

Lorentz Group Equivariant Jet GAN

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Introduction

Goal

• Build a physics-inspired, jet generative deep neural network model that understands and preserves the symmetry of the Lorenz transformation.

Particle Physics

- The study of elementary particles.
- What types of elementary particles exist?
- What are the intrinsic properties of elementary particles?
- How do elementary particles interact with one another?
- Colliders
- Research tool to study particle physics.
- Collides beams of ultra-relativistic particles together.
- Jets
- Collimated sprays of particles.
- Provide key insights into the nature of the underlying dynamics and interactions.
- Central object in the analyses of particle physics experiments
- Simulations
- Important part of data analysis in particle physics experiments.
- Translate the theoretical model into experimental signatures to construct physical objects from raw data.
- Classical simulation programs, such as GEANT4, are accurate but slow, producing data at a rate $\mathcal{O}(1)$ min/event.



Symmetry

- The invariance under some transformations.
- Example: 90° rotation of a square.



- Can be continuous and discrete.
- Example of a discrete symmetry: invariance of a square under a flip along the diagonal line.
- Example of a continuous symmetry: invariance of a circle under a rotation of any angle about the center.
- Naturally described by a mathematical object called groups.
- Ubiquitous and essential in physics.
- Physical laws are the same everywhere in the universe at every moment (spatial and temporal translational symmetry).
- Physical laws do not depend on the choice of reference frames.
- Fundamental interactions in the standard model come from local gauge symmetries.
- Strong interaction: SU(3) gauge symmetry.
- Electroweak interaction: $SU(2) \times U(1)$ gauge symmetry.
- All elementary particles are all described and classified by the Poincaré group.
- All dynamics of elementary particles, as special relativistic objects, obey the Lorentz symmetry, described by the Lorentz group SO(3,1).

Architecture Overview

Deep Neural Network (DNN)

- A machine learning algorithm that employs multiple hidden neural layers with a structure.
- Can be used to approximate the optimal solution for a task.
- Is good at dealing with a large amount of data.
- Could be a good solution to slow generating rate of classical simulation programs.

Autoencoder

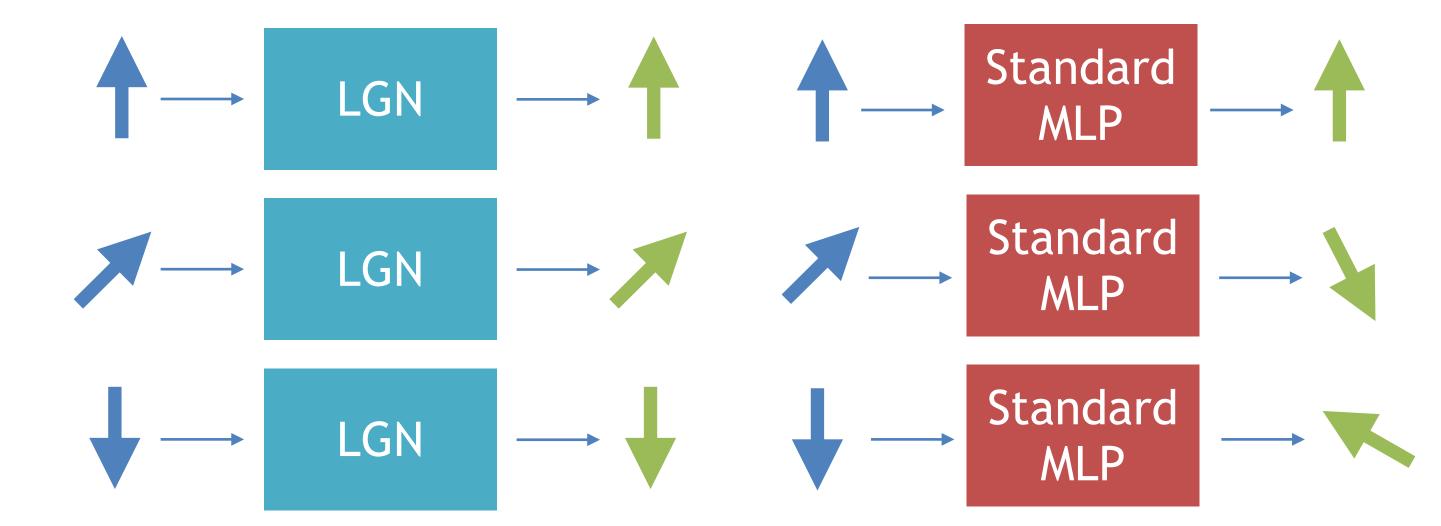
- Consists of an encoder and a decoder.
- Encoder compresses dimension down of the data to the latent space.
- Decoder reconstruct the data from the latent space as much as possible.
- The model is forced to extract the most crucial features in the data.
- The decoder can then be used as a generative model.

Generative Adversarial Networks (GAN)

- Consists of a generator and a discriminator.
- Generator tries to generate data that look as realistic as possible and cheat the discriminator.
- The trained decoder will be used as the generator.
- Discriminator tries to distinguish generated data from real data.

Lorentz Group Network (LGN)

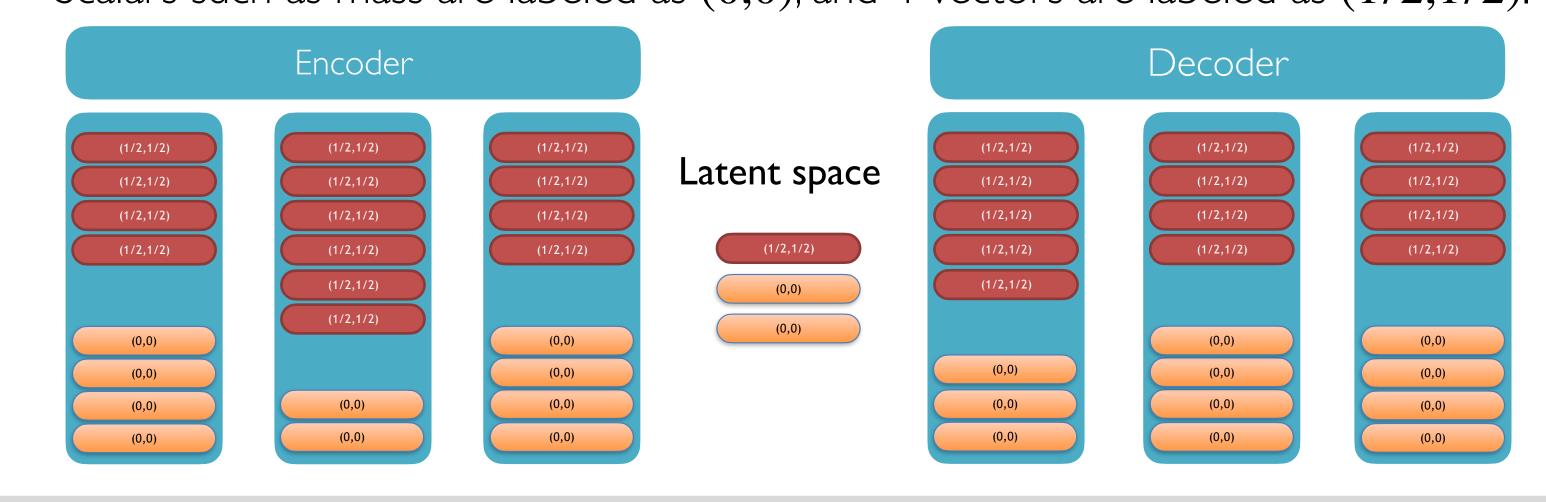
- Built and introduced by Bogatkiy et al. in arXiv:2006.04780.
- Is fully Lorentz group equivariant.



- Has a good performance in jet classification task.
- Needs less parameters because of the constraints of Lorentz group.
- More interpretable because all internal parameters obey the Lorentz symmetry.
- Is not tested for generative task yet.

Lorentz Group Equivariant Jet Autoencoder

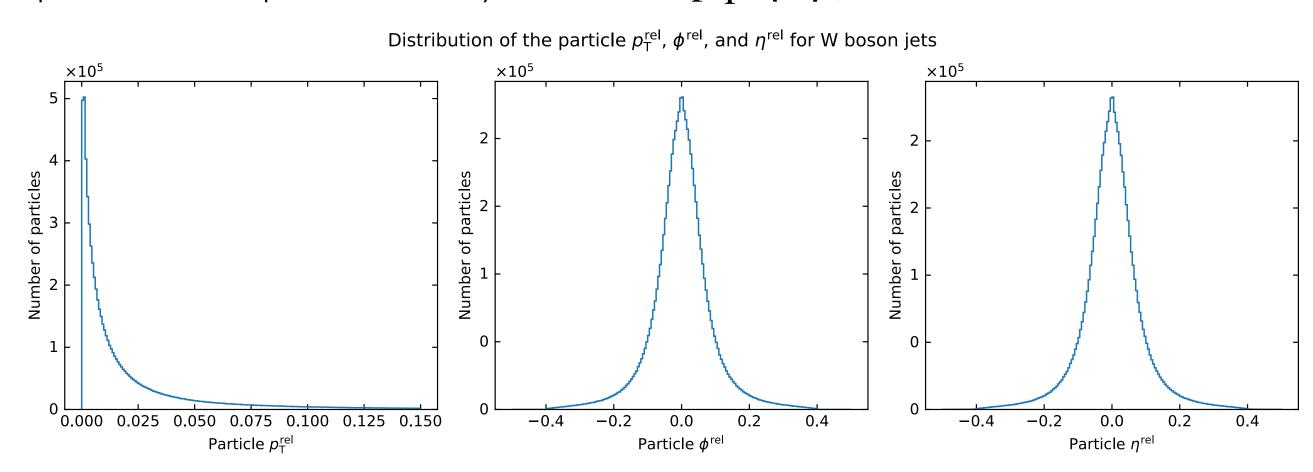
- Built upon the architecture of LGN.
- Takes jets momenta and masses as input.
- Scalars such as mass are labeled as (0,0), and 4-vectors are labeled as (1/2,1/2).



Dataset

Format

- HLS4ML LHC jet dataset preprocessed by Raghav Kansal.
- Each particle is represented by a vector (p_T, η, ϕ) .

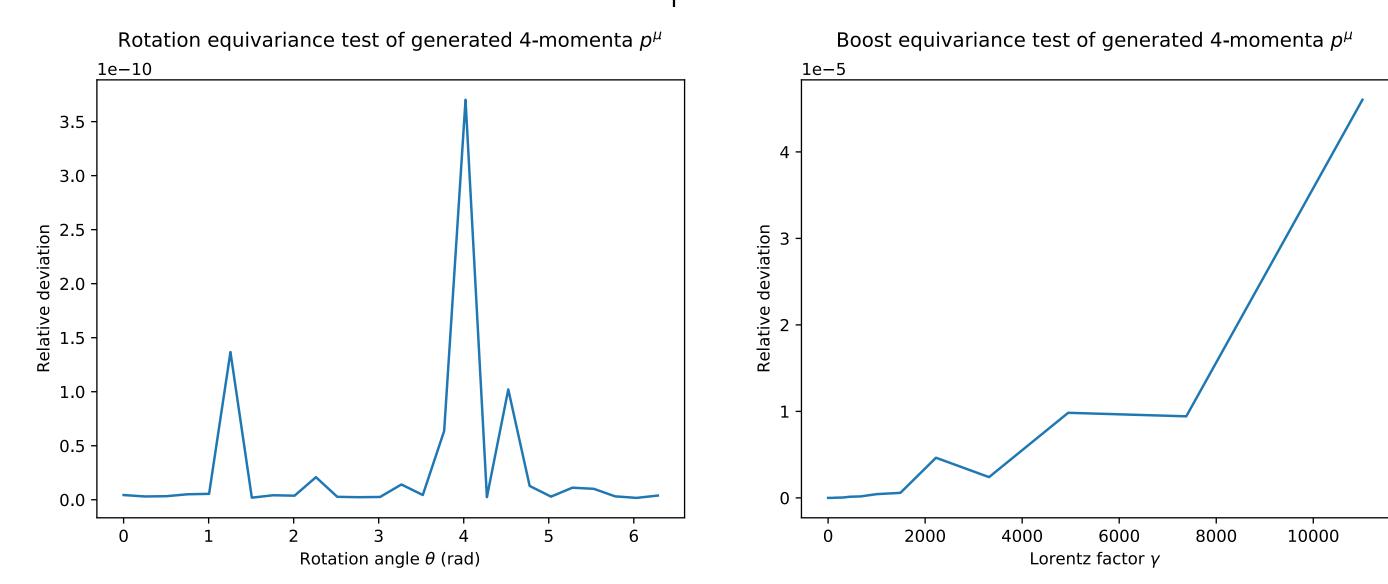


• Converted into Cartesian coordinates $p^{\mu}=(E/c,p_{x},p_{y},p_{z})$.

Evaluation

Equivariance Test

- Rotation of $\theta \in [0,2\pi)$.
- Errors are strictly on the order of $\mathcal{O}(10^{-11})$.
- Boost of $\gamma \in [1,11013.2]$.
- Error increases as γ increases.
- Boost is sensitive to floating point errors.
- Most relevant region for physics: $\gamma \in [1,200]$.
- $\gamma = 11013.2$ corresponds to boosting the reference frame to a speed v = 0.999999992c.
- This model has a **built-in** Lorentz equivariance.



Non-equivariant models produce very large errors:

