Counting Techniques

(1) Product Rule =>

=> n_1 ways to do the first task and n_2 ways to do second task.

$= n_1 \cdot n_2$

eg-1

2					
$n_1 = 1$	$n_2 = 2$	$n_3 = 2$	$n_4 = 2$	$n_5 = 2$	$n_6 = 2$

=) $n_1 \cdot n_2 \cdot n_3 \cdot n_4 \cdot n_5 \cdot n_6$

=) 2^7

~~If $A_1, A_2, \ldots, A_m$ are disjoint finite sets, then no. of elements in union of these sets.~~

(2) Sum Rule =>

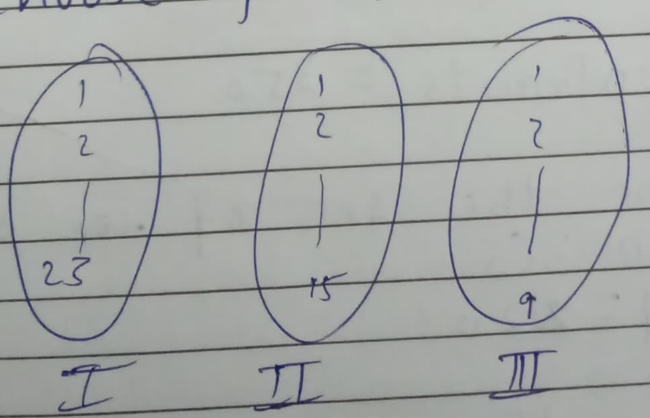
=> Task can be done in n_1 ways or n_2 ways then, complete procedure can be done in "$n_1 + n_2$" ways.

eg→ Suppose that either member of maths faculty or a student who is mathematics major is a choosen as a representative to a university committee, how many different chooses are there for this representative if there are 37 members of mathematics faculty and 83 mathematics majors and no one is both of faculty and student.

→ $83 + 37 \Rightarrow 120$

eg→ Student can choose computer project from one of three lists, The 3 list contain 23, 15 and 19 possible projects, no project is on more than one list, how many possible projects are there to choose from?

→



$\Rightarrow I + II + III$

$\Rightarrow 57$

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$$|A_1 \cup A_2 \cup A_3 \cdots \cup A_m| = |A_1| + |A_2| + \cdots + |A_m|$$

$$|A_1 \times A_2 \times A_3 \cdots \times A_m| = |A_1| \cdot |A_2| \cdot |A_3| \cdots |A_m|$$

$$|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2|$$

eg → A computer company receives 350 applications from computer graduates for a job planning a line for new web servers. Suppose that 220 of these people, major in CS, 147 major in businesses and 51 major both in CS and businesses. How many of these applicants majors neither in CS nor in a business ?

→

Total applicants = 350

Let A_1 be the set of ~~tot~~ students majored in CS.

$$|A_1| = 220$$

Let A_2 be the set of majored in businesses.

$$|A_2| = 147$$

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~~Let A_3 be~~ t

So, $A_1 \cap A_2$ is majored in both CS and businesses.

$|A_1 \cap A_2| = 51$

$A_1 \cup A_2$ = majored in CS or business.

$|A_1 \cup A_2| = ?$

$|A_1 \cup A_2| = |A_1| + |A_2| - (A_1 \cap A_2)$

$= 220 + 147 - 51$

$|A_1 \cup A_2| = 316$

$|(A_1 \cup A_2)'| = |U| - |A_1 \cup A_2|$

$= 350 - 316 \Rightarrow 34.$

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Q1) Find the number of positive integers not exceeding 100 that are ~~not~~ divisible by 5 $\boxed{\text{or}}$ 7.

→

Let A denotes the set of integers which is divisible by 5.

$$|A| = \left\lfloor \frac{100}{5} \right\rfloor = 20$$

→ (Lower funct)

Let B denotes set of int. by 7.

$$|B| = \left\lfloor \frac{100}{7} \right\rfloor = \lfloor 14.2 \rfloor = 14.$$

Let $A \cap B$ denotes —— divisible by 5 and 7.

$$|A \cap B| = \left\lfloor \frac{100}{5 \times 7} \right\rfloor = 2$$

$$|A \cup B| = ?$$

$$|A \cup B| = |A| + |B| - |A \cap B|$$

$$= 20 + 14 - 2$$

$$= \boxed{32}$$

Q2) How many positive integer not exceeding 10,000 are divisible by 5 $\boxed{\text{or}}$ & 11 ?

→ Let A denotes set of integers which are divisible by 5.

$$|A| = \left\lfloor \frac{10000}{5} \right\rfloor = 2000$$

Let B — / — by 11.

$$|B| = \left\lfloor \frac{10000}{11} \right\rfloor = 909$$

Let $A \cap B$ denotes — / — by both 5 and 11.

$$|A \cap B| = \left\lfloor \frac{10000}{55} \right\rfloor = 181$$

$$|A \cup B| = 2000 + 909 - 181$$
$$= \boxed{2728}$$

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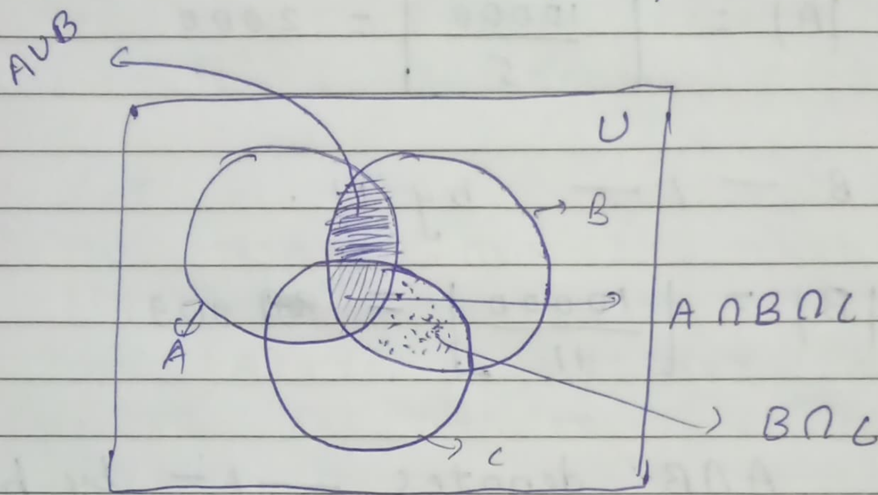
Q3) Let A, B, C be non-empty finite set, then number of elements in A or B or C is
$|A \cup B \cup C| = ?$

→

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$



Q4) A total of 1232 students have taken a course in spanish, 879 taken course in french, and 114 had taken course in Russian. Further 103 had taken course in both spanish and french. 23 have taken course in both spanish and russian. 14 taken in french and russian. if 2092 students have taken atleast one of spanish, french and russian, how many students had taken a course in all three languages?

→ Let A denotes the set of stud. taken course in spanish.

$|A| = 1232$

Let B — // — in french.

$|B| = 879$.

Let C — // — in russian.

$|C| = 114$.

$|A \cap B| = 103$.

$|B \cap C| = 23$

$|A \cap C| = 14$.

$|A \cup B \cup C| = 2092$

$|A \cap B \cap C| = 7$

* The Pigeonhole Principal →

Thm 1 →

If k is positive integer and $k+1$ or more objects are placed into k-boxes, then there's atleast one box containing two or more objects.

Ex How many students must be in class to guarentee that atleast two students receive the same score in final exam ? If exam is graded on scale from 0 to 100.

→ Stud.	1	2	3	———	101	102
marks	0	1	2	———	100	

102 students are required.

* Generalised Pigeonhole Principal →

If suppose n objects are placed into k boxes, then their is atleast one box containing atleast $\lceil N/k \rceil$ objects.

{($\lceil \ \rceil$) → ceiling function}

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Ex-1 What is min. no. of students req. in Discrete Mathematics class to be sure that atleast 6 will receive the same grade if their are 6 possible ways, A, B, C, D, E, F.

→

$k = 6, \quad N = ?$

$$\left\lceil \frac{N}{6} \right\rceil = 6$$

So, $\boxed{N = 31}$

How many cards must be selected from standard deck of 52 cards to guarentee that atleast 3 cards ~~of~~ ~~must be~~ of same suit are choosen?

→

$N = ?, \quad k = 4.$

$$\left\lceil \frac{N}{4} \right\rceil = 3$$

So, $\boxed{N = 9}$

Revision

$+_5$	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

eg → $G = \{1, -1, i, -i\}$

$\times_5$	1	-1	i	-i
1	1	-1	i	-1
-1	-1	1	i	i
i	i	-i	-1	1
-i	-1	i	1	-1

Recurrence Relation :-