MEM 361 Engineering Reliability
Final Exam

DREXEL UNIVERSITY
College of
Engineering
Mechanical Engineering and Mechanics

3/9/2022

<u>Group 26:</u>

Rhys Kawaguchi
John Scully
Brendan Houle
Michael Martinak
Xabi Carneiro

Table of Contents

List of Figures

Problem 1

As part of the quality control, 25 components were subjected to test until failure. The table below displays the data provided to our group for the original time of failure (ToF).

Time of Failure (seconds)				
9.49	11.87	13.17	14.219	15.683
10.193	12.159	13.383	14.446	16.13
10.731	12.435	13.585	14.703	16.642
11.181	12.695	13.782	14.973	17.286
11.527	12.939	13.996	15.281	18.179

Figure 1: Table of original failure times in seconds

The first step taken to analyze the failure time data was sorting the data in ascending order by time of failure. Next, an i value was assigned to each iteration, starting at 1 and ending at N, where for this data N is 25. After creating i values for each data point, the conditional reliability was able to be calculated using the equation provided below:

$$R(t_i|t_{i-1}) = \frac{N+1-i}{N+2-i}$$

After calculating conditional reliability for each data point, the expected reliability was calculated for the data set using the equation provided below:

$$R(t_i) = \frac{N+1-i}{N+2-i}R(t_{i-1})$$

Finally, utilizing the expected reliability values found using the previous equation, the probability of failure for the data points was calculated. The equation below provides the means for calculating failure probability:

$$F(t_i) = 1 - R(t_i)$$

All of the conditional reliability, expected reliability, and failure probability values calculated from the original data set are displayed in Figure 13 in the Appendix.

1a. Determine the two parameters of the pdf (Weibull)

In order to calculate the best fit line for the Weibull distribution, the original data must first be redefined into two variations, x and y, the equations for calculating each are displayed below:

$$x = ln(t)$$

$$y = ln(ln(\frac{1}{1-F(t)}))$$

Using the calculated x and y values for the data set, the parameters necessary for calculating the defining values of the Weibull distribution were able to be found using the data from Figure 14 in the appendix. The necessary parameters are tabulated below:

Parameter	Value
a	6.738056
b	-18.04501
r^2	0.985779
m	6.738056
theta	14.55701

Figure 2: Weibull distribution parameters

After finding all necessary parameters for the data set, the two parameters that define the Weibull distribution which are represented by m and Θ, were calculated using the equations below:

$$m = a$$

$$\theta = exp(\frac{-b}{a})$$

$$m = 6.738$$
$$\theta = 14.557\ seconds$$

Additionally, using parameters a and b in combination with the average of the x values, located in Figure 13 in the Appendix, the data points for a linear trendline were calculated using equation:

$$y = ax + b$$

Combining a plot of the calculated x and y values with the trendline values shows the relationship between the original data set and the best fit line, with the r^2 value representing the accuracy of the trendline. The closer that r^2 gets to 1 the more accurate the fit is, so given our r^2 value was calculated to be 0.985 the best fit line is a reasonably accurate prediction. A plot providing a visual comparison of the original data and trendline for the Weibull distribution is displayed below:

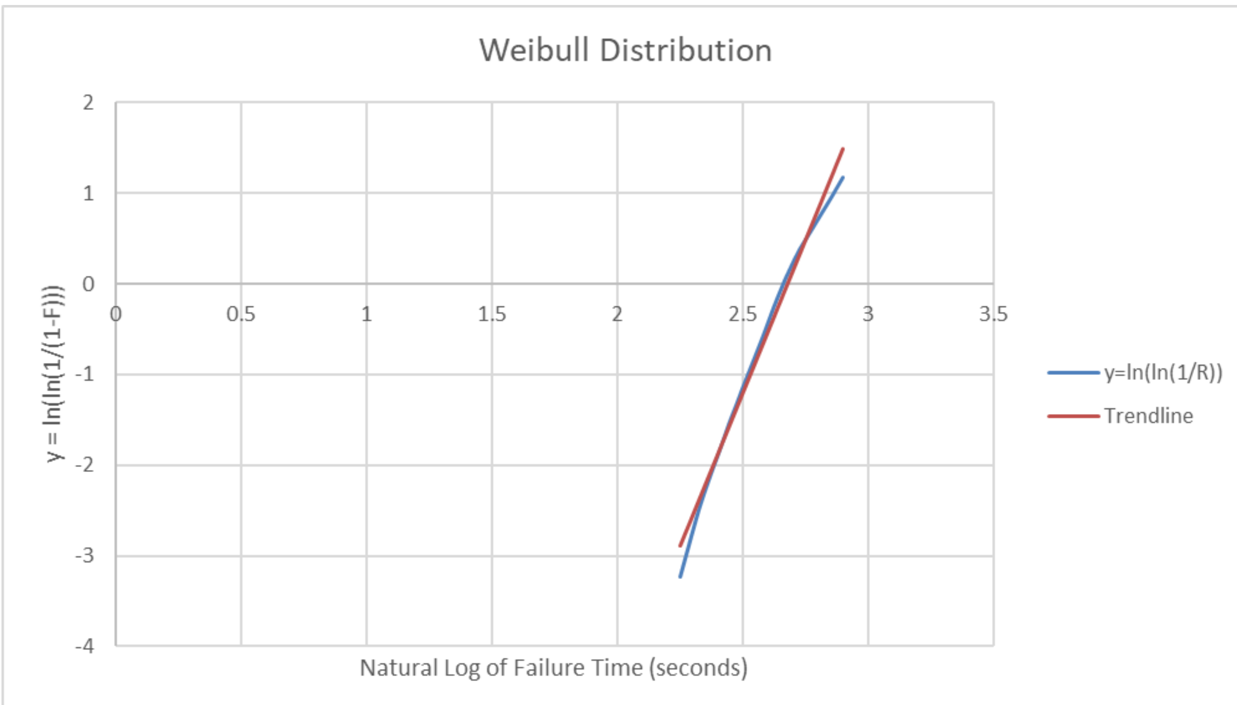

Figure 3: Weibull distribution plot of original data with trendline

1b. Determine its mean and the standard deviation

For a Weibull distribution, the mean and standard deviation can be calculated using the equations below:

$$\mu_W = \theta\mathbf{\Gamma}(1 + \frac{1}{m})$$

$$\sigma_W = \sqrt{\theta^2[\mathbf{\Gamma}(1 + \frac{2}{m}) - \mathbf{\Gamma}(1 + \frac{1}{m})^2]}$$

Using the two equations found above, the values for mean and standard deviation were found to be:

$$\mu_W = 13.59\ seconds$$
$$\sigma_W = 2.365\ seconds$$

1c. If the target life and the Lower Specification Limit (LSL) are 15 seconds and 13 seconds respectively, determine % yield

In order to find the percent yield for the data set, it is first necessary to find the cdf for the Weibull distribution. This is necessary because the Weibull cdf computes the probability of a component failing between time 0 and end time, t. The component reliability at time, t, is represented by the equation:

$$F(t) = 1 - exp[-(\frac{t}{\theta})^m]$$

Percent yield can be defined as the percentage of components that surpass the lower specification limit. Using the Weibull cdf with lower specification limit as time, t, the percent yield was calculated to be:

$$\%\ yield = 0.6271\ or\ 62.71\%$$

Problem 2

The four independent, identical components of a circuit-board in their youth phase have a mean time to failure (MTTF) of 13.59 seconds. This MTTF value is equal to the mean derived using the weibull method which was solved for in problem 1b and is represented by μ_W. The reliability diagram below represents the overall life of the circuit-board.

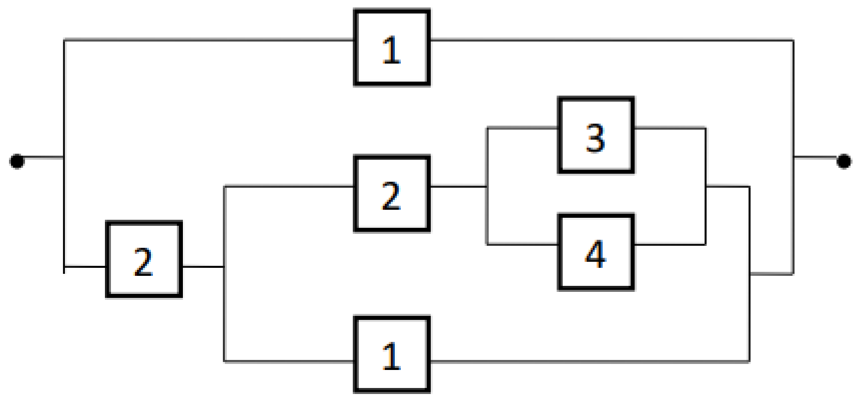

Figure 4: Reliability diagram for circuit-board

2a. Write down its structure function ϕ(x) explicitly in terms of the states x_i

The structure function of a system is used to represent whether or not the system is functioning by using a 1 to represent functioning and a 0 to represent not functioning. Structure functions can be derived from a reliability diagram through reduction processes. The structure function to be created for figure 4 will be formatted as $\phi_i(x_1, x_2, \ldots)$, for which i represents the order that the diagram is reduced. In the case of figure 4, it is most efficient to begin reducing from the most deeply nested portion which is represented by components 3 and 4 in parallel. This is translated into structure function form as $\phi_1(x_3, x_4)$. For this reduction the equation below represents the correlating structure function and figure 5 shows the reduced reliability diagram.

$$\phi_1(x_3, x_4) = 1 - (1 - x_3)(1 - x_4) = x_3 + x_4 - x_3x_4$$

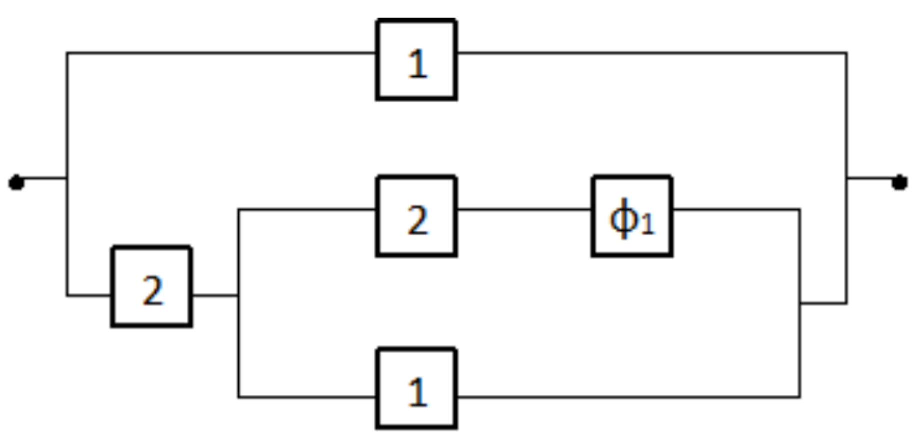

Figure 5: Reliability diagram for circuit-board after first reduction

The next reduction will be the portion containing component 2 and structure function part ϕ_1 in series. Reduction of this section results in the equation below, and the corresponding diagram is displayed in figure 6.

$$\phi_2(x_2, x_3, x_4) = x_2\phi_1 = x_2x_3 + x_2x_4 - x_2x_3x_4$$

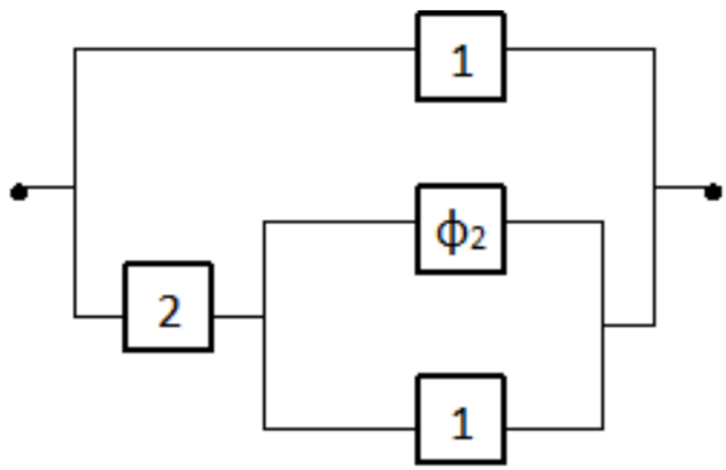

Figure 6: Reliability diagram of circuit-board after second reduction

The next reduction will combine the partial structure function, ϕ_2, with component 1 in parallel. Reduction of this section produces the equation below, and the corresponding reliability diagram is displayed in figure 7.

$$\phi_3(x_1, x_2, x_3, x_4) = 1 - (1 - \phi_2)(1 - x_1)$$

or

$$\phi_3(x_1, x_2, x_3, x_4) = x_1 + x_2x_4 - x_1x_2x_4 + x_2x_3 - x_1x_2x_3 - x_2x_3x_4 + x_1x_2x_3x_4$$

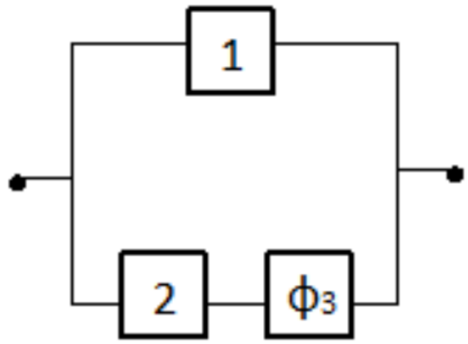

Figure 7: Reliability diagram of circuit-board after third reduction

The next reduction combines the partial structure function ϕ_3 with component 2 in series. Reduction of this section results in the equation below, and the corresponding reliability diagram is displayed in figure 8.

$$\phi_4(x_1, x_2, x_3, x_4) = x_2\phi_3 = x_1x_2 + x_2x_4 - x_1x_2x_4 + x_2x_3 - x_1x_2x_3 - x_2x_3x_4 + x_1x_2x_3x_4$$

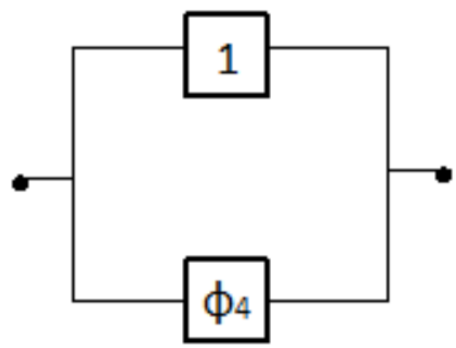

Figure 8: Reliability diagram of circuit-board after fourth reduction

The final reduction of the system combines the partial structure function ϕ_4 with the last component 1 in parallel. The final reduction produces the final structure function for the circuit-board and the resulting equation is displayed below.

$$\phi(x_1, x_2, x_3, x_4) = 1 - (1 - \phi_4)(1 - x_1)$$

or

$$\phi(x_1, x_2, x_3, x_4) = x_1 + x_2x_3 + x_2x_4 - x_1x_2x_3 - x_1x_2x_4 - x_2x_3x_4 + x_1x_2x_3x_4$$

2b. Determine all of its path sets

A path set is defined as a set of components in a system, which by functioning ensures that the system is functioning. For the reliability diagram in figure 4, the path sets are:

{1}, {1,2}, {2,3}, {2,4}, {1, 2, 3}, {1, 2, 4}, {2, 3, 4}, {1, 2, 3, 4}

2c. Identify the minimal path sets

A minimal path set is defined as a path set that cannot be reduced without losing its status, and therefore still allowing the system to function. For the reliability diagram in figure 4, the minimal path sets are:

{1}, {2, 3}, {2, 4}

2d. Redraw the diagram as a parallel structure of the minimum path series structures, and generate its structure function

By redrawing the diagram as a parallel structure of the minimum path series structures figure 9 is obtained.

Reducing the diagram to three partial structure functions (ϕ_1, ϕ_2, and ϕ_3). The reduced reliability diagram is shown in figure 10. The partial structure functions that were used are shown below:

$$\phi_1 = x_1$$

$$\phi_2 = x_2 x_3$$

$$\phi_3 = x_2 x_4$$

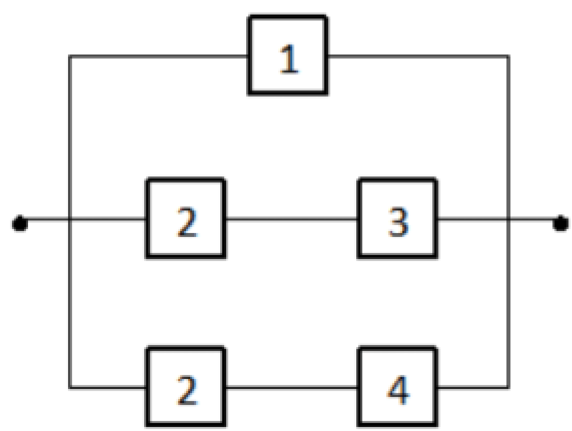

Figure 9: Reliability diagram with minimal path series structures and parallel structure

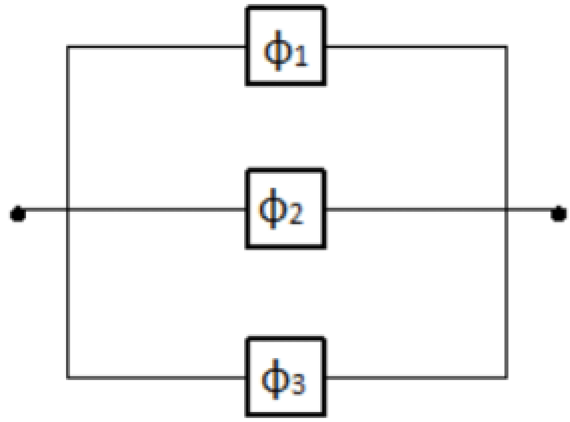

Figure 10: Reliability diagram with minimal path series structures and parallel structure

The diagram can be reduced further. The second equation shown below is a reduction of the first:

$$\phi(x_1, x_2, x_3, x_4) = 1 - (1 - \phi_4)(1 - x_1)$$

$$\phi(x_1, x_2, x_3, x_4) = x_1 + x_2x_3 + x_2x_4 - x_1x_2x_3 - x_1x_2x_4 - x_2x_3x_4 + x_1x_2x_3x_4$$

2e. Determine the reliability function R(t) for each minimal path set

A function of time can be used for the reliability of the component because of the constant failure rate. The components are identical so the same value of λ is used. $\lambda = 1/MTTF = 1/\mu_W$. The equation for R(t) is shown below:

$$R(t) = e^{-\lambda t}$$

The minimal path sets are components in series. The equation below shows that the reliability of components in series is equal to the product of the reliabilities of the individual components. The equation is as follows:

$$R(t) = R_1 R_2 ... R_N$$

Using the equations above and the minimal path sets that were found in part c, the reliabilities of the minimal path sets were found:

$$\text{for } \{1\} \to R_a(t) = R_1(t) = e^{-\lambda t}$$
$$\text{for } \{2,3\} \to R_b(t) = R_2(t)R_3(t) = e^{-2\lambda t}$$
$$\text{for } \{2,4\} \to R_c(t) = R_2(t)R_4(t) = e^{-2\lambda t}$$

2f. Using (e) above, determine the R(t) of the circuit-board, and determine its reliability at 30 seconds

The reliability of the circuit-board is calculated by using the results for e and figure 9. The probability that at least one of the minimal path sets survives needs to be found (R_{tot}(t)=R({1} ∪ {2,3} ∪ {2,4})). This can be found using the equation below:

$$R_{tot}(t) = 1 - [1 - R_a(t)][1 - R_b(t)][1 - R_c(t)]$$

By substituting in knows that were found in part e, the equation below is obtained:

$$R_{tot}(t) = 1 - (1 - e^{-\lambda t})(1 - e^{-2\lambda t})^2$$

The MTTF for this model is:

R_{tot}(30seconds) = 0.131

2g. If the MTTF of each component in the highest reliable path set is increased by 25%, what would be the board's reliability at 30 seconds?

The MTTF of the highest reliable path (a) is increased by 25% the following equation is used:

$$R_{tot}(t) = 1 - (1 - e^{\frac{-\lambda t}{1.25}})(1 - e^{-2\lambda t})^2$$

The reliability of the circuit-board after 30 seconds is:

R_{tot}(30seconds) = 0.191

2h. Determine all of its cut sets

The cut sets of this system are shown below. The cut sets of a system is the components of the set that when they fail the whole system fails.

{1,2}, {1,2,3}, {1,2,4}, {1,3,4}, {1,2,3,4}

2i. Identify the minimal cut sets

The minimal cut sets are those which can not further be reduced so that the system will still fail. Therefore, if a component is removed from a cut set, the system will no longer fail. The minimal cut sets for our system in *Figure 4* are:

$$\{1,2\},\{1,3,4\}$$

2j. Redraw the diagram as a series structure of the minimum cut parallel structures, and generate its structure function

The reliability diagram from Figure 4 can be redrawn as

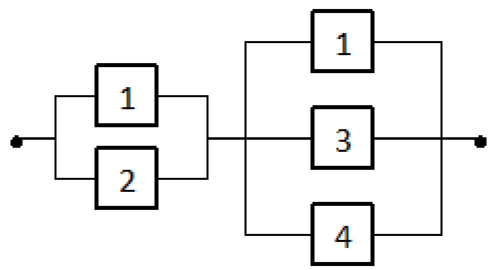

Figure 11: Redrawn reliability diagram using the minimal cut set structures

Using the diagram in Figure 11, we can derive the structure system for the system again. First, reducing the diagram to two structural functions, , φ_1 and φ_2, which are the structure functions for the parallel portions of the diagram in Figure 11.

$$\varphi_1(x_1, x_2) = 1 - (1 - x_1)(1 - x_2) = x_1 + x_2 - x_1x_2$$

$$\varphi_2(x_1, x_3, x_4) = 1 - (1 - x_1)(1 - x_3)(1 - x_4) = x_1 + x_3 + x_4 - x_1x_3 - x_1x_4 - x_3x_4 + x_1x_3x_4$$

Since φ_1 and φ_2 are in series, the final structure is the product of the two functions. The final structure can be seen below.

$$\varphi(x_1, x_2, x_3, x_4) = \varphi_1\varphi_2 = x_1 + x_2x_3 + x_2x_4 - x_1x_2x_3 - x_1x_2x_4 - x_2x_3x_4 + x_1x_2x_3x_4$$

Problem 3

3a. Using the numerical value for μ, determine the MTTF of each unit

From Figure 8, unit 'a' will be defined as the first series of components, which only includes component 1. Unit 'b' will be defined as the second series of components, which includes components 2 and 3. Unit 'c' will be defined as the third series of components, which includes components 2 and 4. In Problem 2 part e, we found the following reliability functions:

$$R_a(t) = e^{-\lambda t}$$
$$R_b(t) = e^{-2\lambda t}$$
$$R_c(t) = e^{-2\lambda t}$$

Equation 6.22 allows us to use these reliability functions directly to calculate the meantime to failure.

$$MTTF = \int_0^\infty R(t)\ dt$$

Using this equation, we found that the MTTF's of each unit are:

$$MTTF_a = 13.59\ seconds$$
$$MTTF_b = 6.795\ seconds$$
$$MTTF_c = 6.795\ seconds$$

3b. Ignore the unit with the least reliability and assume that both of the remaining units can be repaired as and when they occur with a constant repair rate of v = 1.5 / MTTF

3b.i Draw the state transition diagram for this system

We are ignoring unit 'c' so the transition diagram for the system is the transition diagram for a two component load sharing system. The diagram for this state transition can be seen in the figure below, taken from the textbook.

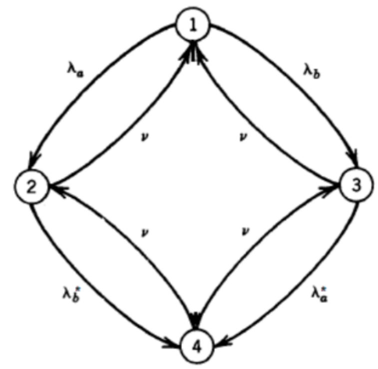

Figure 12: Transition diagram for system consisting of units ‘a’ and ‘b’

3b.ii Determine the transition matrix M in $\dot{\mathbf{P}} = \mathbf{MP}$.

Using the transition diagram we were able to determine the transition matrix of the system. The transition matrix can be seen below. In the matrix (shown below), λ_a and λ_b can be obtained from the solution in part 1. This was found from the relations 1/MTTFa and 1/MTTFb. In this case λ*a and λ*b are the failure rates of unit a and b, respectively, after their component has failed.

$$\mathbf{M} = \begin{bmatrix} -\lambda_a - \lambda_b & \nu & \nu & 0 \\ \lambda_a & -\lambda_b^* - \nu & 0 & \nu \\ \lambda_b & 0 & -\lambda_a^* - \nu & \nu \\ 0 & \lambda_b^* & \lambda_a^* & -2\nu \end{bmatrix} = \begin{bmatrix} -\lambda_a - \lambda_b & \nu & \nu & 0 \\ \lambda_a & -2\lambda_b - \nu & 0 & \nu \\ \lambda_b & 0 & -2\lambda_a - \nu & \nu \\ 0 & 2\lambda_b & 2\lambda_a & -2\nu \end{bmatrix}$$

3b.iii Determine numerically the steady-state probability state vector P(∞)

To numerically determine the steady-state probability vector, the rate of change for the probability of being in any state with respect to time is 0 when time goes to infinity. The equation MP(∞) = 0 can be used to show this. In this specific scenario our equation is to be expanded into four different equations along with four unknowns, as shown below:

$$\sum_i P_i(\infty) = 1$$

$$\dot{P}_1 = (-\lambda_a - \lambda_b)P_1 + \nu P_2 + \nu P_3 = 0$$
$$\dot{P}_2 = \lambda_a P_1 - (2\lambda_b + \nu)P_2 + \nu P_4 = 0$$
$$\dot{P}_3 = \lambda_b P_1 - (2\lambda_a + \nu)P_3 + \nu P_4 = 0$$
$$\dot{P}_4 = 2\lambda_b P_2 + 2\lambda_a P_3 - 2\nu P_4 = 0$$

v = 0.1104

$\lambda_a = 0.0736$
$\lambda_b = 0.1472$
$\lambda_c = 0.1472$

Using these equations with our λa , λb and ν values, the steady-state probability vector can be found as seen below:

$$P(\infty) = \begin{bmatrix} 0.155062 \\ 0.156154 \\ 0.192046 \\ 0.496738 \end{bmatrix}$$

3b.iv Determine the steady-state probability of the system's operation

Our system was found to be operational if either unit a or unit b is operational. The probability of our system to be operational was based on the probability that our system is not in state 4, where both unit a and unit b were both inoperable. Using this information and the steady-state probability state vector, it was found that the steady-state probability of our system's operation is:

$$P_{op}(\infty) = 1 - P_4(\infty) = 1-0.496738 = 0.503262$$

Appendix

i	xi	R(i|i-1)	R(i)	F(i)
0	0		1	0
1	9.489874	0.961538	0.961538	0.038462
2	10.19273	0.96	0.923077	0.076923
3	10.73141	0.958333	0.884615	0.115385
4	11.18072	0.956522	0.846154	0.153846
5	11.52734	0.954545	0.807692	0.192308
6	11.86959	0.952381	0.769231	0.230769
7	12.15858	0.95	0.730769	0.269231
8	12.43538	0.947368	0.692308	0.307692
9	12.69457	0.944444	0.653846	0.346154
10	12.93918	0.941176	0.615385	0.384615
11	13.16967	0.9375	0.576923	0.423077
12	13.38274	0.933333	0.538462	0.461538
13	13.58517	0.928571	0.5	0.5
14	13.78173	0.923077	0.461538	0.538462
15	13.99579	0.916667	0.423077	0.576923
16	14.21877	0.909091	0.384615	0.615385
17	14.44564	0.9	0.346154	0.653846
18	14.7029	0.888889	0.307692	0.692308
19	14.97256	0.875	0.269231	0.730769
20	15.28109	0.857143	0.230769	0.769231
21	15.68331	0.833333	0.192308	0.807692
22	16.1302	0.8	0.153846	0.846154
23	16.64211	0.75	0.115385	0.884615
24	17.28583	0.666667	0.076923	0.923077
25	18.17929	0.5	0.038462	0.961538

Figure 13: Table with original data as well as reliability and failure probabilities

	xi	F(i)	x=ln(ti)	y=ln(ln(1/R	x^2	y^2	xy	Trendline
	9.489874	0.038462	2.250225	-3.23855	5.063514	10.48821	-7.28747	-2.88286
	10.19273	0.076923	2.321674	-2.52519	5.390171	6.376609	-5.86268	-2.40143
	10.73141	0.115385	2.373175	-2.09881	5.631958	4.405001	-4.98084	-2.05442
	11.18072	0.153846	2.414191	-1.78944	5.828317	3.202087	-4.32004	-1.77805
	11.52734	0.192308	2.444721	-1.54377	5.976662	2.38323	-3.77409	-1.57234
	11.86959	0.230769	2.47398	-1.33802	6.120577	1.790301	-3.31024	-1.37519
	12.15858	0.269231	2.498035	-1.15945	6.24018	1.344332	-2.89636	-1.2131
	12.43538	0.307692	2.520545	-1.00042	6.353148	1.000841	-2.52161	-1.06143
	12.69457	0.346154	2.541174	-0.85594	6.457566	0.732635	-2.1751	-0.92243
	12.93918	0.384615	2.56026	-0.72256	6.55493	0.522093	-1.84994	-0.79383
	13.16967	0.423077	2.577916	-0.59775	6.645653	0.357308	-1.54096	-0.67486
	13.38274	0.461538	2.593966	-0.47959	6.72866	0.230003	-1.24403	-0.56672
	13.58517	0.5	2.608979	-0.36651	6.806772	0.134332	-0.95622	-0.46556
	13.78173	0.538462	2.623344	-0.25723	6.881934	0.066168	-0.6748	-0.36877
	13.99579	0.576923	2.638756	-0.15059	6.963034	0.022677	-0.39737	-0.26492
	14.21877	0.615385	2.654563	-0.04551	7.046702	0.002071	-0.12081	-0.15841
	14.44564	0.653846	2.670393	0.059091	7.130998	0.003492	0.157797	-0.05175
	14.7029	0.692308	2.688045	0.164374	7.225583	0.027019	0.441845	0.067189
	14.97256	0.730769	2.706219	0.271695	7.323623	0.073818	0.735266	0.189652
	15.28109	0.769231	2.726616	0.382768	7.434436	0.146511	1.04366	0.327088
	15.68331	0.807692	2.752597	0.499962	7.576792	0.249962	1.376194	0.502149
	16.1302	0.846154	2.780693	0.626902	7.732256	0.393006	1.743221	0.691463
	16.64211	0.884615	2.811936	0.769869	7.906986	0.592699	2.164824	0.901979
	17.28583	0.923077	2.849887	0.941939	8.121857	0.887249	2.684419	1.157695
	18.17929	0.961538	2.900283	1.181143	8.411641	1.395099	3.425649	1.497264
Average	**13.62705**		**2.599287**	**-0.53086**	**6.782158**	**1.47307**	**-1.20559**	

Figure 14: Table with data generated to calculate the best fit parameters for the Weibull distribution