Assignment 1: CS 203 (Fall 2021)

1. (25 marks) There's restaurant named 'Treat'. In this after every meal, the manager picks one from each of the five well shuffled pack of cards having cards labeled A-T. If the picked cards form the restaurant's name (in any order) the meal is on the house.
   (a) What is the probability that a person will enjoy a free meal?
   (b) The manager wants to change the structure of game so that the probability of winning would be less for an expensive meal while it will be more for a cheaper meal. Suggest a way to change the 'winning word' (according to charge of meal), to satisfy manager's needs.

   Solution: □

2. (20 marks) Consider n coding machines M_1, M_2, … M_n producing binary codes 0 and 1. The machine M_1 produces codes 0 and 1 with respective probabilities $\frac{1}{4}$ and $\frac{3}{4}$. The code produced by machine M_k is fed into machine M_{k+1} which may either leave the received code unchanged or may change it. Suppose that each of the subsequent machines change the code with probability $\frac{3}{4}$.
   Given that final produced code is 1. What is the probability that the machine M_1 produced code 0.

   Solution: □

3. (25 marks) Let $\{A_i\}_{i=1}^n$ be a family of sets indexed from 1 to n. Let $I \subseteq [n]$ be a subset of index set. Let B be the event when only and all A_i's from I have happened, i.e.,

$$B = \left(\cap_{i\in I} A_i\right) \cap \left(\cap_{i\notin I} A_i^c\right).$$

   Notice that B is subset of $\cap_{i\in I} A_i$, but need not be equal to it. Show that,

$$P(B) = \sum_{J\supseteq I} (-1)^{|J|-|I|} P(\cap_{i\in J} A_i).$$

   Hint: can you define some new sets in terms of A_i, such that, this problem looks like inclusion-exclusion?

   Solution: □

4. (30 marks) We play a game with a randomly shuffled deck of 52 regular playing cards. Cards are placed face down on table. You have two options, either "take" or "skip" the top card. The skipped card is revealed and game is continued. If only one card is left in deck it is automatically taken. Game stops when you take the top card; you win if the card taken is black, otherwise you lose.
   Prove that no other strategy is better than taking the top card. *Strategy is considered better than the other, if the probability of winning is higher.*
   Hint: Consider the general scenario, suppose there are x black and y red cards, show that whatever you do, the probability of winning is bounded by $\frac{x}{x+y}$.

   Solution: □