

Total No. of Questions—8]

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**S.E. (Information Technology) (I Sem.) EXAMINATION, 2017**  
**DISCRETE STRUCTURE**

**(2015 PATTERN)**

**Time : Two Hours**

**Maximum Marks : 50**

- N.B. :—**
- (i) Answer Q. No. 1 or Q. No. 2, Q. No. 3 or Q. No. 4, Q. No. 5 or Q. No. 6, Q. No. 7 or Q. No. 8.
  - (ii) Neat diagrams must be drawn wherever necessary.
  - (iii) Figures to the right indicate full marks.
  - (iv) Use of calculator is allowed.
  - (v) Assume suitable data, if necessary.

1. (A) Out of a total of 130 students, 60 are wearing hats, 51 are wearing scarves, and 30 are wearing both hats and scarves. Out of 54 students who are wearing sweaters, 26 are wearing hats, 21 are wearing scarves, and 12 are wearing both hats and scarves. Everyone wearing neither a hat nor a scarf is wearing gloves :

- (a) How many students are wearing gloves ?

P.T.O.

- (b) How many students not wearing a sweater are wearing hats but not scarves ?
- (c) How many students not wearing a sweater are wearing neither hat nor a scarf ? [6]
- (B) Tickets numbered 1 to 20 are mixed up and then a ticket is drawn at random. What is the probability that the ticket drawn has a number which is a multiple of 3 or 5 ? [3]
- (C) In a box, there are 8 red, 7 blue and 6 green balls. One ball is picked up randomly. What is the probability that it is neither red nor green ? [3]

Or

2. (A) Prove by induction that the sum of the cubes of three consecutive integers is divisible by 9. [6]
- (B) Two cards are drawn together from a pack of 52 cards. Determine the probability that one is a spade and one is a heart. [4]
- (C) Three unbiased coins are tossed. What is the probability of getting at most two heads ? [2]



3. (A) Solve the following recurrence relation :

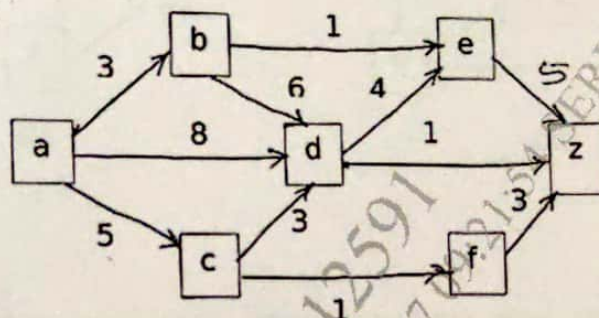
[6]

$$a_r - 7a_{r-1} + 10a_{r-2} = 2^r$$

$$a_1 = 3, a_2 = 21.$$

- (B) Find the shortest path from vertex  $a$  to  $z$  in the following graph :

[6]



Or

4. (A) Functions  $f$ ,  $g$  and  $h$  are defined on the set  $X = \{1, 2, 3\}$  as :

[6]

$$f = \{(1, 3), (2, 1), (3, 2)\};$$

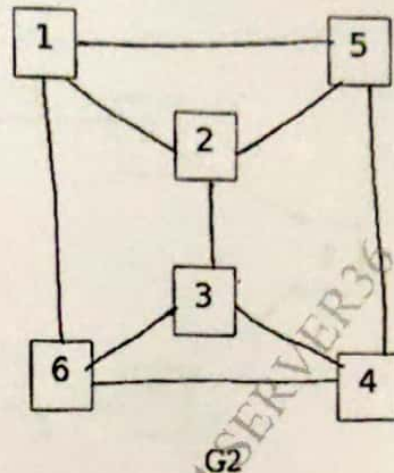
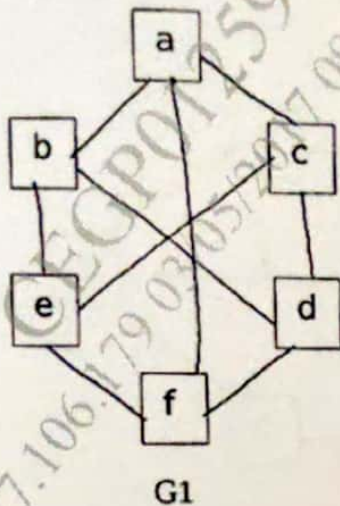
$$g = \{(1, 2), (2, 3), (3, 1)\};$$

$$h = \{(1, 2), (2, 1), (3, 3)\}$$

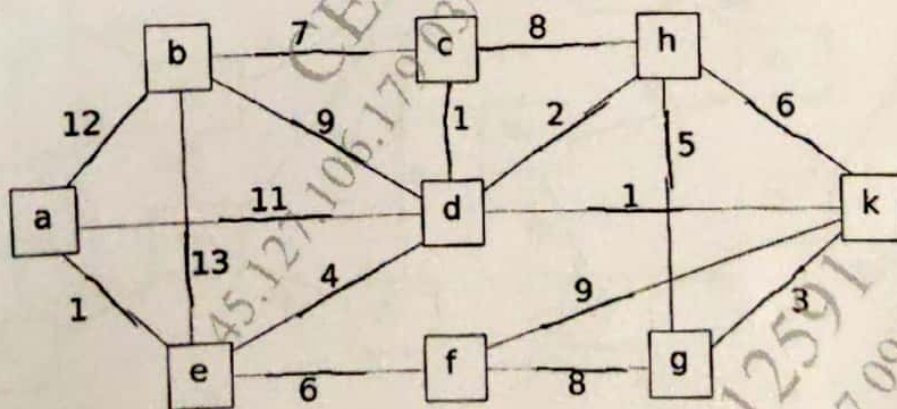
- (i) Find  $f \circ g$  and  $g \circ f$ . Are they equal ?
- (ii) Find  $f \circ g \circ h$  and  $h \circ g \circ f$ .

P.T.O.

- (B) Define Isomorphic Graphs. Show that the following graphs G1 and G2 are isomorphic. [6]



5. (A) Find the minimum spanning tree and weight of it for the given graph using Kruskal's algorithm. [7]



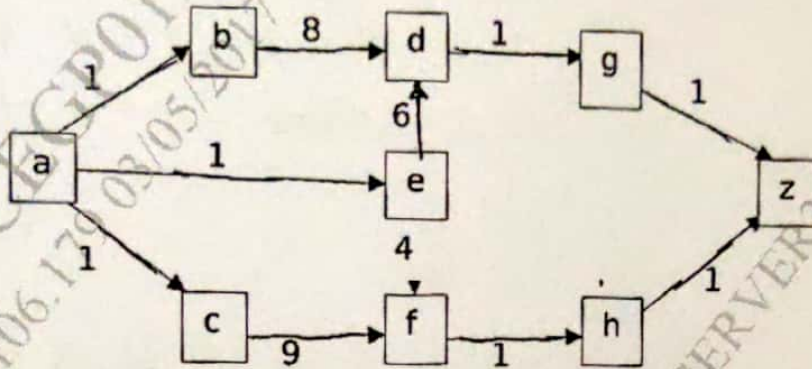
- (B) Define optimal tree. For the following set of weights, construct optimal binary prefix code. For each weight in the set, give corresponding prefix code : [6]

1, 4, 8, 9, 15, 25, 31, 37.

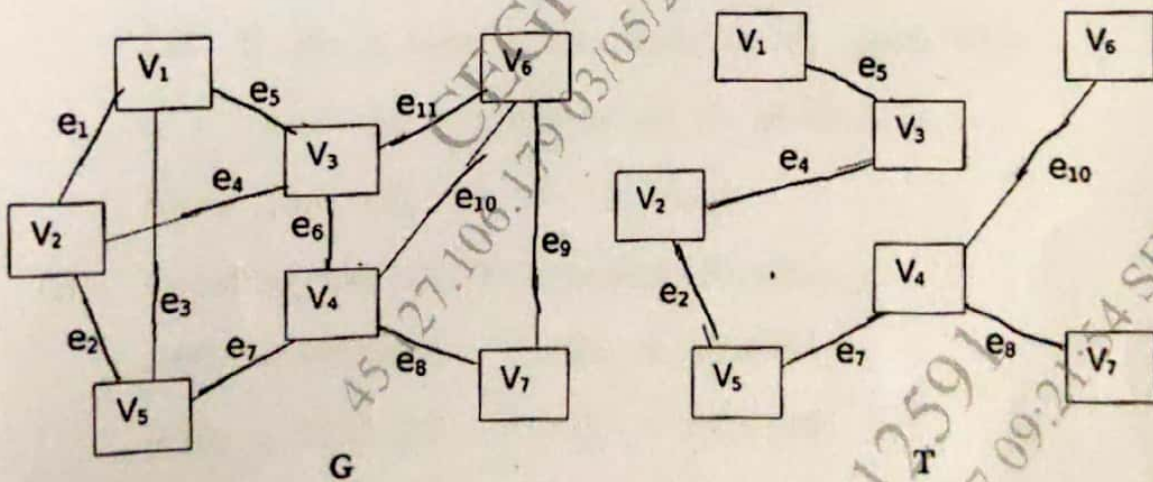


Or

6. (A) Find the maximum flow for the following transport network. [7]



- (B) Find the fundamental system of cut set for the graph G shown below with respect to the spanning tree T. [6]



7. (A) Let  $Q_1$  be the set of all rational numbers other than 1. Show that with operation  $*$  defined on the set  $Q_1$  by  $(a * b = a + b - ab)$  is an Abelian group. [7]

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(B) Let  $I$  be the set of all integers. For each of the following determine whether  $*$  is an associative operation or not : [6]

(1)  $a * b = \max(a, b)$

(2)  $a * b = \min(x + 2, b)$

(3)  $a * b = a - 2b$

(4)  $a * b = \max(2a - b, 2b - a).$

Or

8. (A) Let  $Z_n$  be the set of integers  $\{0, 1, 2, \dots, n - 1\}$ . Let  $\oplus$  be a binary operation on  $Z_n$  such that :

$$a \oplus b = \begin{cases} a + b & \text{if } a + b < n \\ a + b - n & \text{if } a + b \geq n \end{cases}$$

Let  $\odot$  be a binary operation on  $Z_n$  such that :

$a \odot b =$  the remainder of  $ab$  divided by  $n$ .

Show that  $(Z_n, \oplus, \odot)$  is Ring. [7]

(B) Consider the  $(2, 7)$  encoding function  $e$ . [6]

$e(00) = 0000000 \quad e(01) = 1010101$

$e(10) = 0111110 \quad e(11) = 0110110$

(a) Find the minimum distance of  $e$ .

(b) How many errors will  $e$  detect ?