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STAT 608 Homework 07 Summer 2017

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Solution 1)

Assuming unrelated regression lines model
$Y =$ TOTALCWG , $x =$ STAGE

$$M1 \begin{cases} Y = \beta_0 + \beta_1 x + e & TRT = 0 \\ Y = \beta_0 + \beta_2 + (\beta_1 + \beta_3)x + e & TRT = 1 \end{cases}$$

We call model M1 as full model.

Now to test null hypothesis

$H_0 : \beta_3 = 0$ (identical slopes)

$H_A : \beta_3 \neq 0$

The following is the reduced model

$$M2 \begin{cases} Y = \beta_0 + \beta_1 x + e & (TRT = 0) \\ Y = \beta_0 + \beta_2 + \beta_2 x + e & (TRT = 1) \end{cases}$$

The null hypothesis can be tested using partial-F test

$$F = \frac{\dfrac{RSS(\text{reduced}) - RSS(\text{full})}{df(\text{red}) - df(\text{full})}}{RSS(\text{full})/df_{\text{full}}}$$

For model M1

$$Y = 5.4760 + 2.4934x \qquad TRT=0$$

$$Y = 7.7143 + 0.4785x \qquad TRT=1$$

For reduced model

$$Y = 7.2298 + 1.5113x \qquad TRT=0$$

$$Y = 5.91248 + 1.5113x \qquad TRT=1$$

$$F = \frac{\frac{187.59 - 155.22}{1}}{\frac{155.22}{19}} = 3.9623$$

p value $= 0.001786 < \alpha (= 0.05)$

$\Rightarrow$ Reject Null Hypothesis

Solution 2

2)

$$\log \frac{p(x)}{1-p(x)} = \hat{\beta}_0 + \hat{\beta}_1 x$$

$$\Rightarrow \quad p(x) = \exp(\hat{\beta}_0 + \hat{\beta}_1 x) + \exp(\hat{\beta}_0 + \hat{\beta}_1 x)\, p(x)$$

$$\Rightarrow \quad p(x) = \frac{\exp(\hat{\beta}_0 + \hat{\beta}_1 x)}{1 + \exp(\hat{\beta}_0 + \hat{\beta}_1 x)} = \frac{1}{\exp(-\hat{\beta}_0 - \hat{\beta}_1 x) + 1}$$

at $x = 3$

$$p(x) = \frac{1}{\exp[-(-2.643 + 0.674 \times 3)] + 1}$$

$\Rightarrow P(x) = 0.3495$

$\therefore$ probability that the insect will survive $= 1 - p(x)$
$= 0.6504$

Expected no. of surviving insects $= 200 \times (1 - p(x))$
$= 130.$

2.2 Odds on dying $= \dfrac{p(x)}{1-p(x)} = \exp(\hat{\beta}_0 + \hat{\beta}_1 x)$

if x increases by 1 unit, odds on dying increases by $\phi = \exp(0.674)$
$= 1.96207$

2.3 90% CI for ϕ $\exp\left(\hat{\beta}_1 \pm z_{0.05} * s.e(\hat{\beta}_1)\right)$ $\alpha = 0.1$, $\alpha/2 = 0.05$

$\exp(0.674 \pm 1.6448 \times 0.039)$

$(1.840, 2.092)$

Solution 3

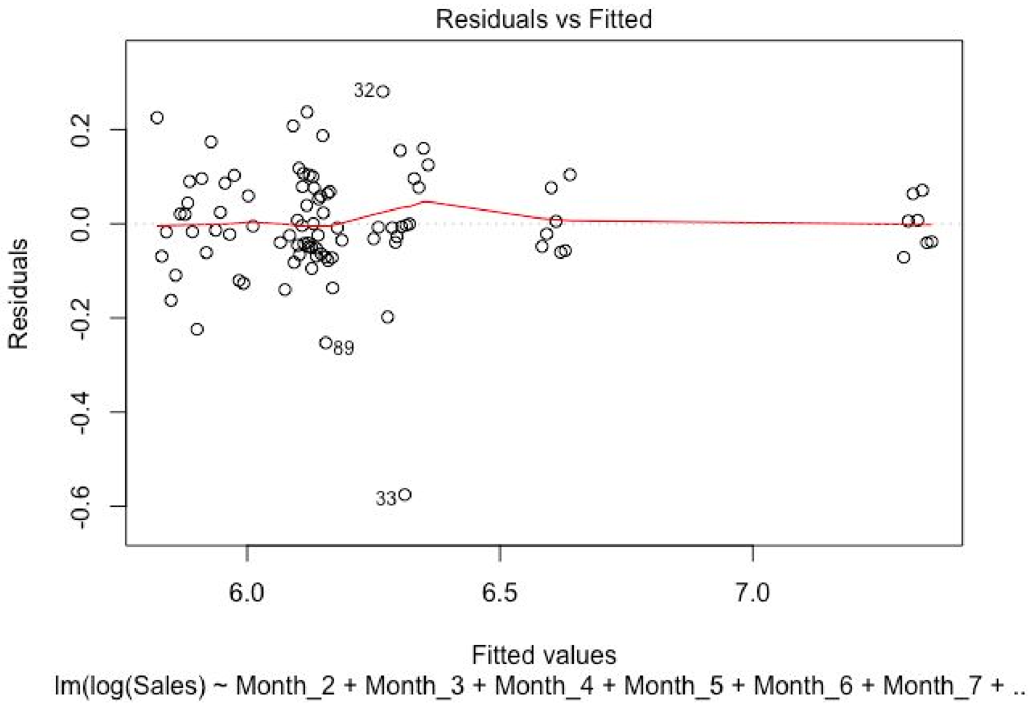
$\log(Sales) = \beta_0 + Time + Month\_i \quad i = 2, \ldots, 12$

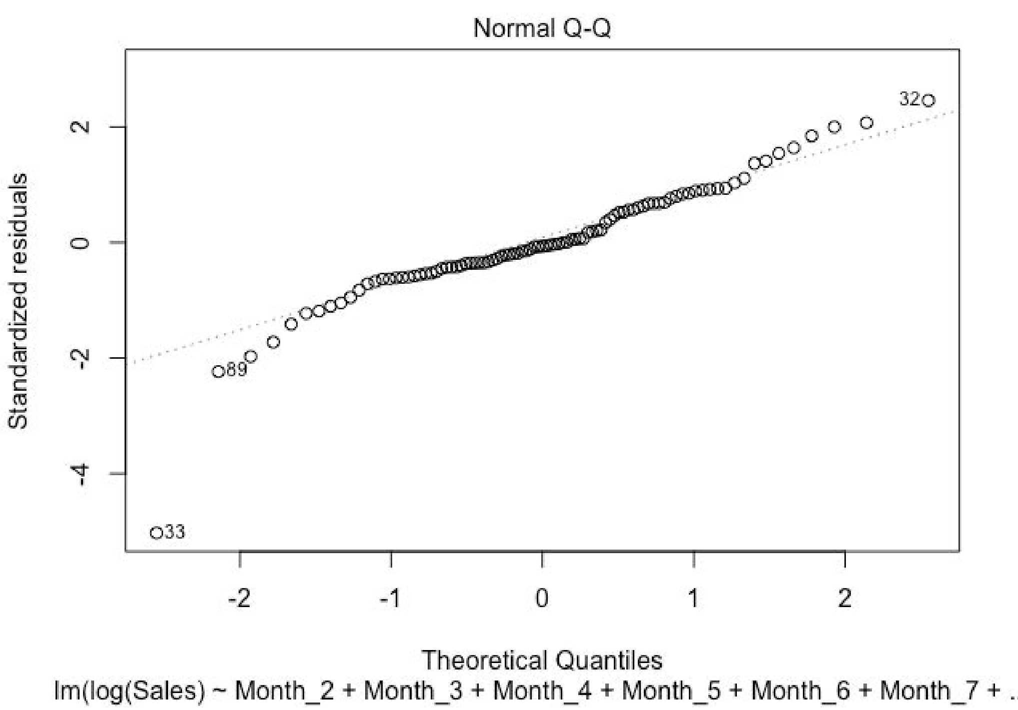
The diagnostic plots and summary from R is attached.

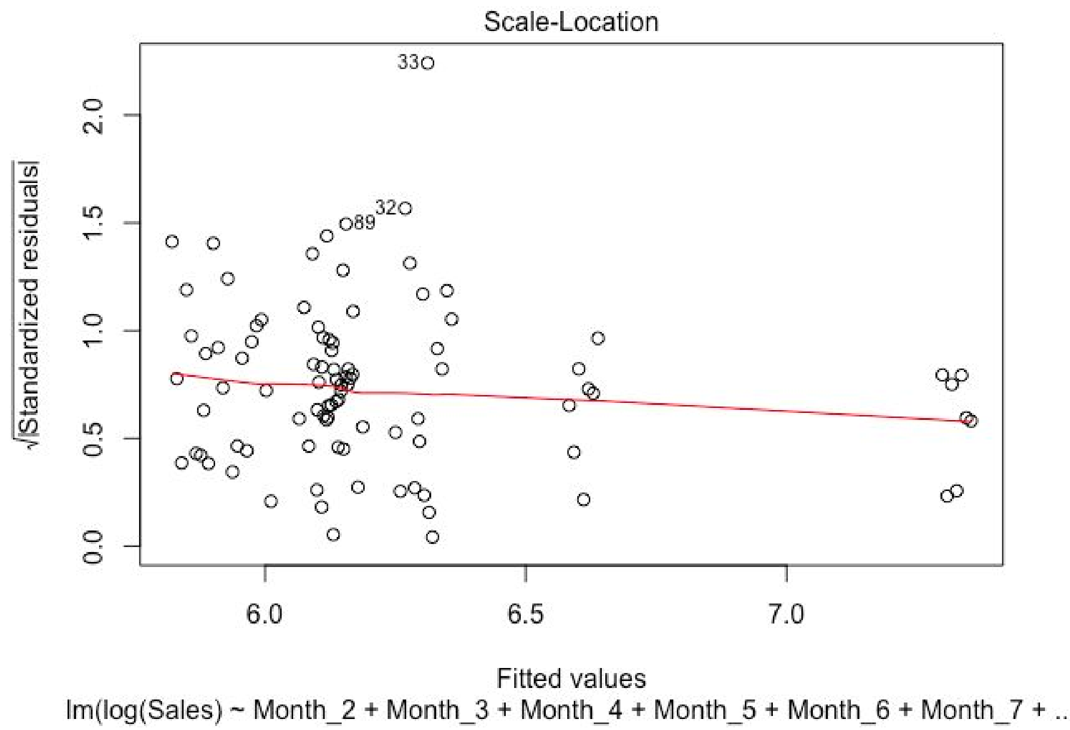
Weakness - Variables Time, Month_2, Month_7 are not statistically significant
- Case 32, 33, 41, 89 have standard resid > 2.

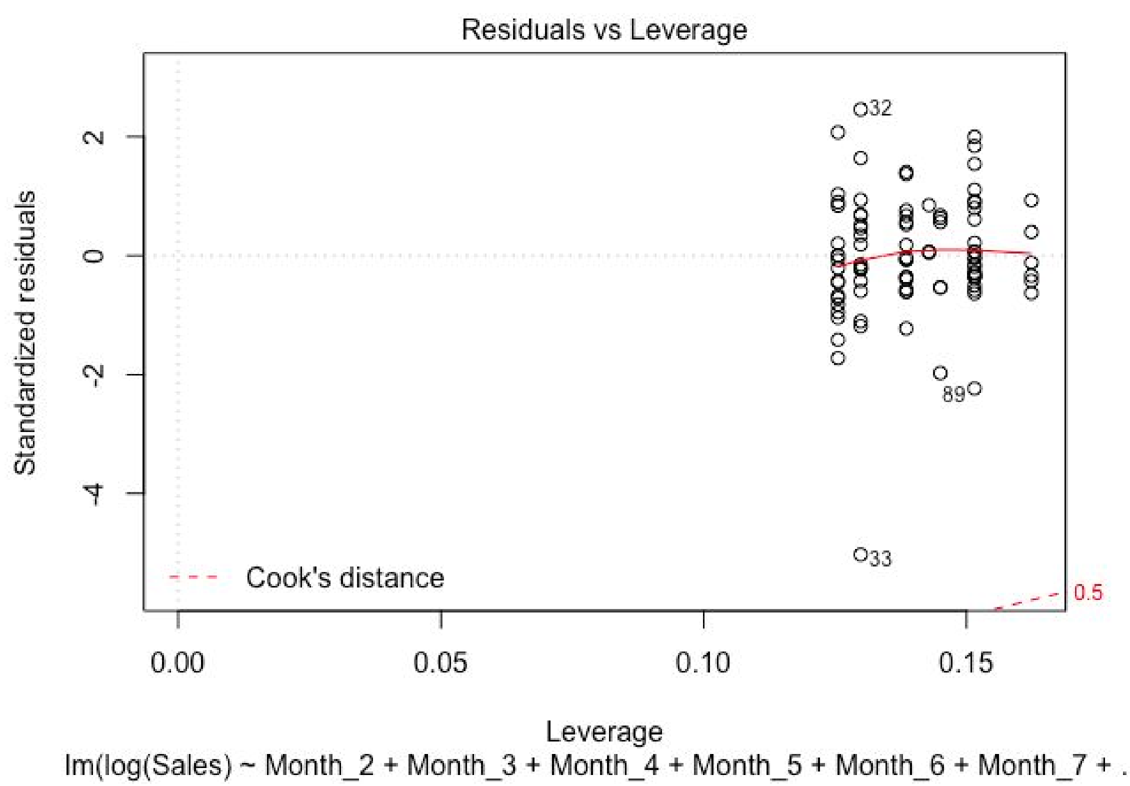
```
Call:
lm(formula = log(Sales) ~ Month_2 + Month_3 + Month_4 + Month_5 +
    Month_6 + Month_7 + Month_8 + Month_9 + Month_10 + Month_11 +
    Month_12 + Time)

Residuals:
     Min       1Q   Median       3Q      Max
-0.57520 -0.05166 -0.00739  0.07133  0.28116

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  5.8723775  0.0518913 113.167  < 2e-16 ***
Month_2     -0.0522214  0.0635314  -0.822 0.413533
Month_3      0.1907035  0.0635154   3.002 0.003572 **
Month_4      0.2474630  0.0635029   3.897 0.000201 ***
Month_5      0.2270750  0.0634940   3.576 0.000595 ***
Month_6      0.2138238  0.0634887   3.368 0.001167 **
Month_7      0.0690719  0.0634869   1.088 0.279874
Month_8      0.2208295  0.0634887   3.478 0.000819 ***
Month_9      0.3708060  0.0634940   5.840 1.07e-07 ***
Month_10     0.4133160  0.0635029   6.509 6.13e-09 ***
Month_11     0.7023337  0.0655759  10.710  < 2e-16 ***
Month_12     1.4169818  0.0655707  21.610  < 2e-16 ***
Time         0.0007670  0.0004756   1.613 0.110755
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.1227 on 80 degrees of freedom
Multiple R-squared:  0.9111,    Adjusted R-squared:  0.8978
F-statistic: 68.33 on 12 and 80 DF,  p-value: < 2.2e-16
```

Diagnostic Plots:


Residuals vs Fitted
0.2
0.0
-0.2
-0.4
-0.6
Residuals
32
89
33
6.0
6.5
7.0
Fitted values
lm(log(Sales) ~ Month_2 + Month_3 + Month_4 + Month_5 + Month_6 + Month_7 + ..


Normal Q-Q
2
0
-2
-4
Standardized residuals
32
89
33
-2
-1
0
1
2
Theoretical Quantiles
lm(log(Sales) ~ Month_2 + Month_3 + Month_4 + Month_5 + Month_6 + Month_7 + ..


Scale-Location
√|Standardized residuals|
Fitted values
lm(log(Sales) ~ Month_2 + Month_3 + Month_4 + Month_5 + Month_6 + Month_7 + ..
0.0
0.5
1.0
1.5
2.0
6.0
6.5
7.0
33
32
89


Residuals vs Leverage
Standardized residuals
Leverage
lm(log(Sales) ~ Month_2 + Month_3 + Month_4 + Month_5 + Month_6 + Month_7 + ..
Cook's distance
0.5
-4
-2
0
2
0.00
0.05
0.10
0.15
32
89
33

Solution 4

(4.1)
$$\underbrace{\begin{bmatrix} e_2 \\ e_3 \\ e_4 \\ \vdots \\ e_n \end{bmatrix}}_{Y} = \rho \underbrace{\begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ \vdots \\ e_{n-1} \end{bmatrix}}_{X} + \begin{bmatrix} \epsilon_2 \\ \epsilon_3 \\ \epsilon_4 \\ \vdots \\ \epsilon_n \end{bmatrix} \quad \text{— (M1)}$$

Clearly a linear regression of Y on X through the origin will estimate ρ. Since all 4 assumptions of Regression Model –

- Y is related to X by simple reg. model ($\because e_t = \rho e_{t-1} + \epsilon_t$)
- ϵ_t are independent of each other
- ϵ_t have common variace
- ϵ_t are normally distributed with mean zero & common variance (=1 $\because$ standard normal) distribution given for ϵ_t (least sq estimate)

$$\hat{\rho} = \text{slope} = 0.412$$

(4.2) YES. the valid s.e. for $\hat{\rho}$ should be 0.133, since Model M1 is valid regression model (linear).