has the smallest area among all rotational surfaces thus the flux density is maximal if the flux remains constant. We expect that a section of a catenoid may be a good candidate for an efficient shaped panel.

Let us consider the following parametrization of a catenoid:

$$\vec{r}(u, v) = \langle \cosh v \sin u, c \cosh v \cos u, -v\rangle, \tag{19}$$

where $u \in [-\frac{\pi}{2} + c, \frac{3\pi}{2} - c]$, $v \in [0, \frac{1}{10}]$, and c is a parameter with the range $c \in [0, \frac{11\pi}{12}]$. Then the tangent vectors are $\vec{r}_u = \langle \cosh v \cos u, -\cosh v \sin u, 0\rangle$ and $\vec{r}_v = \langle \sinh v \sin u, \sinh v \cos u, -1\rangle$ with the normal vector $\vec{n} = \vec{r}_u \times \vec{r}_v = \cosh v \langle \sin u, \cos u, \sinh v\rangle$. Thus,

$$|\vec{n}| = \cosh v \sqrt{1 + \sinh^2 v} = \cosh^2 v \tag{20}$$

and the are can be evaluated as:

$$Area = 2\pi \int_0^1 \cosh^2 v dv = \frac{\pi}{2} \left(2 + \sinh 2\right) \tag{21}$$

Table 4. Catenoidal Segment in NYC latitude

Catenoid	Annual exposure
$c = \frac{7\pi}{12}$	610.842
$c = \frac{8\pi}{12}$	641.702
$c = \frac{9\pi}{12}$	667.103
$c = \frac{10\pi}{12}$	686.022
$c = \frac{11\pi}{12}$	697.692

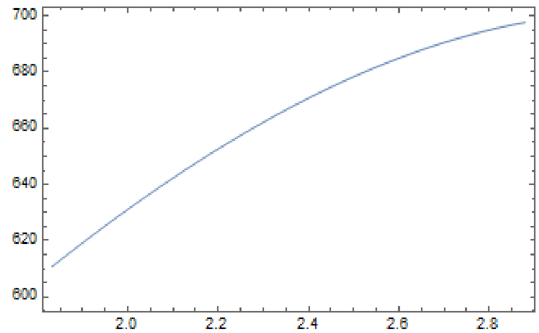

Figure 6. Annual Exposure of a catenoidal segment in NYC

4.3.1 Daily flux

Thus the daily flux is computed as:

$$\mathcal{F}_w = \iiint_E \vec{F}(t) \cdot \vec{n}\, dAdt \tag{22}$$

with

$$\vec{F}(t) \cdot \vec{n} = \langle \cos\delta_w \sin t, \cos\delta_w \cos t, \sin\delta_w \rangle \cdot \cosh v \langle \sin u, \cos u, \sinh v \rangle \tag{23}$$

after simplifications:

$$\vec{F}(t) \cdot \vec{n} = \cosh v \cos\delta_w (\sin t \sin u + \cos t \cos u + \tan\delta_w \sinh v) \tag{24}$$

Thus

$$\mathcal{F}_w = \iiint_E \cosh v \cos\delta_w (\cos(t-u) + \tan\delta_w \sinh v), \tag{25}$$

where E is the solid with (u, v, t) so that $\cos(t-u) > -\tan\delta_w \sinh v$.

4.3.2 Exposure

The annual exposure is computed as the sum with respect to parameter w of daily exposures, i.e.,

$$\mathcal{AE} = \sum_{w=0}^{364} \mathcal{DE}_w \tag{26}$$

The results are obtained numerically from software and presented in Table 4 and Figure 6.

5. SUMMARY OF THE RESULTS AND FUTURE WORK

It is important to recall that all surfaces are placed in the coordinate system, where the z axis is tilted north by the latitude of the location, namely by 40.7128^0. Thus all angles must be modified by this value if considered from the horizontal line. As presented in Table 5 the model provides the best flat panel with parameter $a = 0$, (tilted with the angle equal to the latitude of NYC 40.7128^0) that provides the Annual Exposure of 694.009 units. The best cylindrical segment is in fact a narrow stripe spanned by the angle $\frac{\pi}{6}$ and provides the Annual Exposure 697.913. However, such a small difference in the values, less than 0.5%, may be due to approximations in the model. The best catenoidal segment is spanned by the angle $\frac{\pi}{6}$ and provides the annual exposure 697.692, which is again less than 0.5% better than the flat panel and may be due to the approximations in the model.

In the future work we will perform a comparison of our model with a model described by Duffie and Beckman[6] and using statistical data of the sun irradiance.

Table 5. Comparison of the Annual Exposure

Shape	Parameter	Annual Exposure
Flat Panel	$a = 0$	694.009
Cylindrical Section	$b = \frac{11\pi}{12}$	697.913
Catenoidal Section	$c = \frac{11\pi}{12}$	697.692

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