COVER PAGE

STAT 608 Homework 05 Summer 2017

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NAME: RAJAN KAPOOR

EMAIL: r.kapoor@tamu.edu

HOMEWORK NUMBER: 5

21/21

(Solution 1)

```
adRevenue <- read.csv("AdRevenue.csv", header=TRUE)
#attach(AdRevenue)

library(alr3)
summary(powerTransform(cbind(AdRevenue,Circulation)~1,data=adRevenue)
)
```

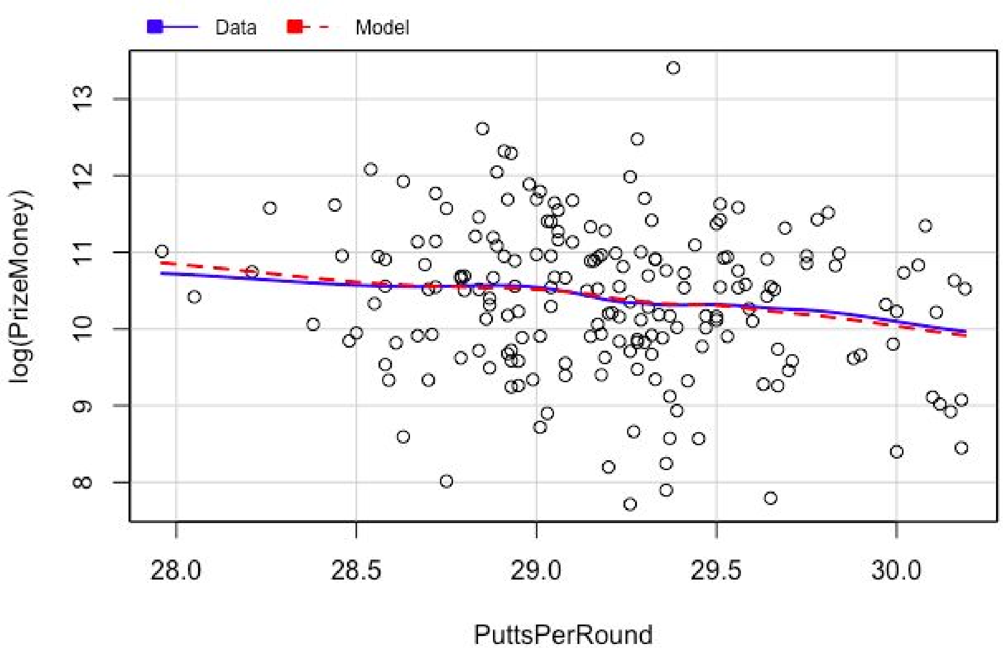
R output:

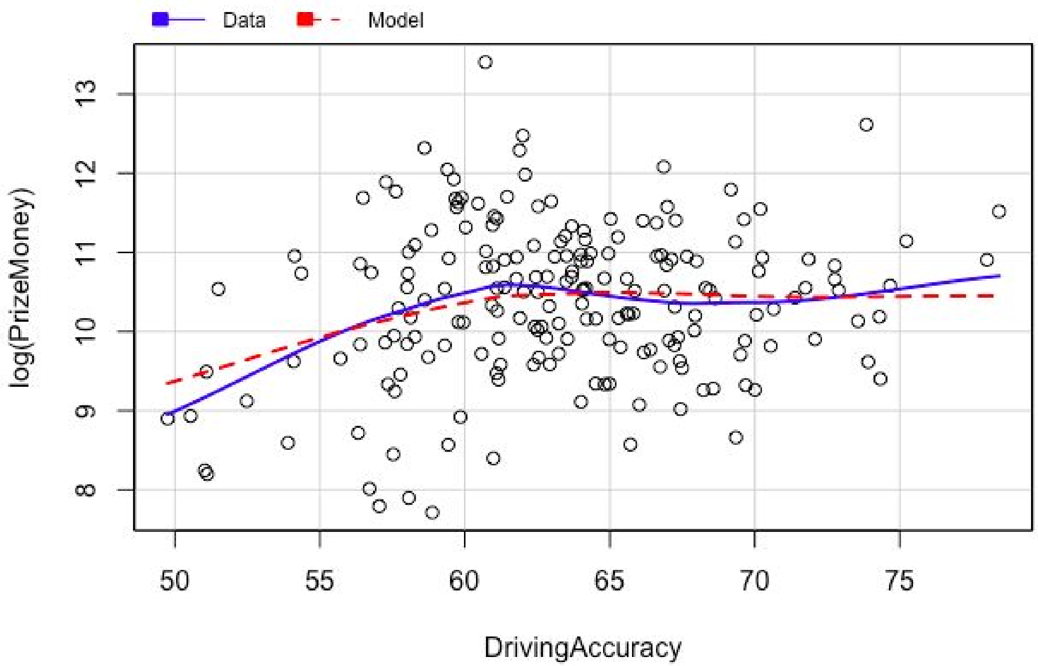
```
bcPower Transformations to Multinormality
            Est.Power Std.Err. Wald Lower Bound Wald Upper Bound
AdRevenue     -0.5873   0.1709          -0.9222          -0.2524
Circulation   -0.2428   0.0828          -0.4051          -0.0805

Likelihood ratio tests about transformation parameters
                                     LRT df        pval
LR test, lambda = (0 0)        13.809568  2 0.001002976
LR test, lambda = (1 1)       249.214715  2 0.000000000
LR test, lambda = (-0.5 -0.33)  4.224511  2 0.120964845
```

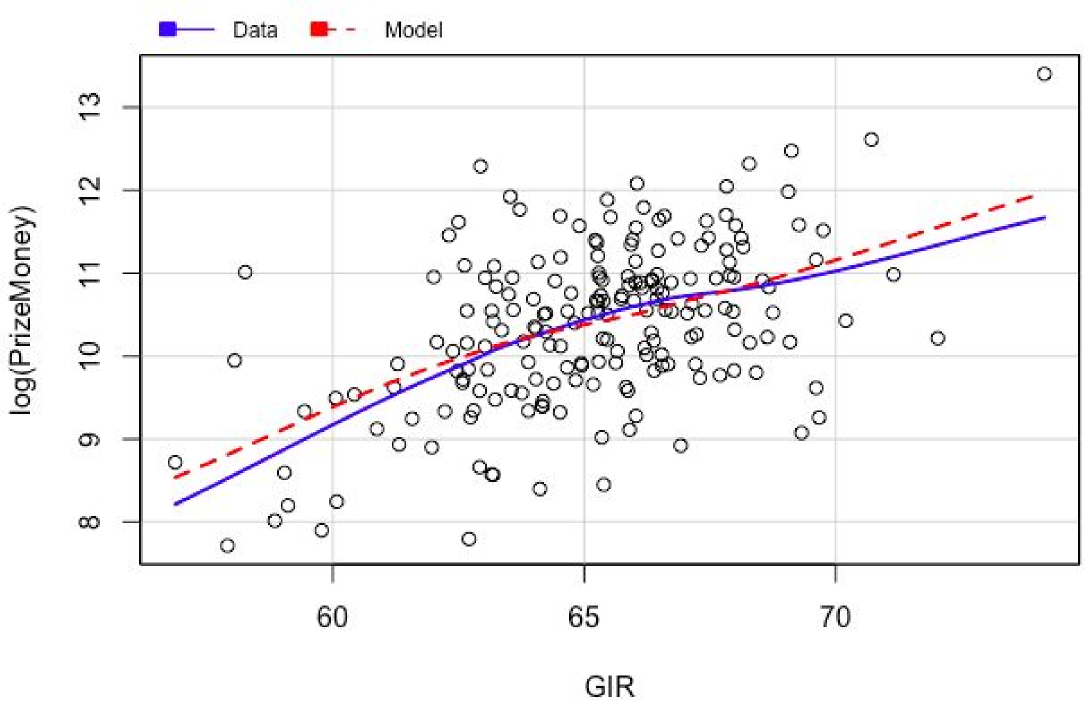
These results indicate `lambda = 0` i.e. log transformation for both Ad Revenue and Circulation is appropriate.

(Solution 2)

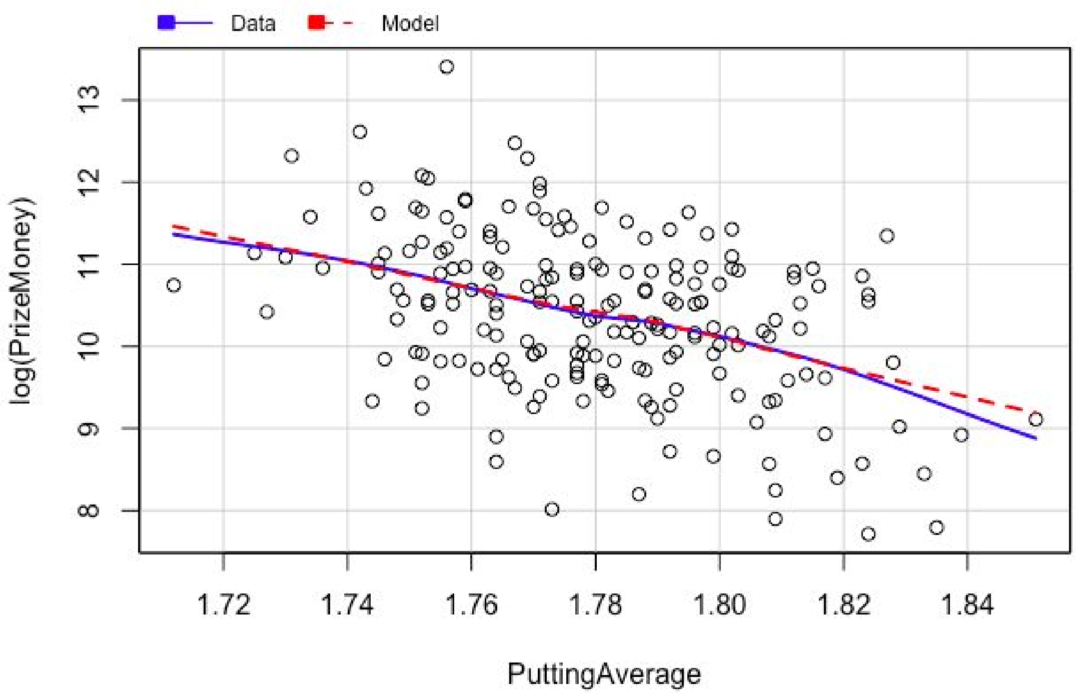


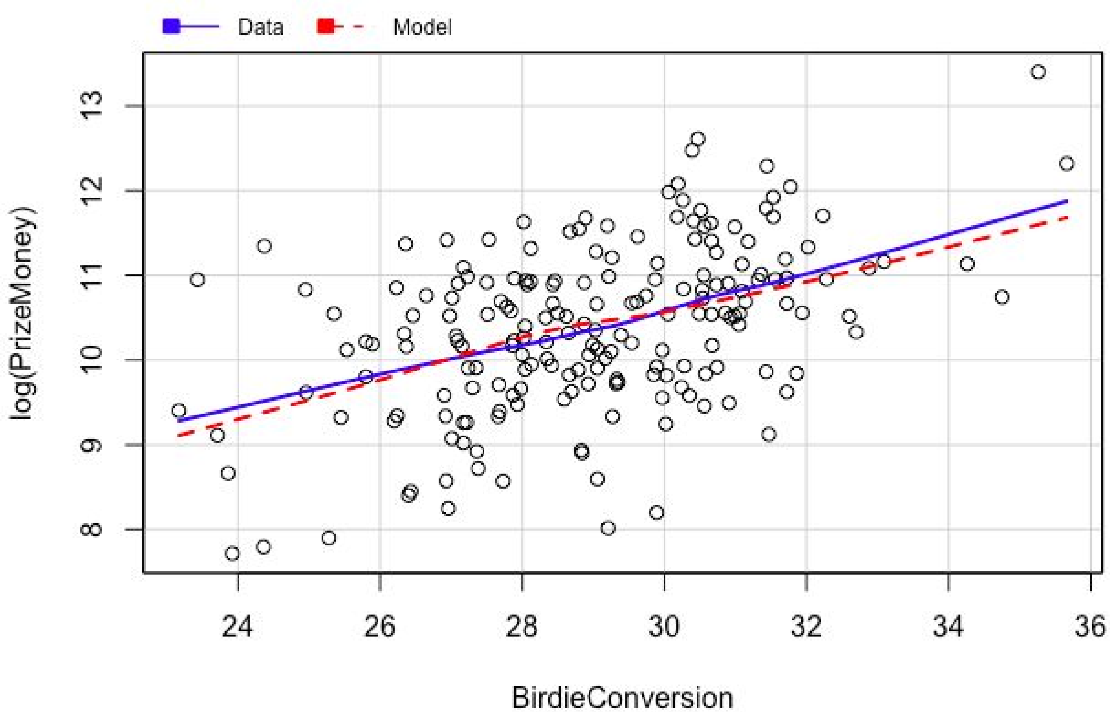


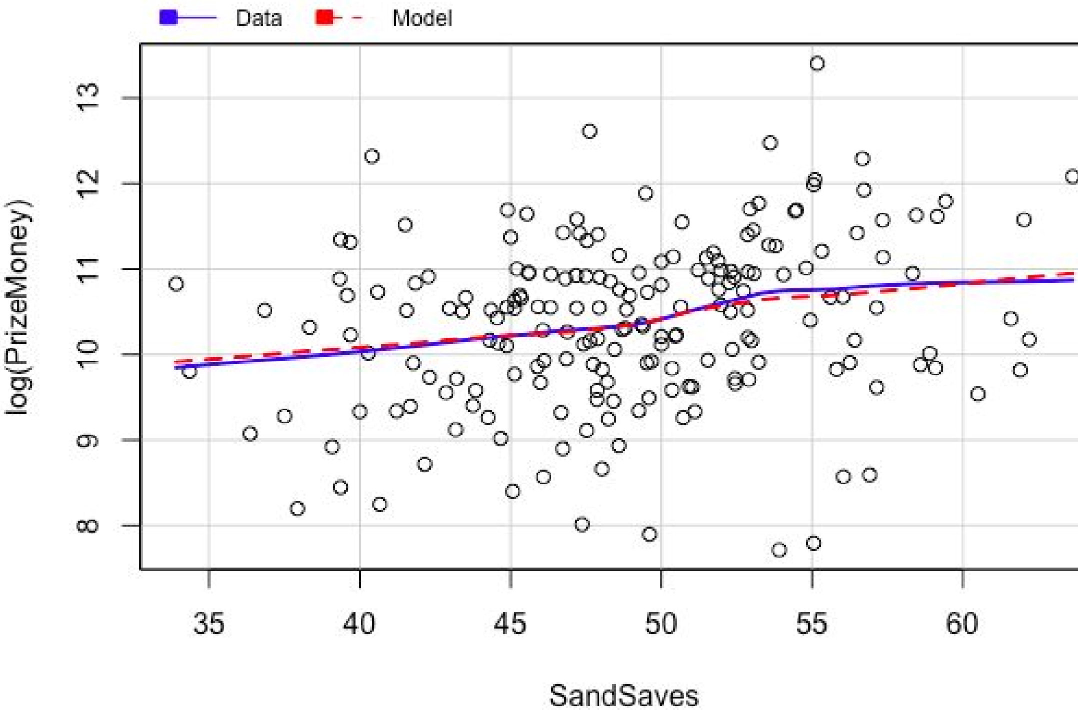
Data
Model
log(PrizeMoney)
8
9
10
11
12
13
50
55
60
65
70
75
DrivingAccuracy



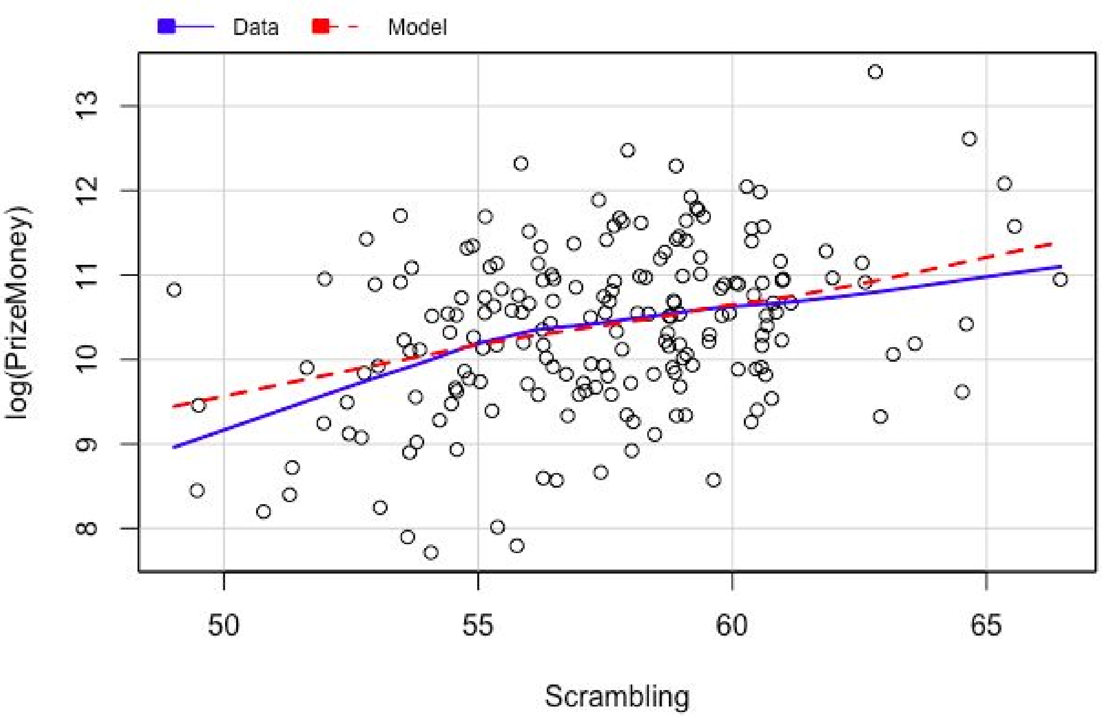
Data
Model
log(PrizeMoney)
8
9
10
11
12
13
60
65
70
GIR


Data
Model
log(PrizeMoney)
8
9
10
11
12
13
1.72
1.74
1.76
1.78
1.80
1.82
1.84
PuttingAverage


Data
Model
log(PrizeMoney)
8
9
10
11
12
13
24
26
28
30
32
34
36
BirdieConversion



Data
Model
log(PrizeMoney)
SandSaves



Data
Model
log(PrizeMoney)
Scrambling

The data and model plots do not perfectly match indicating some transformation of predictor variables is required.

Solution 3

$$y_j = \beta_0 + \beta_1 x_j + e_j \qquad j = 1, \ldots, n$$

$$\hat{y}_j = \hat{\beta}_0 + \hat{\beta}_1 x_j$$

(3.1)

$$\hat{y}_j = \hat{\beta}_0 + \hat{\beta}_1 x_j$$

$$\Rightarrow \sum_{j=1}^{n} \hat{y}_j = \sum_{j=1}^{n} \hat{\beta}_0 + \sum_{j=1}^{n} \hat{\beta}_1 x_j$$

$$\Rightarrow \frac{\sum_{j=1}^{n} \hat{y}_j}{n} = \frac{\sum_{j=1}^{n} \hat{\beta}_0}{n} + \frac{\sum_{j=1}^{n} \hat{\beta}_1 x_j}{n}$$

$$\Rightarrow \bar{\hat{y}} = \hat{\beta}_0 + \hat{\beta}_1 \bar{x}$$

$$= \bar{y}.$$

(3.2) By definition,

$$SS_{reg} = \sum_{i=1}^{n} (\hat{y}_i - \bar{y})^2$$

$$= \sum (\hat{y}_i - \bar{y})(\hat{y}_i - \bar{y})$$

$$= \sum (\hat{y}_i - \bar{y})[(\hat{y}_i - y_i) + (y_i - \bar{y})]$$

$$= \sum (\hat{y}_i - \bar{y})(\hat{y}_i - y_i) + \sum (\hat{y}_i - \bar{y})(y_i - \bar{y})$$

Consider the first term

$$\sum (\hat{y}_i - \bar{y})(\hat{y}_i - y_i)$$

$$= \sum (\hat{\beta}_0 + \hat{\beta}_1 x_i - \hat{\beta}_0 - \hat{\beta}_1 \bar{x})(\hat{\beta}_0 + \hat{\beta}_1 x_i - y_i)$$

$$= \sum \hat{\beta}_1 (x_i - \bar{x})(\bar{y} - \hat{\beta}_1 \bar{x} + \hat{\beta}_1 x_i - y_i)$$

$$= \sum \hat{\beta}_1 (x_i - \bar{x})\{(\bar{y} - y_i) + \hat{\beta}_1 (x_i - \bar{x})\}$$

$$= -\sum \hat{\beta}_1 (x_i - \bar{x})(y_i - \bar{y}) + \sum \hat{\beta}_1^2 (x_i - \bar{x})^2$$

$$= -\hat{\beta}_1 (SXY) + \hat{\beta}_1^2 (SXX)$$

$$= -\hat{\beta}_1 SXY + \beta_1 SXY \qquad \left(\because \hat{\beta}_1 = \frac{SXY}{SXX} \Rightarrow SXX \cdot \hat{\beta}_1 = SXY\right)$$

$$= 0$$

$$\therefore SSreg = \sum_{i=1}^{n} (\hat{y}_i - \bar{\hat{y}})(y_i - \bar{y}) \qquad (\because \bar{\hat{y}} = \bar{y})$$

(3.3)

$$R^2 = \frac{SSreg}{SST}$$

$$= \frac{\sum_i^n (\hat{y}_i - \bar{y})^2}{\sum_i (y_i - \bar{y})^2} \qquad \text{(by definition)} - \text{Eq}①$$

and

$$\rho^2_{(y_i, \hat{y}_i)} = \frac{\left[\sum_{i=1}^{n} (y_i - \bar{y})(\hat{y}_i - \bar{\hat{y}})\right]^2}{\left[\sum_{i=1}^{n} (y_i - \bar{y})^2\right]\left[\sum_{i=1}^{n} (\hat{y}_i - \bar{\hat{y}})^2\right]}$$

Multiplying Eq ① by $\frac{\sum_{i=1}^{n}(\hat{y}_i - \bar{\hat{y}})^2}{\sum_{i=1}^{n}(\hat{y}_i - \bar{\hat{y}})^2}$

$$R^2 = \frac{\left[\sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2\right]^2}{\sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2 \sum_{i=1}^{n}(y_i - \bar{y})^2} \qquad (\because \bar{\hat{y}}_i = \bar{y})$$

$$= \frac{SSreg^2}{\sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2 \sum_{i=1}^{n}(y_i - \bar{y})^2}$$

$$= \frac{\left[\sum_{i=1}^{n}(y_i - \bar{y})(\hat{y} - \bar{\hat{y}})\right]^2}{\sum_{i=1}^{n}(\hat{y}_i - \bar{\hat{y}})^2 \sum_{i=1}^{n}(y_i - \bar{y})^2} \qquad (\text{Using } 3.2)$$

$$= \rho^2$$

(3.4) Adding more covariates reduces RSS so that more of the variation is explained by the regression model, thus increasing SSreg since the sum RSS + SSreg = SST is constant. Since $R^2 = \frac{SSreg}{SST}$, the coefficient of determination increases on adding more covariates and should be maximum at maximum no. of allowable covariates i.e. $m = 20$.