

Introduction

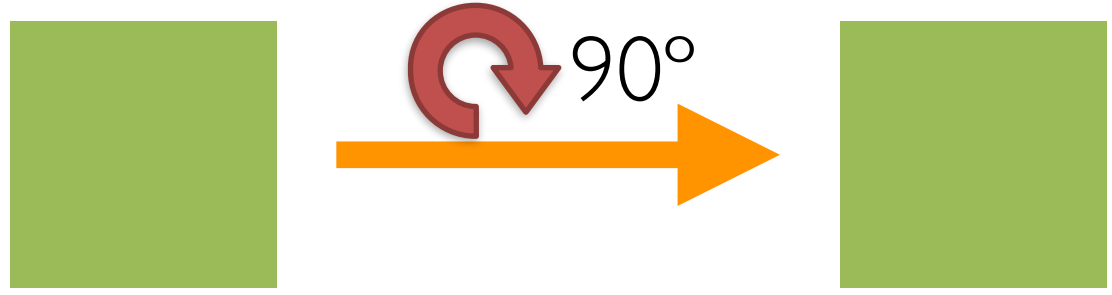
Goal: Design a physics-inspired, jet generative deep neural network model that understands and preserves the symmetry of the Lorentz transformation.

Particle Physics

- The study of elementary particles.
 - What **types** of elementary particles exist?
 - What are the **intrinsic properties** of elementary particles and what are these values?
 - How do elementary particles **interact** with one another?
- Colliders**
 - Research tool to study particle physics.
 - Collides beams of ultra-relativistic particles together.
- Jets**
 - Collimated sprays of particles.
 - Provide key insights into the nature of the underlying dynamics and interactions.
 - Central object in the analyses of particle physics experiments
- Simulations**
 - Important part of data analysis in particle physics experiments.
 - Translate the theoretical model into experimental signatures to construct physical objects from raw data.
 - Classical simulation programs, such as GEANT4, are **accurate but slow**, producing data at a rate $\mathcal{O}(1)$ min/event.



Symmetry

- The invariance under some transformations.
 - Example: 90° rotation of a square.
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- Can be **continuous** and **discrete**.
 - Example of a discrete symmetry: invariance of a square under a flip along the diagonal line.
 - Example of a continuous symmetry: invariance of a circle under a rotation of any angle about the center.
 - Naturally described by a mathematical object called **groups**.
 - Ubiquitous and essential in physics.
 - Physical laws are the same everywhere in the universe at every moment (spatial and temporal translational symmetry).
 - Physical laws do not depend on the choice of reference frames.
 - Fundamental interactions in the standard model come from local gauge symmetries.
 - Strong interaction: **SU(3)** gauge symmetry.
 - Electroweak interaction: **SU(2) × U(1)** gauge symmetry.
 - All elementary particles are all described and classified by the Poincaré group.
 - All dynamics of elementary particles, as special relativistic objects, obey the Lorentz symmetry, described by the **Lorentz group SO(3,1)**.

Architecture Overview

Deep Neural Network (DNN)

- A machine learning algorithm that employs **multiple hidden neural layers** with a **structure**.
- Can be used to approximate the optimal solution for a task.
- Is good at dealing with a large amount of data.
 - Could be a good solution to slow generating rate of classical simulation programs.

Autoencoder

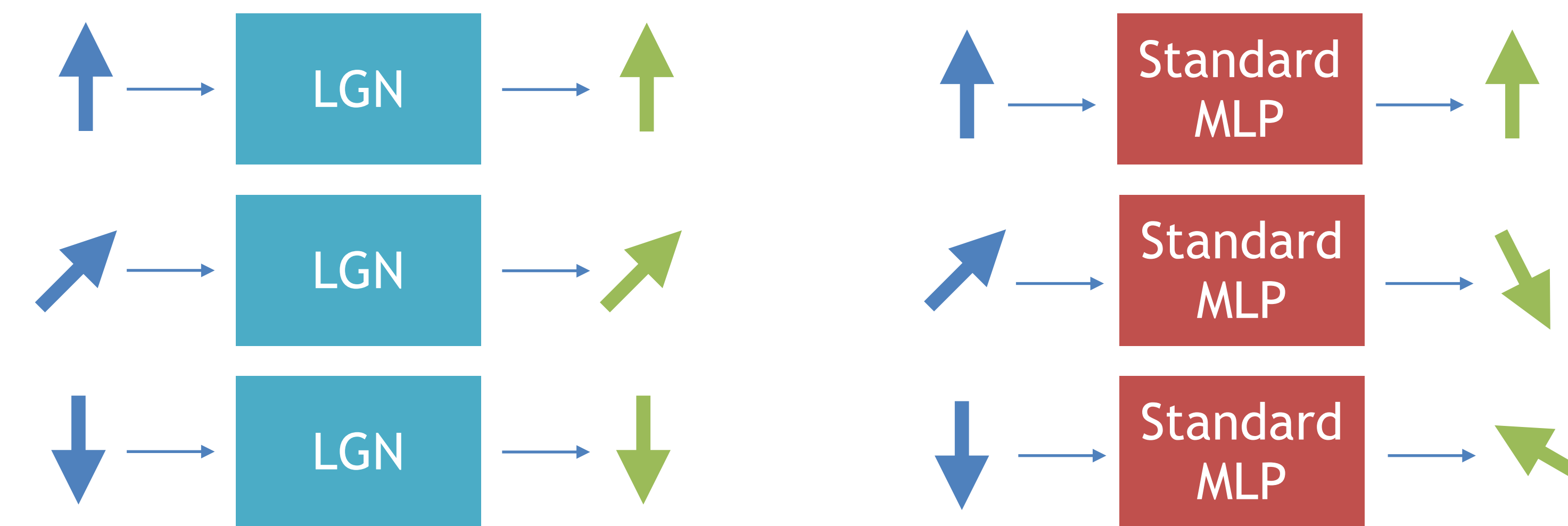
- Consists of an encoder and a decoder.
 - Encoder** compresses dimension down of the data to the **latent space**.
 - Decoder** reconstruct the data from the latent space as much as possible.
- The model is forced to extract the most crucial features in the data.
- The decoder can then be used as a generative model.

Generative Adversarial Networks (GAN)

- Consists of a generator and a discriminator.
 - Generator** tries to generate data that look as realistic as possible and cheat the discriminator.
 - The trained decoder will be used as the generator.
 - Discriminator** tries to distinguish generated data from real data.

Lorentz Group Network (LGN)

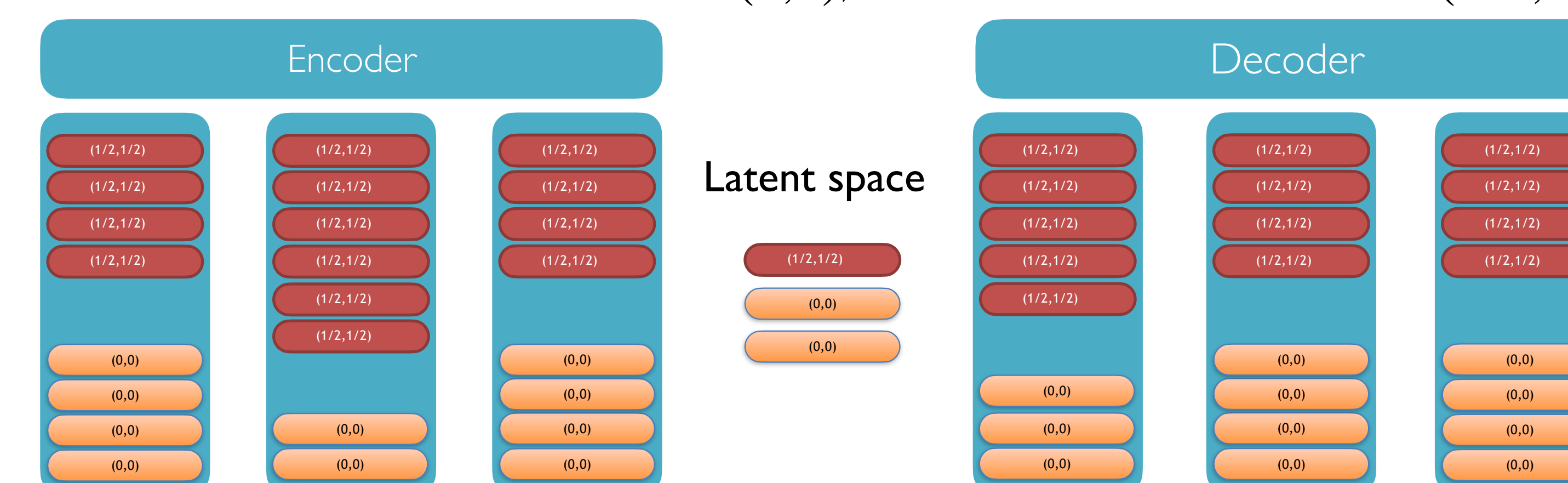
- Built and introduced by Bogatkiy et al. in [arXiv:2006.04780](https://arxiv.org/abs/2006.04780).
- Is fully Lorentz group **equivariant**.



- Has a good performance in jet classification task.
 - Needs less parameters because of the constraints of Lorentz group.
 - More interpretable because all internal parameters obey the Lorentz symmetry.
- Is not tested for generative task yet.

Lorentz Group Equivariant Jet Autoencoder

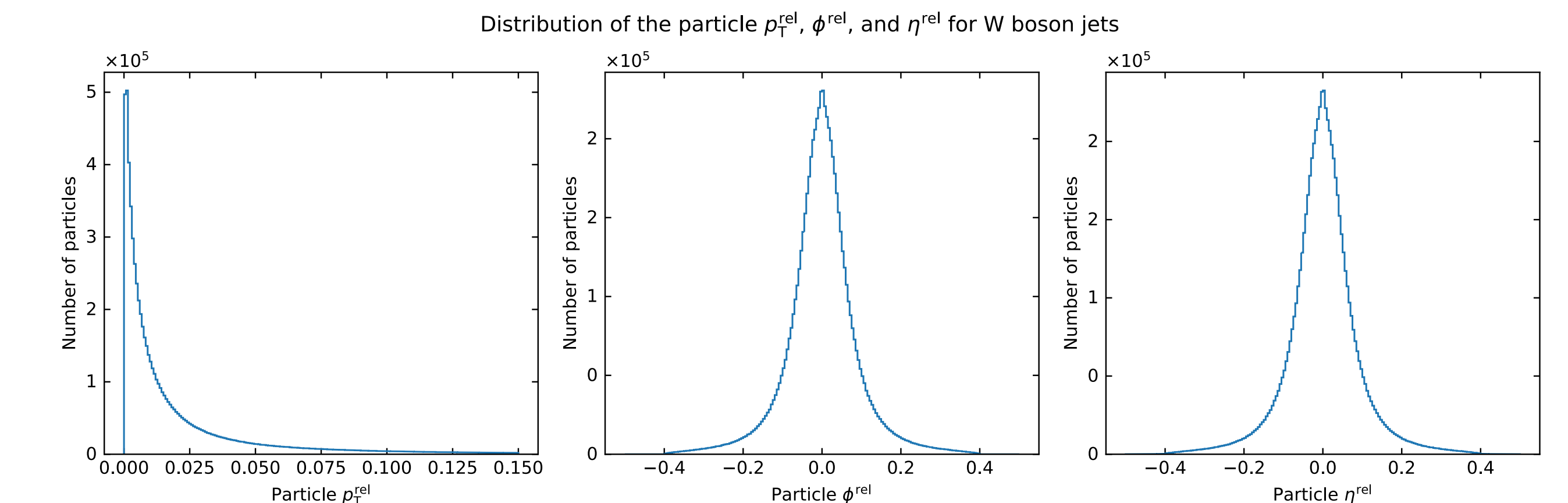
- Built upon the architecture of LGN.
- Takes jets momenta and masses as input.
 - Scalars such as mass are labeled as (0,0), and 4-vectors are labeled as (1/2,1/2).



Dataset

Format

- HLS4ML LHC jet dataset preprocessed by [Raghav Kansal](#).
- Each particle is represented by a vector (p_T, η, ϕ) .

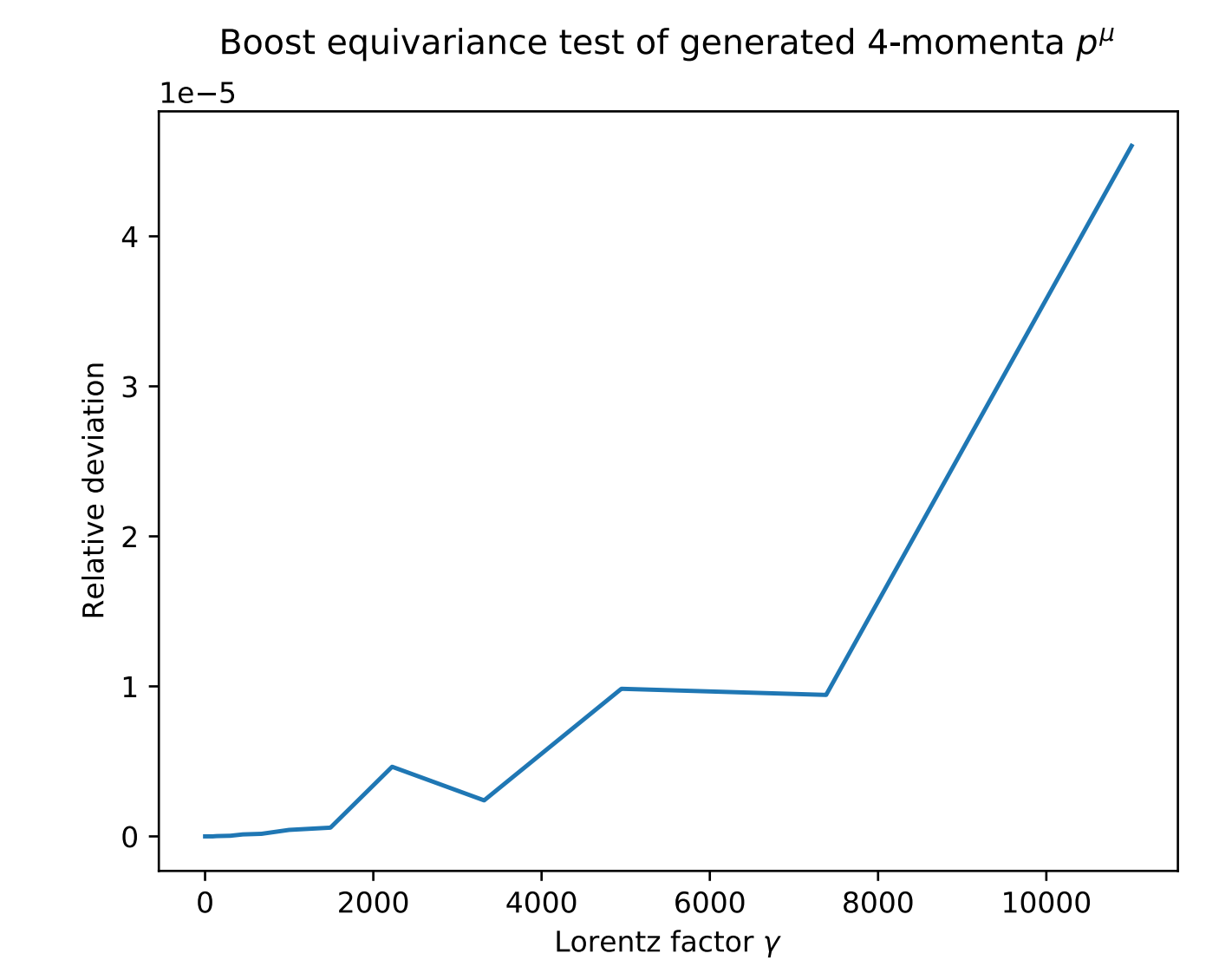
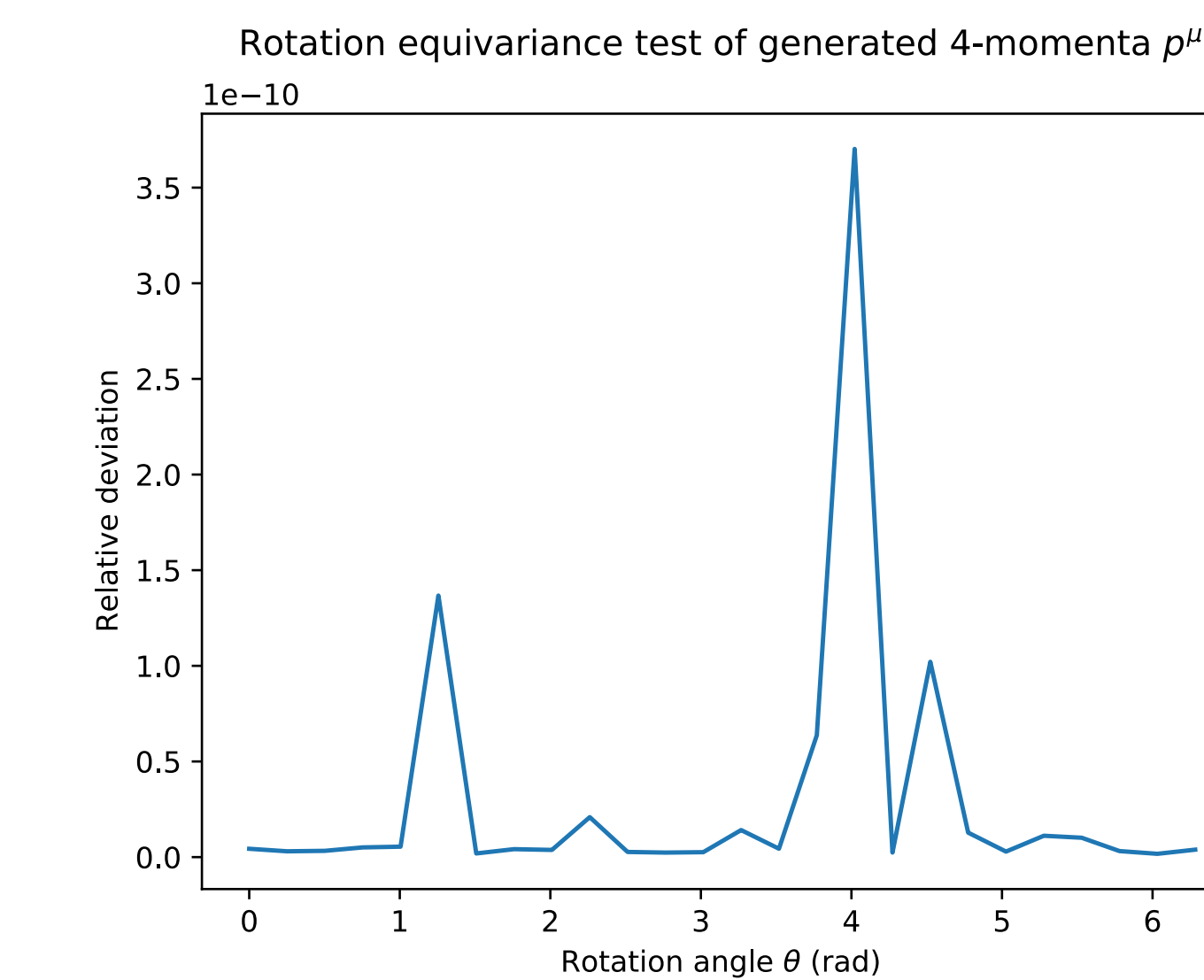


- Converted into Cartesian coordinates $p^\mu = (E/c, p_x, p_y, p_z)$.

Evaluation

Equivariance Test

- Rotation of $\theta \in [0, 2\pi)$.
 - Errors are strictly on the order of $\mathcal{O}(10^{-11})$.
- Boost of $\gamma \in [1, 11013.2]$.
 - Error increases as γ increases.
 - Boost is sensitive to floating point errors.
 - Most relevant region for physics: $\gamma \in [1, 200]$.
 - $\gamma = 11013.2$ corresponds to boosting the reference frame to a speed $v = 0.999999992c$.
- This model has a **built-in** Lorentz equivariance.



- Non-equivariant models produce very large errors:

