

lab3 (1)

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```
[ ]: import numpy as np
import numpy.linalg as la
import random
import matplotlib.pyplot as plt
import math
from sklearn.utils.extmath import randomized_svd
import scipy.sparse as sp
import time
from time import perf_counter
```

0.0.1 Rekurencyjny algorytm dekompresji

```
[ ]: class MatrixTree:
    def __init__(self, matrix, x1, x2, y1, y2):
        self.matrix = matrix
        self.x1 = x1
        self.x2 = x2
        self.y1 = y1
        self.y2 = y2
        self.leaf = False
        self.children = None

    def compress(self, r, eps):
        U, Sigma, V = randomized_svd(self.matrix[self.x1:self.x2, self.y1: self.
→y2], n_components=r+1, random_state=0)
        if self.x1 + r == self.x2 or Sigma[r] <= eps:
            self.leaf = True
            if not self.matrix[self.x1:self.x2, self.y1: self.y2].any():
                self.rank = 0
            else:
                self.rank = len(Sigma)
                self.u = U
                self.s = Sigma
                self.v = V
        else:
            self.children = []
```

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        self.children.append(MatrixTree(self.matrix, self.x1, (self.x1 +
↪self.x2)//2, self.y1, (self.y1 + self.y2)//2))
        self.children.append(MatrixTree(self.matrix, self.x1, (self.x1 +
↪self.x2)//2, (self.y1 + self.y2)//2, self.y2))
        self.children.append(MatrixTree(self.matrix, (self.x1 + self.x2)//
↪2, self.x2, self.y1, (self.y1 + self.y2)//2))
        self.children.append(MatrixTree(self.matrix, (self.x1 + self.x2)//
↪2, self.x2, (self.y1 + self.y2)//2, self.y2))

    for child in self.children:
        child.compress(r, eps)

    def decompress(self, output_matrix):
        if self.leaf:
            if self.rank != 0:
                sigma = np.zeros((self.rank, self.rank))
                np.fill_diagonal(sigma, self.s)
                output_matrix[self.x1:self.x2, self.y1: self.y2] = self.u @
↪sigma @ self.v #zeros
            else:
                output_matrix[self.x1:self.x2, self.y1: self.y2] = self.
↪matrix[self.x1:self.x2, self.y1: self.y2]
        else:
            for child in self.children:
                child.decompress(output_matrix)

```

0.0.2 Funkcja rysująca obraz

```

[ ]: import numpy as np
import matplotlib.pyplot as plt
from collections import deque

def create_tree_image(root, title=''):
    image = np.ones(root.matrix.shape)*255
    Q = deque()
    Q.append(root)
    while Q:
        v = Q.pop()
        if v.leaf:
            image[v.x1:v.x2, v.y1:v.y1+v.rank] = np.zeros((v.x2 - v.x1, v.rank))
            image[v.x1:v.x1+v.rank, v.y1:v.y2] = np.zeros((v.rank , v.y2 - v.
↪y1))

            image[v.x1, v.y1:v.y2] = np.zeros((1,v.y2 - v.y1))
            image[v.x2-1, v.y1:v.y2] = np.zeros((1,v.y2 - v.y1))
            image[v.x1:v.x2,v.y1] = np.zeros(v.x2-v.x1)
            image[v.x1:v.x2,v.y2-1] = np.zeros(v.x2-v.x1)

```

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    else:
        for child in v.children:
            Q.append(child)
    return image

```

0.0.3 Generowanie macierzy dla wybranych wartości procentowych

```

[ ]: k = 2**8

test_matrixes = [sp.random(k,k,0.99).toarray(),
                  sp.random(k,k,0.98).toarray(),
                  sp.random(k,k,0.95).toarray(),
                  sp.random(k,k,0.90).toarray(),
                  sp.random(k,k,0.80).toarray()
                  ]

Vs = []
times = []
for M in test_matrixes:
    S,V,D = randomized_svd(M,n_components=k)
    Vs.append(V)

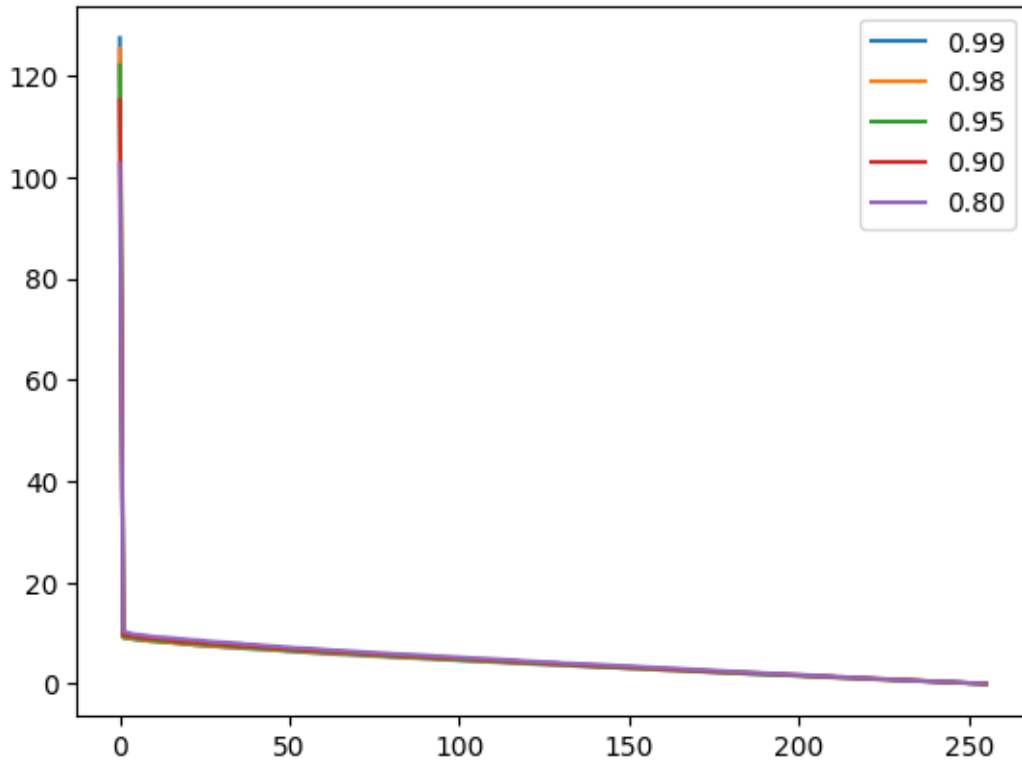
fig, ax = plt.subplots()
ax.plot(range(0,len(Vs[0])),Vs[0],label='0.99')
ax.plot(range(0,len(Vs[1])),Vs[1],label='0.98')
ax.plot(range(0,len(Vs[2])),Vs[2],label='0.95')
ax.plot(range(0,len(Vs[3])),Vs[3],label='0.90')
ax.plot(range(0,len(Vs[4])),Vs[4],label='0.80')
ax.legend()

```

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[ ]: <matplotlib.legend.Legend at 0x1541e9c90>

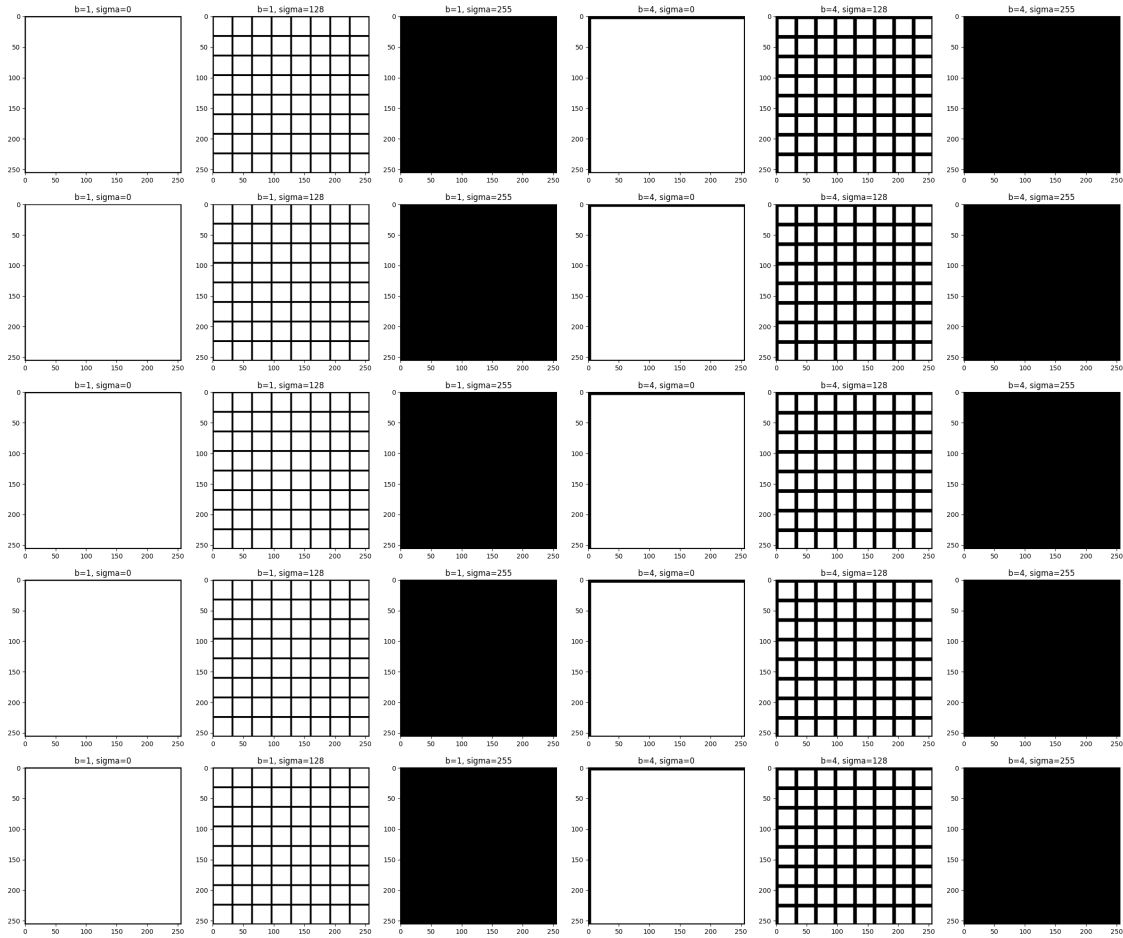
```



0.0.4 Uruchomienie SVD na wybranych macierzach i znalezienie wartości osobliwych

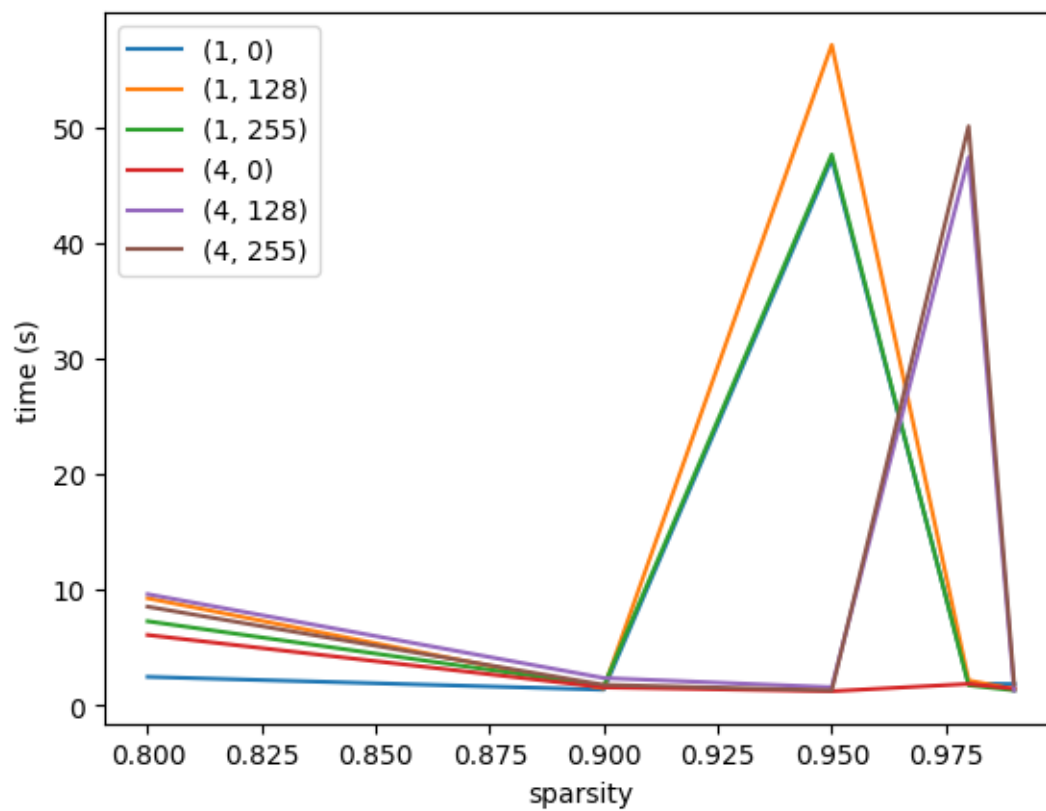
```
[ ]: params = [(1,0),(1,k//2),(1,k-1),(4,0),(4,k//2),(4,k-1)]
sparse = [0.99,0.98,0.95,0.90,0.80]
maps = [[],[],[],[],[]]
times = []
fig, ax = plt.subplots(ncols=6,nrows=5,figsize=(5*6,5*5))
for index,(b,sigma) in enumerate(params):
    for i in range(0,5):
        root = MatrixTree(test_matrixes[i],0,k,0,k)
        start = perf_counter()
        root.compress(b,Vs[i][sigma])
        end = perf_counter()
        maps[i].append(create_tree_image(root))
        times.append(end-start)
        output_matrix = np.zeros((k,k))
        root.decompress(output_matrix)
        error = np.mean(np.square(test_matrixes[i] - output_matrix))
        ax[i,index].imshow(maps[i][index],cmap='gist_gray',vmin=0,vmax=255)
        ax[i,index].set_title(f'b={b}, sigma={sigma}')
        print(f'zeros={sparse[i]}, b={b}, sigma={sigma}, error={error}␣
↳time={end-start}')
```

zeros=0.99, b=1, sigma=0, error=0.08291666804082243 time=1.827944708056748
 zeros=0.98, b=1, sigma=0, error=0.08491797755138637 time=1.38472874998115
 zeros=0.95, b=1, sigma=0, error=0.08904590580552046 time=1.2770081660710275
 zeros=0.9, b=1, sigma=0, error=0.09492717169423259 time=1.4284109170548618
 zeros=0.8, b=1, sigma=0, error=0.10410129579661186 time=1.2648183340206742
 zeros=0.99, b=1, sigma=128, error=0.07040624447159685 time=1.7650969170499593
 zeros=0.98, b=1, sigma=128, error=0.07220010264235983 time=1.8606303751002997
 zeros=0.95, b=1, sigma=128, error=0.07583348705173493 time=2.1357842499855906
 zeros=0.9, b=1, sigma=128, error=0.08062075967964666 time=1.7187488749623299
 zeros=0.8, b=1, sigma=128, error=0.08848280261803702 time=1.8389497089665383
 zeros=0.99, b=1, sigma=255, error=3.738328600091342e-33 time=47.39213562500663
 zeros=0.98, b=1, sigma=255, error=8.61914773060191e-34 time=50.06916133291088
 zeros=0.95, b=1, sigma=255, error=5.834189490652764e-34 time=47.24768545804545
 zeros=0.9, b=1, sigma=255, error=1.8324831885826338e-33 time=57.114699291065335
 zeros=0.8, b=1, sigma=255, error=2.5865363080713707e-33 time=47.63098229188472
 zeros=0.99, b=4, sigma=0, error=0.07933069707905359 time=1.2129682500381023
 zeros=0.98, b=4, sigma=0, error=0.08128439397715985 time=1.5533672920428216
 zeros=0.95, b=4, sigma=0, error=0.08522399617650404 time=1.2757588331587613
 zeros=0.9, b=4, sigma=0, error=0.09081687338761857 time=1.3398307089228183
 zeros=0.8, b=4, sigma=0, error=0.09959000127599227 time=1.450222000014037
 zeros=0.99, b=4, sigma=128, error=0.049033750355197435 time=1.6598619578871876
 zeros=0.98, b=4, sigma=128, error=0.05054699677713905 time=1.541353665990755
 zeros=0.95, b=4, sigma=128, error=0.05297101269993509 time=2.3510497079696506
 zeros=0.9, b=4, sigma=128, error=0.05618789549172549 time=1.7646959170233458
 zeros=0.8, b=4, sigma=128, error=0.061782486558749614 time=2.448532875161618
 zeros=0.99, b=4, sigma=255, error=2.2976037706209566e-31 time=9.248366416199133
 zeros=0.98, b=4, sigma=255, error=2.431024308882479e-31 time=7.256600082851946
 zeros=0.95, b=4, sigma=255, error=2.358175389451866e-31 time=6.068365084007382
 zeros=0.9, b=4, sigma=255, error=2.164465976955343e-31 time=9.604382666992024
 zeros=0.8, b=4, sigma=255, error=2.2189255026712053e-31 time=8.519865750102326



```
[ ]: import pandas as pd
df = pd.DataFrame(np.array(times).round(2).reshape(5,6),columns=[(1,0),(1,k//
↪2),(1,k-1),(4,0),(4,k//2),(4,k-1)],index=[0.99,0.98,0.95,0.90,0.80])
plt.plot(df)
plt.legend(df.columns)
plt.xlabel('sparsity')
plt.ylabel('time (s)')
df
```

```
[ ]:      (1, 0)  (1, 128)  (1, 255)  (4, 0)  (4, 128)  (4, 255)
0.99      1.83      1.38      1.28      1.43      1.26      1.77
0.98      1.86      2.14      1.72      1.84      47.39     50.07
0.95     47.25     57.11     47.63      1.21      1.55      1.28
0.90      1.34      1.45      1.66      1.54      2.35      1.76
0.80      2.45      9.25      7.26      6.07      9.60      8.52
```



0.0.5 Wnioski:

- Zwiększenie b - mniejszy błąd, dłuższy czas
- Zmniejszenie σ - mniejszy błąd, dłuższy czas