COVER PAGE

STAT 608 Homework 03, Summer 2017

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45/49

(1) $H_0 : \gamma_1 = \gamma_2$

$H_A : \gamma_1 \neq \gamma_2$

or

$H_0 : Y_j = \gamma_0 + \gamma_1 z_{j1} + \gamma_1 z_{j2} + \epsilon_j$

$H_A : Y_j = \gamma_0 + \gamma_1 z_{j1} + \gamma_2 z_{j2} + \epsilon_j$

$$F = \frac{RSS_{(reduced)} - RSS_{(full)}}{df_{reduced} - df_{full}} \div \frac{RSS_{full}}{df_{full}}$$

$df_{full} = 60 - 3 = 57$

$df_{reduced} = 60 - 2 = 58$

$F \sim F_{2,57}$

If F is statistically significant, we favor the full model and reject the null hypothesis. i.e. p-value less than threshold (eg 0.001).

(2)

(2.1) Unbiased estimate of the error variance

$$S^2 = \frac{RSS}{df} = \frac{100}{30-4} = \frac{100}{26} = 38.46$$

(2.2) Possible numerical value of $\hat{\beta}_1$

$$t_c = \frac{\hat{\beta}_1 - 0}{se(\hat{\beta}_1)} \sim t\text{-distribution with } df = 26$$

p-value = 0.02

$\Rightarrow P(T > t_c) = 0.02$

For p-value of 0.02, the critical value is given by

$$\Rightarrow t_c = 2.479 \quad \text{or} \quad t_c = -2.479$$

$$\frac{\hat{\beta}_1}{se(\hat{\beta}_1)} = 2.479 \quad \text{or} \quad \frac{\hat{\beta}_1}{se(\hat{\beta}_1)} = -2.479$$

$$\Rightarrow \hat{\beta}_1 = 2.479 \times 0.5 \quad \text{or} \quad \hat{\beta}_1 = -2.479 \times 0.5$$

$$\Rightarrow \hat{\beta}_1 = 1.2395 \quad \text{or} \quad \hat{\beta}_1 = -1.2395$$

(2.3) 95% CI for $\beta_2 = \hat{\beta}_2 \pm t\left(\frac{0.05}{2}, 26\right) \times se(\hat{\beta}_2)$

$$= 1 \pm 0.3385\,(0.25)$$

$$(0.915375,\ 1.084625)$$

(2.4) $R^2 = 1 - \frac{RSS}{SST} = 0.9$

$$\Rightarrow \frac{RSS}{SST} = 0.1 \Rightarrow SST = 1000$$

$$\Rightarrow RSS + SS_{reg} = 1000$$

$$\Rightarrow SS_{reg} = 900 \qquad (\because RSS = 100)$$

(2.5) $$F = \frac{SS_{reg}/p}{RSS/n-p-1}$$

$$= \frac{900}{3} \Big/ \frac{100}{26} = 3 \times 26 = 78 \sim F_{3,26}$$

At 5% Level, $f_c = 2.975$
($\alpha = 0.05$)

F value $> f_c \Rightarrow$ Reject H_0.

(3) (3.1) $H = X(X'X)^{-1}X'$ (uppercase letters denote Matrix)

For idempotent matrix $AA' = A$

$$HH' = X(X'X)^{-1}X' \left[X(X'X)^{-1}X'\right]'$$

$$= X(X'X)^{-1}X' \; X'' \; (X'X)^{-1}X' \qquad [\because (X'X)' = X'X \;\&\; (X^{-1})' = (X')^{-1}]$$

$$= X\underbrace{(X'X)^{-1}X'X}(X'X)^{-1}X' \qquad [\because X'' = X]$$

$$= XI(X'X)^{-1}X' \qquad [\because X^{-1}X = I]$$

$$= X(X'X)^{-1}X' \qquad [\because XI = X]$$

$$= H$$

(3.2) $Var(\vec{\hat{e}}|X) = Var((I-H)\vec{y}|X)$

$$= (I-H)\,Var(\vec{y}|X)(I-H)^T$$

$[\because H = f(X)$ & $Var(A\vec{y}) = A\,Var(\vec{y})A^T]$

$$= (I-H)\Sigma(I-H)'$$

$$= (I-H)\sigma^2 I (I-H)'$$

$$= (I-H)(I-H)'\sigma^2$$

$$= (I-H)(I-H')\sigma^2$$

$$= (I - HI - IH' + HH')\sigma^2$$

$$= (I - H - H + H)\sigma^2 \qquad \because HH' = H$$

$$= (I-H)\sigma^2$$

(4) $\vec{\hat{e}} = (I - H)\vec{y}$

$\vec{\hat{e}} = \left(I - X(X'X)^{-1}X'\right)\vec{y}$ — (1)

To generate given expression, premultiply $\vec{\hat{e}}$ by X'

$$LHS = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \hat{e}_3 \\ \hat{e}_4 \end{bmatrix} = \begin{bmatrix} \hat{e}_1 + \hat{e}_3 \\ \hat{e}_1 + 2\hat{e}_2 + \hat{e}_4 \end{bmatrix}$$

$$RHS = X'\left(I - X(X'X)^{-1}X'\right)\vec{y}$$

$$= \left(X'I - X'X(X'X)^{-1}X'\right)\vec{y}$$

$$= (X' - IX')\vec{y}$$

$$= (X' - X')\vec{y} = 0_{2\times 3}$$

Comparing second row in LHS & RHS.

$\Rightarrow \hat{e}_1 + 2\hat{e}_2 + \hat{e}_4 = 0$. Yes.

(5)

$$\vec{Y} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 2 & 1 \\ 1 & 2 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} \beta_A \\ \beta_B \end{bmatrix} + \vec{\hat{e}}$$

(6) Yes. The Q-Q plots of all predictor variables follow straight line, so no transformation is needed.

(a) for response variable, the Q-Q plot is not a straight line but becomes straight after log transformation.

The Y vs each predictor variable plots show increasing variance without transformation. Also box-cox transformation plot shows $\lambda = 0$ lies within 95% Confidence interval of max. likelihood.

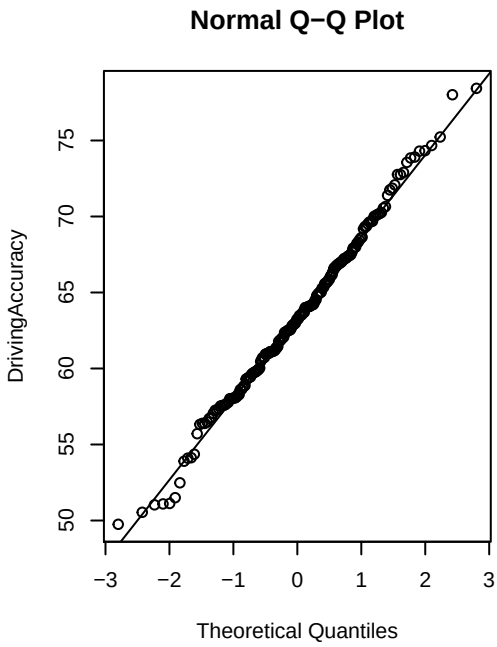
Since $\lambda = 0$ corresponds to log transformation, the recommendation is correct.

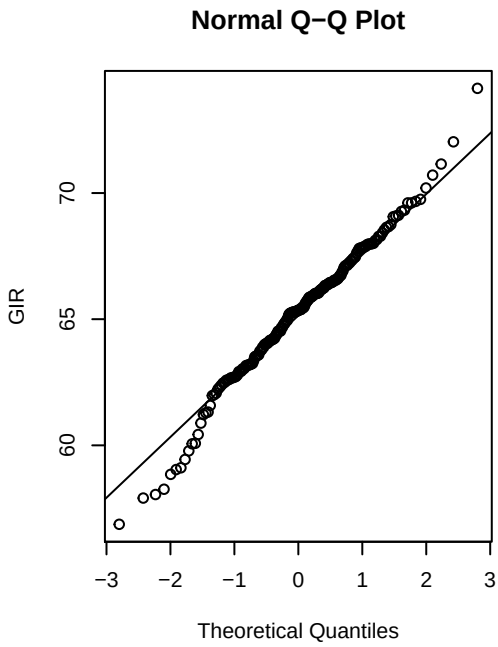
(b) See page -

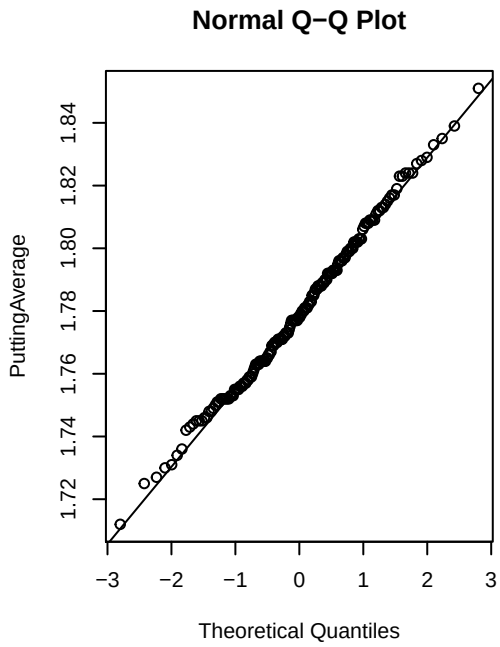
(c) From leverage plot, effects of high leverage points should be looked into before finalizing the model.

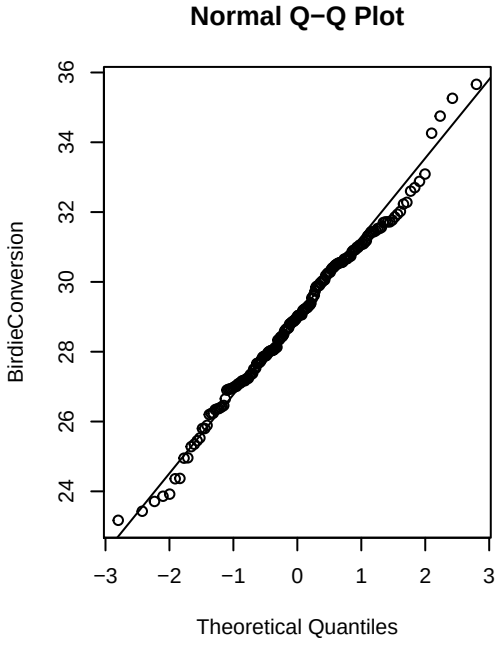
(d) Statistically insignificant predictors can be removed

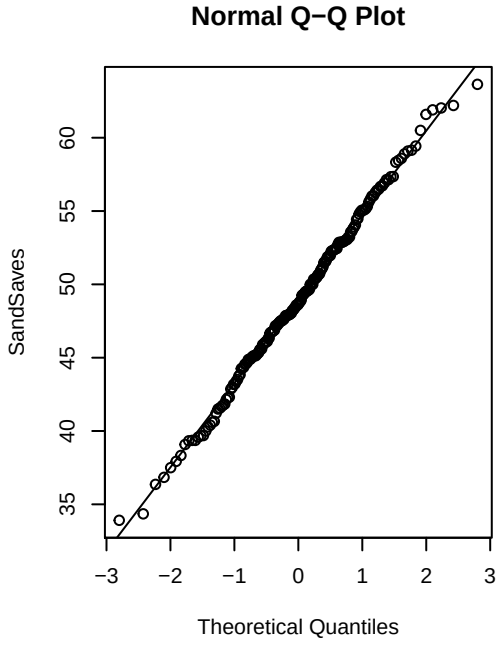
(e) Removing all predictors with insignificant p values at once is not recommended because then we'll be comparing model with subset of predictor variables with original model. p-value can only be used to remove one predictor variable at a time. Partial f-test should be used to decide if a subset of predictor variables should be removed at once.

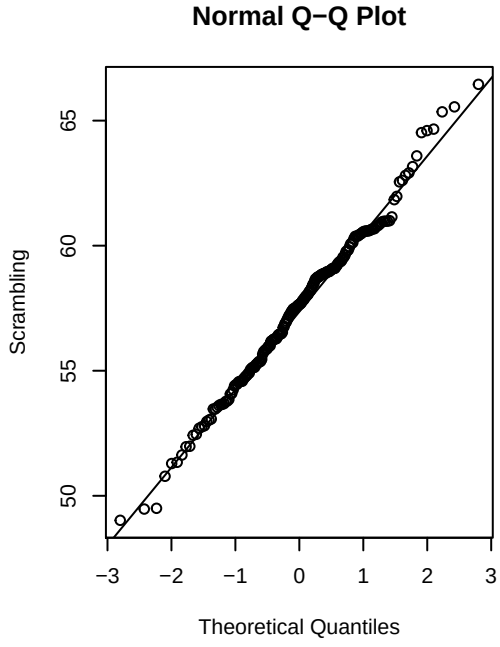

Normal Q−Q Plot
DrivingAccuracy
Theoretical Quantiles

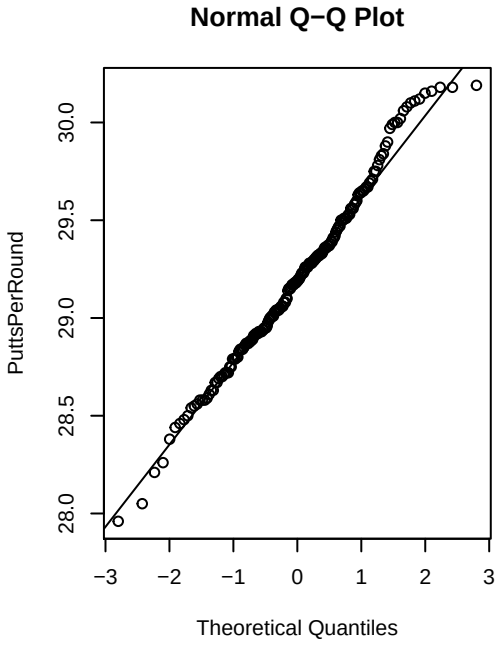

Normal Q−Q Plot
GIR
Theoretical Quantiles

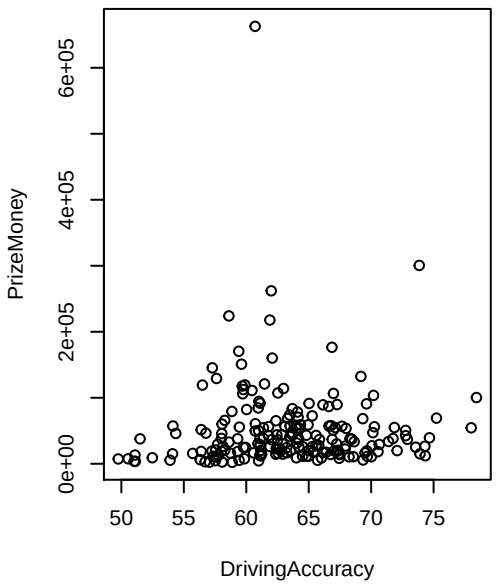

Normal Q−Q Plot
PuttingAverage
Theoretical Quantiles

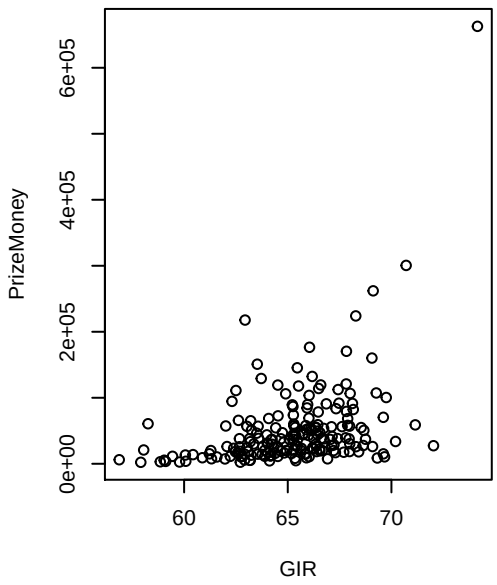

Normal Q−Q Plot
BirdieConversion
Theoretical Quantiles

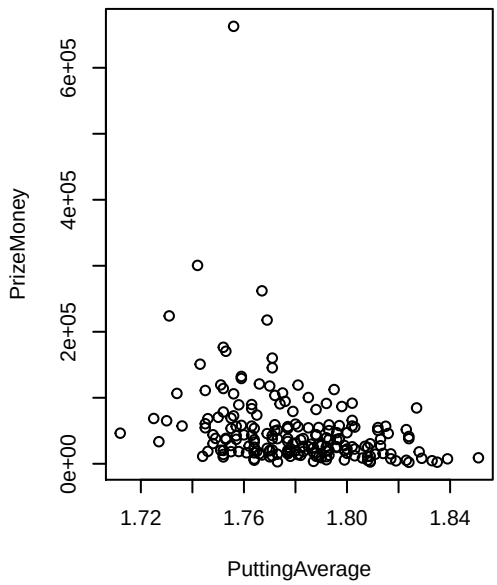

Normal Q−Q Plot
SandSaves
Theoretical Quantiles

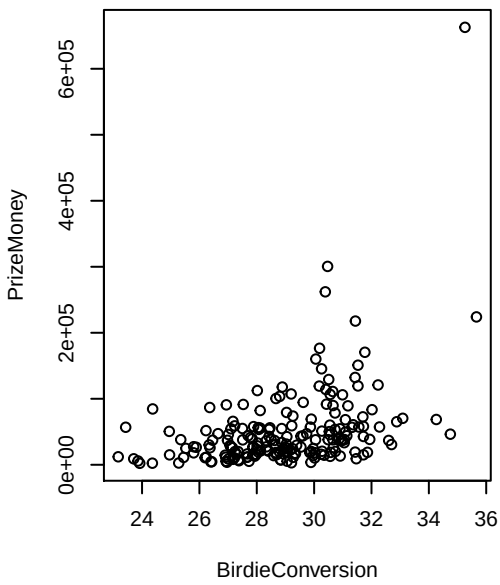

Normal Q−Q Plot
Scrambling
Theoretical Quantiles

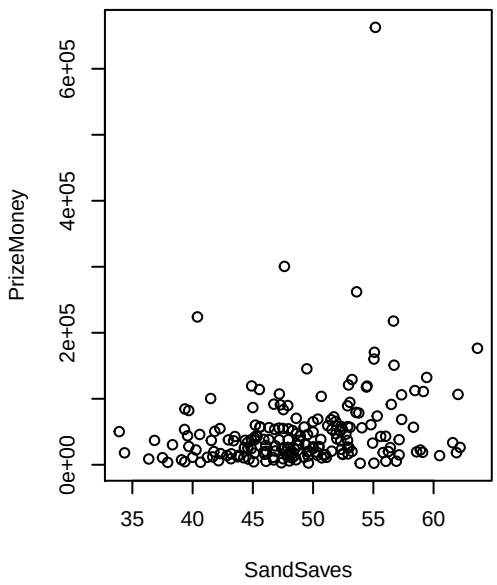

Normal Q−Q Plot
PuttsPerRound
Theoretical Quantiles

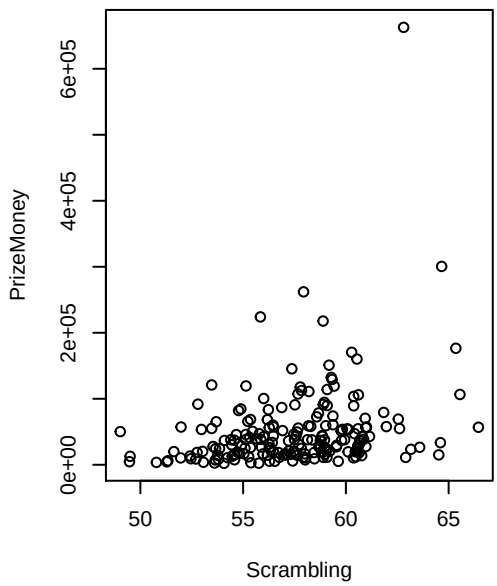

PrizeMoney
0e+00
2e+05
4e+05
6e+05
50
55
60
65
70
75
DrivingAccuracy

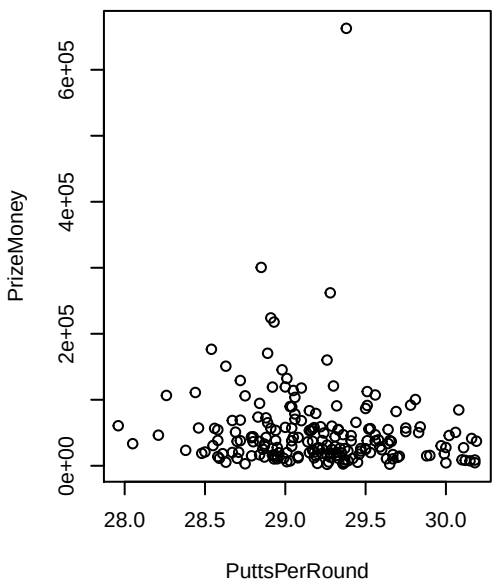

PrizeMoney
0e+00
2e+05
4e+05
6e+05
60
65
70
GIR

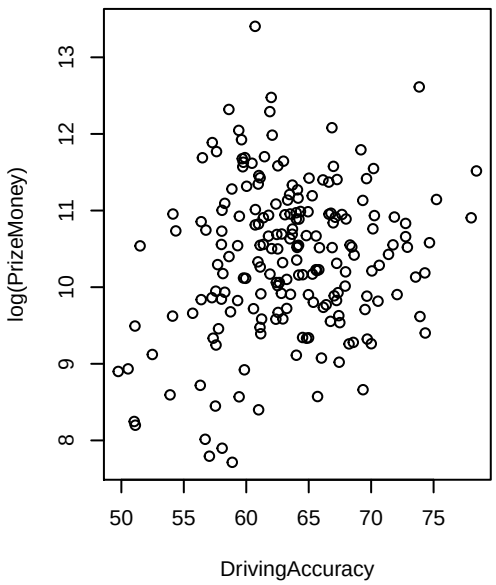

PrizeMoney
0e+00
2e+05
4e+05
6e+05
1.72
1.76
1.80
1.84
PuttingAverage

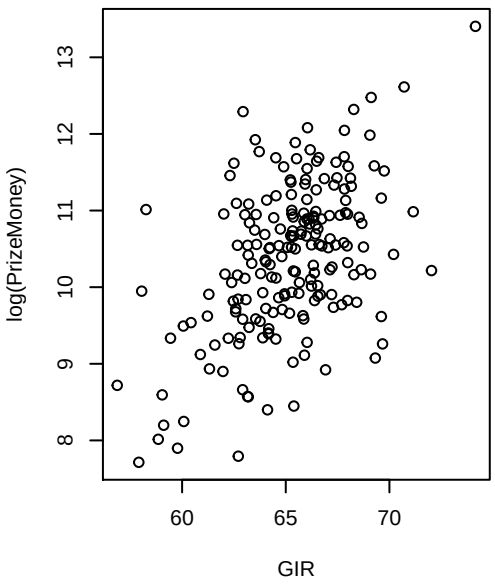

PrizeMoney
0e+00
2e+05
4e+05
6e+05
24
26
28
30
32
34
36
BirdieConversion


PrizeMoney
0e+00
2e+05
4e+05
6e+05
35
40
45
50
55
60
SandSaves


PrizeMoney
0e+00
2e+05
4e+05
6e+05
50
55
60
65
Scrambling


PrizeMoney
0e+00
2e+05
4e+05
6e+05
28.0
28.5
29.0
29.5
30.0
PuttsPerRound


log(PrizeMoney)
8
9
10
11
12
13
50
55
60
65
70
75
DrivingAccuracy


log(PrizeMoney)
8
9
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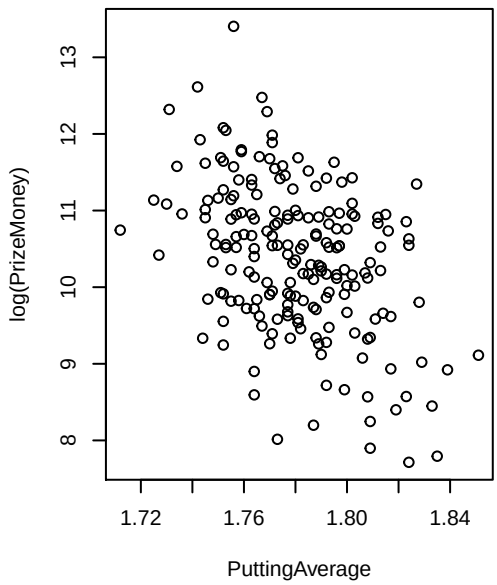
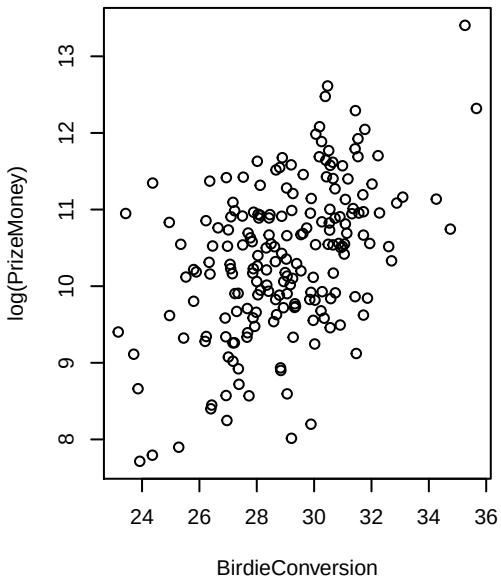
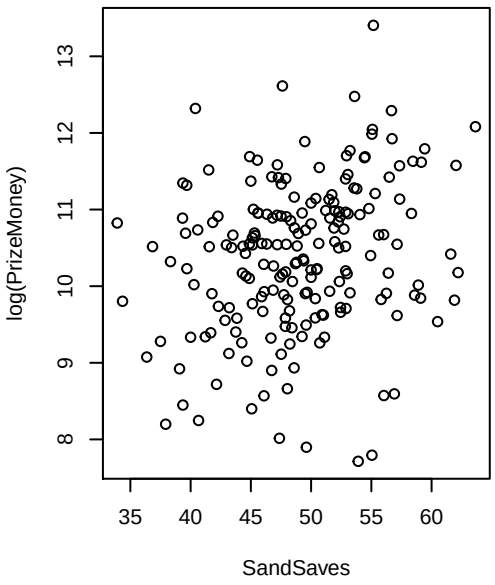
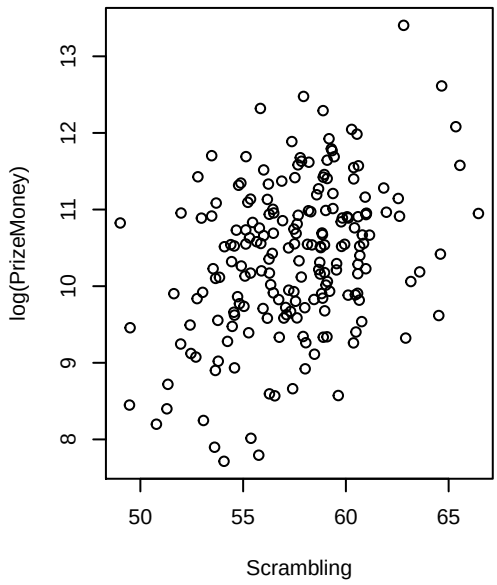
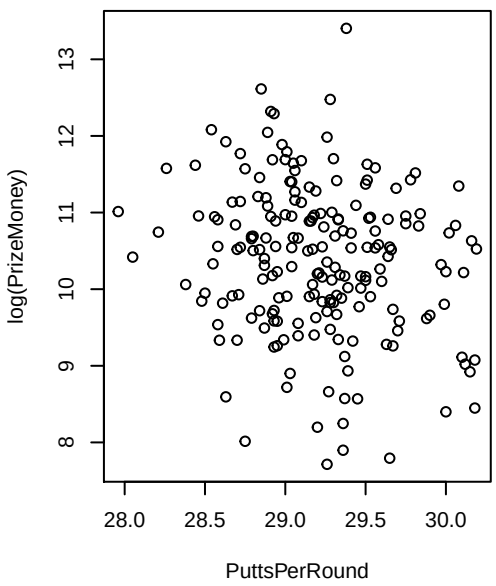
11
12
13
60
65
70
GIR

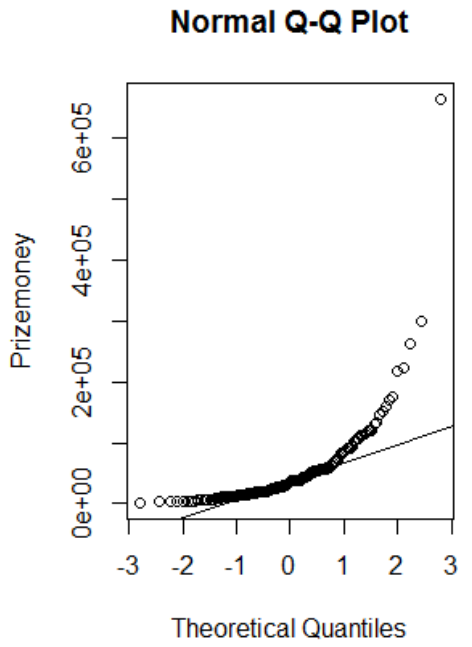

log(PrizeMoney)
8
9
10
11
12
13
1.72
1.76
1.80
1.84
PuttingAverage

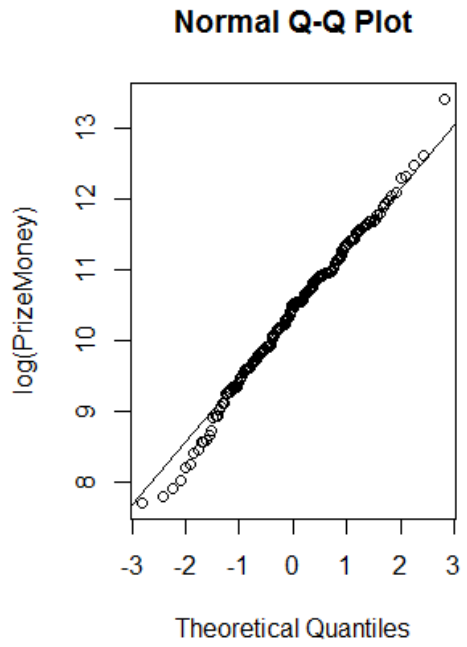

log(PrizeMoney)
8
9
10
11
12
13
24
26
28
30
32
34
36
BirdieConversion

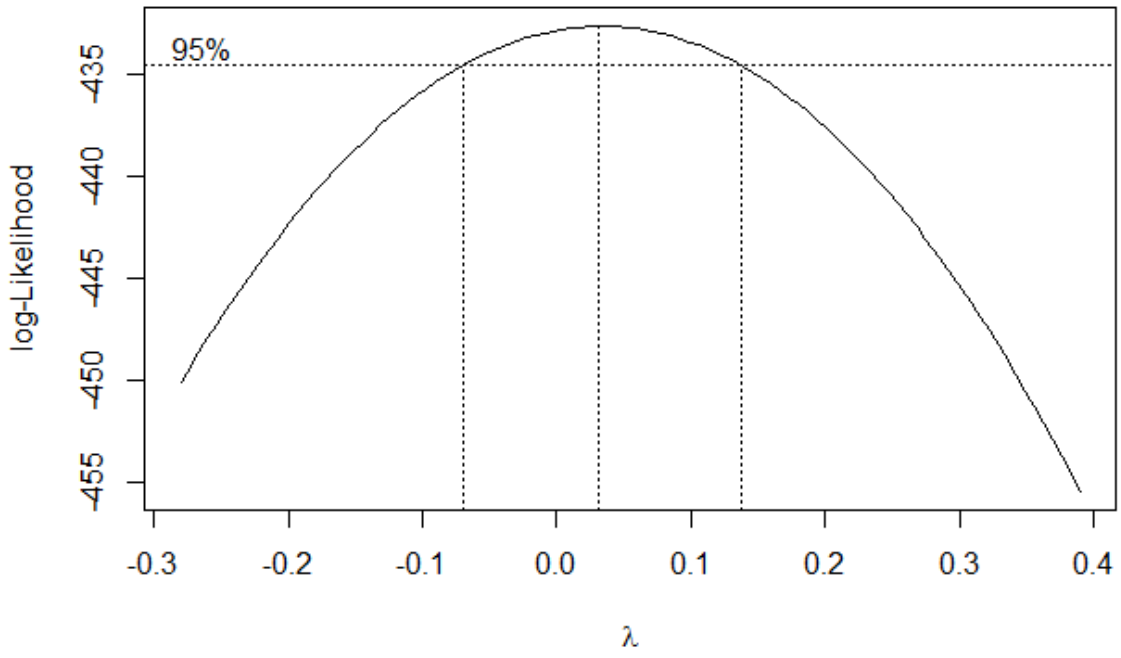

log(PrizeMoney)
8
9
10
11
12
13
35
40
45
50
55
60
SandSaves

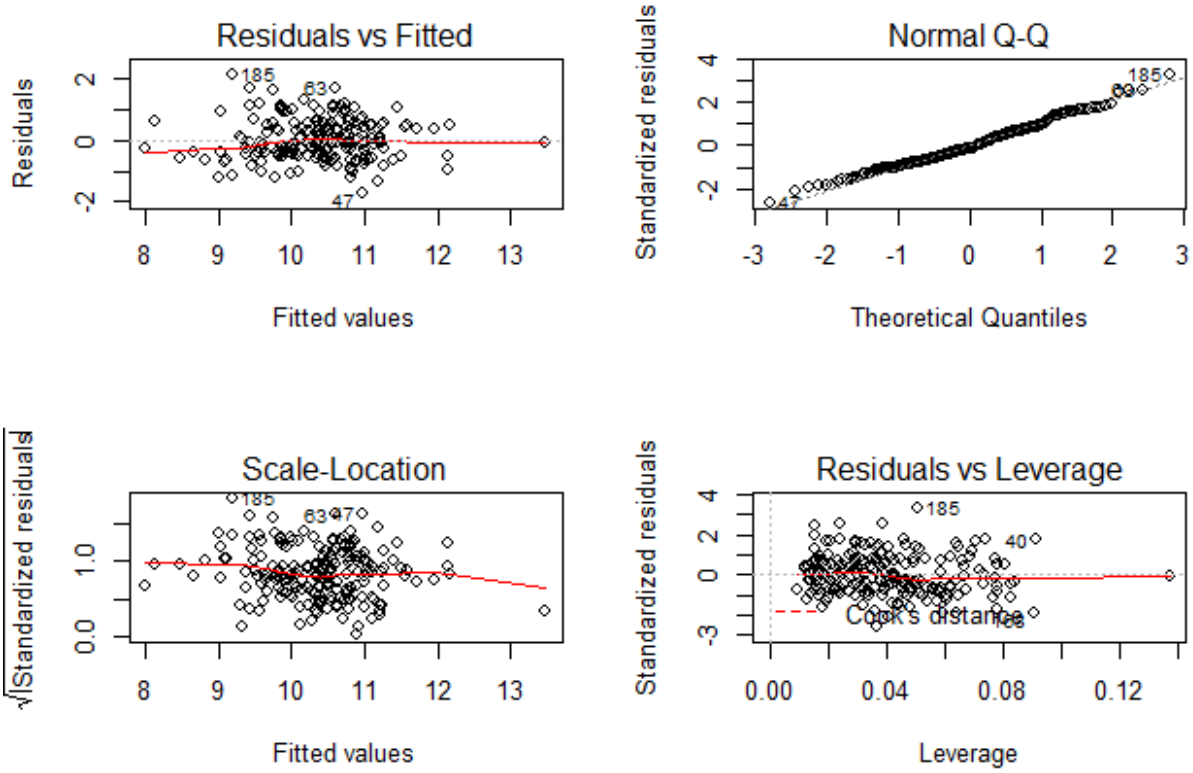

log(PrizeMoney)
8
9
10
11
12
13
50
55
60
65
Scrambling


log(PrizeMoney)
8
9
10
11
12
13
28.0
28.5
29.0
29.5
30.0
PuttsPerRound


Normal Q-Q Plot
Prizemoney
0e+00
2e+05
4e+05
6e+05
-3
-2
-1
0
1
2
3
Theoretical Quantiles


Normal Q-Q Plot
log(PrizeMoney)
8
9
10
11
12
13
-3
-2
-1
0
1
2
3
Theoretical Quantiles


95%
log-Likelihood
-435
-440
-445
-450
-455
-0.3
-0.2
-0.1
0.0
0.1
0.2
0.3
0.4
λ



Call:
lm(formula = log(PrizeMoney) ~ DrivingAccuracy + GIR + PuttingAverage +
BirdieConversion + SandSaves + Scrambling + PuttsPerRound)

Residuals:

Min	1Q	Median	3Q	Max
-1.71949	-0.48608	-0.09172	0.44561	2.14013

Coefficients:

	Estimate	Std. Error	t value	Pr(>|t|)	
(Intercept)	0.194300	7.777129	0.025	0.980095	
DrivingAccuracy	-0.003530	0.011773	-0.300	0.764636	
GIR	0.199311	0.043817	4.549	9.66e-06	***
PuttingAverage	-0.466304	6.905698	-0.068	0.946236	
BirdieConversion	0.157341	0.040378	3.897	0.000136	***
SandSaves	0.015174	0.009862	1.539	0.125551	
Scrambling	0.051514	0.031788	1.621	0.106788	
PuttsPerRound	-0.343131	0.473549	-0.725	0.469601	

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 0.6639 on 188 degrees of freedom
Multiple R-squared: 0.5577, Adjusted R-squared: 0.5412
F-statistic: 33.87 on 7 and 188 DF, p-value: < 2.2e-16