Appendix B

Test Problems*

Test problems are discussed in Section 7.3 and the interested reader should read that section before this one. Moré, Garbow, and Hillstrom [1981] provide a set of approximately 15 test problems each for unconstrained minimization, systems of nonlinear equations, and nonlinear least squares. Many of these problems can be run with various values of n. A standard starting point x_0 is given for each problem, and most are intended to be started from $10 * x_0$ and $100 * x_0$ as well. Thus the number of possible problems provided by Moré, Garbow, and Hillstrom actually is quite large. Below we give a small subset of these problems that might be used in testing a class project or in very preliminary testing of a new method. Anyone interested in comprehensive testing is urged to read Moré, Garbow, and Hillstrom and to use the larger set of test problems contained therein. Hiebert [1982] discusses a comparison of codes for solving systems of nonlinear equations using the test problems in Moré, Garbow and Hillstrom, and includes suggestions for modifying these problems into problems with poor scaling or problems where the objective function is noisy.

Problems 1-4 below are standard test problems for both nonlinear equations and unconstrained minimization. In each case, a set of n single-valued functions of n unknowns, $f_i(x)$, $i=1, \cdots, n$, is given. The nonlinear equations function vector is

$$F(x) = \begin{bmatrix} f_1(x) \\ \cdots \\ f_n(x) \end{bmatrix}$$

while the unconstrained minimization objective function is

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$$f(x) = \sum_{i=1}^{n} f_i(x)^2$$

Problems 1-3 may be run with various values of n. The standard dimensions for problems 1 and 2 are $n=2$ and $n=4$ respectively, and these should be tried first. A reasonable first choice for problem 3 is $n=10$. Problem 5 is a difficult and often used test problem for unconstrained minimization only.

We copy Moré, Garbow, and Hillstrom and list for each problem: a) the dimension n; b) tne nonlinear function(s); c) the starting point x_0; d) the root and minimizer x_* if it is available in closed form. The minimum value of $f(x)$ is zero in all cases. All these problems may be started from x_0, $10 * x_0$, or $100 * x_0$. For the original source of each problem, see Moré, Garbow, and Hillstrom.

1) *Extended Rosenbrock Function*

a) n = any positive multiple of 2

b) for $i=1, \cdots, n/2$:

$$f_{2i-1}(x) = 10(x_{2i} - x_{2i-1}^2)$$
$$f_{2i}(x) = 1 - x_{2i-1}$$

c) $x_0 = (-1.2, 1, \cdots, -1.2, 1)$

d) $x_* = (1, 1, \cdots, 1, 1)$

2) *Extended Powell Singular Function*

a) n = any positive multiple of 4

b) for $i=1, \cdots, n/4$:

$$f_{4i-3}(x) = x_{4i-3} + 10x_{4i-2}$$
$$f_{4i-2}(x) = \sqrt{5}(x_{4i-1} - x_{4i})$$
$$f_{4i-1}(x) = (x_{4i-2} - 2x_{4i-1})^2$$
$$f_{4i}(x) = \sqrt{10}(x_{4i-3} - x_{4i})^2$$

c) $x_0 = (3, -1, 0, 1, \cdots, 3, -1, 0, 1)$

d) $x_* = (0, 0, 0, 0, \cdots, 0, 0, 0, 0)$

note: both $F'(x_*)$ and $\nabla^2 f(x_*)$ are singular

3) *Trigonometric Function*

a) n = any positive integer

b) for $i=1, \cdots, n$:

$$f_i(x) = n - \sum_{j=1}^{n}(\cos x_j + i(1 - \cos x_i) - \sin x_i)$$

c) $x_0 = (1/n, 1/n, \cdots, 1/n)$

4) *Helical Valley Function*

a) $n = 3$

b) $f_1(x) = 10(x_3 - 10\Theta(x_1, x_2))$

$$f_2(x) = 10((x_1^2 + x_2^2)^{1/2} - 1)$$
$$f_3(x) = x_3$$

where

$$\Theta(x_1, x_2) = (1/2\pi)\arctan(x_2/x_1) \quad \text{if } x_1 > 0$$

$$\Theta(x_1, x_2) = (1/2\pi)\arctan(x_2/x_1) + 0.5 \quad \text{if } x_1 < 0$$

c) $x_0 = (-1, 0, 0)$

d) $x_* = (1, 0, 0)$

5) *Wood Function*

a) $n = 4$

b) $$f(x) = 100(x_1^2 - x_2)^2 + (1 - x_1)^2 + 90(x_3^2 - x_4)^2 + (1 - x_3)^2 + 10.1((1 - x_2)^2 + (1 - x_4)^2) + 19.8(1 - x_2)(1 - x_4)$$

c) $x_0 = (-3, -1, -3, -1)$

d) $x_* = (1, 1, 1, 1)$