90/100

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STATISTICS 610-601: HW # 1

SEPTEMBER 14, 2017

DISTRIBUTION THEORY

Prof. Irina Gaynanova

Homework 1 - Due Thu, Sep 14th, in class

Problem 1: Consider the sets

$$A_1 = \{1,2\} \times \{2,3\}, \quad A_2 = \{(i,j) \in \{1,2,3\}^2 : i < j\}, \quad A_3 = \{H \subset \{1,2,3\} : |H| = 2\},$$
$$B_1 = \{\{1,2\},\{1,3\},\{2,3\}\}, \quad B_2 = \{(1,2),(1,3),(2,2),(2,3)\}, \quad B_3 = \{(1,2),(1,3),(2,3)\}.$$

Each A set equals one of the B sets. Find the matching pairs.

Problem 2: Consider two events such as $P(A) = 0.33$ and $P(B^c) = 0.25$. Is it possible for A and B to be disjoint? Justify your answer.

Problem 3: 1.24 from the book

Problem 4: 1.26 from the book

Problem 5: 1.34 from the book

Problem 6: You roll two fair dice hoping for a total of 7 (probability 1/6). After the roll one die is hidden, but you see that the other one shows four dots. What is the (updated) chance that you have a total of 7? What is the (updated) chance that the total is n ($n = 1, 2, \ldots$)?

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Please label each page of your homework clearly with your name IN BLOCK CAPITALS and your UIN. If you use more than one sheet of paper, please staple the sheets together.

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Solution 1:

$$A_2 = \{(i,j) \in \{1,2,3\}^2 : i<j\}$$

$$= \{(i,j) : i \in \{1,2,3\} \ \& \ j \in \{1,2,3\} \ \& \ i<j\}$$

$$= \{(1,2), (1,3), (2,3)\}$$

$$= B_3$$

$$A_1 = \{1,2\} \times \{2,3\}$$

= set of all ordered pairs from $\{1,2\}$ and $\{2,3\}$

$$= \{(1,2), (1,3), (2,2), (2,3)\}$$

$$= B_2$$

$$A_3 = \{H \subset \{1,2,3\} : |H| = 2\}$$

= a set of subsets of $\{1,2,3\}$ in which cardinality of each subset is 2

$= B_1$

since $|\{1,2\}| = |\{1,3\}| = |\{2,3\}| = 2$

and $\{1,2\} \subset \{1,2,3\}$, $\{1,3\} \subset \{1,2,3\}$ and $\{2,3\} \subset \{1,2,3\}$

$A_2 = B_3$ $\quad A_1 = B_2$ $\quad A_3 = B_1$

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Solution 2:

We have, $P(S) = 1$ and $P(C) \leq 1$ for any event C in S.

Consider the event $C = A \cup B$

If A and B are disjoint events,

$$\begin{aligned} P(C) &= P(A \cup B) \\ &= P(A) + P(B) \\ &= P(A) + 1 - P(B^c) \\ &= 0.33 + 0.75 \end{aligned}$$

$$\Rightarrow P(C) > 1$$

Which violates the condition $P(C) \leq 1$ to be satisfied by a valid probability function defined under the Kolmogorov axioms.

$\therefore$ A and B cannot be disjoint events.

Solution 4 :

let A_n = event that 6 doesn't appear in nth throw

$$P(A_n) = \frac{5}{6}$$

$$P(A = \text{No six in 5 throws}) = P\left(\bigcap_{n=1}^{5} A_n\right)$$

$$= \prod_{n=1}^{5} P(A_n)$$

(since A_n are independent events)

$$= \left(\frac{5}{6}\right)^5$$

Solution 5 :

(a)

litter 1	litter 2
2 B	3 B
1 G	2 G

litter 1 and litter 2 form a partition of the sample space

$$\therefore P(\text{Brown-haired}) = P(\text{Brown-haired} \cap \text{litter } 1) + P(\text{Brown-haired} \cap \text{litter } 2)$$

$$= \sum_{i=1}^{2} P(\text{Brown-haired} \mid \text{litter } i)\, P(\text{litter } i)$$

$$= \frac{2}{3} \times \frac{1}{2} + \frac{3}{5} \times \frac{1}{2} = \frac{1}{3} + \frac{3}{10} = \frac{10+9}{30}$$

$$= \frac{19}{30}$$

(b) P(litter 1 | Brown-haired) = ?

We know P(Brown-haired | litter 1) = 2/3

Using flipping rule (Bayes Rule),

$$P(B|A) = P(A|B) \frac{P(B)}{P(A)}$$

P(litter 1 | Brown-haired) =

$$P(\text{Brown-haired} | \text{litter } 1) \frac{P(\text{litter } 1)}{P(\text{Brown-haired})}$$

$$= \frac{\frac{2}{3} \times \frac{1}{2}}{\frac{19}{30}} = \frac{10}{19}$$

Solution 6:

(a) $P(\text{total is } 7) = \frac{6}{36}$

$P(\text{total is } 7 \mid \text{one dice shows } 4) = P(\text{other die shows } 3)$

$= \frac{\cancel{6}}{36} = \frac{1}{6}$

or

$P(\text{total is } 7 \mid \text{one die shows } 4)$

$= \frac{P(\text{total is } 7 \cap \text{dice 1 shows } 4)}{P(\text{die 1 shows } 4)}$

$= \frac{\frac{1}{36}}{\frac{6}{36}} = \frac{1}{6}$

(b) let X = random variable denoting the sum

n	$P(X=n \mid \text{one die is } 4)$	Favorable events	
≤ 4	0	No event	
5	1/6	other dice shows 1	(4,1)
6	1/6	2	(4,2)
7	1/6	3	(4,3)
8	1/6	4	(4,4)
9	1/6	5	(4,5)
10	1/6	6	(4,6)
≥ 11	0	No event	

Solution 3

(a) Let event A_n = A gets head in nth toss
= A wins in nth toss

event A = A wins
$$= \bigcup_{n=1}^{\infty} A_n$$

Also events A_n are disjoint because A can win in nth toss only if no one won in previous tosses, which in turn is superset of the events A_K - A didn't win in previous k^{th} toss $k<n$.

Also, n can only take values of the form $2k+1$ $k = \{0, 1, 2, \ldots\}$ for non-zero probability since A only tosses on odd flips.

$$P(A_n) \begin{cases} = \left(\frac{1}{2}\right)^{n-1} \left(\frac{1}{2}\right) & \text{if } n \text{ is odd} \\ = 0 & \text{if } n \text{ is even} \end{cases}$$

$$P(A \text{ wins}) = P\left(\bigcup_{n=1}^{\infty} A_n\right) = \sum_{n=1}^{\infty} P(A_n)$$

$$= \sum_{k=0}^{\infty} P(A_{2k+1})$$

$$= \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^{2k} \left(\frac{1}{2}\right) = \frac{\frac{1}{2}}{1-\frac{1}{2^2}} = \frac{\frac{1}{2}}{\frac{3}{4}} = 2/3$$

(b) $P(\text{head}) = p \quad \Rightarrow P(\text{tail}) = 1-p$

$$P(A \text{ wins}) = \sum_{k=0}^{\infty} (1-p)^{2k}\, p$$

$$= \frac{p}{1-(1-p)^2} = \frac{p}{1-1-p^2+2p}$$

$$= \frac{1}{2-p}$$

(c) The R Markdown document on the next page contains the plot of $P(A \text{ wins}) = f(p)$.

The slope of the curve is given by

$$\frac{df}{dp} = \frac{-1}{(2-p)^2}(-1) = \frac{1}{(2-p)^2} > 0 \quad \forall\, p \in (0,1]$$

Also $P(A \text{ wins})_{p=0} = \frac{1}{2}$

$\because$ f is a continuous and increasing (strictly) function for given range of p,

$$P(A \text{ wins}) > \frac{1}{2} \quad \forall\, p \in (0,1]$$

Stat610Hw1Prob3c

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September 11, 2017

R Markdown

This is an R Markdown document for Problem 3 (c) in Homework #1 for STAT 610-601: DISTRIBUTION THEORY. The R code chunk shown below generates the plot of the probability that A wins, y, as a function of probability of getting head, p.

```
p = seq(from = 0, to = 1, by = 0.001)
y = 1/(2-p)
```

Plots

It can be seen that y is an increasing continuous function of p:

```
plot(p,y)
```