



RAJAN KAPOOR

STATISTICS 610-601: HW # 8



NOVEMBER 9, 2017
DISTRIBUTION THEORY
Prof. Irina Gaynanova

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Homework 8 (due on Thursday November 9, 2017)

Problem 1: Consider independent random variables X and Y with joint pdf $f_{X,Y}$. Show that the independence of X and Y implies E(X|Y) = EX, and, by an example in a sample space of three points, that the reverse implication is false.

Problem 2: Independent random variables X and Y (with finite second moments) are always uncorrelated but the reverse implication is not necessarily true. Illustrate this statement with the following example.

Problem 3: Find E(V) and Var(V) if

- (a) $W \sim \text{exponential}(\beta)$ (with $\beta > 0$) and $V|W \sim \text{normal}(W, 1)$,
- **(b)** $W \sim \text{gamma}(\alpha, 1)$ (with $\alpha > 0$) and $V|W \sim \text{uniform}(-W, W)$,
- (c) $W \sim \text{beta}(\alpha, \beta)$ (with $\alpha > 0, \beta > 0$) and $V|W \sim \text{Bernoulli}(W)$.

Problems 4 to 5: numbers 4.30 and 4.47 from the textbook.

The genwork & John on Thursday Nevember 9, 2017;

It condends the Consider such generalization variables X and Y with joint pdf f_{XY} . Show that the content is X and, by some ample in a sample space of three contents the remaining limits X.

Paraharen de Enders et eo eo adom variables N and V (with finite se ond moments) are always conserved to every exercise enquication a not necessarily true. Historia this statement with a college exercise.

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Solution 3

$$E(N) = E(E(N|M))$$

$$= B(N)$$

$$Var(V) = E(Var(VIW)) + Var(E(VIW))$$

= $E(I) + Var(W)$
= $I + \beta^2$

$$E(N) = E(E(NW))$$

$$= E(0)$$

$$= 0$$

$$Var(v) = E(Var(V1W)) + Var(E(V1W))$$

$$= E\left(\frac{4W^{2}}{12}\right) + Var(0)$$

$$= \frac{1}{3}E(W^{2})$$

$$= \frac{1}{3} \left(Var \left(w \right) + E^{2}(w) \right)$$

$$= \frac{1}{3} \left(x + x^{2} \right)$$

Solution

$$Cov(X,Y) = E(XY) - \mu_{X}\mu_{Y}$$

$$E(XY) = \sum_{Y(X,Y)} f_{X,Y}(X,Y)$$

$$= O + 1 \times O \times \frac{1}{4} + 1 \times 1 \times O + 1 \times 2 \times \frac{1}{4}$$

$$= \frac{1}{2}$$

$$M_{X} = O \times \left(\frac{10}{24}\right) + 1 \times \left(\frac{1}{6}\right) + 2 \times \left(\frac{10}{24}\right)$$

$$= \frac{1}{6} + \frac{20}{24} = \frac{1}{6} + \frac{2}{6} = 1$$

$$\mu_{Y} = O + 1 \times \frac{1}{2} = 0$$

$$\Rightarrow Cost(X,Y) = \frac{1}{2} - 1 \times \frac{1}{2} = 0$$

$$\Rightarrow Cost(X,Y) = 0$$

But X and Y are not independent
$$f_{XY}(0,0) = \frac{1}{6} \neq f_{X}(0)f_{Y}(0)$$

$$= \frac{10}{24} \times \frac{1}{2} = \frac{5}{24}$$

× 0 1/6 1/4 10/24 1 1/6 0 1/6 2 1/6 1/24 1 1/24 UIN: 225001987

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$$E(V) = E(E(V|W))$$

$$= E(P(W=1|W \sim beta(X, \beta)))$$

$$V_{os}(V) = V_{os}(E(V|W)) + E(V_{os}(V|W))$$

Solution 4

$$=2(\frac{1}{12})+\frac{1}{4}$$

$$=\frac{1}{6}+\frac{1}{4}=\frac{10}{24}=\frac{5}{12}$$

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$$Cov(X,Y) = E(XY) - \mu_{X}\mu_{Y}$$

$$E(XY) = E(E(XY|X))$$

$$= E(X = (Y|X))$$

$$= E(X = (X|X))$$

$$= E(X^{2}) = Var(X) + E(X)$$

$$Cer(X,Y) = E(X,Y) - \mu_{X}\mu_{Y}$$

$$= \frac{1}{3} - \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{3} - \frac{1}{4} \times \frac{1}{2}$$

(b)
$$\frac{Y}{X}|X \sim \mathcal{N}(1,1)$$

2: which is independent of X Steps! S

Solution 1