Lorentz Group Equivariant Jet GAN

Zichun Hao, Javier Duarte, Raghav Kansal, Farouk Mokhtar
Department of Physics, University of California, San Diego

Introduction

Goal

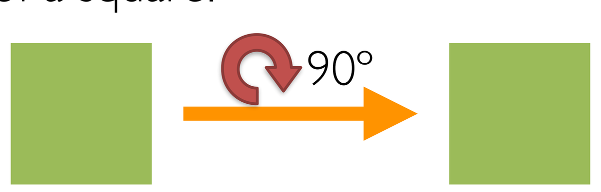
- Build a **physics-inspired**, jet generative deep neural network model that **understands and preserves the symmetry** of the Lorenz transformation.

Particle Physics

- The study of elementary particles.
  - What **types** of elementary particles exist?
  - What are the **intrinsic properties** of elementary particles?
  - How do elementary particles **interact** with one another?
- **Colliders**
  - Research tool to study particle physics.
  - Collides beams of ultra-relativistic particles together.
- **Jets**
  - Collimated sprays of particles.
  - Provide key insights into the nature of the underlying dynamics and interactions.
  - Central object in the analyses of particle physics experiments
- **Simulations**
  - Important part of data analysis in particle physics experiments.
  - Translate the theoretical model into experimental signatures to construct physical objects from raw data.
  - Classical simulation programs, such as `GEANT4`, are **accurate but slow**, producing data at a rate $\mathcal{O}(1)$ min/event.

Symmetry

- The invariance under some transformations.
  - Example: 90° rotation of a square.



- Can be **continuous** and **discrete**.
  - Example of a discrete symmetry: invariance of a square under a flip along the diagonal line.
  - Example of a continuous symmetry: invariance of a circle under a rotation of any angle about the center.
- Naturally described by a mathematical object called groups.
- Ubiquitous and essential in physics.
  - Physical laws are the same everywhere in the universe at every moment (spatial and temporal translational symmetry).
  - Physical laws do not depend on the choice of reference frames.
  - Fundamental interactions in the standard model come from local gauge symmetries.
    - Strong interaction: $\mathrm{SU}(3)$ gauge symmetry.
    - Electroweak interaction: $\mathrm{SU}(2) \times \mathrm{U}(1)$ gauge symmetry.
  - All elementary particles are all described and classified by the Poincaré group.
  - All dynamics of elementary particles, as special relativistic objects, obey the Lorentz symmetry, described by the Lorentz group $\mathrm{SO}(3,1)$.

Architecture Overview

Deep Neural Network (DNN)

- A machine learning algorithm that employs **multiple hidden neural layers** with a **structure**.
- Can be used to approximate the optimal solution for a task.
- Is good at dealing with a large amount of data.
  - Could be a good solution to slow generating rate of classical simulation programs.

Autoencoder

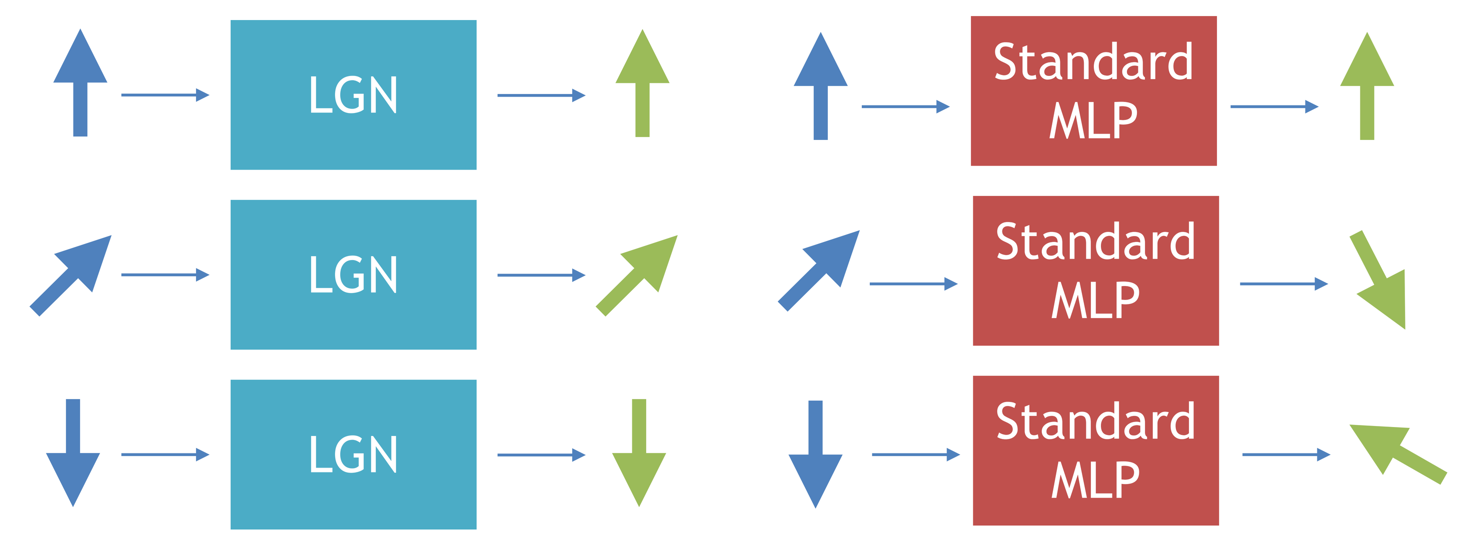
- Consists of an encoder and a decoder.
  - **Encoder** compresses dimension down of the data to the **latent space**.
  - **Decoder** reconstruct the data from the latent space as much as possible.
- The model is forced to extract the most crucial features in the data.
- The decoder can then be used as a generative model.

Generative Adversarial Networks (GAN)

- Consists of a generator and a discriminator.
  - **Generator** tries to generate data that look as realistic as possible and cheat the discriminator.
    - The trained decoder will be used as the generator.
  - **Discriminator** tries to distinguish generated data from real data.

Lorentz Group Network (LGN)

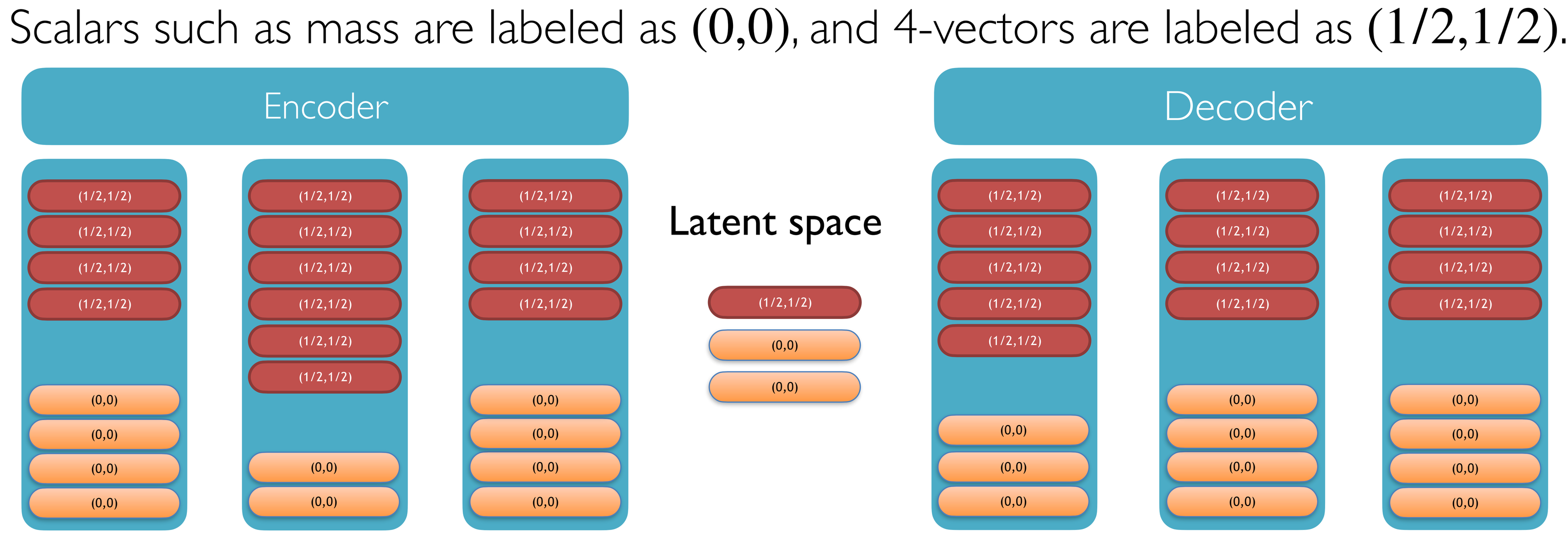
- Built and introduced by Bogatkiy et al. in arXiv:2006.04780.
- Is fully Lorentz group equivariant.



- Has a good performance in jet classification task.
  - Needs less parameters because of the constraints of Lorentz group.
  - More interpretable because all internal parameters obey the Lorentz symmetry.
- Is not tested for generative task yet.

Lorentz Group Equivariant Jet Autoencoder

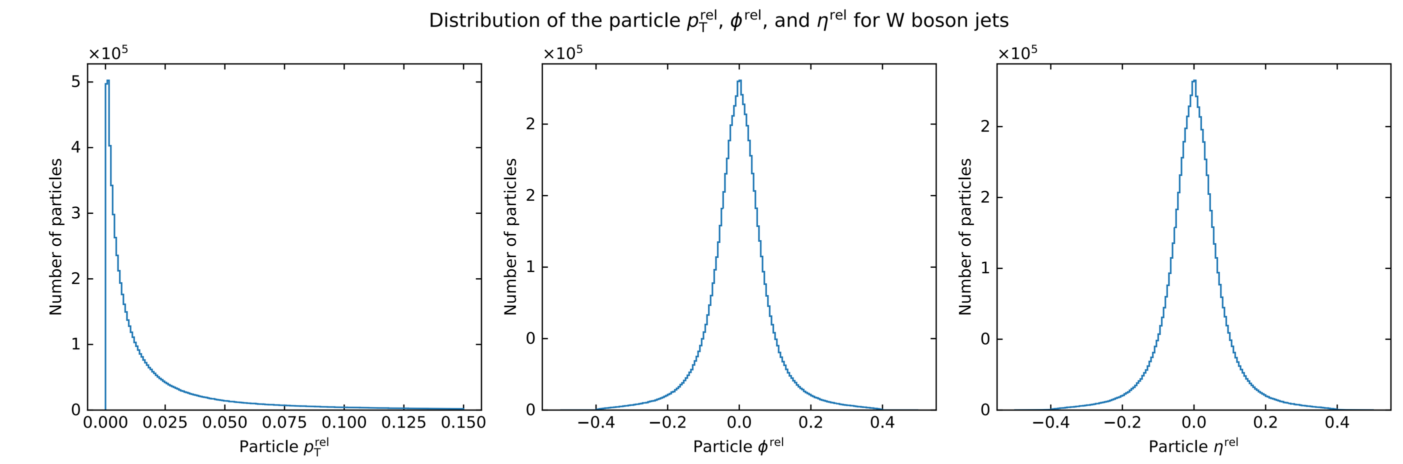
- Built upon the architecture of LGN.
- Takes jets momenta and masses as input.
  - Scalars such as mass are labeled as $(0,0)$, and 4-vectors are labeled as $(1/2,1/2)$.



Dataset

Format

- HLS4ML LHC jet dataset preprocessed by Raghav Kansal.
- Each particle is represented by a vector $(p_\mathrm{T}, \eta, \phi)$.

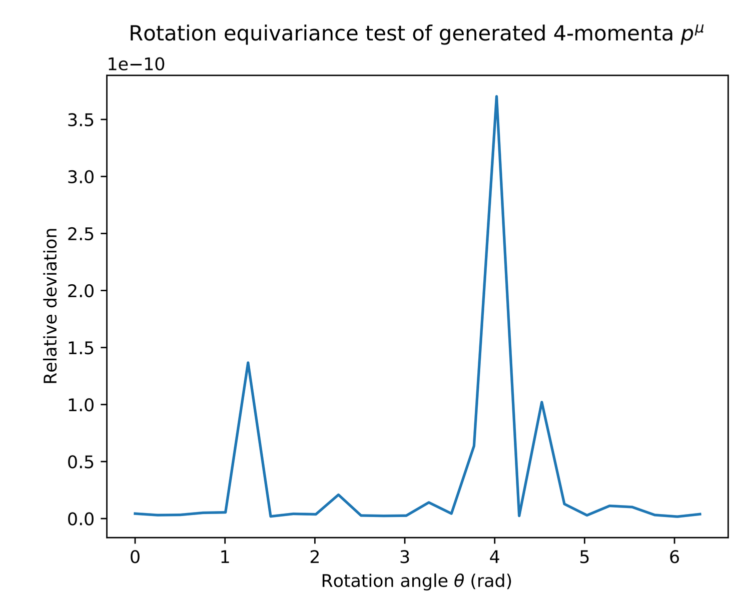


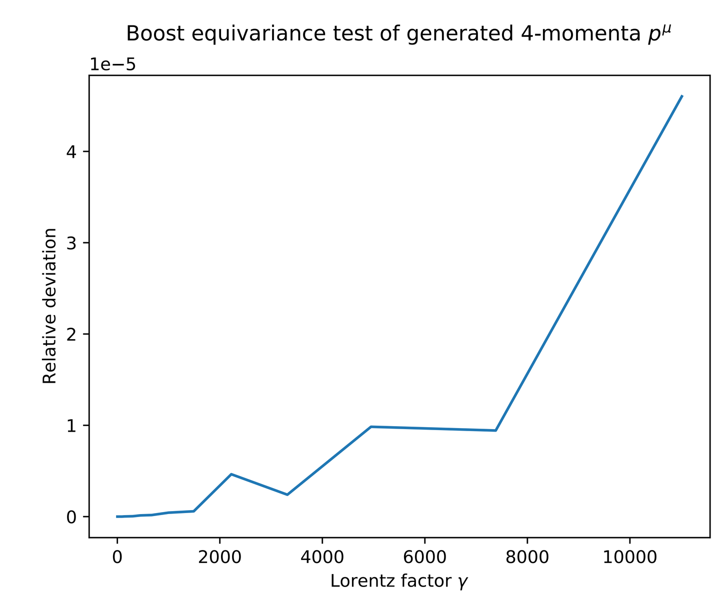
- Converted into Cartesian coordinates $p^\mu = (E/c, p_x, p_y, p_z)$.

Evaluation

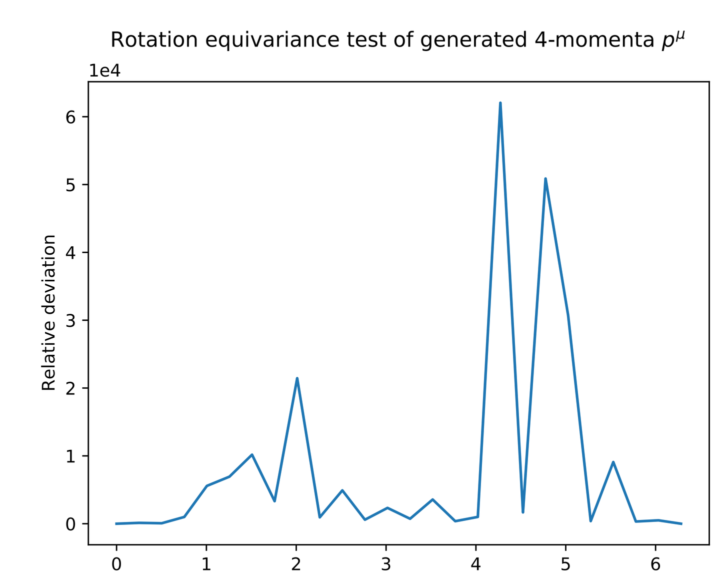
Equivariance Test

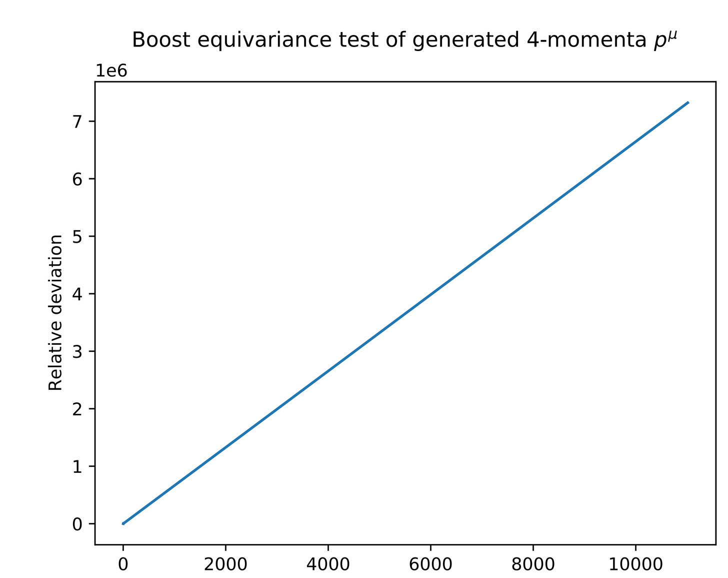
- Rotation of $\theta \in [0, 2\pi)$.
  - Errors are strictly on the order of $\mathcal{O}(10^{-11})$.
- Boost of $\gamma \in [1, 11013.2]$.
  - Error increases as γ increases.
    - Boost is sensitive to floating point errors.
  - Most relevant region for physics: $\gamma \in [1, 200]$.
  - $\gamma = 11013.2$ corresponds to boosting the reference frame to a speed $v = 0.999999992c$.
- This model has a **built-in** Lorentz equivariance.





- Non-equivariant models produce very large errors:





https://github.com/zichunhao/lgn-autoencoder