

## Introduction

### Goal

- Build a **physics-inspired**, jet generative deep neural network model that **understands and preserves the symmetry** of the Lorentz transformation.

### Particle Physics

- The study of elementary particles.
  - What **types** of elementary particles exist?
  - What are the **intrinsic properties** of elementary particles?
  - How do elementary particles **interact** with one another?
- Colliders**
  - Research tool to study particle physics.
  - Collides beams of ultra-relativistic particles together.
- Jets**
  - Collimated sprays of particles.
  - Provide key insights into the nature of the underlying dynamics.
- Simulations**
  - Important part of data analysis in particle physics experiments.
  - Translate the theoretical model into experimental signatures to construct physical objects from raw data.
  - Classical simulation programs, such as GEANT4, are **accurate but slow**, producing data at a rate  $\mathcal{O}(1)$  min/event.



### Symmetry

- The invariance under some transformations.
  - Example: 90° rotation of a square.



- Can be **continuous** and **discrete**.
  - Example of a discrete symmetry: invariance of a square under a flip along the diagonal line.
  - Example of a continuous symmetry: invariance of a circle under a rotation of any angle about the center.
- Naturally described by a mathematical object called **groups**.
- Ubiquitous and essential in physics.
  - Physical laws have space and time translational symmetry)
  - Physical laws do not depend on the choice of reference frames.
  - Fundamental interactions in the standard model come from local gauge symmetries.
    - Strong interaction: **SU(3)** gauge symmetry.
    - Electroweak interaction: **SU(2) × U(1)** gauge symmetry.
- All elementary particles are all described and classified by the Poincaré group.
- All dynamics of elementary particles, as special relativistic objects, obey the Lorentz symmetry, described by the **Lorentz group SO(3,1)**.
- Conventional neural networks do not learn symmetry very well.
  - A model will not yield the same result if a picture of a cat is rotated.

## Architecture Overview

### Deep Neural Network (DNN)

- A machine learning algorithm that employs **multiple hidden neural layers** with a **structure**.
- Can be used to approximate the optimal solution for a task.
- Is good at dealing with a large amount of data.
  - Could be a good solution to slow generating rate of classical simulation programs.

### Autoencoder

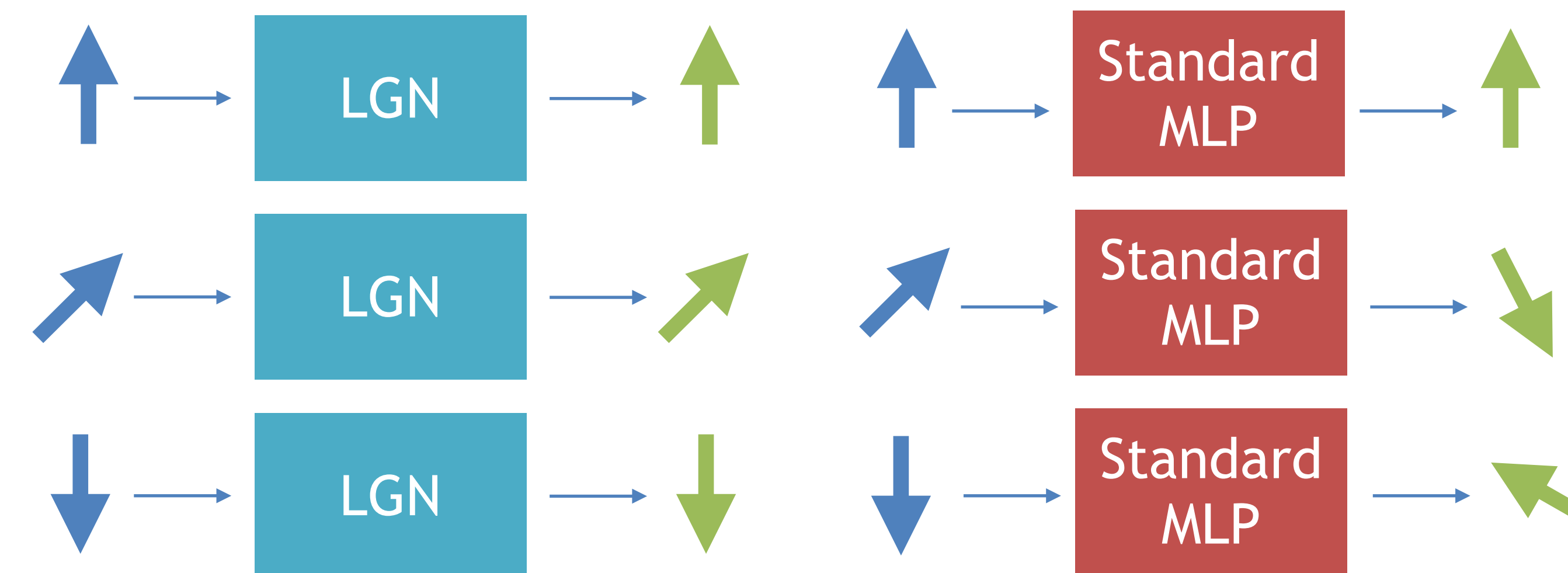
- Consists of an encoder and a decoder:
  - Encoder** compresses dimension down of the data to the **latent space**.
  - Decoder** reconstruct the data from the latent space as much as possible.
- The model is forced to extract the most crucial features in the data.
- The decoder can then be used as a generative model.

### Generative Adversarial Networks (GAN)

- Consists of a generator and a discriminator:
  - Generator** tries to generate data that look as realistic as possible and cheat the discriminator.
    - The trained decoder will be used as the generator.
  - Discriminator** tries to distinguish generated data from real data.

### Lorentz Group Network (LGN)

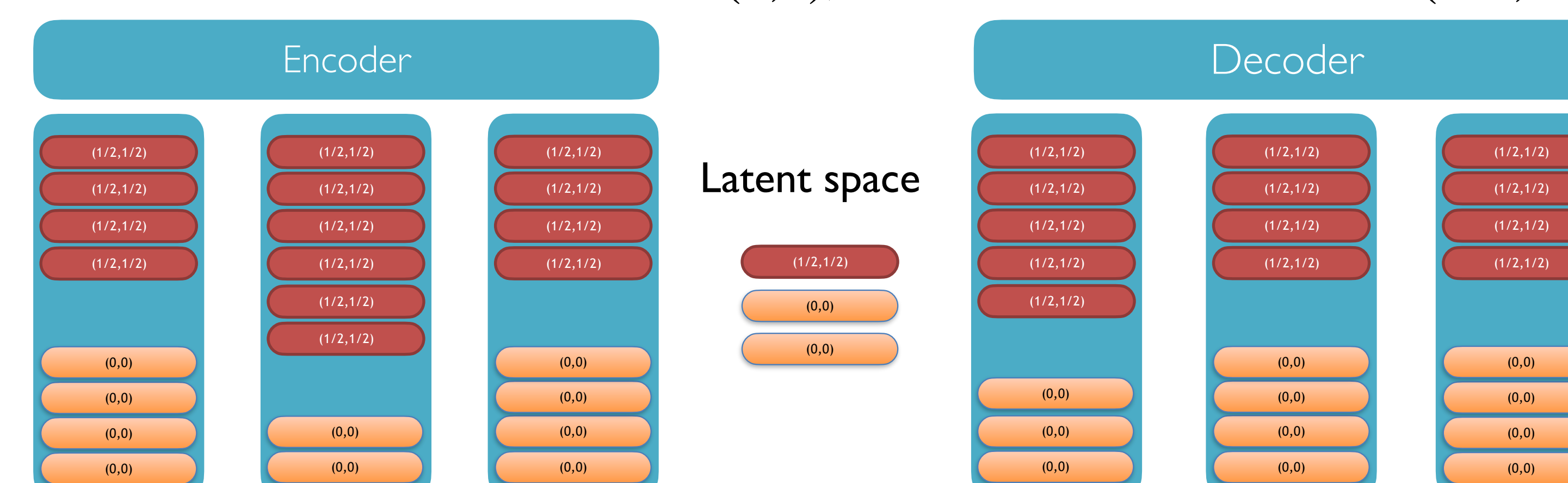
- Built and introduced by Bogatkiy et al. in [arXiv:2006.04780](https://arxiv.org/abs/2006.04780).
- Is fully Lorentz group **equivariant**.



- Has a good performance in jet classification task.
  - Needs less parameters because of the constraints of Lorentz group.
  - More interpretable because all internal parameters obey the Lorentz symmetry.
- Is not tested for generative task yet.

### Lorentz Group Equivariant Jet Autoencoder

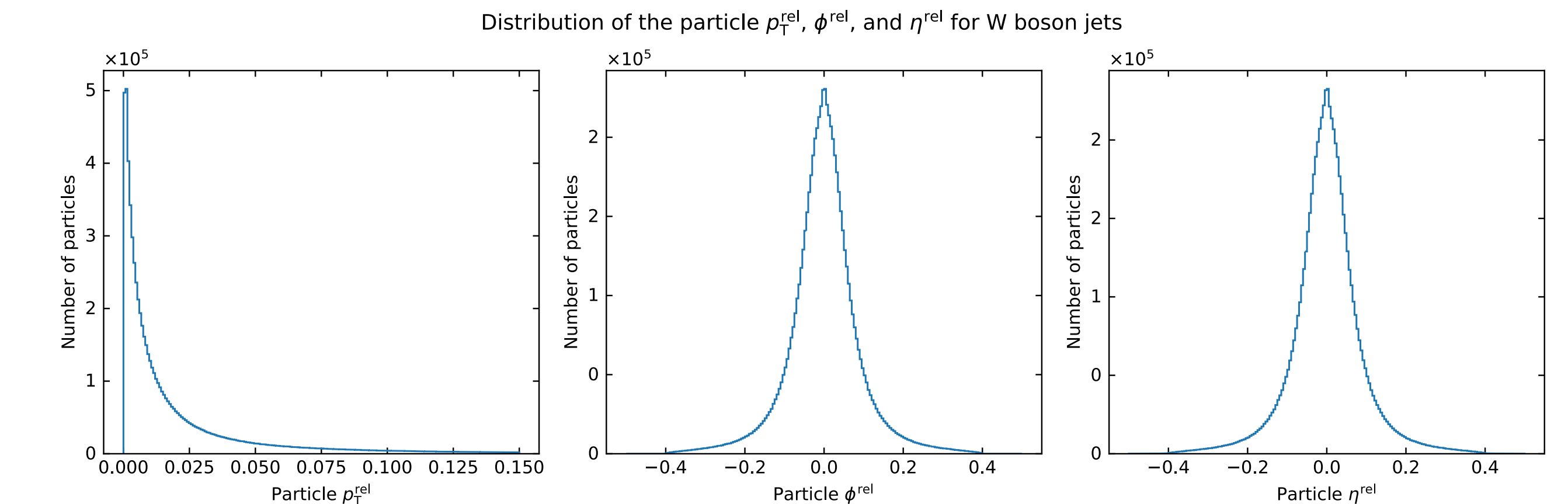
- Built upon the architecture of LGN.
- Takes jets momenta and masses as input.
  - Scalars such as mass are labeled as (0,0), and 4-vectors are labeled as (1/2,1/2).



## Dataset

### Format

- HLS4ML LHC jet dataset preprocessed by [Raghav Kansal](#).
- Each particle is represented by a vector  $(p_T, \eta, \phi)$ .

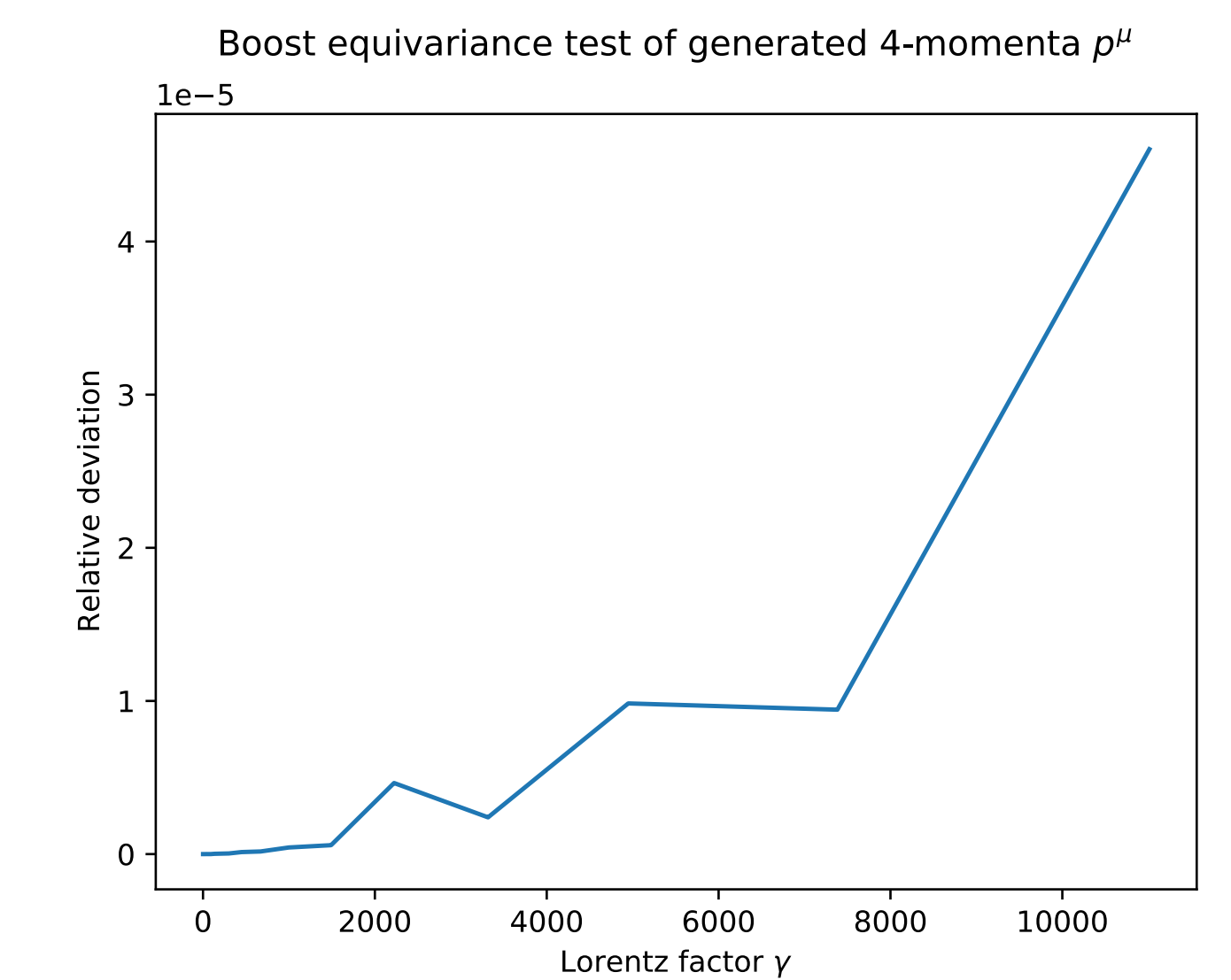
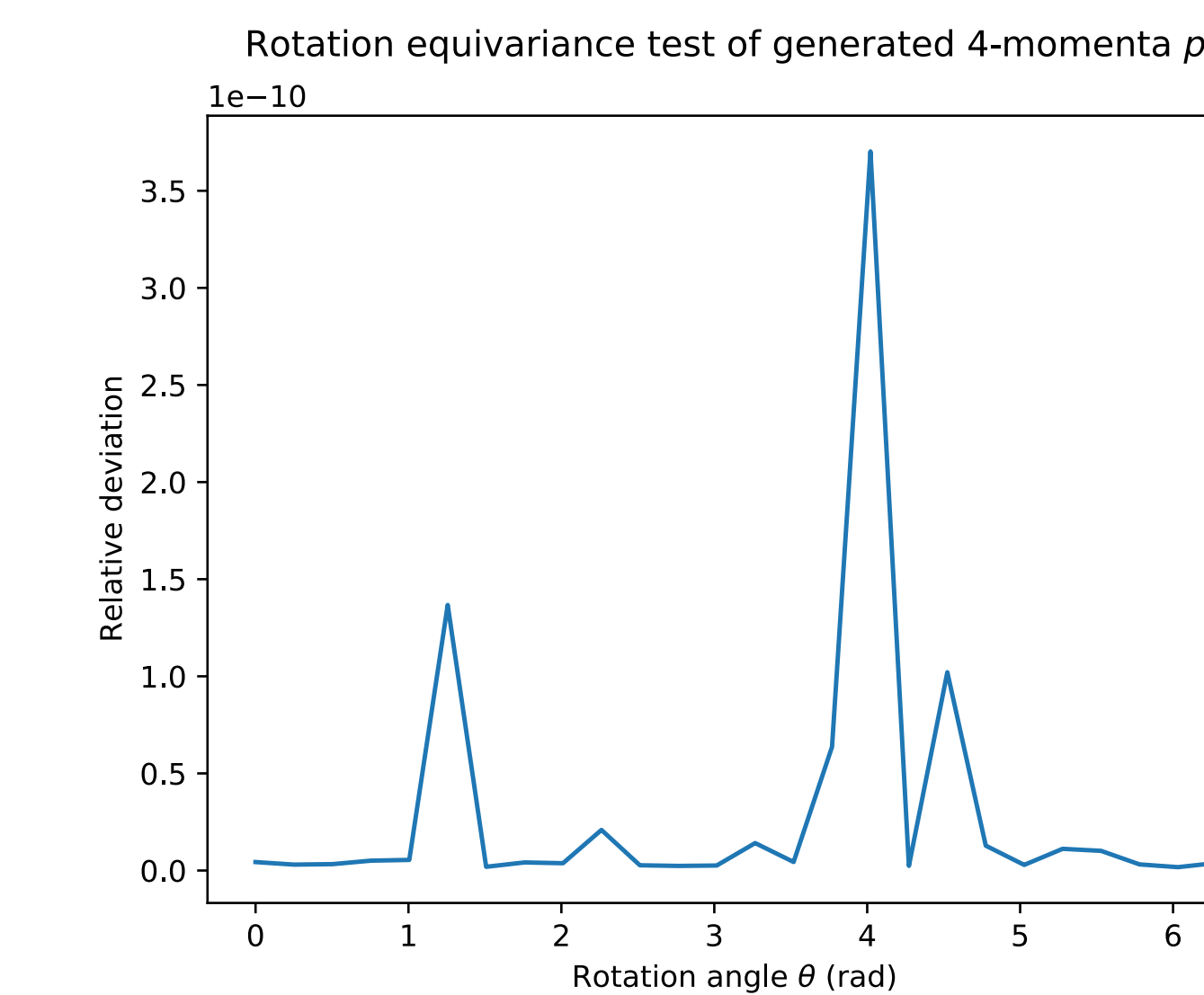


- Converted into Cartesian coordinates  $p^\mu = (E/c, p_x, p_y, p_z)$ .

## Evaluation

### Equivariance Test

- Rotation of  $\theta \in [0, 2\pi)$ .
  - Errors are strictly on the order of  $\mathcal{O}(10^{-11})$ .
- Boost of  $\gamma \in [1, 11013.2]$ .
  - Error increases as  $\gamma$  increases.
  - Boost is sensitive to floating point errors.
  - Most relevant region for physics:  $\gamma \in [1, 200]$ .
  - $\gamma = 11013.2$  corresponds to boosting the reference frame to a speed  $v = 0.999999992c$ .
- This model has a **built-in** Lorentz equivariance.



- Non-equivariant models produce very large errors:

