Impulse Excitation Formulas

Below are the explicit relations used by ModuliCalc to determine the characteristic engineering moduli for both rectangular and cylindrical specimens. Note that these formulas are derived empirically, so the constants are only valid for the given units. For situations where explicit means cannot be employed (e.g. when the geometry yields variable correction factors), a guess-and-check iterative scheme is used.

Rectangular Specimens

Young's Modulus

$$E = .9465 \left(\frac{m f_f^2}{b}\right) (L^3/t^3)\, T_1,$$

where

E = Young's (Elastic) Modulus, Pa
m = mass of the bar, g
b = width of the bar, mm
L = length of the bar, mm
t = thickness of the bar, mm
f_f = fundamental frequency (longitudinal), Hz
T_1 = correction factor.

T_1 is defined by

$$\begin{aligned} T_1 = {} & 1 + 6.585(1 + .0752\nu + .8109\nu^2)(t/L)^2 - .868(t/L)^4 \\ & - \frac{8.340(1 + .2023\nu + 2.173\nu^2)(t/L)^2}{1.000 + 6.338(1 + .1408\nu + 1.536\nu^2)(t/L)^2}, \end{aligned}$$

with ν as the Poisson Ratio. If $L/t \geq 20$ then T_1 can be reasonably approximated to simply

$$T_1 = 1.000 + 6.585(t/L)^2,$$

which does not depend on ν allowing E to be calculated directly.

Shear Modulus

$$G = \frac{4 L m f_t^2}{bt}(B/(1 + A)),$$

where

G = Shear Modulus, Pa
L = length of bar, mm
f_t = torsional frequency, Hz
b = width of the bar, mm
t = thickness of the bar, mm
A = a correction factor (defined below)
B = another correction factor (defined below)

$$A = \frac{.5062 - .8776(b/t) + .3504(b/t)^2 - .0078(b/t)^3}{12.03(b/t) + 9.892(b/t)^2}$$

$$B = \frac{b/t + t/b}{4(t/b) - 2.52(t/b)^2 + .21(t/b)^6}$$

Poisson Ratio

Lastly, to relate E and G, the well-known Poisson ratio is used:

$$\nu = \frac{E}{2G} - 1.$$

Cylindrical Specimens

Young's Modulus

$$E = 1.6067\,(L^3/D^4)(mf_f)^2\,T_1',$$

where

E = Young's Modulus, Pa
L = length of the rod, mm
D = diameter of the rod, mm
m = mass of the rod, g
f_f = fundamental frequency, Hz
T_1' = a correction factor, defined by

$$T_1' = 1 + 4.939(1 + .0752\nu + .8109\nu^2)(D/L)^2 - .4833(D/L)^4 - \left(\frac{4.691(1 + .2023\nu + 2.173\nu^2)(D/L)^4}{1.000 + 4.754(1 + .1408\nu + 1.536\nu^2)(D/L)^2}\right).$$

If $L/D \geq 20$ then a simplified version of T_1' may be used, namely

$$T_1' = 1.000 + 4.939\,(D/L)^2.$$

Shear Modulus

$$G = 16 m f_t^2 \frac{L}{\pi D^2},$$

with

G = Shear Modulus, Pa
m = mass, g
f_t = torsional frequency, Hz
L = length, mm
D = diameter, mm.

References

[1] "Standard Test Method for Dynamic Young's Modulus, Shear Modulus, and Poisson's Ratio by Impulse Excitation of Vibration," `ASTM International Standard E 1876 - 01 (2005). (Link)`