APPENDIX: Test Problems of the First Collection

Purpose of this appendix is to list a detailed description of all test problems published in the monograph [4], which is out of print. We proceed from the nonlinear program (1) and list the following data of an example:

PROBLEM:	test problem number
CLASSIFICATION:	classification number in the form OCD-Kr-s according to the scheme given below
NUMBER OF VARIABLES:	number of variables n
NUMBER OF CONSTRAINTS:	number of inequality constraints, m_1, number of equality constraints, i.e., $m - m_1$, and number of variable bounds of variables, b
OBJECTIVE FUNCTION:	analytical expressions for objective function $f(x)$
CONSTRAINTS:	analytical expressions for constraints $g_j(x), j = 1, \ldots, m$
START:	starting values for variables, x_0, and corresponding objective function value, $f(x_0)$, together with an information whether x_0 is feasible or not
SOLUTION:	information about optimal solution $x^\star$, i.e., - objective function value $f(x^\star)$ - constraint violation, $r(x^\star)$ - norm of gradient of Lagrange function - number of active constraints, μ - active constraints, $I(x^\star)$ - degree of degeneracy, $u^\star_{max}/u^\star_{min}$ - condition number of projected Hessian of Lagrange function, $\lambda^\star_{max}/\lambda^\star_{min}$

The general form of the classification scheme is

OCD-Kr-s

with

O	-	objective function
C	-	constraints
D	-	regularity
K	-	information about solution, i.e., whether an exact solution is known or not
r	-	order of partial derivatives
s	-	serial number within a class

The purpose of the classification scheme is to characterize the mathematical structure of objective function and constraints, and to give more information about the implementation and the solution. A problem is called a regular one, if first and second derivatives exist in the feasible region for all problem functions, otherwise an irregular one. The subsequent abbreviations are used:

class	key	description
O	C	constant function
	L	linear function
	Q	quadratic function
	S	sum of squares
	P	generalized polynomial function
	G	general function
C	U	unconstrained problem
	B	only upper and lower bounds
	L	linear functions
	Q	quadratic functions
	P	generalized polynomial functions
	G	general functions
D	R	regular problem
	I	irregular problem
K	T	exact solution known (*theoretical problem*)
	P	exact solution not known (*practical problem*)
r	0	derivatives not implemented
	1	first derivatives implemented

For some test problems, we cannot describe objective or constraint functions just by a few analytical expressions. In these cases, program fragments are attached at the end of this section together with more extensive information about a test problem, e.g., constant data, starting or solution values.

The subsequent pages are xeroxed copies of the original publication.

PROBLEM:	1
CLASSIFICATION:	PBR-T1-1
SOURCE:	Betts [8]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 0$, $b = 1$

OBJECTIVE FUNCTION:

$$f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

CONSTRAINTS:

$$-1.5 \le x_2$$

START: $x_o = (-2 , 1)$ (feasible)

$f(x_o) = 909$

SOLUTION: $x^* = (1 , 1)$

$f(x^*) = 0$

$r(x^*) = 0$

$e(x^*) = -$

$\mu = 0$

$I(x^*) = -$

$u^*_{max}/u^*_{min} = -$

$\lambda^*_{max}/\lambda^*_{min} = .25E\ 4$

PROBLEM:	2
CLASSIFICATION:	PBR-T1-2
SOURCE:	Betts [8]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 0$, $b = 1$

OBJECTIVE FUNCTION:

$$f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

CONSTRAINTS:

$$1.5 \le x_2$$

START: $x_0 = (-2, 1)$ (not feasible)

$f(x_0) = 909$

SOLUTION: $x^* = (2a\cos(\frac{1}{3}\arccos\frac{1}{b}), 1.5)$

$f(x^*) = .05042\ 61879$

$r(x^*) = 0$

$e(x^*) = .13E-7$

$\mu = 1$

$I(x^*) = (1)$

$u^*_{max}/u^*_{min} = .1833/.1833 = 1$

$\lambda^*_{max}/\lambda^*_{min} = 200/200 = 1$

$a = (598/1200)^{1/2}$

$b = 400\ a^3$

PROBLEM:	3
CLASSIFICATION:	QBR-T1-1
SOURCE:	Schuldt [56]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 0$, $b = 1$

OBJECTIVE FUNCTION:

$$f(x) = x_2 + 10^{-5}(x_2 - x_1)^2$$

CONSTRAINTS:

$$0 \le x_2$$

START:	x_o	=	(10 , 1)	(feasible)
	$f(x_o)$	=	1.00081	
SOLUTION:	x^*	=	(0 , 0)	
	$f(x^*)$	=	0	
	$r(x^*)$	=	0	
	$e(x^*)$	=	0	
	μ	=	1	
	$I(x^*)$	=	(1)	
	u^*_{max}/u^*_{min}	=	1.0000/1.0000 = 1	
	$\lambda^*_{max}/\lambda^*_{min}$	=	.20E-4/.20E-4 = 1	

PROBLEM:	4
CLASSIFICATION:	PBR-T1-3
SOURCE:	Asaadi [1]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 0$, $b = 2$

OBJECTIVE FUNCTION:

$$f(x) = \frac{1}{3}(x_1 + 1)^3 + x_2$$

CONSTRAINTS:

$$1 \le x_1$$

$$0 \le x_2$$

START:	x_o	=	(1.125 , .125)	(feasible)
	$f(x_o)$	=	3.323568	
SOLUTION:	x^*	=	(1 , 0)	
	$f(x^*)$	=	8/3	
	$r(x^*)$	=	0	
	$e(x^*)$	=	0	
	μ	=	2	
	$I(x^*)$	=	(1 , 2)	
	u^*_{max}/u^*_{min}	=	4.0000/1.0000 = 4.00	
	$\lambda^*_{max}/\lambda^*_{min}$	=	-	

PROBLEM:	5
CLASSIFICATION:	GBR-T1-1
SOURCE:	McCormick [41]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 0$, $b = 4$

OBJECTIVE FUNCTION:

$$f(x) = \sin(x_1 + x_2) + (x_1 - x_2)^2 - 1.5x_1 + 2.5x_2 + 1$$

CONSTRAINTS:

$$-1.5 \leq x_1 \leq 4$$

$$-3 \leq x_2 \leq 3$$

START:	x_o	=	$(0\ ,\ 0)$	(feasible)
	$f(x_o)$	=	1	
SOLUTION:	x^*	=	$(-\frac{\pi}{3} + \frac{1}{2}\ ,\ -\frac{\pi}{3} - \frac{1}{2})$	
	$f(x^*)$	=	$-\frac{1}{2}\sqrt{3} - \frac{\pi}{3}$	
	$r(x^*)$	=	0	
	$e(x^*)$	=	-	
	μ	=	0	
	$I(x^*)$	=	-	
	u^*_{max}/u^*_{min}	=	-	
	$\lambda^*_{max}/\lambda^*_{min}$	=	$4.00/1.73 = 2.31$	

PROBLEM:	6
CLASSIFICATION:	QQR-T1-1
SOURCE:	Betts [8]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 1$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = (1 - x_1)^2$$

CONSTRAINTS:

$$10(x_2 - x_1^2) = 0$$

START: $x_o = (-1.2 , 1)$ (not feasible).

$f(x_o) = 4.84$

SOLUTION:

$x^* = (1 , 1)$

$f(x^*) = 0$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 0$

$I(x^*) = -$

$u^*_{max}/u^*_{min} = 0$

$\lambda^*_{max}/\lambda^*_{min} = .40/.40 = 1$

PROBLEM:	7
CLASSIFICATION:	GPR-T1-1
SOURCE:	Miele e.al. [44,45]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 1$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = \ln(1 + x_1^2) - x_2$$

CONSTRAINTS:

$$(1 + x_1^2)^2 + x_2^2 - 4 = 0$$

START: $x_o = (2 , 2)$ (not feasible)

$f(x_o) = \ln 5 - 2$

SOLUTION: $x^* = (0 , \sqrt{3})$

$f(x^*) = -\sqrt{3}$

$r(x^*) = 0$

$e(x^*) = .21E-24$

$\mu = 0$

$I(x^*) = -$

$u^*_{max}/u^*_{min} = .2887/.2887 = 1$

$\lambda^*_{max}/\lambda^*_{min} = 3.15/3.15 = 1$

PROBLEM:	8
CLASSIFICATION:	CQR-T1-1
SOURCE:	Betts [8]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 2$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = -1$$

CONSTRAINTS:

$$x_1^2 + x_2^2 - 25 = 0$$

$$x_1 x_2 - 9 = 0$$

START:

$x_o = (2 , 1)$ (not feasible)

$f(x_o) = -1$

SOLUTION:

$x^* = (a , \frac{9}{a}) , (-a , -\frac{9}{a}) , (b , \frac{9}{b}) , (-b , -\frac{9}{b})$

$f(x^*) = -1$

$r(x^*) = 0$ $\qquad a = \sqrt{\dfrac{25 + \sqrt{301}}{2}}$

$e(x^*) = 0$

$\mu = 0$ $\qquad b = \sqrt{\dfrac{25 - \sqrt{301}}{2}}$

$I(x^*) = -$

$u^*_{max}/u^*_{min} = 0$

$\lambda^*_{max}/\lambda^*_{min} = -$

PROBLEM:	9
CLASSIFICATION:	GLR-T1-1
SOURCE:	Miele e.al. [44]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 1(1)$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = \sin(\pi x_1/12)\cos(\pi x_2/16)$$

CONSTRAINTS:

$$4x_1 - 3x_2 = 0$$

START:

$x_o = (0 , 0)$ (feasible)

$f(x_o) = 0$

SOLUTION:

$x^* = (12k - 3 , 16k - 4)$, $k=0, \pm 1, \pm 2, \ldots$

$f(x^*) = -.5$

$r(x^*) = 0$

$e(x^*) = .73E-12$

$\mu = 0$

$I(x^*) = -$

$u^*_{max}/u^*_{min} = .03272/.03272 = 1$

$\lambda^*_{max}/\lambda^*_{min} = .049/.049 = 1$

PROBLEM:	10
CLASSIFICATION:	LQR-T1-1
SOURCE:	Biggs [10]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 1$, $m-m_1 = 0$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = x_1 - x_2$$

CONSTRAINTS:

$$-3x_1^2 + 2x_1x_2 - x_2^2 + 1 \geq 0$$

START: $x_o = (-10\ ,\ 10)$ (not feasible)

$f(x_o) = -20$

SOLUTION: $x^* = (0\ ,\ 1)$

$f(x^*) = -1$

$r(x^*) = 0$

$e(x^*) = .92E-11$

$\mu = 1$

$I(x^*) = (1)$

$u^*_{max}/u^*_{min} = .5000/.5000 = 1$

$\lambda^*_{max}/\lambda^*_{min} = 1.00/1.00 = 1$

PROBLEM:	11
CLASSIFICATION:	QQR-T1-2
SOURCE:	Biggs [10]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 1$, $m-m_1 = 0$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = (x_1 - 5)^2 + x_2^2 - 25$$

CONSTRAINTS:

$$-x_1^2 + x_2 \geq 0$$

START: $x_0 = (4.9 , .1)$. (not feasible)

$f(x_0) = -24.98$

SOLUTION: $x^* = ((a - \frac{1}{a})/\sqrt{6} , (a^2 - 2 + a^{-2})/6)$

$f(x^*) = -8.49846\ 4223$

$r(x^*) = 0$ $\qquad a = 7.5\sqrt{6} + \sqrt{338.5}$

$e(x^*) = .17E-9$

$\mu = 1$

$I(x^*) = (1)$

$u^*_{max}/u^*_{min} = 3.0493/3.0493 = 1$

$\lambda^*_{max}/\lambda^*_{min} = 2.86/2.86 = 1$

PROBLEM:	12
CLASSIFICATION:	QQR-T1-3
SOURCE:	Mine e.al. [46]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 1$, $m-m_1 = 0$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = .5x_1^2 + x_2^2 - x_1x_2 - 7x_1 - 7x_2$$

CONSTRAINTS:

$$25 - 4x_1^2 - x_2^2 \geq 0$$

START: $x_o = (0 , 0)$ (feasible)

$f(x_o) = 0$

SOLUTION:

$x^* = (2 , 3)$

$f(x^*) = -30$

$r(x^*) = 0$

$e(x^*) = .81E-10$

$\mu = 1$

$I(x^*) = (1)$

$u^*_{max}/u^*_{min} = .5000/.5000 = 1$

$\lambda^*_{max}/\lambda^*_{min} = 3.90/3.90 = 1$

PROBLEM:	13
CLASSIFICATION:	QPR-T1-1
SOURCE:	Betts [8], Kuhn, Tucker [38]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 1$, $m-m_1 = 0$, $b = 2$

OBJECTIVE FUNCTION:

$$f(x) = (x_1 - 2)^2 + x_2{}^2$$

CONSTRAINTS:

$$(1 - x_1)^3 - x_2 \geq 0$$

$$0 \leq x_1$$

$$0 \leq x_2$$

START: $x_o = (-2 , -2)$ (not feasible)

$f(x_o) = 20$

SOLUTION: $x^* = (1 , 0)$

$f(x^*) = 1$

$r(x^*) = 0$

$e(x^*) = 2$ (constraint qualification not satisfied)

$\mu = 2$

$I(x^*) = (1,3)$

$u^*_{max}/u^*_{min} = 0/0$

$\lambda^*_{max}/\lambda^*_{min} = -$

PROBLEM:	14
CLASSIFICATION:	QQR-T1-4
SOURCE:	Bracken, McCormick [13], Himmelblau [29]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 1$, $m-m_1 = 1(1)$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = (x_1 - 2)^2 + (x_2 - 1)^2$$

CONSTRAINTS:

$$-.25x_1^2 - x_2^2 + 1 \geq 0$$

$$x_1 - 2x_2 + 1 = 0$$

START: $x_o = (2\ ,\ 2)$ (not feasible)

$f(x_o) = 1$

SOLUTION:

$x^* = (.5(\sqrt{7} - 1)\ ,\ .25(\sqrt{7} + 1))$

$f(x^*) = 9 - 2.875\sqrt{7}$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 1$

$I(x^*) = (1)$

$u^*_{max}/u^*_{min} = 1.8466/1.5945 = 1.15$

$\lambda^*_{max}/\lambda^*_{min} = -$

PROBLEM:	15
CLASSIFICATION:	PQR-T1-1
SOURCE:	Betts [8]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 2$, $m-m_1 = 0$, $b = 1$

OBJECTIVE FUNCTION:

$$f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

CONSTRAINTS:

$$x_1 x_2 - 1 \geq 0$$

$$x_1 + x_2^2 \geq 0$$

$$x_1 \leq .5$$

START: $x_0 = (-2 , 1)$ (not feasible)

$f(x_0) = 909$

SOLUTION:

$x^* = (.5 , 2)$

$f(x^*) = 306.5$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 2$

$I(x^*) = (1 , 3)$

$u^*_{max}/u^*_{min} = 1751/700 = 2.50$

$\lambda^*_{max}/\lambda^*_{min} = -$

PROBLEM:	16
CLASSIFICATION:	PQR-T1-2
SOURCE:	Betts [8]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 2$, $m-m_1 = 0$, $b = 3$

OBJECTIVE FUNCTION:

$$f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

CONSTRAINTS:

$$x_1 + x_2^2 \geq 0$$

$$x_1^2 + x_2 \geq 0$$

$$-2 \quad -\cancel{.5} \leq x_1 \leq .5$$

$$x_2 \leq 1$$

START:

$x_o = (-2 , 1)$ (not feasible)

$f(x_o) = 909$

SOLUTION:

$x^* = (.5 , .25)$

$f(x^*) = .25$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 1$

$I(x^*) = (4)$

$u^*_{max}/u^*_{min} = 1.0000/1.0000 = 1$

$\lambda^*_{max}/\lambda^*_{min} = 200/200 = 1$

PROBLEM:	17
CLASSIFICATION:	PQR-T1-3
SOURCE:	Betts [8]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 2$, $m-m_1 = 0$, $b = 3$

OBJECTIVE FUNCTION:

$$f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

CONSTRAINTS:

$$x_2^2 - x_1 \geq 0$$

$$x_1^2 - x_2 \geq 0$$

$$-.5 \leq x_1 \leq .5$$

$$x_2 \leq 1$$

START:	x_o	=	$(-2 , 1)$	(not feasible)
	$f(x_o)$	=	909	

SOLUTION:	x^*	=	$(0 , 0)$
	$f(x^*)$	=	1
	$r(x^*)$	=	0
	$e(x^*)$	=	0
	μ	=	2
	$I(x^*)$	=	$(1 , 2)$
	u^*_{max}/u^*_{min}	=	2.0000/0
	$\lambda^*_{max}/\lambda^*_{min}$	=	–

PROBLEM:	18
CLASSIFICATION:	QQR-T1-5
SOURCE:	Betts [8]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 2$, $m-m_1 = 0$, $b = 4$

OBJECTIVE FUNCTION:

$$f(x) = .01x_1^2 + x_2^2$$

CONSTRAINTS:

$$x_1x_2 - 25 \geq 0$$

$$x_1^2 + x_2^2 - 25 \geq 0$$

$$2 \leq x_1 \leq 50$$

$$0 \leq x_2 \leq 50$$

START: $x_o = (2\ ,\ 2)$ (not feasible)

$f(x_o) = 4.04$

SOLUTION:

$x^* = (\sqrt{250}\ ,\ \sqrt{2.5})$

$f(x^*) = 5$

$r(x^*) = 0$

$e(x^*) = .24E-9$

$\mu = 1$

$I(x^*) = (1)$

$u^*_{max}/u^*_{min} = .2000/.2000 = 1$

$\lambda^*_{max}/\lambda^*_{min} = .079/.079 = 1$

PROBLEM:	19
CLASSIFICATION:	PQR-T1-4
SOURCE:	Betts [8], Gould [27]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 2$, $m-m_1 = 0$, $b = 4$

OBJECTIVE FUNCTION:

$$f(x) = (x_1 - 10)^3 + (x_2 - 20)^3$$

CONSTRAINTS:

$$(x_1 - 5)^2 + (x_2 - 5)^2 - 100 \geq 0$$

$$-(x_2 - 5)^2 - (x_1 - 6)^2 + 82.81 \geq 0$$

$$13 \leq x_1 \leq 100$$

$$0 \leq x_2 \leq 100$$

START:	x_o	=	(20.1 , 5.84) (not feasible)
	$f(x_o)$	=	-1808.858296
SOLUTION:	x^*	=	(14.095 , .84296079)
	$f(x^*)$	=	-6961.81381
	$r(x^*)$	=	0
	$e(x^*)$	=	0
	μ	=	2
	$I(x^*)$	=	(1 , 2)
	u^*_{max}/u^*_{min}	=	1229.5/1097.1 = 1.12
	$\lambda^*_{max}/\lambda^*_{min}$	=	-

PROBLEM:	20		
CLASSIFICATION:	PQR-T1-5		
SOURCE:	Betts [8]		
NUMBER OF VARIABLES:	$n = 2$		
NUMBER OF CONSTRAINTS:	$m_1 = 3$	, $m-m_1 = 0$	, $b = 2$

OBJECTIVE FUNCTION:

$$f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

CONSTRAINTS:

$$x_1 + x_2^2 \geq 0$$

$$x_1^2 + x_2 \geq 0$$

$$x_1^2 + x_2^2 - 1 \geq 0$$

$$-.5 \leq x_1 \leq .5$$

START: $x_0 = (-2 , 1)$ (not feasible)

$f(x_0) = 909$

SOLUTION: $x^* = (.5 , .5\sqrt{3})$

$f(x^*) = 81.5 - 25\sqrt{3}$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 2$

$I(x^*) = (3 , 5)$

$u^*_{max}/u^*_{min} = 195.34/71.132 = 2.75$

$\lambda^*_{max}/\lambda^*_{min} = -$

PROBLEM:	21
CLASSIFICATION:	QLR-T1-1
SOURCE:	Betts [8]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 1(1)$, $m-m_1 = 0$, $b = 4$

OBJECTIVE FUNCTION:

$$f(x) = .01x_1^2 + x_2^2 - 100$$

CONSTRAINTS:

$$10x_1 - x_2 - 10 \geq 0$$

$$2 \leq x_1 \leq 50$$

$$-50 \leq x_2 \leq 50$$

START:

$x_0 = (-1 , -1)$ (not feasible)

$f(x_0) = -98.99$

SOLUTION:

$x^* = (2 , 0)$

$f(x^*) = -99.96$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 1$

$I(x^*) = (2)$

$u^*_{max}/u^*_{min} = .04/.04 = 1$

$\lambda^*_{max}/\lambda^*_{min} = -$

PROBLEM:	22
CLASSIFICATION:	QQR-T1-6
SOURCE:	Bracken, McCormick [13], Himmelblau [29], Sheela [57]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 2(1)$, $m-m_1 = 0$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = (x_1 - 2)^2 + (x_2 - 1)^2$$

CONSTRAINTS:

$$-x_1 - x_2 + 2 \geq 0$$

$$-x_1^2 + x_2 \geq 0$$

START:

$x_o = (2 , 2)$ (not feasible)

$f(x_o) = 1$

SOLUTION:

$x^* = (1 , 1)$

$f(x^*) = 1$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 2$

$I(x^*) = (1 , 2)$

$u^*_{max}/u^*_{min} = .6666/.6666 = 1$

$\lambda^*_{max}/\lambda^*_{min} = -$

PROBLEM:	23
CLASSIFICATION:	QQR-T1-7
SOURCE:	Betts [8]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 5(1)$, $m-m_1 = 0$, $b = 4$

OBJECTIVE FUNCTION:

$$f(x) = x_1^2 + x_2^2$$

CONSTRAINTS:

$$x_1 + x_2 - 1 \geq 0$$

$$x_1^2 + x_2^2 - 1 \geq 0$$

$$9x_1^2 + x_2^2 - 9 \geq 0$$

$$x_1^2 - x_2 \geq 0$$

$$x_2^2 - x_1 \geq 0$$

$$-50 \leq x_i \leq 50 \quad , \quad i=1,2$$

START: $x_o = (3 , 1)$ (not feasible)

$f(x_o) = 10$

SOLUTION:

$x^* = (1 , 1)$

$f(x^*) = 2$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 2$

$I(x^*) = (4 , 5)$

$u^*_{max}/u^*_{min} = 2/2 = 1$

$\lambda^*_{max}/\lambda^*_{min} = -$

PROBLEM:	24
CLASSIFICATION:	PLR-T1-1
SOURCE:	Betts [8], Box [12]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 3(3)$, $m-m_1 = 0$, $b = 2$

OBJECTIVE FUNCTION:

$$f(x) = \frac{1}{27\sqrt{3}} ((x_1 - 3)^2 - 9)\, x_2{}^3$$

CONSTRAINTS:

$$x_1/\sqrt{3} - x_2 \geq 0$$

$$x_1 + \sqrt{3}x_2 \geq 0$$

$$-x_1 - \sqrt{3}x_2 + 6 \geq 0$$

$$0 \leq x_1$$

$$0 \leq x_2$$

START: $x_0 = (1 , .5)$ (feasible).

$f(x_0) = -.01336459$

SOLUTION:

$x^* = (3 , \sqrt{3})$

$f(x^*) = -1$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 2$

$I(x^*) = (1 , 3)$

$u^*_{max}/u^*_{min} = .86603/.5 = 1.73$

$\lambda^*_{max}/\lambda^*_{min} = -$

PROBLEM:	25
CLASSIFICATION:	SBR-T1-1
SOURCE:	Holzmann [32], Himmelblau [29]
NUMBER OF VARIABLES:	$n = 3$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 0$, $b = 6$

OBJECTIVE FUNCTION:

$$f(x) = \sum_{i=1}^{99} (f_i(x))^2$$

$$f_i(x) = -.01i + \exp\left(-\frac{1}{x_1}(u_i - x_2)^{x_3}\right)$$

$$u_i = 25 + (-50 \ln(.01i))^{2/3}$$

$$i = 1,\ldots,99$$

CONSTRAINTS:

$$.1 \le x_1 \le 100$$

$$0 \le x_2 \le 25.6$$

$$0 \le x_3 \le 5$$

START: $x_o = (100 , 12.5 , 3)$ $f(x_o) = 32.835$ (feasible)

SOLUTION: $f(x^*) = 0$

$x^* = (50 , 25 , 1.5)$

$r(x^*) = 0$ $e(x^*) = -$

$\mu = 0$ $I(x^*) = -$

$u^*_{max}/u^*_{min} = -$

$\lambda^*_{max}/\lambda^*_{min} = 94.7/.14E-4 = .70E7$

PROBLEM:	26
CLASSIFICATION:	PPR-T1-1
SOURCE:	Huang, Aggerwal [34], Miele e.al. [43]
NUMBER OF VARIABLES:	$n = 3$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 1$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = (x_1 - x_2)^2 + (x_2 - x_3)^4$$

CONSTRAINTS:

$$(1 + x_2{}^2)x_1 + x_3{}^4 - 3 = 0$$

START: $x_o = (-2.6 , 2 , 2)$ (feasible)

$f(x_o) = 21.16$

SOLUTION: $x^* = (1 , 1 , 1) , (a , a , a)$

$f(x^*) = 0$

$r(x^*) = 0$ $\quad a = \sqrt[3]{\alpha-\beta} - \sqrt[3]{\alpha+\beta} - 2/3$

$e(x^*) = 0$ $\quad \alpha = \sqrt{139/108}$

$\mu = 0$ $\quad \beta = 61/54$

$I(x^*) = -$

$u^*_{max}/u^*_{min} = 0$

$\lambda^*_{max}/\lambda^*_{min} = 4/0$

PROBLEM:	27
CLASSIFICATION:	PQR-T1-6
SOURCE:	Miele e.al. [44,45]
NUMBER OF VARIABLES:	$n = 3$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 1$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = .01(x_1 - 1)^2 + (x_2 - x_1^2)^2$$

CONSTRAINTS:

$$x_1 + x_3^2 + 1 = 0$$

START: $x_o = (2 , 2 , 2)$ (not feasible)

$f(x_o) = 4.01$

SOLUTION: $x^* = (-1 , 1 , 0)$

$f(x^*) = .04$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 0$

$I(x^*) = -$

$u^*_{max}/u^*_{min} = .04/.04 = 1$

$\lambda^*_{max}/\lambda^*_{min} = 2/.08 = 25$

PROBLEM:	28
CLASSIFICATION:	QLR-T1-2
SOURCE:	Huang, Aggerwal [34]
NUMBER OF VARIABLES:	$n = 3$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 1(1)$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = (x_1 + x_2)^2 + (x_2 + x_3)^2$$

CONSTRAINTS:

$$x_1 + 2x_2 + 3x_3 - 1 = 0$$

START: $x_o = (-4\ ,\ 1\ ,\ 1)$ (feasible)

$f(x_o) = 13$

SOLUTION: $x^* = (.5\ ,\ -.5\ ,\ .5)$

$f(x^*) = 0$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 0$

$I(x^*) = -$

$u^*_{max}/u^*_{min} = 0$

$\lambda^*_{max}/\lambda^*_{min} = 2.72/.42 = 6.45$

PROBLEM:	29
CLASSIFICATION:	PQR-T1-7
SOURCE:	Biggs [10]
NUMBER OF VARIABLES:	$n = 3$
NUMBER OF CONSTRAINTS:	$m_1 = 1$, $m-m_1 = 0$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = -x_1x_2x_3$$

CONSTRAINTS:

$$-x_1^2 - 2x_2^2 - 4x_3^2 + 48 \geq 0$$

START:

$x_o = (1 , 1 , 1)$ (feasible)

$f(x_o) = -1$

SOLUTION:

$x^* = (a,b,c)$, $(a,-b,-c)$, $(-a,b,-c)$, $(-a,-b,c)$

$f(x^*) = -16\sqrt{2}$

$r(x^*) = 0$

$e(x^*) = .19E-9$

$\mu = 1$

$I(x^*) = (1)$

$u^*_{max}/u^*_{min} = .7071/.7071 = 1$

$\lambda^*_{max}/\lambda^*_{min} = 7.79/3.52 = 2.22$

$a = 4$

$b = 2\sqrt{2}$

$c = 2$

PROBLEM:	30
CLASSIFICATION:	QQR-T1-8
SOURCE:	Betts [8]
NUMBER OF VARIABLES:	$n = 3$
NUMBER OF CONSTRAINTS:	$m_1 = 1$, $m-m_1 = 0$, $b = 6$

OBJECTIVE FUNCTION:

$$f(x) = x_1^2 + x_2^2 + x_3^2$$

CONSTRAINTS:

$$x_1^2 + x_2^2 - 1 \geq 0$$

$$1 \leq x_1 \leq 10$$

$$-10 \leq x_2 \leq 10$$

$$-10 \leq x_3 \leq 10$$

START: $x_o = (1 , 1 , 1)$ (feasible)

$f(x_o) = 3$

SOLUTION: $x^* = (1 , 0 , 0)$

$f(x^*) = 1$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 2$

$I(x^*) = (1 , 2)$

$u^*_{max}/u^*_{min} = 1/0$

$\lambda^*_{max}/\lambda^*_{min} = 2/2 = 1$

PROBLEM:	31
CLASSIFICATION:	QQR-T1-9
SOURCE:	Betts [8]
NUMBER OF VARIABLES:	$n = 3$
NUMBER OF CONSTRAINTS:	$m_1 = 1$, $m-m_1 = 0$, $b = 6$

OBJECTIVE FUNCTION:

$$f(x) = 9x_1^2 + x_2^2 + 9x_3^2$$

CONSTRAINTS:

$$x_1x_2 - 1 \geq 0$$

$$-10 \leq x_1 \leq 10$$

$$1 \leq x_2 \leq 10$$

$$-10 \leq x_3 \leq 1$$

START:	x_o	=	(1 , 1 , 1) (feasible)
	$f(x_o)$	=	19
SOLUTION:	x^*	=	$(1/\sqrt{3}\ ,\ \sqrt{3}\ ,\ 0)$
	$f(x^*)$	=	6
	$r(x^*)$	=	0
	$e(x^*)$	=	.57E-10
	μ	=	1
	$I(x^*)$	=	(1)
	u^*_{max}/u^*_{min}	=	6/6 = 1
	$\lambda^*_{max}/\lambda^*_{min}$	=	18/7.2 = 2.5

PROBLEM:	32
CLASSIFICATION:	QPR-T1-2
SOURCE:	Evtushenko [25]
NUMBER OF VARIABLES:	$n = 3$
NUMBER OF CONSTRAINTS:	$m_1 = 1$, $m-m_1 = 1(1)$, $b = 3$

OBJECTIVE FUNCTION:

$$f(x) = (x_1 + 3x_2 + x_3)^2 + 4(x_1 - x_2)^2$$

CONSTRAINTS:

$$6x_2 + 4x_3 - x_1^3 - 3 \geq 0$$

$$1 - x_1 - x_2 - x_3 = 0$$

$$0 \leq x_i \quad , \quad i = 1,2,3$$

START:

$x_o = (.1 \; , \; .7 \; , \; .2)$ (feasible)

$f(x_o) = 7.2$

SOLUTION:

$x^* = (0 \; , \; 0 \; , \; 1)$

$f(x^*) = 1$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 2$

$I(x^*) = (2 \; , \; 3)$

$u^*_{max}/u^*_{min} = 4/0$

$\lambda^*_{max}/\lambda^*_{min} = -$

PROBLEM:	33
CLASSIFICATION:	PQR-T1-8
SOURCE:	Beltrami [6], Hartmann [28]
NUMBER OF VARIABLES:	$n = 3$
NUMBER OF CONSTRAINTS:	$m_1 = 2$, $m-m_1 = 0$, $b = 4$

OBJECTIVE FUNCTION:

$$f(x) = (x_1 - 1)(x_1 - 2)(x_1 - 3) + x_3$$

CONSTRAINTS:

$$x_3^2 - x_2^2 - x_1^2 \geq 0$$

$$x_1^2 + x_2^2 + x_3^2 - 4 \geq 0$$

$$0 \leq x_1$$

$$0 \leq x_2$$

$$0 \leq x_3 \leq 5$$

START: $x_o = (0 , 0 , 3)$ (feasible)

$f(x_o) = -3$

SOLUTION: $x^* = (0 , \sqrt{2} , \sqrt{2})$

$f(x^*) = \sqrt{2} - 6$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 3$

$I(x^*) = (1 , 2 , 3)$

$u^*_{max}/u^*_{min} = 11/.17678 = 62.23$

$\lambda^*_{max}/\lambda^*_{min} = -$

PROBLEM:	34
CLASSIFICATION:	LGR-T1-1
SOURCE:	Eckhardt [24]
NUMBER OF VARIABLES:	$n = 3$
NUMBER OF CONSTRAINTS:	$m_1 = 2$, $m-m_1 = 0$, $b = 6$

OBJECTIVE FUNCTION:

$$f(x) = -x_1$$

CONSTRAINTS:

$$x_2 - \exp(x_1) \geq 0$$

$$x_3 - \exp(x_2) \geq 0$$

$$0 \leq x_1 \leq 100$$

$$0 \leq x_2 \leq 100$$

$$0 \leq x_3 \leq 10$$

START:

$x_o = (0 , 1.05 , 2.9)$ (feasible)

$f(x_o) = 0$

SOLUTION:

$x^* = (\ln(\ln 10) , \ln 10 , 10)$

$f(x^*) = -\ln(\ln 10)$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 3$

$I(x^*) = (1 , 2 , 8)$

$u^*_{max}/u^*_{min} = .4343/.04343 = 10$

$\lambda^*_{max}/\lambda^*_{min} = -$

PROBLEM:	35 (Beale's problem)
CLASSIFICATION:	QLR-T1-3
SOURCE:	Asaadi [1], Charalambous [18], Dimitru [23], Sheela [57]
NUMBER OF VARIABLES:	$n = 3$
NUMBER OF CONSTRAINTS:	$m_1 = 1(1)$, $m-m_1 = 0$, $b = 3$

OBJECTIVE FUNCTION:

$$f(x) = 9 - 8x_1 - 6x_2 - 4x_3 + 2x_1^2 + 2x_2^2 + x_3^2 + 2x_1x_2 + 2x_1x_3$$

CONSTRAINTS:

$$3 - x_1 - x_2 - 2x_3 \geq 0$$

$$0 \leq x_i \quad , \quad i=1,2,3$$

START:

$x_o = (.5 , .5 , .5)$ (feasible)

$f(x_o) = 2.25$

SOLUTION:

$x^* = (4/3 , 7/9 , 4/9)$

$f(x^*) = 1/9$

$r(x^*) = 0$

$e(x^*) = .49E-10$

$\mu = 1$

$I(x^*) = (1)$

$u^*_{max}/u^*_{min} = .2222/.2222 = 1$

$\lambda^*_{max}/\lambda^*_{min} = 3.72/1.61 = 2.31$

PROBLEM:	36
CLASSIFICATION:	PLR-T1-2
SOURCE:	Biggs [10]
NUMBER OF VARIABLES:	$n = 3$
NUMBER OF CONSTRAINTS:	$m_1 = 1(1)$, $m-m_1 = 0$, $b = 6$

OBJECTIVE FUNCTION:

$$f(x) = -x_1x_2x_3$$

CONSTRAINTS:

$$72 - x_1 - 2x_2 - 2x_3 \geq 0$$

$$0 \leq x_1 \leq 20$$

$$0 \leq x_2 \leq 11$$

$$0 \leq x_3 \leq 42$$

START:

$x_o = (10 , 10 , 10)$ (feasible)

$f(x_o) = -1000$

SOLUTION:

$x^* = (20 , 11 , 15)$

$f(x^*) = -3300$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 3$

$I(x^*) = (1 , 5 , 6)$

$u^*_{max}/u^*_{min} = 110/55 = 2$

$\lambda^*_{max}/\lambda^*_{min} = -$

PROBLEM:	37
CLASSIFICATION:	PLR-T1-3
SOURCE:	Betts [8], Box [12]
NUMBER OF VARIABLES:	$n = 3$
NUMBER OF CONSTRAINTS:	$m_1 = 2(2)$, $m-m_1 = 0$, $b = 6$

OBJECTIVE FUNCTION:

$$f(x) = -x_1x_2x_3$$

CONSTRAINTS:

$$72 - x_1 - 2x_2 - 2x_3 \geq 0$$

$$x_1 + 2x_2 + 2x_3 \geq 0$$

$$0 \leq x_i \leq 42 \quad , \quad i=1,2,3$$

START: $x_o = (10\ ,\ 10\ ,\ 10)$ (feasible)

$f(x_o) = -1000$

SOLUTION: $x^* = (24\ ,\ 12\ ,\ 12)$

$f(x^*) = -3456$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 1$

$I(x^*) = (1)$

$u^*_{max}/u^*_{min} = 144/144 = 1$

$\lambda^*_{max}/\lambda^*_{min} = 24/8 = 3$

PROBLEM:	38 (Colville No.4)
CLASSIFICATION:	PBR-T1-4
SOURCE:	Colville [20], Himmelblau [29]
NUMBER OF VARIABLES:	$n = 4$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 0$, $b = 8$

OBJECTIVE FUNCTION:

$$f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2 + 90(x_4 - x_3^2)^2 + (1-x_3)^2$$
$$+ 10.1((x_2 - 1)^2 + (x_4 - 1)^2) + 19.8(x_2 - 1)(x_4 - 1)$$

CONSTRAINTS:

$$-10 \le x_i \le 10 \quad , \quad i=1,...,4$$

START:	x_o	=	(-3 , -1 , -3 , -1) (feasible)
	$f(x_o)$	=	19192
SOLUTION:	x^*	=	(1 , 1 , 1 , 1)
	$f(x^*)$	=	0
	$r(x^*)$	=	0
	$e(x^*)$	=	-
	μ	=	0
	$I(x^*)$	=	-
	u^*_{max}/u^*_{min}	=	-
	$\lambda^*_{max}/\lambda^*_{min}$	=	.10E4/.72 = .14E4

PROBLEM:	39
CLASSIFICATION:	LPR-T1-1
SOURCE:	Miele e.al. [44,45]
NUMBER OF VARIABLES:	$n = 4$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 2$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = -x_1$$

CONSTRAINTS:

$$x_2 - x_1^3 - x_3^2 = 0$$

$$x_1^2 - x_2 - x_4^2 = 0$$

START: $x_o = (2 , 2 , 2 , 2)$ (not feasible)

$f(x_o) = -2$

SOLUTION: $x^* = (1 , 1 , 0 , 0)$

$f(x^*) = -1$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 0$

$I(x^*) = -$

$u^*_{max}/u^*_{min} = 1/1 = 1$

$\lambda^*_{max}/\lambda^*_{min} = 2/2 = 1$

PROBLEM:	40
CLASSIFICATION:	PPR-T1-2
SOURCE:	Beltrami [6], Indusi [35]
NUMBER OF VARIABLES:	$n = 4$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m - m_1 = 3$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = -x_1 x_2 x_3 x_4$$

CONSTRAINTS:

$$x_1^3 + x_2^2 - 1 = 0$$

$$x_1^2 x_4 - x_3 = 0$$

$$x_4^2 - x_2 = 0$$

START: $x_o = (.8\ ,\ .8\ ,\ .8\ ,\ .8)$ (not feasible)

$f(x_o) = -.4096$

SOLUTION: $x^* = (2^a\ ,\ 2^{2b}\ ,\ (-1)^i 2^c\ ,\ (-1)^i 2^b)$

$f(x^*) = -.25$ $\quad i = 1,2$

$r(x^*) = 0$ $\quad a = -1/3$

$e(x^*) = .80E-11$ $\quad b = -1/4$

$\mu = 0$ $\quad c = -11/12$

$I(x^*) = -$

$u^*_{max}/u^*_{min} = .5/.3536 = 1.41$

$\lambda^*_{max}/\lambda^*_{min} = 1.74/1.74 = 1$

PROBLEM:	41
CLASSIFICATION:	PLR-T1-4
SOURCE:	Betts [8], Miele e.al. [42]
NUMBER OF VARIABLES:	$n = 4$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 1(1)$, $b = 8$

OBJECTIVE FUNCTION:

$$f(x) = 2 - x_1 x_2 x_3$$

CONSTRAINTS:

$$x_1 + 2x_2 + 2x_3 - x_4 = 0$$

$$0 \le x_i \le 1 \quad , \quad i=1,2,3$$

$$0 \le x_4 \le 2$$

START: $x_0 = (2 , 2 , 2 , 2)$ (not feasible)

$f(x_0) = -6$

SOLUTION: $x^* = (2/3 , 1/3 , 1/3 , 2)$

$f(x^*) = 52/27$

$r(x^*) = 0$

$e(x^*) = .13E-10$

$\mu = 1$

$I(x^*) = (8)$

$u^*_{max}/u^*_{min} = .1111/.1111 = 1$

$\lambda^*_{max}/\lambda^*_{min} = .67/.22 = 3$

PROBLEM:	42	
CLASSIFICATION:	QQR-T1-10	
SOURCE:	Brusch [14]	
NUMBER OF VARIABLES:	$n = 4$	
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 2(1)$, $b = 0$	

OBJECTIVE FUNCTION:

$$f(x) = (x_1 - 1)^2 + (x_2 - 2)^2 + (x_3 - 3)^2 + (x_4 - 4)^2$$

CONSTRAINTS:

$$x_1 - 2 = 0$$

$$x_3^2 + x_4^2 - 2 = 0$$

START: $x_0 = (1, 1, 1, 1)$ (not feasible)

$f(x_0) = 14$

SOLUTION:

$x^* = (2, 2, .6\sqrt{2}, .8\sqrt{2})$

$f(x^*) = 28 - 10\sqrt{2}$

$r(x^*) = 0$

$e(x^*) = .2E-23$

$\mu = 0$

$I(x^*) = -$

$u^*_{max}/u^*_{min} = 2.5355/2.0000 = 1.26$

$\lambda^*_{max}/\lambda^*_{min} = 7.07/2.00 = 3.54$

PROBLEM:	43 (Rosen-Suzuki)
CLASSIFICATION:	QQR-T1-11
SOURCE: Betts [8], Charalambous [18], Gould [27], Sheela [57]	
NUMBER OF VARIABLES:	$n = 4$
NUMBER OF CONSTRAINTS:	$m_1 = 3$, $m-m_1 = 0$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = x_1^2 + x_2^2 + 2x_3^2 + x_4^2 - 5x_1 - 5x_2 - 21x_3 + 7x_4$$

CONSTRAINTS:

$$8 - x_1^2 - x_2^2 - x_3^2 - x_4^2 - x_1 + x_2 - x_3 + x_4 \geq 0$$

$$10 - x_1^2 - 2x_2^2 - x_3^2 - 2x_4^2 + x_1 + x_4 \geq 0$$

$$5 - 2x_1^2 - x_2^2 - x_3^2 - 2x_1 + x_2 + x_4 \geq 0$$

START: $x_o = (0 , 0 , 0 , 0)$ (feasible)

$f(x_o) = 0$

SOLUTION: $x^* = (0 , 1 , 2 , -1)$

$f(x^*) = -44$

$r(x^*) = 0$

$e(x^*) = .21E-9$

$\mu = 2$

$I(x^*) = (1 , 3)$

$u^*_{max}/u^*_{min} = 2/1 = 2$

$\lambda^*_{max}/\lambda^*_{min} = 9/8.07 = 1.12$

PROBLEM:	44
CLASSIFICATION:	QLR-T1-4
SOURCE:	Konno [37]
NUMBER OF VARIABLES:	$n = 4$
NUMBER OF CONSTRAINTS:	$m_1 = 6(6)$, $m-m_1 = 0$, $b = 4$

OBJECTIVE FUNCTION:

$$f(x) = x_1 - x_2 - x_3 - x_1x_3 + x_1x_4 + x_2x_3 - x_2x_4$$

CONSTRAINTS:

$$8 - x_1 - 2x_2 \geq 0$$

$$12 - 4x_1 - x_2 \geq 0$$

$$12 - 3x_1 - 4x_2 \geq 0$$

$$8 - 2x_3 - x_4 \geq 0$$

$$8 - x_3 - 2x_4 \geq 0$$

$$5 - x_3 - x_4 \geq 0 \quad , \quad 0 \leq x_i \quad , \quad i=1,\ldots,4$$

START:	x_o	=	(0 , 0 , 0 , 0) (feasible)
	$f(x_o)$	=	0
SOLUTION:	x^*	=	(0 , 3 , 0 , 4)
	$f(x^*)$	=	-15
	$r(x^*)$	=	0
	$e(x^*)$	=	0
	μ	=	4
	$I(x^*)$	=	(3 , 5 , 7 , 9)
	u^*_{max}/u^*_{min}	=	8.75/1.25 = 7
	$\lambda^*_{max}/\lambda^*_{min}$	=	-

PROBLEM:	45
CLASSIFICATION:	PBR-T1-5
SOURCE:	Betts [8], Miele e.al. [42]
NUMBER OF VARIABLES:	$n = 5$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 0$, $b = 10$

OBJECTIVE FUNCTION:

$$f(x) = 2 - \frac{1}{120} x_1 x_2 x_3 x_4 x_5$$

CONSTRAINTS:

$$0 \le x_i \le i \quad , \quad i=1,\ldots,5$$

START:

$x_o = (2 , 2 , 2 , 2)$ (not feasible)

$f(x_o) = 26/15$

SOLUTION:

$x^* = (1 , 2 , 3 , 4 , 5)$

$f(x^*) = 1$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 5$

$I(x^*) = (6 , 7 , 8 , 9 , 10)$

$u^*_{max}/u^*_{min} = 1/.2 = 5$

$\lambda^*_{max}/\lambda^*_{min} = -$

PROBLEM:	47
CLASSIFICATION:	PPR-T1-3
SOURCE:	Huang, Aggerwal [34], Miele e.al. [43]
NUMBER OF VARIABLES:	$n = 5$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 3$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = (x_1 - x_2)^2 + (x_2 - x_3)^3 + (x_3 - x_4)^4 + (x_4 - x_5)^4$$

CONSTRAINTS:

$$x_1 + x_2^2 + x_3^3 - 3 = 0$$

$$x_2 - x_3^2 + x_4 - 1 = 0$$

$$x_1 x_5 - 1 = 0$$

START: $x_o = (2 , \sqrt{2} , -1 , 2-\sqrt{2} , .5)$ (feasible)

$f(x_o) = 12.4954368$

SOLUTION:

$x^* = (1 , 1 , 1 , 1 , 1)$

$f(x^*) = 0$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 0$

$I(x^*) = -$

$u^*_{max}/u^*_{min} = 0/0$

$\lambda^*_{max}/\lambda^*_{min} = 2.08/.53 = 3.92$

PROBLEM:	48
CLASSIFICATION:	QLR-T1-5
SOURCE:	Huang, Aggerwal [34], Miele e.al. [43]
NUMBER OF VARIABLES:	$n = 5$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 2(2)$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = (x_1 - 1)^2 + (x_2 - x_3)^2 + (x_4 - x_5)^2$$

CONSTRAINTS:

$$x_1 + x_2 + x_3 + x_4 + x_5 - 5 = 0$$

$$x_3 - 2(x_4 + x_5) + 3 = 0$$

START: $x_o = (3 , 5 , -3 , 2 , -2)$ (feasible)

$f(x_o) = 84$

SOLUTION: $x^* = (1 , 1 , 1 , 1 , 1)$

$f(x^*) = 0$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 0$

$I(x^*) = -$

$u^*_{max}/u^*_{min} = 0/0$

$\lambda^*_{max}/\lambda^*_{min} = 4/1.49 = 2.69$

PROBLEM:	49
CLASSIFICATION:	PLR-T1-5
SOURCE:	Huang, Aggerwal [34]
NUMBER OF VARIABLES:	$n = 5$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 2(2)$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = (x_1 - x_2)^2 + (x_3 - 1)^2 + (x_4 - 1)^4 + (x_5 - 1)^6$$

CONSTRAINTS:

$$x_1 + x_2 + x_3 + 4x_4 - 7 = 0$$

$$x_3 + 5x_5 - 6 = 0$$

START: $x_o = (10\ ,\ 7\ ,\ 2\ ,\ -3\ ,\ .8)$ (feasible)

$f(x_o) = 266$

SOLUTION:

$x^* = (1\ ,\ 1\ ,\ 1\ ,\ 1\ ,\ 1)$

$f(x^*) = 0$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 0$

$I(x^*) = -$

$u^*_{max}/u^*_{min} = 0/0$

$\lambda^*_{max}/\lambda^*_{min} = 4/.70E-10 = .57E11$

PROBLEM:	50
CLASSIFICATION:	PLR-T1-6
SOURCE:	Huang, Aggerwal [34]
NUMBER OF VARIABLES:	$n = 5$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 3(3)$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = (x_1 - x_2)^2 + (x_2 - x_3)^2 + (x_3 - x_4)^4 + (x_4 - x_5)^2$$

CONSTRAINTS:

$$x_1 + 2x_2 + 3x_3 - 6 = 0$$

$$x_2 + 2x_3 + 3x_4 - 6 = 0$$

$$x_3 + 2x_4 + 3x_5 - 6 = 0$$

START: $x_o = (35 , -31 , 11 , 5 , -5)$ (feasible)

$f(x_o) = 17416$

SOLUTION: $x^* = (1 , 1 , 1 , 1 , 1)$

$f(x^*) = 0$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 0$

$I(x^*) = -$

$u^*_{max}/u^*_{min} = 0/0$

$\lambda^*_{max}/\lambda^*_{min} = 5.89/1.64 = 3.6$

PROBLEM:	51
CLASSIFICATION:	QLR-T1-6
SOURCE:	Huang, Aggerwal [34]
NUMBER OF VARIABLES:	$n = 5$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 3(3)$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = (x_1 - x_2)^2 + (x_2 + x_3 - 2)^2 + (x_4 - 1)^2 + (x_5 - 1)^2$$

CONSTRAINTS:

$$x_1 + 3x_2 - 4 = 0$$

$$x_3 + x_4 - 2x_5 = 0$$

$$x_2 - x_5 = 0$$

START: $x_o = (2.5\ ,\ .5\ ,\ 2\ ,\ -1\ ,\ .5)$ (feasible)

$f(x_o) = 8.5$

SOLUTION:

$x^* = (1\ ,\ 1\ ,\ 1\ ,\ 1\ ,\ 1)$

$f(x^*) = 0$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 0$

$I(x^*) = -$

$u^*_{max}/u^*_{min} = 0/0$

$\lambda^*_{max}/\lambda^*_{min} = 3.49/1.90 = 1.84$

PROBLEM:	52
CLASSIFICATION:	QLR-T1-7
SOURCE:	Miele e.al. [44,45]
NUMBER OF VARIABLES:	$n = 5$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 3(3)$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = (4x_1 - x_2)^2 + (x_2 + x_3 - 2)^2 + (x_4 - 1)^2 + (x_5 - 1)^2$$

CONSTRAINTS:

$$x_1 + 3x_2 = 0$$

$$x_3 + x_4 - 2x_5 = 0$$

$$x_2 - x_5 = 0$$

START:	x_0	=	(2 , 2 , 2 , 2 , 2) (not feasible)
	$f(x_0)$	=	42
SOLUTION:	x^*	=	(-33 , 11 , 180 , -158 , 11)/349
	$f(x^*)$	=	1859/349
	$r(x^*)$	=	0
	$e(x^*)$	=	.14E-9
	μ	=	0
	$I(x^*)$	=	-
	u^*_{max}/u^*_{min}	=	7.7479/2.9054 = 2.6667
	$\lambda^*_{max}/\lambda^*_{min}$	=	26.93/1.99 = 13.51

PROBLEM:	53
CLASSIFICATION:	QLR-T1-8
SOURCE:	Betts [8], Miele e.al. [42,43]
NUMBER OF VARIABLES:	$n = 5$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 3(3)$, $b = 10$

OBJECTIVE FUNCTION:

$$f(x) = (x_1 - x_2)^2 + (x_2 + x_3 - 2)^2 + (x_4 - 1)^2 + (x_5 - 1)^2$$

CONSTRAINTS:

$$x_1 + 3x_2 = 0$$

$$x_3 + x_4 - 2x_5 = 0$$

$$x_2 - x_5 = 0$$

$$-10 \le x_i \le 10 \quad , \quad i=1,\ldots,5$$

START: $x_o = (2 , 2 , 2 , 2 , 2)$ (not feasible)

$f(x_o) = 6$

SOLUTION: $x^* = (-33 , 11 , 27 , -5 , 11)/43$

$f(x^*) = 176/43$

$r(x^*) = 0$

$e(x^*) = .28E-9$

$\mu = 0$

$I(x^*) = -$

$u^*_{max}/u^*_{min} = 5.9535/2.0465 = 1.84$

$\lambda^*_{max}/\lambda^*_{min} = 3.49/1.90 = 1.84$

PROBLEM:	54
CLASSIFICATION:	GLR-T1-2
SOURCE:	Betts [8], Picket [50]
NUMBER OF VARIABLES:	$n = 6$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 1(1)$, $b = 12$

OBJECTIVE FUNCTION:

$$f(x) = -\exp(-h(x)/2)$$

$$\begin{aligned} h(x) = & ((x_1 - 1.E6)^2/6.4E7 + (x_1 - 1.E4)(x_2 - 1)/2.E4 \\ & + (x_2 - 1)^2)(x_3 - 2.E6)^2/(.96 \cdot 4.9E13) \\ & + (x_4 - 10)^2/2.5E3 + (x_5 - 1.E-3)^2/2.5E-3 \\ & + (x_6 - 1.E8)^2/2.5E17 \end{aligned}$$

CONSTRAINTS:

$$x_1 + 4.E3x_2 - 1.76E4 = 0$$

$$0 \le x_1 \le 2.E4 \qquad -10 \le x_2 \le 10 \qquad 0 \le x_3 \le 1.E7$$

$$0 \le x_4 \le 20 \qquad -1 \le x_5 \le 1 \qquad 0 \le x_6 \le 2.E8$$

START:

$x_0 = (6E3 , 1.5 , 4E6 , 2 , 3E-3 , 5E7)$

$f(x_0) = -.7651$ (not feasible)

SOLUTION:

$x^* = (91600/7, 79/70, 2E6, 10, 1E-3, 1E8)$

$f(x^*) = -\exp(-27/280)$

$r(x^*) = 0$

$e(x^*) = .20E-10$

$\mu = 0$

$I(x^*) = -$

$u^*_{max}/u^*_{min} = .4865E-4/.4865E-4 = 1$

$\lambda^*_{max}/\lambda^*_{min} = 362.9/.36E-17 = .10E21$

PROBLEM:	55
CLASSIFICATION:	GLR-T1-3
SOURCE:	Hsia [33]
NUMBER OF VARIABLES:	$n = 6$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 6(6)$, $b = 8$

OBJECTIVE FUNCTION:

$$f(x) = x_1 + 2x_2 + 4x_5 + \exp(x_1 x_4)$$

CONSTRAINTS:

$$x_1 + 2x_2 + 5x_5 - 6 = 0$$

$$x_1 + x_2 + x_3 - 3 = 0$$

$$x_4 + x_5 + x_6 - 2 = 0$$

$$x_1 + x_4 - 1 = 0$$

$$x_2 + x_5 - 2 = 0$$

$$x_3 + x_6 - 2 = 0$$

$$0 \le x_1 \;,\; i=1,\ldots,6 \;,\; x_1 \le 1 \;,\; x_4 \le 1$$

START: $x_o = (1 , 2 , 0 , 0 , 0 , 2)$ (not feasible)

$f(x_o) = 6$

SOLUTION: $x^* = (0 , 4/3 , 5/3 , 1 , 2/3 , 1/3)$

$f(x^*) = 19/3$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 8$

$I(x^*) = (1 , 8)$

$u^*_{max}/u^*_{min} = -$

$\lambda^*_{max}/\lambda^*_{min} = -$

PROBLEM:	56
CLASSIFICATION:	PGR-T1-2
SOURCE:	Brusch [15]
NUMBER OF VARIABLES:	$n = 7$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 4$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = -x_1x_2x_3$$

CONSTRAINTS:

$$x_1 - 4.2\sin^2 x_4 = 0$$

$$x_2 - 4.2\sin^2 x_5 = 0$$

$$x_3 - 4.2\sin^2 x_6 = 0$$

$$x_1 + 2x_2 + 2x_3 - 7.2\sin^2 x_7 = 0$$

START: $x_0 = (1, 1, 1, a, a, a, b)$

$f(x_0) = -1$ (feasible)

SOLUTION: $x^* = (2.4, 1.2, 1.2, \pm c+j\pi, \pm d+k\pi, \pm d+l\pi, (r+.5)\pi)$

$f(x^*)$	= -3.456		$a = \arcsin\sqrt{1/4.2}$
$r(x^*)$	= 0		$b = \arcsin\sqrt{5/7.2}$
$e(x^*)$	= .67E-10		$c = \arcsin\sqrt{4/7}$
μ	= 0		$d = \arcsin\sqrt{2/7}$
$I(x^*)$	= -		$j,k,l,r = 0, \pm 1, \pm 2, \ldots$
u^*_{max}/u^*_{min}	= 1.44/.68E-11 = .21E12		
$\lambda^*_{max}/\lambda^*_{min}$	= 20.74/.76 = 27.45		

PROBLEM:	57
CLASSIFICATION:	SQR-P1-1
SOURCE:	Betts [8], Gould [27]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 1$, $m-m_1 = 0$, $b = 2$

OBJECTIVE FUNCTION:

$$f(x) = \sum_{i=1}^{44} f_i(x)^2$$

$$f_i(x) = b_i - x_1 - (.49 - x_1)\exp(-x_2(a_i - 8))$$

$$i=1,\ldots,44$$

a_i, b_i: cf. Appendix A

CONSTRAINTS:

$$.49x_2 - x_1x_2 - .09 \geq 0$$

$$.4 \leq x_1 \quad , \quad -4 \leq x_2$$

START:	x_0	=	(.42 , 5) (feasible)
	$f(x_0)$	=	.030798602
SOLUTION:	x^*	=	(.419952675 , 1.284845629)
	$f(x^*)$	=	.02845966972
	$r(x^*)$	=	0
	$e(x^*)$	=	.98E-7
	μ	=	1
	$I(x^*)$	=	(1)
	u^*_{max}/u^*_{min}	=	.06671/.06671 = 1
	$\lambda^*_{max}/\lambda^*_{min}$	=	.23/.23 = 1

PROBLEM:	59
CLASSIFICATION:	GQR-P1-1
SOURCE:	Barnes [3], Himmelblau [29]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 3$, $m-m_1 = 0$, $b = 4$

OBJECTIVE FUNCTION:

$$\begin{aligned} f(x) = & -75.196 + 3.8112x_1 + .0020567x_1^3 - 1.0345E\text{-}5x_1^4 \\ & + 6.8306x_2 - .030234x_1x_2 + 1.28134E\text{-}3x_2x_1^2 \\ & + 2.266E\text{-}7x_1^4x_2 - .25645x_2^2 + .0034604x_2^3 - 1.3514E\text{-}5x_2^4 \\ & + 28.106/(x_2 + 1) + 5.2375E\text{-}6x_1^2x_2^2 + 6.3E\text{-}8x_1^3x_2^2 \\ & - 7E\text{-}10x_1^3x_2^3 - 3.405E\text{-}4x_1x_2^2 + 1.6638E\text{-}6x_1x_2^3 \\ & + 2.8673\exp(.0005x_1x_2) - 3.5256E\text{-}5x_1^3x_2 \end{aligned}$$

CONSTRAINTS:

$$x_1x_2 - 700 \geq 0$$

$$x_2 - x_1^2/125 \geq 0 \qquad 0 \leq x_1 \leq 75$$

$$(x_2 - 50)^2 - 5(x_1 - 55) \geq 0 \qquad 0 \leq x_2 \leq 65$$

START: $x_o = (90 , 10)$ (not feasible)

$f(x_o) = 86.878639$

SOLUTION:

$x^* = (13.55010424 , 51.66018129)$

$f(x^*) = -7.804226324$

$r(x^*) = 0$

$e(x^*) = .27E\text{-}6$

$\mu = 1$

$I(x^*) = (1)$

$u^*_{max}/u^*_{min} = .01142/.01142 = 1$

$\lambda^*_{max}/\lambda^*_{min} = .13/.13 = 1$

PROBLEM:	60
CLASSIFICATION:	PPR-P1-1
SOURCE:	Betts [8], Miele e.al. [42,44]
NUMBER OF VARIABLES:	$n = 3$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 1$, $b = 6$

OBJECTIVE FUNCTION:

$$f(x) = (x_1 - 1)^2 + (x_1 - x_2)^2 + (x_2 - x_3)^4$$

CONSTRAINTS:

$$x_1(1 + x_2^2) + x_3^4 - 4 - 3\sqrt{2} = 0$$

$$-10 \le x_i \le 10 \quad , \quad i=1,2,3$$

START: $x_o = (2 , 2 , 2)$ (not feasible)

$f(x_o) = 1$

SOLUTION:

$x^* = (1.104859024, 1.196674194, 1,535262257)$

$f(x^*) = .03256820025$

$r(x^*) = .23E-9$

$e(x^*) = .38E-7$

$\mu = 1$

$I(x^*) = (1)$

$u^*_{max}/u^*_{min} = .01073/.01073 = 1$

$\lambda^*_{max}/\lambda^*_{min} = 5.72/2.07 = 2.76$

PROBLEM:	61		
CLASSIFICATION:	QQR-P1-1		
SOURCE:	Fletcher, Lill [26]		
NUMBER OF VARIABLES:	$n = 3$		
NUMBER OF CONSTRAINTS:	$m_1 = 0$	, $m-m_1 = 2$	, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = 4x_1^2 + 2x_2^2 + 2x_3^2 - 33x_1 + 16x_2 - 24x_3$$

CONSTRAINTS:

$$3x_1 - 2x_2^2 - 7 = 0$$

$$4x_1 - x_3^2 - 11 = 0$$

START:	x_o	=	(0 , 0 , 0) (not feasible)
	$f(x_o)$	=	0
SOLUTION:	x^*	=	(5.326770157,-2.118998639,3.210464239)
	$f(x^*)$	=	-143.6461422
	$r(x^*)$	=	.29E-9
	$e(x^*)$	=	.21E-6
	μ	=	0
	$I(x^*)$	=	-
	u^*_{max}/u^*_{min}	=	1.7378/.8877 = 1.96
	$\lambda^*_{max}/\lambda^*_{min}$	=	7.83/7.83 = 1

PROBLEM:	62
CLASSIFICATION:	GLR-P1-1
SOURCE:	Betts [8], Picket [50]
NUMBER OF VARIABLES:	$n = 3$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 1(1)$, $b = 6$

OBJECTIVE FUNCTION:

$$f(x) = -32.174(255\ \ln((x_1+x_2+x_3+.03)/(.09x_1+x_2+x_3+.03))$$
$$+ 280\ \ln((x_2+x_3+.03)/(.07x_2+x_3+.03))$$
$$+ 290\ \ln((x_3+.03)/(.13x_3+.03)))$$

CONSTRAINTS:

$$x_1 + x_2 + x_3 - 1 = 0$$
$$0 \le x_i \le 1 \quad , \quad i=1,2,3$$

START: x_o = (.7 , .2 , .1) (feasible)

$f(x_o)$ = -25698.3

SOLUTION: x^* = (.6178126908,.328202223,.5398508606E-1)

$f(x^*)$ = -26272.51448

$r(x^*)$ = 0

$e(x^*)$ = .20E-5

μ = 0

$I(x^*)$ = -

u^*_{max}/u^*_{min} = 6387/6387 = 1

$\lambda^*_{max}/\lambda^*_{min}$ = .32E6/.72E4 = 44.9

PROBLEM:	63
CLASSIFICATION:	QQR-P1-2
SOURCE:	Himmelblau [29], Paviani [48], Sheela [57]
NUMBER OF VARIABLES:	$n = 3$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 2(1)$, $b = 3$

OBJECTIVE FUNCTION:

$$f(x) = 1000 - x_1^2 - 2x_2^2 - x_3^2 - x_1x_2 - x_1x_3$$

CONSTRAINTS:

$$8x_1 + 14x_2 + 7x_3 - 56 = 0$$

$$x_1^2 + x_2^2 + x_3^2 - 25 = 0$$

$$0 \le x_i \quad , \quad i=1,2,3$$

START:	x_o	=	(2 , 2 , 2) (not feasible)
	$f(x_o)$	=	976
SOLUTION:	x^*	=	(3.512118414,.2169881741,3.552174034)
	$f(x^*)$	=	961.7151721
	$r(x^*)$	=	0
	$e(x^*)$	=	.62E-5
	μ	=	0
	$I(x^*)$	=	-
	u^*_{max}/u^*_{min}	=	1.223/.2749 = 4.45
	$\lambda^*_{max}/\lambda^*_{min}$	=	1.52/1.52 = 1

PROBLEM:	64
CLASSIFICATION:	PPR-P1-2
SOURCE:	Best [7]
NUMBER OF VARIABLES:	$n = 3$
NUMBER OF CONSTRAINTS:	$m_1 = 1$, $m-m_1 = 0$, $b = 3$

OBJECTIVE FUNCTION:

$$f(x) = 5x_1 + 50000/x_1 + 20x_2 + 72000/x_2 + 10x_3 + 144000/x_3$$

CONSTRAINTS:

$$1 - 4/x_1 - 32/x_2 - 120/x_3 \geq 0$$

$$1.E-5 \leq x_i \quad , \quad i=1,2,3$$

START:	x_o	=	(1 , 1 , 1) (not feasible)
	$f(x_o)$	=	266035
SOLUTION:	x^*	=	(108.7347175,85.12613942,204.3247078)
	$f(x^*)$	=	6299.842428
	$r(x^*)$	=	0
	$e(x^*)$	=	.28E-4
	μ	=	1
	$I(x^*)$	=	(1)
	u^*_{max}/u^*_{min}	=	2279/2279 = 1
	$\lambda^*_{max}/\lambda^*_{min}$	=	.21/.092 = 2.28

PROBLEM:	65
CLASSIFICATION:	QQR-P1-3
SOURCE:	Murtagh, Sargent [47]
NUMBER OF VARIABLES:	$n = 3$
NUMBER OF CONSTRAINTS:	$m_1 = 1$, $m-m_1 = 0$, $b = 6$

OBJECTIVE FUNCTION:

$$f(x) = (x_1 - x_2)^2 + (x_1 + x_2 - 10)^2/9 + (x_3 - 5)^2$$

CONSTRAINTS:

$$48 - x_1^2 - x_2^2 - x_3^2 \geq 0$$

$$-4.5 \leq x_i \leq 4.5 \quad , \quad i=1,2$$

$$-5 \leq x_3 \leq 5$$

START: $x_o = (-5 , 5 , 0)$ (not feasible)

$f(x_o) = 1225/9$

SOLUTION:

$x^* = (3.650461821,3.65046168,4.6204170507)$

$f(x^*) = .9535288567$

$r(x^*) = 0$

$e(x^*) = .40E-6$

$\mu = 1$

$I(x^*) = (1)$

$u^*_{max}/u^*_{min} = .08215/.08215 = 1$

$\lambda^*_{max}/\lambda^*_{min} = 1.95/1.68 = 1.16$

PROBLEM:	66
CLASSIFICATION:	LGR-P1-1
SOURCE:	Eckhardt [24]
NUMBER OF VARIABLES:	$n = 3$
NUMBER OF CONSTRAINTS:	$m_1 = 2$, $m-m_1 = 0$, $b = 6$

OBJECTIVE FUNCTION:

$$f(x) = .2x_3 - .8x_1$$

CONSTRAINTS:

$$x_2 - \exp(x_1) \geq 0$$

$$x_3 - \exp(x_2) \geq 0$$

$$0 \leq x_1 \leq 100$$

$$0 \leq x_2 \leq 100$$

$$0 \leq x_3 \leq 10$$

START:	x_o	$= (0 , 1.05 , 2.9)$	(feasible)
	$f(x_o)$	$= .58$	
SOLUTION:	x^*	$= (.1841264879, 1.202167873, 3.327322322)$	
	$f(x^*)$	$= .5181632741$	
	$r(x^*)$	$= .58E-10$	
	$e(x^*)$	$= .86E-11$	
	μ	$= 2$	
	$I(x^*)$	$= (1 , 2)$	
	u^*_{max}/u^*_{min}	$= .6654/.2 = 3.33$	
	$\lambda^*_{max}/\lambda^*_{min}$	$= .096/.096 = 1$	

PROBLEM:	67 (Colville No.8)
CLASSIFICATION:	GGI-P1-1
SOURCE:	Colville [20], Himmelblau [29]
NUMBER OF VARIABLES:	$n = 3$
NUMBER OF CONSTRAINTS:	$m_1 = 14$, $m-m_1 = 0$, $b = 6$

OBJECTIVE FUNCTION:

$$f(x) = -(.063y_2(x)y_5(x) - 5.04x_1 - 3.36y_3(x) - .035x_2 - 10x_3)$$

$y_i(x)$: cf. Appendix A

CONSTRAINTS:

$$y_{i+1}(x) - a_i \geq 0 \quad , \quad i=1,\ldots,7$$

$$a_i - y_{i-6}(x) \geq 0 \quad , \quad i=8,\ldots,14$$

$$1.E-5 \leq x_1 \leq 2.E3$$

$$1.E-5 \leq x_2 \leq 1.6E4$$

$$1.E-5 \leq x_3 \leq 1.2E2$$

a_i : cf. Appendix A

START: $x_0 = (1745 , 12000 , 110)$ (feasible)

$f(x_0) = 868.6458$

SOLUTION:

$x^* = (1728.371286, 16000.00000, 98.14151402)$

$f(x^*) = -1162.036507$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 3$

$I(x^*) = (9 , 11 , 19)$

$u^*_{max}/u^*_{min} = 1.5872/.03403 = 46.6$

$\lambda^*_{max}/\lambda^*_{min} = -$

PROBLEM:	68 , 69 (cost optimal inspection plan)
CLASSIFICATION:	GGR-P1-(1,2)
SOURCE:	Collani [19]
NUMBER OF VARIABLES:	$n = 4$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 2$, $b = 8$

OBJECTIVE FUNCTION:

$$f(x) = \left(a_i n_i - \frac{b_i(\exp(x_1) - 1) - x_3}{\exp(x_1) - 1 + x_4} x_4\right)/x_1 \quad , \quad i=1,2$$

No. 68 : $a_1 = .0001$, $b_1 = 1$, $d_1 = 1$, $n_1 = 24$

No. 69 : $a_2 = .1$, $b_2 = 1000$, $d_2 = 1$, $n_2 = 4$

CONSTRAINTS:

$$x_3 - 2\Phi(-x_2) = 0$$

$$x_4 - \Phi(-x_2 + d_i\sqrt{n_i}) - \Phi(-x_2 - d_i\sqrt{n_i}) = 0$$

$$\Phi(x) = \int_{-\infty}^{x} \exp(-y^2/2)/\sqrt{2\pi} \; dy$$

$.0001 \le x_1 \le 100$, $0 \le x_2 \le 100$, $0 \le x_3 \le 2$, $0 \le x_4 \le 2$

START: x_o = (1 , 1 , 1 , 1) (not feasible) – for both problems

		No. 68	No. 69
$f(x_o)$	=	-.2618407	-631.3525

SOLUTION:

		No. 68	No. 69
x^*	=	(.06785874,3.6461717,.00026617,.8948622)	(.02937141,1.1902534,.23394676,.7916678)
$f(x^*)$	=	-.920425026	-956.71288
$r(x^*)$	=	.54E-7	.44E-10
$e(x^*)$	=	.14E-4	.33E-4
μ	=	0	0
$I(x^*)$	=	–	–
u^*_{max}/u^*_{min}	=	13.66/.0777 = 176	44.47/32.81 = 1.3
$\lambda^*_{max}/\lambda^*_{min}$	=	16.4/.062 = .26E3	.26E5/19.6 = .1E4

PROBLEM:	70
CLASSIFICATION:	SQR-P1-1
SOURCE:	Himmelblau [29,30]
NUMBER OF VARIABLES:	$n = 4$
NUMBER OF CONSTRAINTS:	$m_1 = 1$, $m-m_1 = 0$, $b = 8$

OBJECTIVE FUNCTION:

$$f(x) = \sum_{i=1}^{19} (y_{i,cal} - y_{i,obs})^2$$

$$y_{i,cal} = (1 + \frac{1}{12x_2})[x_3 b^{x_2} (x_2/6.2832)^{.5}(c_i/7.685)^{x_2-1}$$

$$\exp(x_2 - bc_i x_2/7.658)] + (1 + \frac{1}{12x_1})[(1 - x_3)(b/x_4)^{x_1}$$

$$(x_1/6.2832)^{.5}(c_i/7.658)^{x_1-1} \exp(x_1 - bc_i x_1/(7.658x_4))]$$

$$b = x_3 + (1 - x_3)x_4$$

c_i, $y_{i,obs}$: cf. Appendix A

CONSTRAINTS:

$$x_3 + (1 - x_3)x_4 \geq 0$$

$$.00001 \leq x_i \leq 100 \quad , \quad i=1,2,4$$

$$.00001 \leq x_3 \leq 1$$

START:	x_0	=	(2 , 4 , .04 , 2) (feasible)
	$f(x_0)$	=	.9818596
SOLUTION:	x^*	=	(12.27695,4.631788,.3128625,2.029290)
	$f(x^*)$	=	.007498464
	$r(x^*)$	=	0
	$e(x^*)$	=	0
	μ	=	0
	$I(x^*)$	=	-
	u^*_{max}/u^*_{min}	=	-
	$\lambda^*_{max}/\lambda^*_{min}$	=	18.1/.91E-3 = .20E5

PROBLEM:	71
CLASSIFICATION:	PPR-P1-3
SOURCE:	Bartholomew-Biggs [4]
NUMBER OF VARIABLES:	$n = 4$
NUMBER OF CONSTRAINTS:	$m_1 = 1$, $m-m_1 = 1$, $b = 8$

OBJECTIVE FUNCTION:

$$f(x) = x_1x_4(x_1 + x_2 + x_3) + x_3$$

CONSTRAINTS:

$$x_1x_2x_3x_4 - 25 \geq 0$$

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 - 40 = 0$$

$$1 \leq x_i \leq 5 \ , \quad i=1,\ldots,4$$

START:	x_o	=	(1 , 5 , 5 , 1) (feasible)
	$f(x_o)$	=	16
SOLUTION:	x^*	=	(1 , 4.7429994 , 3.8211503 , 1.3794082)
	$f(x^*)$	=	17.0140173
	$r(x^*)$	=	0
	$e(x^*)$	=	.51E-6
	μ	=	2
	$I(x^*)$	=	(1 , 2)
	u^*_{max}/u^*_{min}	=	1.0879/.1615 = 6.74
	$\lambda^*_{max}/\lambda^*_{min}$	=	1.18/1.18 = 1

PROBLEM:	72 (optimal sample size)
CLASSIFICATION:	LPR-P1-1
SOURCE:	Bracken, McCormick [13]
NUMBER OF VARIABLES:	$n = 4$
NUMBER OF CONSTRAINTS:	$m_1 = 2$, $m-m_1 = 0$, $b = 8$

OBJECTIVE FUNCTION:

$$f(x) = 1 + x_1 + x_2 + x_3 + x_4$$

CONSTRAINTS:

$$.0401 - 4/x_1 - 2.25/x_2 - 1/x_3 - .25/x_4 \geq 0$$

$$.010085 - .16/x_1 - .36/x_2 - .64/x_3 - .64/x_4 \geq 0$$

$$.001 \leq x_i \leq (5 - i)E5 \quad , \quad i=1,\ldots,4$$

START:	x_o	=	(1 , 1 , 1 , 1) (not feasible)
	$f(x_o)$	=	5
SOLUTION:	x^*	=	(193.4071,179.5475,185.0186,168.7062)
	$f(x^*)$	=	727.67937
	$r(x^*)$	=	0
	$e(x^*)$	=	.11E-4
	μ	=	2
	$I(x^*)$	=	(1 , 2)
	u^*_{max}/u^*_{min}	=	.4147E5/.7693E4 = 5.39
	$\lambda^*_{max}/\lambda^*_{min}$	=	.011/.011 = 1.02

PROBLEM:	73 (cattle-feed)
CLASSIFICATION:	LGI-P1-1
SOURCE:	Biggs [10], Bracken, McCormick [13]
NUMBER OF VARIABLES:	$n = 4$
NUMBER OF CONSTRAINTS:	$m_1 = 2(1)$, $m-m_1 = 1(1)$, $b = 4$

OBJECTIVE FUNCTION:

$$f(x) = 24.55x_1 + 26.75x_2 + 39x_3 + 40.50x_4$$

CONSTRAINTS:

$$2.3x_1 + 5.6x_2 + 11.1x_3 + 1.3x_4 - 5 \geq 0$$

$$12x_1 + 11.9x_2 + 41.8x_3 + 52.1x_4 - 21$$
$$- 1.645(.28x_1^2 + .19x_2^2 + 20.5x_3^2 + .62x_4^2)^{1/2} \geq 0$$

$$x_1 + x_2 + x_3 + x_4 - 1 = 0$$

$$0 \leq x_i \quad , \quad i=1,\ldots,4$$

START: $x_o = (1 , 1 , 1 , 1)$ (not feasible)

$f(x_o) = 130.8$

SOLUTION:

$x^* = (.6355216,-.12E-11,.3127019,.05177655)$

$f(x^*) = 29.894378$

$r(x^*) = .99E-10$

$e(x^*) = 0$

$\mu = 3$

$I(x^*) = (1 , 2 , 4)$

$u^*_{max}/u^*_{min} = 18.37/.2433 = 75.5$

$\lambda^*_{max}/\lambda^*_{min} = -$

PROBLEM:	74 , 75
CLASSIFICATION:	PGR-P1-(1,2)
SOURCE:	Beuneu [9]
NUMBER OF VARIABLES:	n = 4
NUMBER OF CONSTRAINTS:	$m_1 = 2(2)$, $m-m_1 = 3$, $b = 8$

OBJECTIVE FUNCTION:

$$f(x) = 3x_1 + 1.E-6x_1^3 + 2x_2 + \tfrac{2}{3}E-6x_2^3$$

CONSTRAINTS:

$$x_4 - x_3 + a_j \geq 0$$
$$x_3 - x_4 + a_j \geq 0$$
$$1000\sin(-x_3 - .25) + 1000\sin(-x_4 - .25) + 894.8 - x_1 = 0$$
$$1000\sin(x_3 - .25) + 1000\sin(x_3 - x_4 - .25) + 894.8 - x_2 = 0$$
$$1000\sin(x_4 - .25) + 1000\sin(x_4 - x_3 - .25) + 1294.8 = 0$$
$$0 \leq x_i \leq 1200 \quad , \quad i=1,2$$
$$-a_j \leq x_i \leq a_j \quad , \quad i=3,4$$

No. 74 : $a_1 = .55$ No.75 : $a_2 = .48$

START:	x_o	=	(0 , 0 , 0 , 0) – for both problems	(not feasible)
	$f(x_o)$	=	0	
SOLUTION:	x^*	=	(679.9453,1026.067,.1188764,-.3962336) (776.1592,925.1949,.05110879, -.4288911)	
	$f(x^*)$	=	5126.4981	5174.4129
	$r(x^*)$	=	.75E-7	.30E-7
	$e(x^*)$	=	.52E-7	0
	μ	=	0	1
	$I(x^*)$	=	-	(1)
	u^*_{max}/u^*_{min}	=	5.46/4.11 = 1.33	2779/3.712 = 748.7
	$\lambda^*_{max}/\lambda^*_{min}$	=	.49E-2/.49E-2 = 1	-

PROBLEM:	76
CLASSIFICATION:	QLR-P1-1
SOURCE:	Murtagh, Sargent [47]
NUMBER OF VARIABLES:	$n = 4$
NUMBER OF CONSTRAINTS:	$m_1 = 3(3)$, $m-m_1 = 0$, $b = 4$

OBJECTIVE FUNCTION:

$$f(x) = x_1^2 + .5x_2^2 + x_3^2 + .5x_4^2 - x_1x_3 + x_3x_4 - x_1 - 3x_2 + x_3 - x_4$$

CONSTRAINTS:

$$5 - x_1 - 2x_2 - x_3 - x_4 \geq 0$$

$$4 - 3x_1 - x_2 - 2x_3 + x_4 \geq 0$$

$$x_2 + 4x_3 - 1.5 \geq 0$$

$$0 \leq x_i \ , \quad i=1,\dots,4$$

START: $x_o = (.5\ ,\ .5\ ,\ .5\ ,\ .5)$ (feasible)

$f(x_o) = -1.25$

SOLUTION:

$x^* = (.2727273, 2.090909, -.26E-10, .5454545)$

$f(x^*) = -4.681818181$

$r(x^*) = .84E-10$

$e(x^*) = .15E-10$

$\mu = 2$

$I(x^*) = (1\ ,\ 6)$

$u^*_{max}/u^*_{min} = 1.7272/.4545 = 3.8$

$\lambda^*_{max}/\lambda^*_{min} = 1.83/1 = 1.83$

PROBLEM:	77
CLASSIFICATION:	PGR-P1-3
SOURCE:	Betts [8], Miele e.al. [42,44,45]
NUMBER OF VARIABLES:	$n = 5$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 2$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = (x_1 - 1)^2 + (x_1 - x_2)^2 + (x_3 - 1)^2 + (x_4 - 1)^4 + (x_5 - 1)^6$$

CONSTRAINTS:

$$x_1^2 x_4 + \sin(x_4 - x_5) - 2\sqrt{2} = 0$$

$$x_2 + x_3^4 x_4^2 - 8 - \sqrt{2} = 0$$

START: $x_o = (2 , 2 , 2 , 2 , 2 , 2)$

$f(x_o) = 4$ (not feasible)

SOLUTION:

$x^* = (1.166172, 1.182111, 1.380257, 1.506036, .6109203)$

$f(x^*) = .24150513$

$r(x^*) = .12E-9$

$e(x^*) = .53E-7$

$\mu = 0$

$I(x^*) = -$

$u^*_{max}/u^*_{min} = .08554/.03188 = 2.68$

$\lambda^*_{max}/\lambda^*_{min} = 3.92/.75 = 5.25$

PROBLEM:	78
CLASSIFICATION:	PPR-P1-4
SOURCE:	Asaadi [1], Powell [51]
NUMBER OF VARIABLES:	$n = 5$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 3$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = x_1x_2x_3x_4x_5$$

CONSTRAINTS:

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 - 10 = 0$$

$$x_2x_3 - 5x_4x_5 = 0$$

$$x_1^3 + x_2^3 + 1 = 0$$

START: $x_o = (-2 , 1.5 , 2 , -1 , -1)$

$f(x_o) = -6$ (not feasible)

SOLUTION: $x^* = (-1.717142, 1.595708, 1.827248, -.7636429, -.7636435)$

$f(x^*) = -2.91970041$

$r(x^*) = .35E-9$

$e(x^*) = .91E-5$

$\mu = 0$

$I(x^*) = -$

$u^*_{max}/u^*_{min} = .7444/.09681 = 7.69$

$\lambda^*_{max}/\lambda^*_{min} = 3.04/2.98 = 1.02$

PROBLEM:	79
CLASSIFICATION:	PPR-P1-5
SOURCE:	Betts [8], Miele e.al. [42,44,45]
NUMBER OF VARIABLES:	$n = 5$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 3$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = (x_1 - 1)^2 + (x_1 - x_2)^2 + (x_2 - x_3)^2 + (x_3 - x_4)^4 + (x_4 - x_5)^4$$

CONSTRAINTS:

$$x_1 + x_2^2 + x_3^3 - 2 - 3\sqrt{2} = 0$$

$$x_2 - x_3^2 + x_4 + 2 - 2\sqrt{2} = 0$$

$$x_1 x_5 - 2 = 0$$

START: $x_0 = (2 , 2 , 2 , 2 , 2)$ (not feasible)

$f(x_0) = 1$

SOLUTION: $x^* = (1.191127, 1.362603, 1.472818, 1.635017, 1.679081)$

$f(x^*) = .0787768209$

$r(x^*) = .58E-9$

$e(x^*) = .71E-10$

$\mu = 0$

$I(x^*) = -$

$u^*_{max}/u^*_{min} = .3882E-1/.2873E-3 = 135.1$

$\lambda^*_{max}/\lambda^*_{min} = 2.03/.70 = 2.88$

PROBLEM:	80
CLASSIFICATION:	GPR-P1-1
SOURCE:	Powell [52]
NUMBER OF VARIABLES:	$n = 5$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 3$, $b = 10$

OBJECTIVE FUNCTION:

$$f(x) = \exp(x_1x_2x_3x_4x_5)$$

CONSTRAINTS:

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 - 10 = 0$$

$$x_2x_3 - 5x_4x_5 = 0$$

$$x_1^3 + x_2^3 + 1 = 0$$

$$-2.3 \le x_i \le 2.3 \quad , \quad i=1,2$$

$$-3.2 \le x_i \le 3.2 \quad , \quad i=3,4,5$$

START: $x_o = (-2 , 2 , 2 , -1 , -1)$ (not feasible)

$f(x_o) = 3.3546E-4$

SOLUTION:

$x^* = (-1.717143, 1.595709, 1.827247, -.7636413, -.7636450)$

$f(x^*) = .0539498478$

$r(x^*) = .41E-9$

$e(x^*) = .49E-6$

$\mu = 0$

$I(x^*) = -$

$u^*_{max}/u^*_{min} = .04016/.005222 = 7.69$

$\lambda^*_{max}/\lambda^*_{min} = .16/.16 = 1.02$

PROBLEM:	81
CLASSIFICATION:	GPR-P1-2
SOURCE:	Powell [52]
NUMBER OF VARIABLES:	$n = 5$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 3$, $b = 10$

OBJECTIVE FUNCTION:

$$f(x) = \exp(x_1x_2x_3x_4x_5) - .5(x_1^3 + x_2^3 + 1)^2$$

CONSTRAINTS:

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 - 10 = 0$$

$$x_2x_3 - 5x_4x_5 = 0$$

$$x_1^3 + x_2^3 + 1 = 0$$

$$-2.3 \le x_i \le 2.3 \quad , \quad i=1,2$$

$$-3.2 \le x_i \le 3.2 \quad , \quad i=3,4,5$$

START: $x_o = (-2 , 2 , 2 , -1 , -1)$ (not feasible)

$f(x_o) = -.49966$

SOLUTION: $x^* = (-1.717142, 1.159571, 1.827248, -.7636474, -.7636390)$

$f(x^*) = .0539498478$

$r(x^*) = .21E-9$

$e(x^*) = .11E-5$

$\mu = 0$

$I(x^*) = -$

$u^*_{max}/u^*_{min} = .04016/.005223 = 7.69$

$\lambda^*_{max}/\lambda^*_{min} = .16/.16 = 1.02$

PROBLEM:	83 (Colville No.3)
CLASSIFICATION:	QQR-P1-4
SOURCE:	Colville [20], Dembo [22], Himmelblau [29]
NUMBER OF VARIABLES:	$n = 5$
NUMBER OF CONSTRAINTS:	$m_1 = 6$, $m-m_1 = 0$, $b = 10$

OBJECTIVE FUNCTION:

$$f(x) = 5.3578547x_3^2 + .8356891x_1x_5 + 37.293239x_1 - 40792.141$$

CONSTRAINTS:

$$92 \ge a_1 + a_2x_2x_5 + a_3x_1x_4 - a_4x_3x_5 \ge 0$$

$$20 \ge a_5 + a_6x_2x_5 + a_7x_1x_2 + a_8x_3^2 - 90 \ge 0$$

$$5 \ge a_9 + a_{10}x_3x_5 + a_{11}x_1x_3 + a_{12}x_3x_4 - 20 \ge 0$$

$$78 \le x_1 \le 102$$

$$33 \le x_2 \le 45$$

$$27 \le x_i \le 45 \quad , \quad i=3,4,5$$

a_i : cf. Appendix A

START: $x_o = (78 , 33 , 27 , 27 , 27)$ (not feasible)

$f(x_o) = -32217$

SOLUTION:

$x^* = (78,33,29.99526,45,36.77581)$

$f(x^*) = -30665.53867$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 5$

$I(x^*) = (3 , 4 , 7 , 8 , 15)$

$u^*_{max}/u^*_{min} = 809.4/26.64 = 30.4$

$\lambda^*_{max}/\lambda^*_{min} = -$

PROBLEM:	84
CLASSIFICATION:	QQR-P1-5
SOURCE:	Betts [8], Box [11,12], Himmelblau [29]
NUMBER OF VARIABLES:	$n = 5$
NUMBER OF CONSTRAINTS:	$m_1 = 6$, $m-m_1 = 0$, $b = 10$

OBJECTIVE FUNCTION:

$$f(x) = -a_1 - a_2x_1 - a_3x_1x_2 - a_4x_1x_3 - a_5x_1x_4 - a_6x_1x_5$$

CONSTRAINTS:

$$294000 \geq a_7x_1 + a_8x_1x_2 + a_9x_1x_3 + a_{10}x_1x_4 + a_{11}x_1x_5 \geq 0$$

$$294000 \geq a_{12}x_1 + a_{13}x_1x_2 + a_{14}x_1x_3 + a_{15}x_1x_4 + a_{16}x_1x_5 \geq 0$$

$$277200 \geq a_{17}x_1 + a_{18}x_1x_2 + a_{19}x_1x_3 + a_{20}x_1x_4 + a_{21}x_1x_5 \geq 0$$

$$0 \leq x_1 \leq 1000$$

$$1.2 \leq x_2 \leq 2.4$$

$$20 \leq x_3 \leq 60$$

$$9 \leq x_4 \leq 9.3$$

$$6.5 \leq x_5 \leq 7$$

a_i : cf. Appendix A

START: $x_0 = (2.52 , 2 , 37.5 , 9.25 , 6.8)$

$f(x_0) = -2351243.5$ (feasible)

SOLUTION: $x^* = (4.53743097 , 2.4 , 60 , 9.3 , 7)$

$f(x^*) = -5280335.133$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 5$

$I(x^*) = (6 , 13 , 14 , 15 , 16)$

$u^*_{max}/u^*_{min} = .7168E6/.1914E2 = .37E5$

$\lambda^*_{max}/\lambda^*_{min} = -$

PROBLEM:	85
CLASSIFICATION:	GGI-P1-2
SOURCE:	Barness [2], Caroll [17], Himmelblau [29]
NUMBER OF VARIABLES:	$n = 5$
NUMBER OF CONSTRAINTS:	$m_1 = 38(3)$, $m-m_1 = 0$, $b = 10$

OBJECTIVE FUNCTION:

$$f(x) = -5.843E-7y_{17}(x) + 1.17E-4y_{14}(x) + 2.358E-5y_{13}(x)$$
$$+ 1.502E-6y_{16}(x) + .0321y_{12}(x) + .00423y_5(x)$$
$$+ 1.E-4c_{15}(x)/c_{16}(x) + 37.48y_2(x)/c_{12}(x) - .1365$$

CONSTRAINTS:

$$1.5x_2 - x_3 \geq 0 \qquad 704.4148 \leq x_1 \leq 906.3855$$
$$y_1(x) - 213.1 \geq 0 \qquad 68.6 \leq x_2 \leq 288.88$$
$$405.23 - y_1(x) \geq 0 \qquad 0 \leq x_3 \leq 134.75$$
$$y_{j-2}(x) - a_{j-2} \geq 0 \ , \ j=4,\dots,19 \qquad 193 \leq x_4 \leq 287.0966$$
$$b_{j-18} - y_{j-18}(x) \geq 0 \ , \ j=20,\dots,35 \qquad 25 \leq x_5 \leq 84.1988$$
$$y_4(x) - .28/.72y_5(x) \geq 0$$
$$21 - 3496y_2(x)/c_{12}(x) \geq 0$$
$$62212/c_{17}(x) - 110.6 - y_1(x) \geq 0$$

$y_j(x)$, $c_j(x)$, a_j , b_j : cf. Appendix A

START:	x_o	=	(900 , 80 , 115 , 267 , 27)
	$f(x_o)$	=	-.939 (feasible)
SOLUTION:	x^*	=	(705.1803,68.60005,102.90001,282.324999, 37.5850413)
	$f(x^*)$	=	-1.90513375
	$r(x^*)$	=	0
	$e(x^*)$	=	0
	μ	=	0
	$I(x^*)$	=	-
	u^*_{max}/u^*_{min}	=	-
	$\lambda^*_{max}/\lambda^*_{min}$	=	.56E-3/.46E-6 = .12E4

PROBLEM:	86 (Colville No.1)
CLASSIFICATION:	PLR-P1-1
SOURCE:	Colville [20], Himmelblau [29], Murthagh, Sargent [47]
NUMBER OF VARIABLES:	$n = 5$
NUMBER OF CONSTRAINTS:	$m_1 = 10(10)$, $m-m_1 = 0$, $b = 5$

OBJECTIVE FUNCTION:

$$f(x) = \sum_{j=1}^{5} e_j x_j + \sum_{i=1}^{5} \sum_{j=1}^{5} c_{ij} x_i x_j + \sum_{j=1}^{5} d_j x_j^3$$

CONSTRAINTS:

$$\sum_{j=1}^{5} a_{ij} x_j - b_i \geq 0 \quad , \quad i=1,\ldots,10$$

$$0 \leq x_i \quad , \quad i=1,\ldots,5$$

$a_{ij}, b_i, c_{ij}, d_j, e_j$: cf. Appendix A

START:	x_o	=	(0 , 0 , 0 , 0 , 1) (feasible)
	$f(x_o)$	=	20
SOLUTION:	x^*	=	(.3,.33346761,.4,.42831010,.22396487)
	$f(x^*)$	=	-32.34867897
	$r(x^*)$	=	.70E-9
	$e(x^*)$	=	.94E-8
	μ	=	4
	$I(x^*)$	=	(3 , 5 , 6 , 9)
	u^*_{max}/u^*_{min}	=	11.84/.1039 = 113.9
	$\lambda^*_{max}/\lambda^*_{min}$	=	68.1/68.1 = 1

PROBLEM:	87 (Colville No.6)
CLASSIFICATION:	GGI-P1-3
SOURCE:	Colville [20], Himmelblau [29]
NUMBER OF VARIABLES:	n = 6
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 4$, $b = 12$

OBJECTIVE FUNCTION:

$$f(x) = f_1(x) + f_2(x)$$

$$f_1(x) = \begin{cases} 30x_1 , & 0 \le x_1 < 300 \\ 31x_1 , & 300 \le x_1 \le 400 \end{cases} \qquad f_2(x) = \begin{cases} 28x_2 , & 0 \le x_2 < 100 \\ 29x_2 , & 100 \le x_2 < 200 \\ 30x_2 , & 200 \le x_2 \le 1000 \end{cases}$$

CONSTRAINTS:

$$300 - x_1 - \frac{1}{a} x_3 x_4 \cos(b - x_6) + \frac{c}{a} d x_3^2 = 0 \qquad (a = 131.078)$$

$$- x_2 - \frac{1}{a} x_3 x_4 \cos(b + x_6) + \frac{c}{a} d x_4^2 = 0 \qquad (b = 1.48577)$$

$$- x_5 - \frac{1}{a} x_3 x_4 \sin(b + x_6) + \frac{c}{a} e x_4^2 = 0 \qquad (c = .90798)$$

$$200 - \frac{1}{a} x_3 x_4 \sin(b - x_6) + \frac{c}{a} e x_3^2 = 0 \qquad (d = \cos 1.47588) \quad (e = \sin 1.47588)$$

$$0 \le x_1 \le 400 \qquad 340 \le x_3 \le 420 \qquad -1000 \le x_5 \le 10000$$

$$0 \le x_2 \le 1000 \qquad 340 \le x_4 \le 420 \qquad 0 \le x_6 \le .5236$$

START: x_o = (390,1000,419.5,340.5,198.175,.5)

$f(x_o)$ = 42090 (not feasible)

SOLUTION: x^* = (107.8119,196.3186,373.8307,420, 213.0713,.1532920)

$f(x^*)$ = 8927.5977

$r(x^*)$ = .10E-6

$e(x^*)$ = .74E-6

μ = 1

$I(x^*)$ = (10)

u^*_{max}/u^*_{min} = 30/.23E-6 = .13E9

$\lambda^*_{max}/\lambda^*_{min}$ = .017/.017 = 1

PROBLEM:	88 - 92 (time-optimal heat conduction)
CLASSIFICATION:	QGR-P1-(1,...,5)
SOURCE:	Schittkowski [54]
NUMBER OF VARIABLES:	$n = 2,\ldots,6$
NUMBER OF CONSTRAINTS:	$m_1 = 1$, $m-m_1 = 0$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = \sum_{i=1}^{n} x_i^2$$

CONSTRAINTS:

$$\epsilon^2 - h(x) \geq 0$$

$$\epsilon = .01$$

$$h(x) = \int_0^1 \left(\sum_{i=1}^{30} \alpha_j(s)\rho_j(x) - k_o(s)\right)^2 ds$$

$$\alpha_j(s) = \mu_j^2 A_j \cos(\mu_j s)$$

$$\rho_j(x) = -\mu_j^{-2}\left(\exp\left(-\mu_j^2 \sum_{i=1}^{n} x_i^2\right) - 2\exp\left(-\mu_j^2 \sum_{i=2}^{n} x_i^2\right) + \ldots\right.$$

$$\left. + (-1)^{n-1} 2\exp(-\mu_j^2 x_n^2 + (-1)^n\right)$$

$$k_o(s) = .5(1 - s^2)$$

$$A_j = 2\sin\mu_j/(\mu_j + \sin\mu_j\cos\mu_j)\ , \quad \mu_j : \mu \tan\mu = 1$$

START:	x_o	=	$(.5\ ,\ -.5\ ,\ \ldots\ ,\ (-1)^{n+1}.5)$
	$f(x_o)$	=	.25n (not feasible)
SOLUTION:	x^*	=	(cf. Appendix A)
	$f(x^*)$	=	1.36265681
	$r(x^*)$	$\leq$	.30E-10 (cf. Appendix A)
	$e(x^*)$	$\leq$	.16E-2 (cf. Appendix A)
	μ	=	1
	$I(x^*)$	=	(1)
	u^*_{max}/u^*_{min}	=	1059.8/1059.8 = 1
	$\lambda^*_{max}/\lambda^*_{min}$	$\leq$	.11E9 (cf. Appendix A)

PROBLEM:	93 (transformer design)
CLASSIFICATION:	PPR-P1-6
SOURCE:	Bartholomew-Biggs [4]
NUMBER OF VARIABLES:	$n = 6$
NUMBER OF CONSTRAINTS:	$m_1 = 2$, $m-m_1 = 0$, $b = 6$

OBJECTIVE FUNCTION:

$$f(x) = .0204x_1x_4(x_1 + x_2 + x_3) + .0187x_2x_3(x_1 + 1.57x_2 + x_4)$$
$$+ .0607x_1x_4x_5^2(x_1 + x_2 + x_3)$$
$$+ .0437x_2x_3x_6^2(x_1 + 1.57x_2 + x_4)$$

CONSTRAINTS:

$$.001x_1x_2x_3x_4x_5x_6 - 2.07 \geq 0$$

$$1 - .00062x_1x_4x_5^2(x_1 + x_2 + x_3)$$
$$- .00058x_2x_3x_6^2(x_1 + 1.57x_2 + x_4) \geq 0$$

$$0 \leq x_i \quad , \quad i=1,\ldots,6$$

START: x_0 = (5.54,4.4,12.02,11.82,.702,.852)

$f(x_0)$ = 137.066 (feasible)

SOLUTION: x^* = (5.332666,4.656744,10.43299,12.08230,
.7526074,.87865084)

$f(x^*)$ = 135.075961

$r(x^*)$ = .77E-7

$e(x^*)$ = .77E-6

μ = 2

$I(x^*)$ = (1 , 2)

u^*_{max}/u^*_{min} = 71.46/62.15 = 1.15

$\lambda^*_{max}/\lambda^*_{min}$ = 118.9/.21 = 562.9

PROBLEM:	95 - 98
CLASSIFICATION:	LQR-P1-(1,...,4)
SOURCE:	Himmelblau [29], Holzman [32]
NUMBER OF VARIABLES:	$n = 6$
NUMBER OF CONSTRAINTS:	$m_1 = 4$, $m-m_1 = 0$, $b = 12$

OBJECTIVE FUNCTION:

$$f(x) = 4.3x_1 + 31.8x_2 + 63.3x_3 + 15.8x_4 + 68.5x_5 + 4.7x_6$$

CONSTRAINTS:

$$17.1x_1 + 38.2x_2 + 204.2x_3 + 212.3x_4 + 623.4x_5 + 1495.5x_6 - 169x_1x_3 - 3580x_3x_5 - 3810x_4x_5 - 18500x_4x_6 - 24300x_5x_6 \geq b_1$$

$$17.9x_1 + 36.8x_2 + 113.9x_3 + 169.7x_4 + 337.8x_5 + 1385.2x_6 - 139x_1x_3 - 2450x_4x_5 - 16600x_4x_6 - 17200x_5x_6 \geq b_2$$

$$-273x_2 - 70x_4 - 819x_5 + 26000x_4x_5 \geq b_3$$

$$159.9x_1 - 311x_2 + 587x_4 + 391x_5 + 2198x_6 - 14000x_1x_6 \geq b_4$$

$$0 \leq x_1 \leq .31 \;,\; 0 \leq x_3 \leq .068 \;,\; 0 \leq x_5 \leq .028$$

$$0 \leq x_2 \leq .046 \;,\; 0 \leq x_4 \leq .042 \;,\; 0 \leq x_6 \leq .0134$$

4 different data vectors b : cf. Appendix A

START: $x_0 = (0 , 0 , 0 , 0 , 0 , 0)$ (not feasible)

$f(x_0) = 0$

SOLUTION: $x^* = (0,0,0,0,0,.0033233033)$ (95,96)

$= (.2685649,0,0,0,.028,.0134)$ (97,98)

$f(x^*) = .015619514$ (95,96) 3.1358091 (97,98)

$r(x^*) = .21E-9$ (95,96) 0

$e(x^*) = 0$

$\mu = 6$

$I(x^*) = (1,5,6,7,8,9)$ (95,96) (1,6,7,8,15,16)

$u^*_{max}/u^*_{min} = 66.8/.003=.2E5$ (95,96) 200/.251=.8E3

$\lambda^*_{max}/\lambda^*_{min} = -$

PROBLEM:	99
CLASSIFICATION:	GGR-P1-3
SOURCE:	Betts [8]
NUMBER OF VARIABLES:	$n = 7$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 2$, $b = 14$

OBJECTIVE FUNCTION:

$$f(x) = -r_8(x)^2$$

$$r_1(x) = 0 \ , \quad r_i(x) = a_i(t_i - t_{i-1})\cos x_{i-1} + r_{i-1}(x), \ i=2,..,8$$

CONSTRAINTS:

$$q_8(x) - 1.E5 = 0$$

$$s_8(x) - 1.E3 = 0$$

$$0 \le x_i \le 1.58 \ , \ i=1,\dots,7$$

$$q_1(x) = s_1(x) = 0$$

$$q_i(x) = .5(t_i - t_{i-1})^2(a_i \sin x_{i-1} - b) + (t_i - t_{i-1})s_{i-1}(x) + q_{i-1}(x)$$

$$s_i(x) = (t_i - t_{i-1})(a_i \sin x_{i-1} - b) + s_{i-1}(x) \ , \quad i=2,\dots,8$$

a_i, t_i, b : cf. Appendix A

START:

$x_0 = (.5 , .5 , .5 , .5 , .5 , .5 , .5)$

$f(x_0) = -.7763605E9$ (not feasible)

SOLUTION:

$x^* = (.5424603, .5290159, .5084506, .4802693, .4512352, .4091878, .3527847)$

$f(x^*) = -.831079892E9$

$r(x^*) = .30E-7$

$e(x^*) = .31E4$, $\|\nabla f(x^*)\| = .32E9$

$\mu = 0$

$I(x^*) = -$

$u^*_{max}/u^*_{min} = .1934E5/.4194E2 = .46E3$

$\lambda^*_{max}/\lambda^*_{min} = .50E9/.84E8 = 5.95$

PROBLEM:	100
CLASSIFICATION:	PPR-P1-7
SOURCE:	Asaadi [1], Charalambous [18], Wong [59]
NUMBER OF VARIABLES:	$n = 7$
NUMBER OF CONSTRAINTS:	$m_1 = 4$, $m-m_1 = 0$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = (x_1 - 10)^2 + 5(x_2 - 12)^2 + x_3^4 + 3(x_4 - 11)^2 + 10x_5^6 + 7x_6^2 + x_7^4 - 4x_6x_7 - 10x_6 - 8x_7$$

CONSTRAINTS:

$$127 - 2x_1^2 - 3x_2^4 - x_3 - 4x_4^2 - 5x_5 \geq 0$$

$$282 - 7x_1 - 3x_2 - 10x_3^2 - x_4 + x_5 \geq 0$$

$$196 - 23x_1 - x_2^2 - 6x_6^2 + 8x_7 \geq 0$$

$$-4x_1^2 - x_2^2 + 3x_1x_2 - 2x_3^2 - 5x_6 + 11x_7 \geq 0$$

START:	x_o	=	(1 , 2 , 0 , 4 , 0 , 1 , 1)
	$f(x_o)$	=	714 (feasible)
SOLUTION:	x^*	=	(2.330499,1.951372,-.4775414,4.365726, -.6244870,1.038131,1.594227)
	$f(x^*)$	=	680.6300573
	$r(x^*)$	=	.90E-7
	$e(x^*)$	=	.36E-8
	μ	=	2
	$I(x^*)$	=	(1 , 4)
	u^*_{max}/u^*_{min}	=	1.140/.3686 = 3.09
	$\lambda^*_{max}/\lambda^*_{min}$	=	46.6/4.34 = 10.7

PROBLEM:	101 - 103
CLASSIFICATION:	PPR-P1-(8,9,10)
SOURCE:	Beck, Ecker [5], Dembo [22]
NUMBER OF VARIABLES:	$n = 7$
NUMBER OF CONSTRAINTS:	$m_1 = 6$, $m-m_1 = 0$, $b = 14$

OBJECTIVE FUNCTION: 101 : a=-.25 , 102 : a=.125 , 103 : a=.5

$$f(x) = 10x_1x_2^{-1}x_4^2x_6^{-3}x_7^a + 15x_1^{-1}x_2^{-2}x_3x_4x_5^{-1}x_7^{-.5} + 20x_1^{-2}x_2x_4^{-1}x_5^{-2}x_6 + 25x_1^2x_2^2x_3^{-1}x_5^{.5}x_6^{-2}x_7$$

CONSTRAINTS:

$$1 - .5x_1^{.5}x_3^{-1}x_6^{-2}x_7 - .7x_1^3x_2x_3^{-2}x_6x_7^{.5} - .2x_2^{-1}x_3x_4^{-.5}x_6^{2/3}x_7^{1/4} \geq 0$$

$$1 - 1.3x_1^{-.5}x_2x_3^{-1}x_5^{-1}x_6 - .8x_3x_4^{-1}x_5^{-1}x_6^2 - 3.1x_1^{-1}x_2^{.5}x_4^{-2}x_5^{-1}x_6^{1/3} \geq 0$$

$$1 - 2x_1x_3^{-1.5}x_5x_6^{-1}x_7^{1/3} - .1x_2x_3^{-.5}x_5x_6^{-1}x_7^{-.5} - x_1^{-1}x_2x_3^{.5}x_5 - .65x_2^{-2}x_3x_5x_6^{-1}x_7 \geq 0$$

$$1 - .2x_1^{-2}x_2x_4^{-1}x_5^{.5}x_7^{1/3} - .3x_1^{.5}x_2^2x_3x_4^{1/3}x_7^{1/4}x_5^{-2/3} - .4x_1^{-3}x_2^{-2}x_3x_5x_7^{3/4} - .5x_3^{-2}x_4x_7^{.5} \geq 0$$

$$100 \leq f(x) \leq 3000 \ , \ .1 \leq x_i \leq 10 \ , \ i=1,\ldots,6, \ .01 \leq x_7 \leq 10$$

START: x_0 = (6 , 6 , 6 , 6 , 6 , 6 , 6) (not feas.)

$f(x_0)$ = 2205.868 , 2206.889 , 2208.886

SOLUTION: x^* = (cf. Appendix A)

		101	102	103
$f(x^*)$	=	1809.76476	911.880571	543.667958
$r(x^*)$	=	.10E-10	.27E-10	.61E-11
$e(x^*)$	=	.59E-6	.14E-6	.92E-7
μ	=	3	3	4
$I(x^*)$	=	(2,3,13)	(1,2,3)	(1,2,3,4)
u^*_{max}/u^*_{min}	=	4567/.81E-6	2173/21.13	1287/38.71
$\lambda^*_{max}/\lambda^*_{min}$	=	.43E4/.51E2=85	.23E4/.32E2=73	.46E3/.88E2=5.2

PROBLEM:	104 (optimal reactor design)
CLASSIFICATION:	PPR-P1-11
SOURCE:	Dembo [22], Rijckaert [53]
NUMBER OF VARIABLES:	$n = 8$
NUMBER OF CONSTRAINTS:	$m_1 = 6$, $m-m_1 = 0$, $b = 16$

OBJECTIVE FUNCTION:

$$f(x) = .4x_1^{.67}x_7^{-.67} + .4x_2^{.67}x_8^{-.67} + 10 - x_1 - x_2$$

CONSTRAINTS:

$$1 - .0588x_5x_7 - .1x_1 \geq 0$$

$$1 - .0588x_6x_8 - .1x_1 - .1x_2 \geq 0$$

$$1 - 4x_3x_5^{-1} - 2x_3^{-.71}x_5^{-1} - .0588x_3^{-1.3}x_7 \geq 0$$

$$1 - 4x_4x_6^{-1} - 2x_4^{-.71}x_6^{-1} - .0588x_4^{-1.3}x_8 \geq 0$$

$$1 \leq f(x) \leq 4.2$$

$$.1 \leq x_i \leq 10 \quad , \quad i=1,\ldots,8$$

START: $x_o = (6,3,.4,.2,6,6,1,.5)$ (not feasible)

$f(x_o) = 3.65$

SOLUTION: $x^* = (6.465114, 2.232709, .6673975, .5957564, 5.932676, 5.527235, 1.013322, .4006682)$

$f(x^*) = 3.9511634396$

$r(x^*) = .58E-10$

$e(x^*) = .31E-10$

$\mu = 4$

$I(x^*) = (1, 2, 3, 4)$

$u^*_{max}/u^*_{min} = 6.206/.8472 = 7.32$

$\lambda^*_{max}/\lambda^*_{min} = 1.87/.043 = 43.2$

PROBLEM:	105 (maximum-likelihood estimation)
CLASSIFICATION:	GLR-P1-2
SOURCE:	Bracken, McCormick [13]
NUMBER OF VARIABLES:	n = 8
NUMBER OF CONSTRAINTS:	$m_1 = 1(1)$, $m-m_1 = 0$, $b = 16$

OBJECTIVE FUNCTION:

$$f(x) = -\sum_{i=1}^{235} \ln((a_i(x) + b_i(x) + c_i(x))/\sqrt{2\pi})$$

$$a_i(x) = x_1/x_6 \exp(-(y_i - x_3)^2/(2x_6^2))$$

$$b_i(x) = x_2/x_7 \exp(-(y_i - x_4)^2/(2x_7^2)) \quad , \quad i=1,\ldots,235$$

$$c_i(x) = (1 - x_2 - x_1)/x_8 \exp(-(y_i - x_5)^2/(2x_8^2))$$

y_i : cf. Appendix A

CONSTRAINTS:

$$1 - x_1 - x_2 \geq 0$$

$.001 \leq x_i \leq .499$, $i=1,2$ $\qquad 100 \leq x_3 \leq 180$

$130 \leq x_4 \leq 210$ $\qquad 170 \leq x_5 \leq 240$ $\qquad 5 \leq x_i \leq 25$, $i=6,7,8$

START: $x_o = (.1,.2,100,125,175,11.2,13.2,15.8)$

$f(x_o) = 1297.6693$ (feasible)

SOLUTION: $x^* = (.4128928,.4033526,131.2613,164.3135,$
$217.4222,12.28018,15.77170,20.74682)$

$f(x^*) = 1138.416240$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 0$

$I(x^*) = -$

$u^*_{max}/u^*_{min} = -$

$\lambda^*_{max}/\lambda^*_{min} = .26E4/.28E-2 = .92E6$

PROBLEM:	106 (heat exchanger design)
CLASSIFICATION:	LQR-P1-5
SOURCE:	Avriel, Williams [2], Dembo [22]
NUMBER OF VARIABLES:	$n = 8$
NUMBER OF CONSTRAINTS:	$m_1 = 6(3)$, $m-m_1 = 0$, $b = 16$

OBJECTIVE FUNCTION:

$$f(x) = x_1 + x_2 + x_3$$

CONSTRAINTS:

$$1 - .0025(x_4 + x_6) \geq 0$$

$$1 - .0025(x_5 + x_7 - x_4) \geq 0$$

$$1 - .01(x_8 - x_5) \geq 0$$

$$x_1x_6 - 833.33252x_4 - 100x_1 + 83333.333 \geq 0$$

$$x_2x_7 - 1250x_5 - x_2x_4 + 1250x_4 \geq 0$$

$$x_3x_8 - 1250000 - x_3x_5 + 2500x_5 \geq 0$$

$$100 \leq x_1 \leq 10000$$

$$1000 \leq x_i \leq 10000 \quad , \quad i=2,3$$

$$10 \leq x_i \leq 1000 \quad , \quad i=4,\ldots,8$$

START:

x_o = (5000,5000,5000,200,350,150,225,425)

$f(x_o)$ = 15000 (not feasible)

SOLUTION:

x^* = (579.3167,1359.943,5110.071,182,0174, 295.5985,217.9799,286.4162,395,5979)

$f(x^*)$ = 7049.330923

$r(x^*)$ = 0

$e(x^*)$ = .19E-4

μ = 6

$I(x^*)$ = (1 , 2 , 3 , 4 , 5 , 6)

u^*_{max}/u^*_{min} = 5210/.00848 = .61E6

$\lambda^*_{max}/\lambda^*_{min}$ = .81E-3/.38E-3 = 2.11

PROBLEM:	107 (static power scheduling)
CLASSIFICATION:	PGR-P1-4
SOURCE:	Bartholomew-Biggs [4]
NUMBER OF VARIABLES:	$n = 9$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 6$, $b = 8$

OBJECTIVE FUNCTION:

$$f(x) = 3000x_1 + 1000x_1^3 + 2000x_2 + 666.667x_2^3$$

CONSTRAINTS:

$$.4 - x_1 + 2cx_5^2 - x_5x_6(dy_1 + cy_2) - x_5x_7(dy_3 + cy_4) = 0$$
$$.4 - x_2 + 2cx_6^2 + x_5x_6(dy_1 - cy_2) + x_6x_7(dy_5 - cy_6) = 0$$
$$.8 + 2cx_7^2 + x_5x_7(dy_3 - cy_4) - x_6x_7(dy_5 + cy_6) = 0$$
$$.2 - x_3 + 2dx_5^2 + x_5x_6(cy_1 - dy_2) + x_5x_7(cy_3 - dy_4) = 0$$
$$.2 - x_4 + 2dx_6^2 - x_5x_6(cy_1 + dy_2) - x_6x_7(cy_5 + dy_6) = 0$$
$$-.337 + 2dx_7^2 - x_5x_7(cy_3 + dy_4) + x_6x_7(cy_5 - dy_6) = 0$$
$$0 \le x_i \ , \ i=1,2 \quad , \quad .90909 \le x_i \le 1.0909 \ , \ i=5,6,7$$

$$y_1 = \sin x_8 \ , \quad y_2 = \cos x_8 \ , \quad y_3 = \sin x_9$$
$$y_4 = \cos x_9 \ , \quad y_5 = \sin(x_8-x_9) \ , \quad y_6 = \cos(x_8-x_9)$$
$$c = (48.4/50.176)\sin .25 \ , \quad d = (48.4/50.176)\cos .25$$

START:	x_o	=	(.8,.8,.2,.2,1.0454,1.0454,0,0)
	$f(x_o)$	=	4853.3335 (not feasible)
SOLUTION:	x^*	=	(.6670095,1.022388,.2282879,.1848217, 1.090900,1.090900,1.069036,.1066126, -.3387867)
	$f(x^*)$	=	5055.011803
	$r(x^*)$	=	.18E-9
	$e(x^*)$	=	0
	μ	=	3
	$I(x^*)$	=	(6 , 7 , 8)
	u^*_{max}/u^*_{min}	=	5208/0
	$\lambda^*_{max}/\lambda^*_{min}$	=	-

PROBLEM:	108
CLASSIFICATION:	QQR-P1-6
SOURCE:	Himmelblau [29], Pearson [49]
NUMBER OF VARIABLES:	$n = 9$
NUMBER OF CONSTRAINTS:	$m_1 = 13$, $m-m_1 = 0$, $b = 1$

OBJECTIVE FUNCTION:

$$f(x) = -.5(x_1x_4 - x_2x_3 + x_3x_9 - x_5x_9 + x_5x_8 - x_6x_7)$$

CONSTRAINTS:

$$1 - x_3^2 - x_4^2 \geq 0 \qquad 1 - x_9^2 \geq 0$$

$$1 - x_5^2 - x_6^2 \geq 0 \qquad 1 - x_1^2 - (x_2 - x_9)^2 \geq 0$$

$$1 - (x_1 - x_5)^2 - (x_2 - x_6)^2 \geq 0$$

$$1 - (x_1 - x_7)^2 - (x_2 - x_8)^2 \geq 0$$

$$1 - (x_3 - x_5)^2 - (x_4 - x_6)^2 \geq 0$$

$$1 - (x_3 - x_7)^2 - (x_4 - x_8)^2 \geq 0$$

$$1 - x_7^2 - (x_8 - x_9)^2 \geq 0 \qquad x_1x_4 - x_2x_3 \geq 0$$

$$x_3x_9 \geq 0 \qquad -x_5x_9 \geq 0$$

$$x_5x_8 - x_6x_7 \geq 0 \qquad 0 \leq x_9$$

START: $x_0 = (1,1,1,1,1,1,1,1,1)$ (not feasible)

$f(x_0) = 0$

SOLUTION: $x^* = (.8841292, .4672425, .03742076, .9992996,$
$.8841292, .4672424, .03742076, .9992996,$
$.26E-19)$

$f(x^*) = -.8660254038$

$r(x^*) = .39E-9$ $\qquad e(x^*) = .33E-11$

$u = 9$ $\qquad I(x^*) = (1,3,4,6,7,9,11,12,14)$

$u^*_{max}/u^*_{min} = .1443/0$

$\lambda^*_{max}/\lambda^*_{min} = -$

PROBLEM:	109
CLASSIFICATION:	PGR-P1-5
SOURCE:	Beuneu [9]
NUMBER OF VARIABLES:	$n = 9$
NUMBER OF CONSTRAINTS:	$m_1 = 4(2)$, $m-m_1 = 6$, $b = 16$

OBJECTIVE FUNCTION:

$$f(x) = 3x_1 + 1.E\text{-}6x_1^3 + 2x_2 + .522074E\text{-}6x_2^3$$

CONSTRAINTS:

$$x_4 - x_3 + .55 \geq 0 \qquad x_3 - x_4 + .55 \geq 0$$

$$2250000 - x_1^2 - x_8^2 \geq 0 \qquad 2250000 - x_2^2 - x_9^2 \geq 0$$

$$x_5x_6\sin(-x_3 - \tfrac{1}{4}) + x_5x_7\sin(-x_4 - \tfrac{1}{4}) + 2bx_5^2 - ax_1 + 400a = 0$$

$$x_5x_6\sin(x_3 - \tfrac{1}{4}) + x_6x_7\sin(x_3 - x_4 - \tfrac{1}{4}) + 2bx_6^2 - ax_2 + 400a = 0$$

$$x_5x_7\sin(x_4 - \tfrac{1}{4}) + x_6x_7\sin(x_4 - x_3 - \tfrac{1}{4}) + 2bx_7^2 + 881.779a = 0$$

$$ax_8 + x_5x_6\cos(-x_3 - \tfrac{1}{4}) + x_5x_7\cos(-x_4 - \tfrac{1}{4}) - 200a - 2cx_5^2 + .7533E\text{-}3ax_5^2 = 0$$

$$ax_9 + x_5x_6\cos(x_3 - \tfrac{1}{4}) + x_6x_7\cos(x_3 - x_4 - \tfrac{1}{4}) - 2cx_6^2 + .7533E\text{-}3ax_6^2 - 200a = 0$$

$$x_5x_7\cos(x_4 - \tfrac{1}{4}) + x_6x_7\cos(x_4 - x_3 - \tfrac{1}{4}) - 2cx_7^2 + 22.938a + .7533E\text{-}3ax_7^2 = 0$$

$$0 \leq x_i \;,\; i=1,2 \qquad -.55 \leq x_i \leq .55 \;,\; i=3,4$$

$$196 \leq x_i \leq 252 \;,\; i=5,6,7 \qquad -400 \leq x_i \leq 800 \;,\; i=8,9$$

$$a = 50.176 \;,\quad b = \sin .25 \;,\quad c = \cos .25$$

START: $x_0 = (0,\ldots,0)$ $\qquad f(x_0) = 0$ (not feasible)

SOLUTION: $\qquad f(x^*) = 5362.06928$

$x^* =$ (cf. Appendix A)

$r(x^*) = .36E\text{-}7 \qquad e(x^*) = 0$

$\mu = 3 \qquad I(x^*) = (1 , 16 , 17)$

$u^*_{max}/u^*_{min} = 12.53/.13E\text{-}10 = .95E12$

$\lambda^*_{max}/\lambda^*_{min} = -$

PROBLEM:	110
CLASSIFICATION:	GBR-P1-1
SOURCE:	Himmelblau [29], Paviani [48]
NUMBER OF VARIABLES:	$n = 10$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 0$, $b = 20$

OBJECTIVE FUNCTION:

$$f(x) = \sum_{i=1}^{10} [(\ln(x_i - 2))^2 + (\ln(10 - x_i))^2 - (\prod_{i=1}^{10} x_i)^{.2}$$

CONSTRAINTS:

$$2.001 \leq x_i \leq 9.999 \quad , \quad i=1,\ldots,10$$

START:	x_o	=	$(9,\ldots,9)$ (feasible)
	$f(x_o)$	=	-43.134337
SOLUTION:	x^*	=	$(9.35025655,\ldots,9.35025655)$
	$f(x^*)$	=	-45.77846971
	$r(x^*)$	=	0
	$e(x^*)$	=	0
	μ	=	0
	$I(x^*)$	=	-
	u^*_{max}/u^*_{min}	=	-
	$\lambda^*_{max}/\lambda^*_{min}$	=	$6.92/6.52 = 1.06$

PROBLEM:	111
CLASSIFICATION:	GGR-P1-4
SOURCE:	Bracken, McCormick [13], Himmelblau [29], White [58]
NUMBER OF VARIABLES:	$n = 10$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 3$, $b = 20$

OBJECTIVE FUNCTION:

$$f(x) = \sum_{j=1}^{10} \exp(x_j)(c_j + x_j - \ln(\sum_{k=1}^{10} \exp(x_k)))$$

c_j : cf. Appendix A

CONSTRAINTS:

$$\exp(x_1) + 2\exp(x_2) + 2\exp(x_3) + \exp(x_6) + \exp(x_{10}) - 2 = 0$$

$$\exp(x_4) + 2\exp(x_5) + \exp(x_6) + \exp(x_7) - 1 = 0$$

$$\exp(x_3) + \exp(x_7) + \exp(x_8) + 2\exp(x_9) + \exp(x_{10}) - 1 = 0$$

$$-100 \le x_i \le 100 \quad , \quad i=1,\dots,10$$

START: $x_0 = (-2.3 , \dots ,-2.3)$ (not feasible)

$f(x_0) = -21.015$

SOLUTION: $x^* = (-3.201212,-1.912060,-.2444413,-6.537489,$
$-.7231524,-7.267738,-3.596711,-4.017769,$
$-3.287462,-2.335582)$

$f(x^*) = -47.76109026$

$r(x^*) = .34E-9$ $\qquad e(x^*) = .14E-3$

$u = 0$ $\qquad I(x^*) = -$

$u^*_{max}/u^*_{min} = 15.22/9.785 = 1.56$

$\lambda^*_{max}/\lambda^*_{min} = .11/.70E-3 = 160.1$

PROBLEM:	112 (chemical equilibrium)
CLASSIFICATION:	GLR-P1-3
SOURCE:	Bracken, McCormick [13], Himmelblau [29], White [58]
NUMBER OF VARIABLES:	$n = 10$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 3(3)$, $b = 10$

OBJECTIVE FUNCTION:

$$f(x) = \sum_{j=1}^{10} x_j \left(c_j + \ln \frac{x_j}{x_1 + \ldots + x_{10}}\right)$$

c_j : cf. Appendix A

CONSTRAINTS:

$$x_1 + 2x_2 + 2x_3 + x_6 + x_{10} - 2 = 0$$

$$x_4 + 2x_5 + x_6 + x_7 - 1 = 0$$

$$x_3 + x_7 + x_8 + 2x_9 + x_{10} = 0$$

$$1.E-6 \leq x_i \quad , \quad i=1,\ldots,10$$

START: $x_o = (.1 , \ldots , .1)$ (not feasible)

$f(x_o) = -20.961$

SOLUTION: $x^* = (.01773548,.08200180,.8825646,.7233256E-3,$
$.4907851,.4335469E-3,.01727298,$
$.007765639,.01984929,.05269826)$

$f(x^*) = -47.707579$

$r(x^*) = .23E-7$ $\quad e(x^*) = .43E-6$

$u = 2$ $\quad I(x^*) = (4 , 6)$

$u^*_{max}/u^*_{min} = 15.02/.262E-3 = .57E5$

$\lambda^*_{max}/\lambda^*_{min} = 191/8.98 = 21.3$

PROBLEM:	113 (Wong No.2)
CLASSIFICATION:	QQR-P1-7
SOURCE:	Asaadi [1], Charalambous [18], Wong [59]
NUMBER OF VARIABLES:	$n = 10$
NUMBER OF CONSTRAINTS:	$m_1 = 8(3)$, $m-m_1 = 0$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = x_1^2 + x_2^2 + x_1x_2 - 14x_1 - 16x_2 + (x_3 - 10)^2 + 4(x_4 - 5)^2 + (x_5 - 3)^2 + 2(x_6 - 1)^2 + 5x_7^2 + 7(x_8 - 11)^2 + 2(x_9 - 10)^2 + (x_{10} - 7)^2 + 45$$

CONSTRAINTS:

$$105 - 4x_1 - 5x_2 + 3x_7 - 9x_8 \geq 0$$
$$-10x_1 + 8x_2 + 17x_7 - 2x_8 \geq 0$$
$$8x_1 - 2x_2 - 5x_9 + 2x_{10} + 12 \geq 0$$
$$-3(x_1 - 2)^2 - 4(x_2 - 3)^2 - 2x_3^2 + 7x_4 + 120 \geq 0$$
$$-5x_1^2 - 8x_2 - (x_3 - 6)^2 + 2x_4 + 40 \geq 0$$
$$-.5(x_1 - 8)^2 - 2(x_2 - 4)^2 - 3x_5^2 + x_6 + 30 \geq 0$$
$$-x_1^2 - 2(x_2 - 2)^2 + 2x_1x_2 - 14x_5 + 6x_6 \geq 0$$
$$3x_1 - 6x_2 - 12(x_9 - 8)^2 + 7x_{10} \geq 0$$

START: $x_o = (2,3,5,5,1,2,7,3,6,10)$ (feasible)

$f(x_o) = 753$

SOLUTION: $x^* = (2.171996, 2.363683, 8.773926, 5.095984, .9906548, 1.430574, 1.321644, 9.828726, 8.280092, 8.375927)$

$f(x^*) = 24.3062091$

$r(x^*) = .12E-8$ $\quad e(x^*) = .46E-9$

$u = 6$ $\quad I(x^*) = (1,2,3,4,5,7)$

$u^*_{max}/u^*_{min} = 1.717/.02055 = 83.5$

$\lambda^*_{max}/\lambda^*_{min} = 7.79/2.24 = 3.48$

PROBLEM:	114 (alkylation process)
CLASSIFICATION:	QGR-P1-6
SOURCE:	Bracken, McCormick [13]
NUMBER OF VARIABLES:	$n = 10$
NUMBER OF CONSTRAINTS:	$m_1 = 8(4)$, $m-m_1 = 3(1)$, $b = 20$

OBJECTIVE FUNCTION:

$$f(x) = 5.04x_1 + .035x_2 + 10x_3 + 3.36x_5 - .063x_4x_7$$

CONSTRAINTS:

$$g_1(x) = 35.82 - .222x_{10} - bx_9 \geq 0$$
$$g_2(x) = -133 + 3x_7 - ax_{10} \geq 0$$
$$g_3(x) = -g_1(x) + x_9(1/b - b) \geq 0$$
$$g_4(x) = -g_2(x) + (1/a - a)x_{10} \geq 0$$
$$g_5(x) = 1.12x_1 + .13167x_1x_8 - .00667x_1x_8^2 - ax_4 \geq 0$$
$$g_6(x) = 57.425 + 1.098x_8 - .038x_8^2 + .325x_6 - ax_7 \geq 0$$
$$g_7(x) = -g_5(x) + (1/a - a)x_4 \geq 0$$
$$g_8(x) = -g_6(x) + (1/a - a)x_7 \geq 0$$
$$g_9(x) = 1.22x_4 - x_1 - x_5 = 0$$
$$g_{10}(x) = 98000x_3/(x_4x_9 + 1000x_3) - x_6 = 0 \qquad a = .99$$
$$g_{11}(x) = (x_2 + x_5)/x_1 - x_8 = 0 \qquad b = .9$$

bounds: cf. Appendix A

START: $x_o = (1745, 12000, 110, 3048, 1974, 89.2, 92.8, 8, 3.6, 145)$ (not feasible)

$f(x_o) = -872.3872$

SOLUTION: $x^* = (1698.096, 15818.73, 54.10228, 3031.226, 2000, 90.11537, 95, 10.49336, 1.561636, 153.53535)$

$f(x^*) = -1768.80696$

$r(x^*) = 0$ $\qquad e(x^*) = .16E-5$

$u = 6$ $\qquad I(x^*) = (2,3,5,6,23,25)$

$u^*_{max}/u^*_{min} = 311.8/.6778 = 460$

$\lambda^*_{max}/\lambda^*_{min} = .14E-4/.14E-4 = 1$

PROBLEM:	116 (3-stage membrane separation)
CLASSIFICATION:	LQR-P1-6
SOURCE:	Dembo [21,22]
NUMBER OF VARIABLES:	$n = 13$
NUMBER OF CONSTRAINTS:	$m_1 = 15(5)$, $m-m_1 = 0$, $b = 26$

OBJECTIVE FUNCTION:

$$f(x) = x_{11} + x_{12} + x_{13}$$

CONSTRAINTS:

$$x_3 - x_2 \geq 0 \qquad x_2 - x_1 \geq 0$$

$$1 - .002x_7 + .002x_8 \geq 0 \qquad 50 \leq f(x) \leq 250$$

$$x_{13} - 1.262626x_{10} + 1.231059x_3x_{10} \geq 0$$

$$x_5 - .03475x_2 - .975x_2x_5 + .00975x_2^2 \geq 0$$

$$x_6 - .03475x_3 - .975x_3x_6 + .00975x_3^2 \geq 0$$

$$x_5x_7 - x_1x_8 - x_4x_7 + x_4x_8 \geq 0$$

$$1 - .002(x_2x_9 + x_5x_8 - x_1x_8 - x_6x_9) - x_5 - x_6 \geq 0$$

$$x_2x_9 - x_3x_{10} - x_6x_9 - 500x_2 + 500x_6 + x_2x_{10} \geq 0$$

$$x_2 - .9 - .002(x_2x_{10} - x_3x_{10}) \geq 0$$

$$x_4 - .03475x_1 - .975x_1x_4 + .00975x_1^2 \geq 0$$

$$x_{11} - 1.262626x_8 + 1.231059x_1x_8 \geq 0$$

$$x_{12} - 1.262626x_9 + 1.231059x_2x_9 \geq 0$$

bounds: cf. Appendix A

START: $x_0 = (.5,.8,.9,.1,.14,.5,489,80,650,450,150,150,150)$ (not feasible)

$f(x_0) = 450$

SOLUTION: $x^* = (.8037703,.8999860,.9709724,.09999952,.1908154,.4605717,574.0803,74.08043,500.0162,.1,20.23413,77.34755,.00673039)$

$f(x^*) = 97.588409$

$r(x^*) = 0 \qquad e(x^*) = 0$

$u = 14 \qquad I(x^*) = (3,6,\ldots,15,25,28,32)$

$u^*_{max}/u^*_{min} = 2088/.423E-3 = .49E7$

$\lambda^*_{max}/\lambda^*_{min} = -$

PROBLEM:	117 (Colville No.2, Shell Dual)
CLASSIFICATION:	PQR-P1-1
SOURCE:	Colville [20], Himmelblau [29]
NUMBER OF VARIABLES:	$n = 15$
NUMBER OF CONSTRAINTS:	$m_1 = 5$, $m-m_1 = 0$, $b = 15$

OBJECTIVE FUNCTION:

$$f(x) = -\sum_{j=1}^{10} b_j x_j + \sum_{j=1}^{5} \sum_{k=1}^{5} c_{kj} x_{10+k} x_{10+j} + 2 \sum_{j=1}^{5} d_j x_{10+j}^3$$

CONSTRAINTS:

$$2 \sum_{k=1}^{5} c_{kj} x_{10+k} + 3 d_j x_{10+j}^2 + e_j - \sum_{k=1}^{10} a_{kj} x_k \geq 0 \quad , \quad j=1,\ldots,5$$

$$0 \leq x_i \quad , \quad i=1,\ldots,15$$

$a_{ij}, b_j, c_{ij}, d_j, e_j$: cf. Appendix A

START: $x_o = .001(1,1,1,1,1,1,60000,1,1,1,1,1,1,1,1)$

$f(x_o) = 2400.1053$ (feasible)

SOLUTION: $x^* = (0,0,5.174136,0,3.061093,11.83968,0,0,$
$.1039071,0,.2999929,.3334709,.3999910,$
$.4283145,.2239607)$

$f(x^*) = 32.348679$

$r(x^*) = 0$ $\quad$ $e(x^*) = .35E-4$

$u = 11$ $\quad$ $I(x^*) = (1,\ldots,7,9,12,13,15)$

$u^*_{max}/u^*_{min} = 56.75/.2240 = 253$

$\lambda^*_{max}/\lambda^*_{min} = 3.30/.10 = 32.3$

PROBLEM:	118
CLASSIFICATION:	QLR-P1-2
SOURCE:	Bartholomew-Biggs [4]
NUMBER OF VARIABLES:	$n = 15$
NUMBER OF CONSTRAINTS:	$m_1 = 29(29)$, $m-m_1 = 0$, $b = 30$

OBJECTIVE FUNCTION:

$$f(x) = \sum_{k=0}^{4} (2.3x_{3k+1} + .0001x_{3k+1}^2 + 1.7x_{3k+2} + .0001x_{3k+2}^2 + 2.2x_{3k+3} + .00015x_{3k+3}^2)$$

CONSTRAINTS:

$$0 \le x_{3j+1} - x_{3j-2} + 7 \le 13 \qquad 0 \le x_{3j+2} - x_{3j-1} + 7 \le 14$$

$$0 \le x_{3j+3} - x_{3j} + 7 \le 13 \qquad j=1,\ldots,4$$

$$x_1 + x_2 + x_3 - 60 \ge 0 \qquad x_4 + x_5 + x_6 - 50 \ge 0$$

$$x_7 + x_8 + x_9 - 70 \ge 0 \qquad x_{10} + x_{11} + x_{12} - 85 \ge 0$$

$$x_{13} + x_{14} + x_{15} - 100 \ge 0$$

$$8 \le x_1 \le 21 \qquad 43 \le x_2 \le 57 \qquad 3 \le x_3 \le 16$$

$$0 \le x_{3k+1} \le 90 \qquad 0 \le x_{3k+2} \le 120 \qquad 0 \le x_{3k+3} \le 60$$

$k=1,\ldots,4$

START: x_o = (20,55,15,20,60,20,20,60,20,20,60,20,20,60,20) (feasible)

$f(x_o)$ = 664.82045000

SOLUTION: x^* = (8,49,3,1,56,0,1,63,6,3,70,12,5,77,18)

$f(x^*)$ = 664.8204500

$r(x^*)$ = 0

$e(x^*)$ = 0

μ = 15

$I(x^*)$ = (1,17,18,19,20,22,23,24,25,27,28,29,30,32,35)

u^*_{max}/u^*_{min} = 2.941/.04860 = 60.5

$\lambda^*_{max}/\lambda^*_{min}$ = -

PROBLEM:	119 (Colville No.7)
CLASSIFICATION:	PLR-P1-2
SOURCE:	Colville [20], Himmelblau [29]
NUMBER OF VARIABLES:	$n = 16$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 8(8)$, $b = 32$

OBJECTIVE FUNCTION:

$$f(x) = \sum_{i=1}^{16} \sum_{j=1}^{16} a_{ij}(x_i^2 + x_i + 1)(x_j^2 + x_j + 1)$$

CONSTRAINTS:

$$\sum_{j=1}^{16} b_{ij}x_j - c_i = 0 \quad , \quad i=1,\ldots,8$$

$$0 \le x_i \le 5 \quad , \quad i=1,\ldots,16$$

a_{ij}, b_{ij}, c_i : cf. Appendix A

START: $x_o = (10,\ldots,10)$ (not feasible)

$f(x_o) = 566766$

SOLUTION: $x^* = (.03984735,.7919832,.2028703,.8443579,$
$1.126991,.9347387,1.681962,.1553009,$
$1.567870,0,0,0,.6602041,0,.6742559,0)$

$f(x^*) = 244.899698$

$r(x^*) = .26E-9$ $e(x^*) = .36E-8$

$u = 5$ $I(x^*) = (10,11,12,14,16)$

$u^*_{max}/u^*_{min} = 95.99/4.201 = 22.9$

$\lambda^*_{max}/\lambda^*_{min} = 39.2/25.1 = 1.56$