ITM 517 Algorithm

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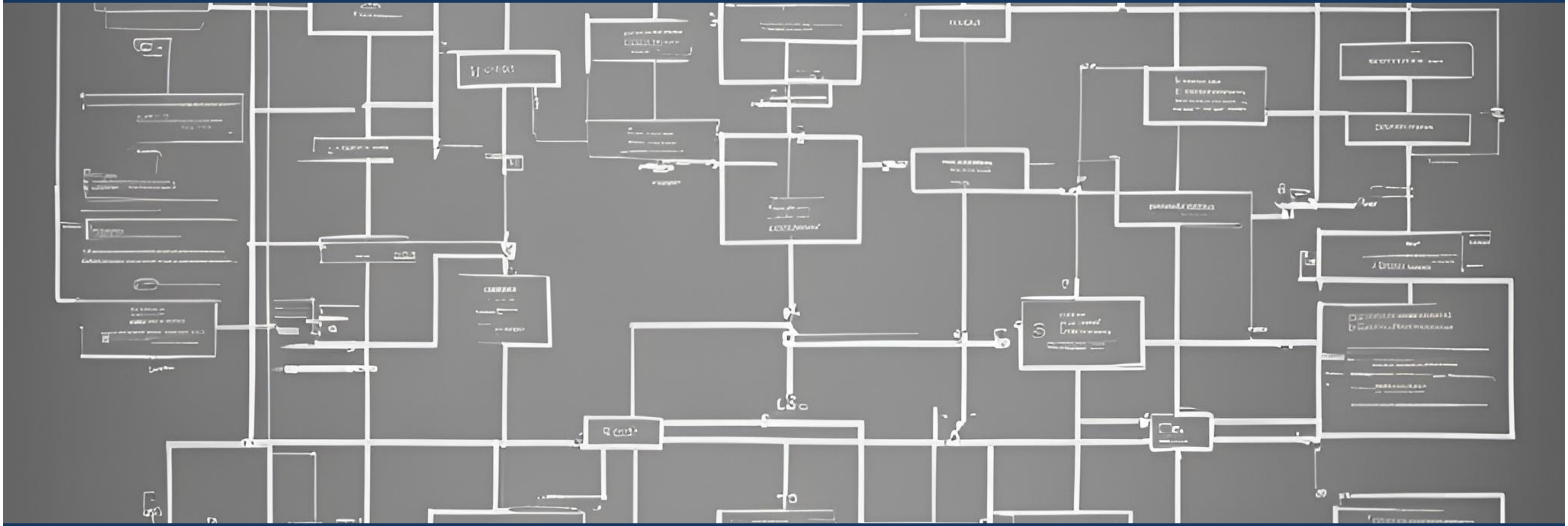
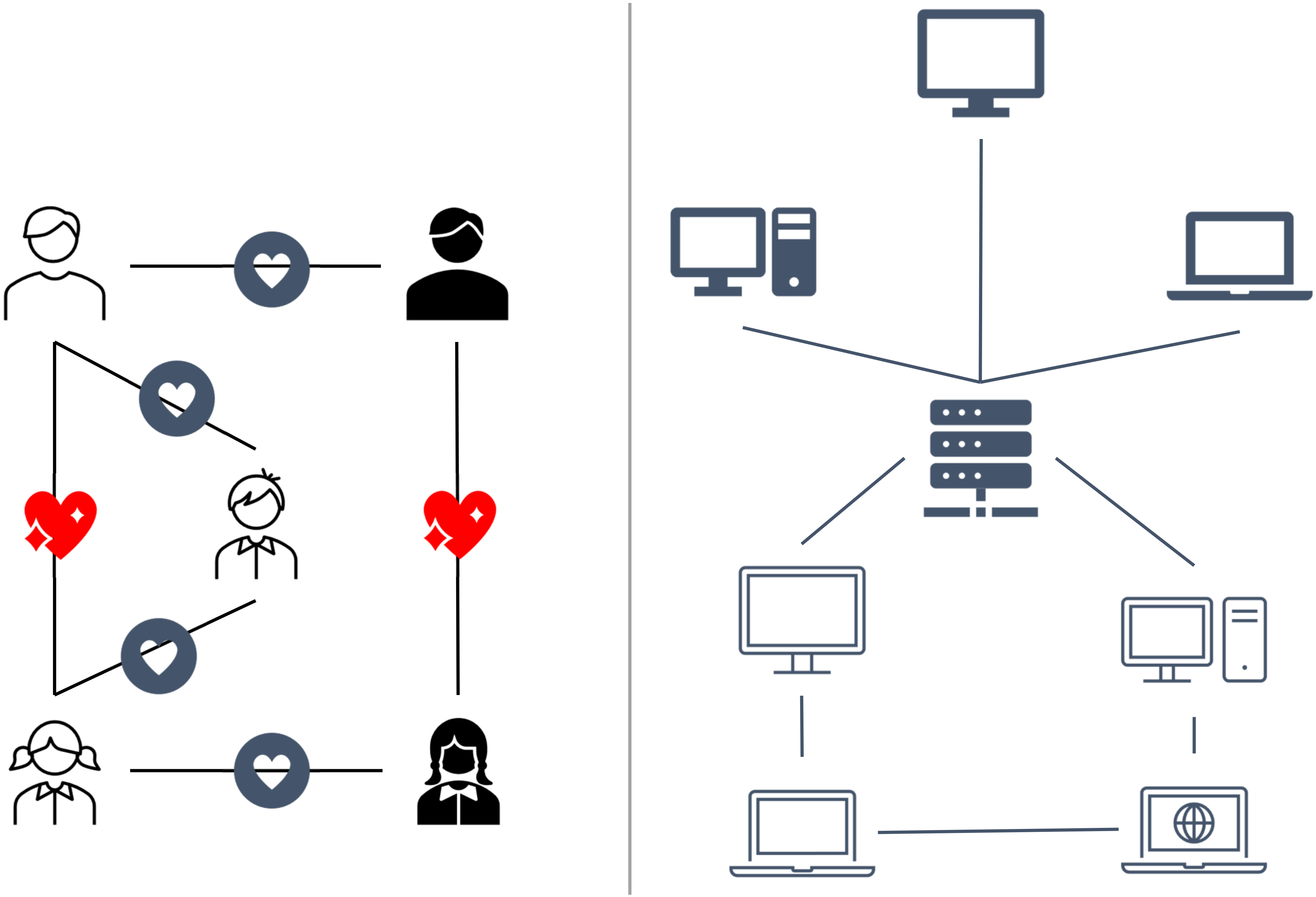
Graph



Introduction to Graph

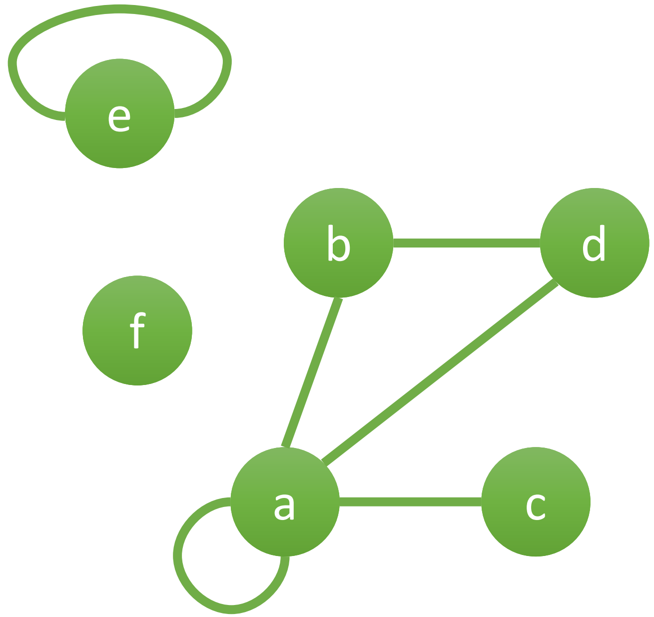
Example of Graph



Graph

- An **undirected graph**, **or graph** is a couple $G = (V, E)$ consists of
  - V: a nonempty set of vertices
  - E: a set of **unordered pairs** of distinct elements of V called edges.
  - For each $e \in E$, $e = \{u, v\}$ where $u, v \in V$.

- Example
  - V={a, b, c, d, e, f}
  - E = {{a, a}, {a, b}, {a, c}, {a, d}, {b, d}, {e, e}}



Terminology

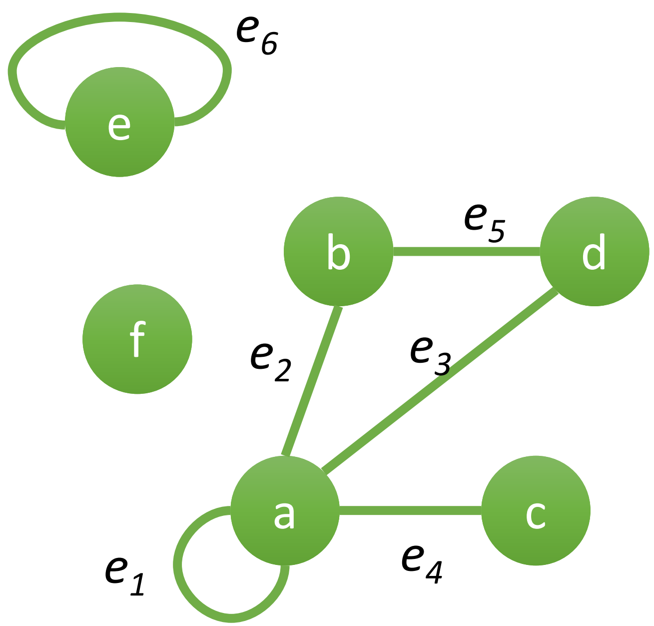
- Ends of edge
- Adjacent, Incident
- Loop, link, simple graph
- Degree, pendant, isolated
- path, cycle, circuit
- Connectivity
- Tree
- Identical, isomorphic
- Complete graph
- Subgraph
- Weighted graph
- Directed Graph

Ends of edge

- Condition
  - e is an edge {u, v}
  - u, v: vertices

- Definition
  - Join: e is said to join u and v
  - End: u and v are called the ends of e.

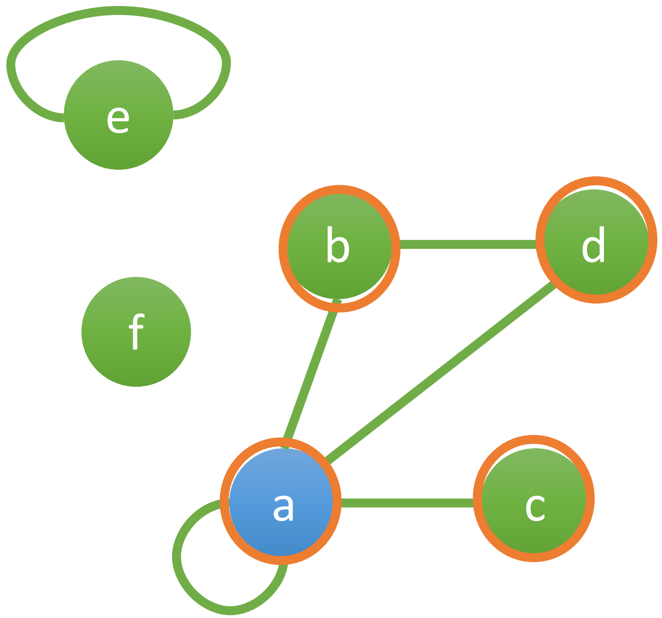
- Example
  - e_1 joins a and a
  - The ends of e_3 are a and d.



adjacent and **incident**

- Two vertices u and v in an undirected graph *G* are called **adjacent** (or **neighbors**) in *G* if *{u, v}* is an edge in *G*.
- If e = {u, v}, the edge e is called **incident with** the vertices u and v. The edge e is also said to **connect** u and v.

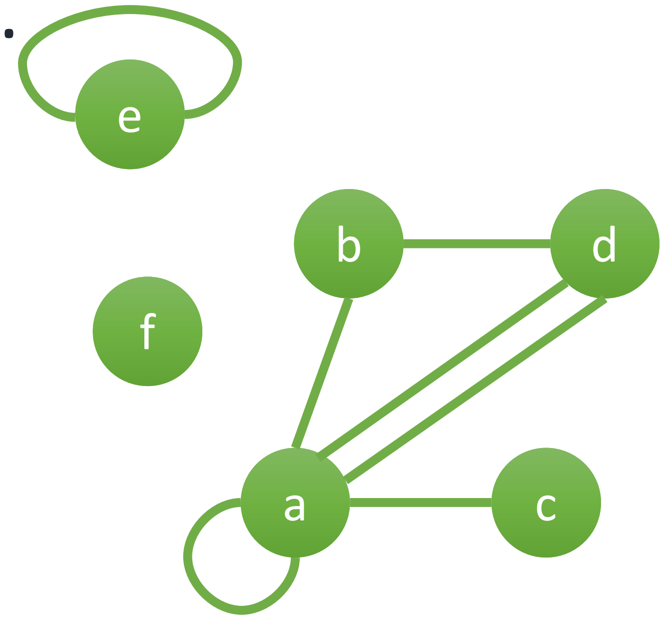
- Ex: adjacent of *a*
- Edge *{a,d}* incidents with *a* and *d*
- Edge *{a,d}* connects *a* and *d*
- *a* and *d* are the end points or ends of edge *{a,d}*



Loop

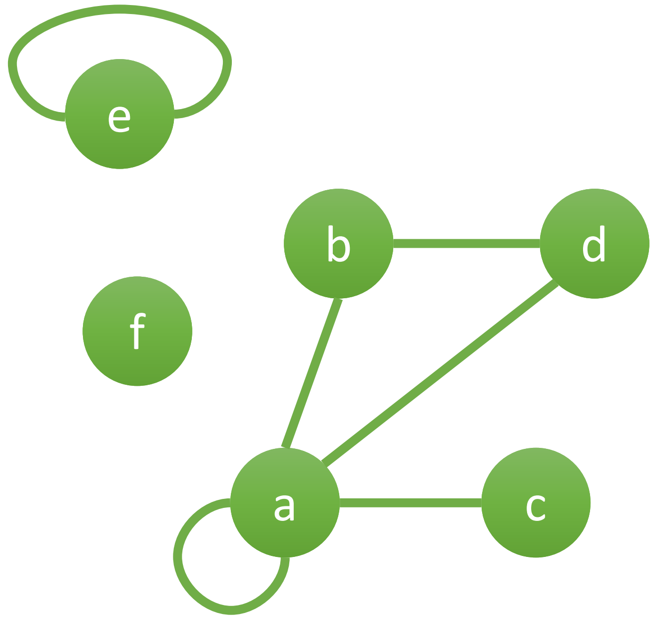
- If there is an edge incidenting to itself, this is a **loop**.
  - $e=(u,u) \in E$
- An edge with distinct eds is called as a **link**.
- Question: How many does this graph have a loop?

- A **simple graph** a graph with no loop and no multiple edges with the same ends.



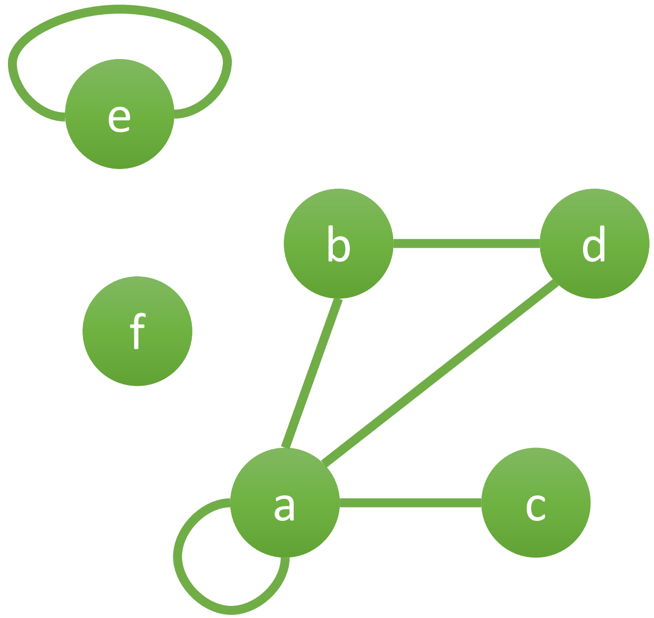
degree for an undirected graph

- The **degree** of a vertex in an undirected graph is the number of edges incident with it
  - a loop at a vertex contributes twice to the degree of that vertex
- **counting the lines** that touch it
- denoted ***deg(v)***
- Question
  - deg(a):
  - deg(b):
  - deg(c):
  - deg(f):



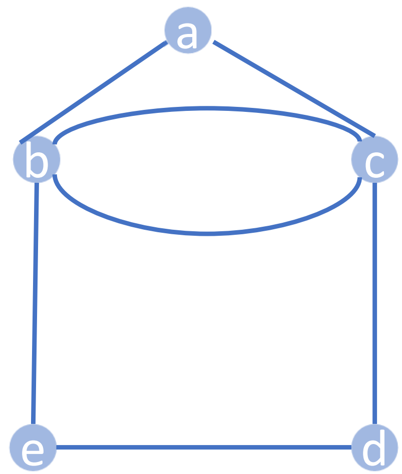
pendant and isolated

- pendant:
  - A vertex of degree 1 is called **pendant**. It is adjacent to exactly one other vertex.
- isolated:
  - A vertex of degree 0 is called **isolated**, since it is not adjacent to any vertex.
- Question
  - Which is a pendant? c
  - Which is isolated? f



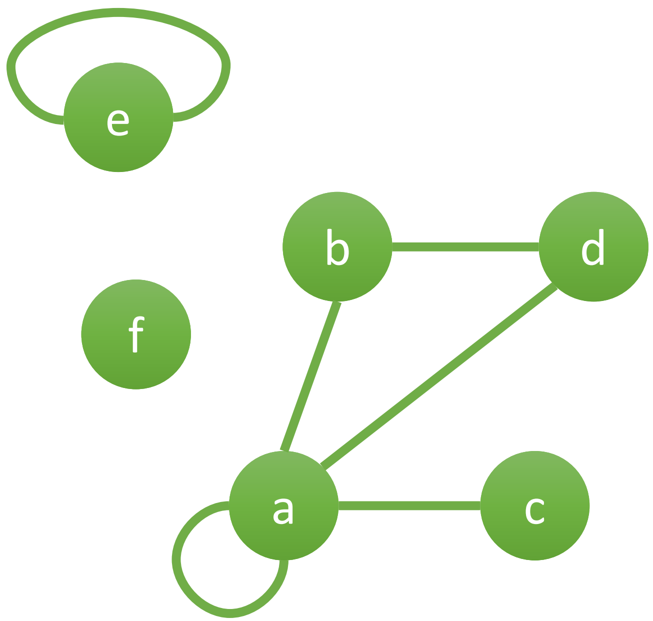
Path

- A **path** of length n from u to v, where n is a positive integer, in an **undirected graph** is a sequence of edges $e_1, e_2, \ldots, e_n$ of the graph such that $e_1 = \{x_0, x_1\}$, $e_2 = \{x_1, x_2\}, \ldots, e_n = \{x_{n-1}, x_n\}$, where $x_0 = u$ and $x_n = v$. The path is a **circuit(cycle)** if it begins and ends at the same vertex, that is, if $\boldsymbol{u = v}$.
- A path or cycle is **simple** if it does not contain the same vertex more than once.



Connectivity

- An undirected graph is called **connected** if there is a path between every pair of distinct vertices in the graph.



V, E)
={a, c, e, f}
E {{a,a b}, {a, c}, {a,d},{b, d}, {e,e}}

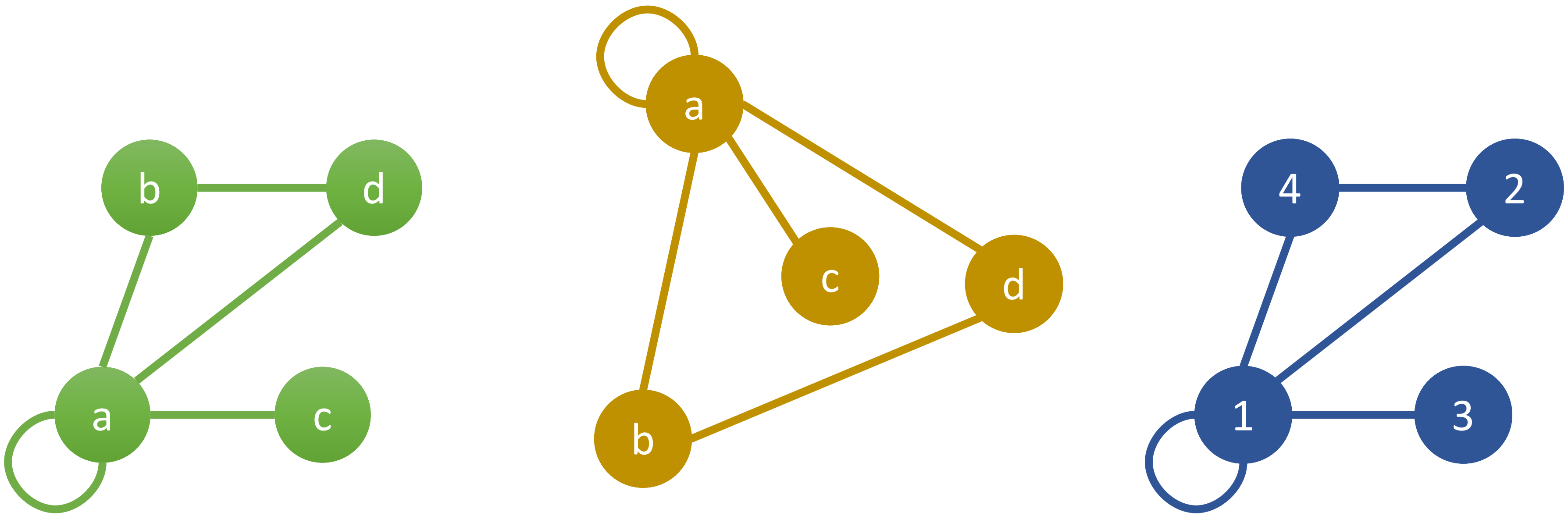
V, E)
{a, b, c, d}
a}, {a,b}, {a, c}, {a,d},{b, d

Tree

- a **tree** is an undirected graph in which
  - any two vertices are connected by exactly one path
  - or equivalently **a connected acyclic** undirected graph.
- A **forest** is an undirected graph in which
  - any two vertices are connected by at most one path
  - equivalently an **acyclic** undirected graph, or equivalently a disjoint union of trees.

Identical vs isomorphism

- Two graphs G and H are **identical** if V(G) = V(H) and E(G)=E(H)
- If there is a mapping V(G)→V(H) and E(G)→E(H), we say the mapping is an **isomorphism** between G and H

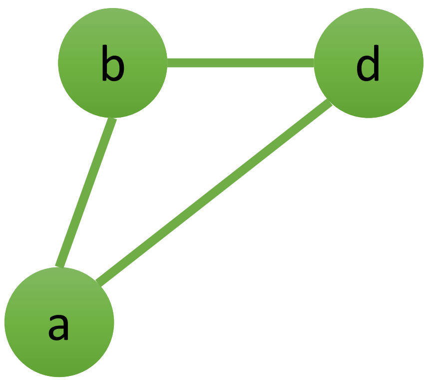


G1 G2 G3

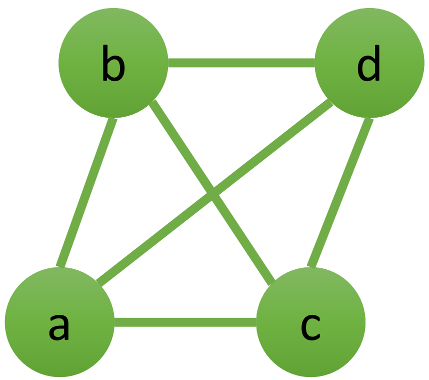
complete graph

- The **complete graph** on *n* vertices, denoted by K_n, is the simple graph that contains exactly one edge between each pair of distinct vertices.

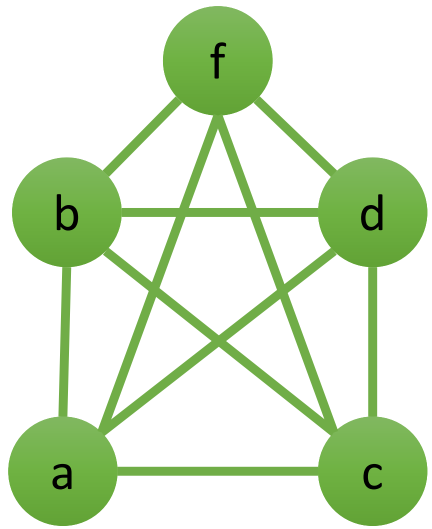
- An **empty graph** is one with no edge.



K_3



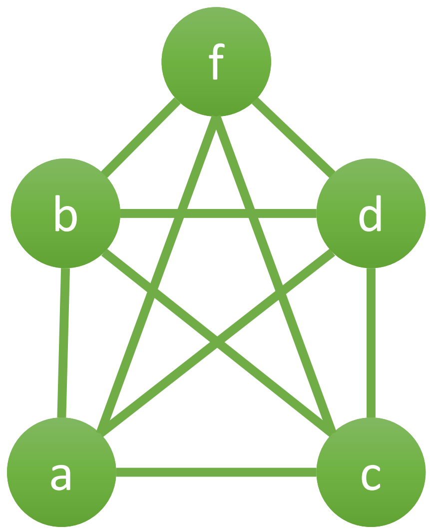
K_4

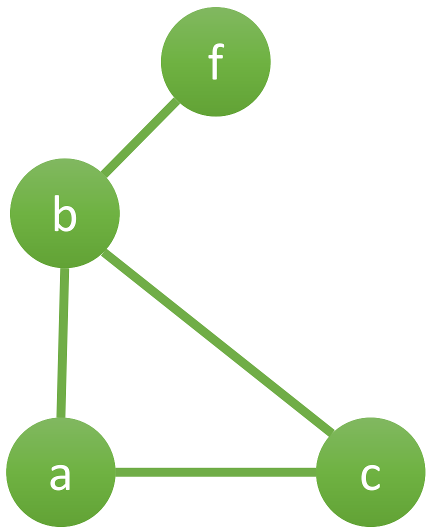


K_5

subgraph

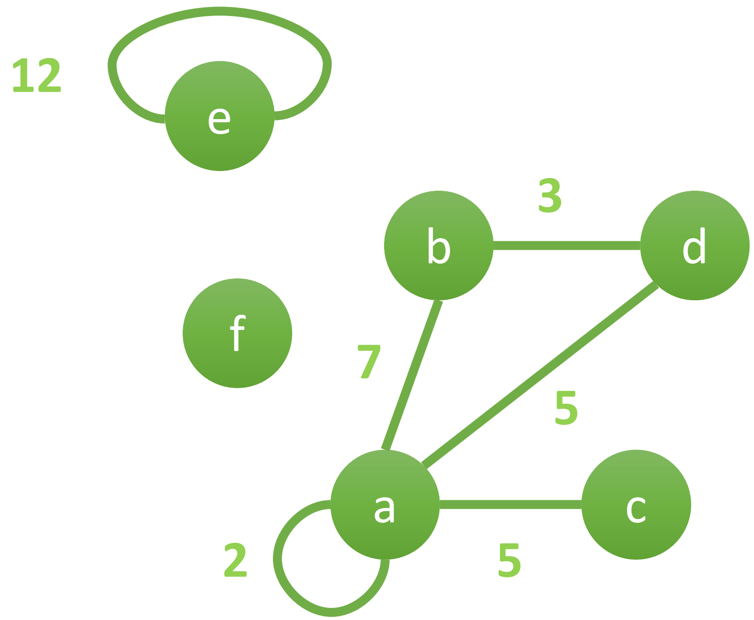
- A **subgraph** of a graph $G = (V, E)$ is a graph $H = (W, F)$ where $W \subseteq V$ and $F \subseteq E$. Of course, H is a valid graph, so we cannot remove any endpoints of remaining edges when creating H.





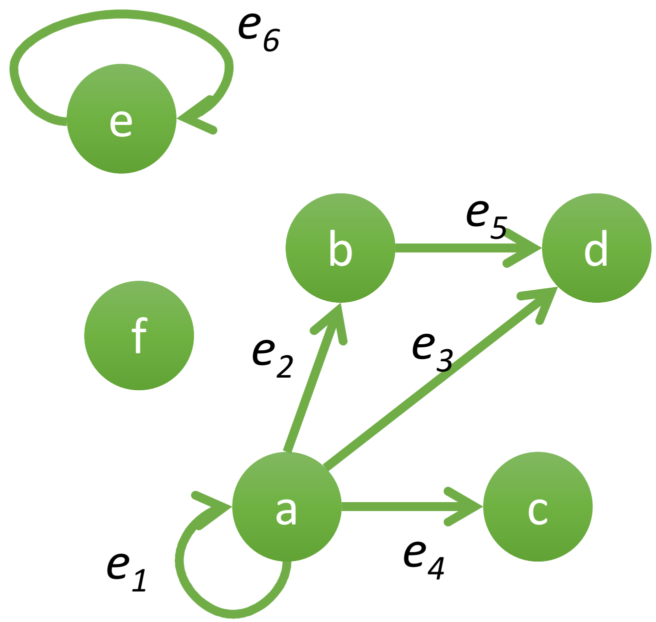
weighted graph

- A **weighted graph** is a graph in which a number (the weight) is assigned to each edge.
  - weights might represent for example costs, lengths or capacities, depending on the problem at hand.
  - *G = (V, E, W)*
    - *W:E→Z, where Z is a real number.*



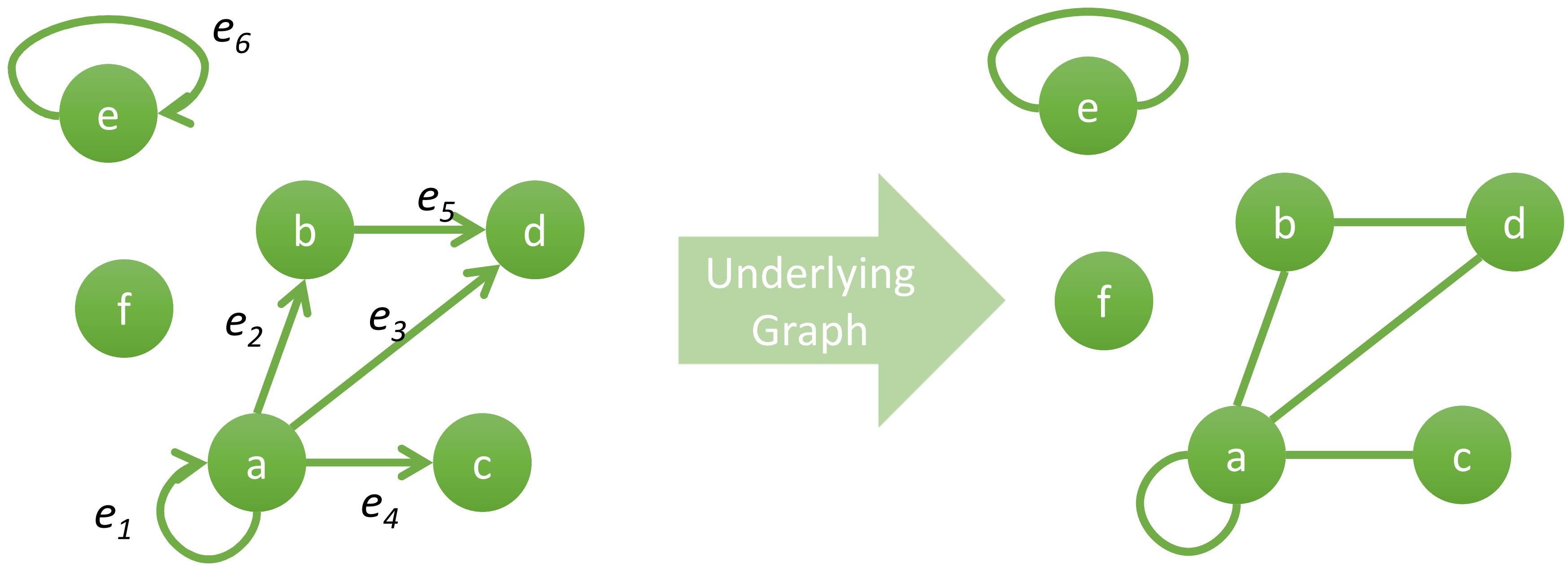
Directed graph

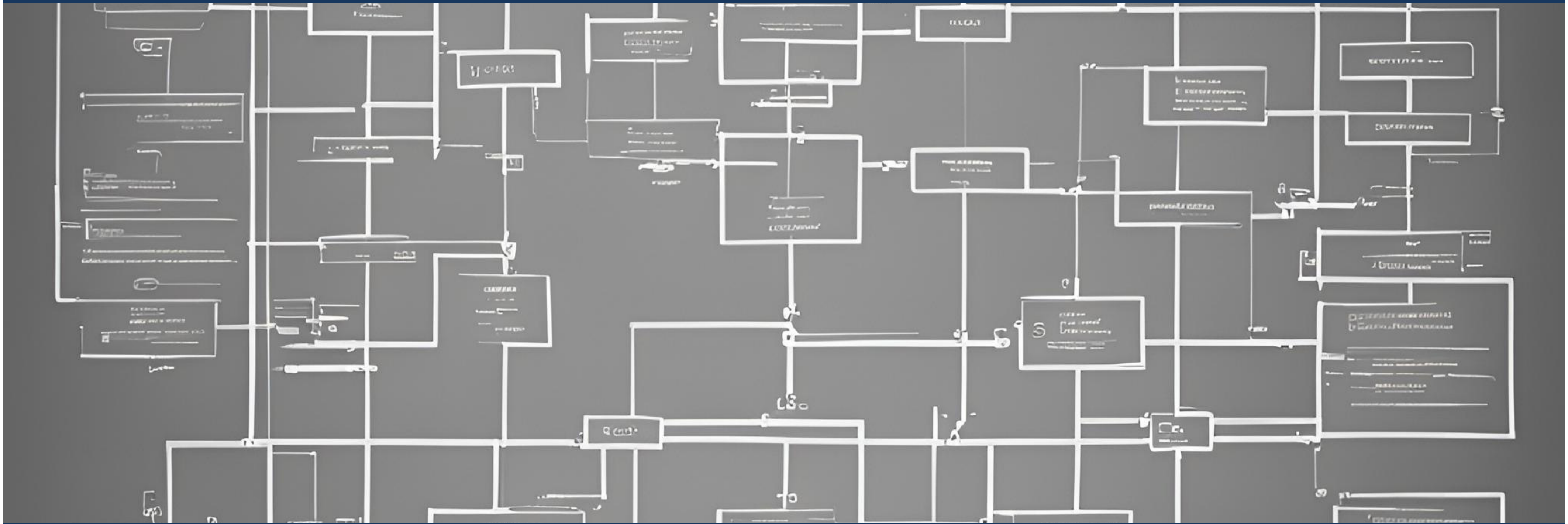
- **Directed graph (Digraph)** *D* is an ordered pair *(V(D), A(D))*
  - *V(D)*: a nonempty set of vertices
  - *A(D)*: a set of **arcs**. Each arc of *A* is an ordered pair of vertices of *D*
- If *a* is an arc and *u* and *v* are vertices of *a*: *(u, v)*
  - *u*: **tail** of *a*
  - *v*: **head** of *a*



Underlying graph

- The **underlying graph** *G* of a digraph *D*
  - With each digraph *D* we can associate a graph *G* on the same vertex set;
  - corresponding to each arc of *D* there is an edge of *G* with the same ends.
  - *D* is an **orientation** of *G*





Implementations

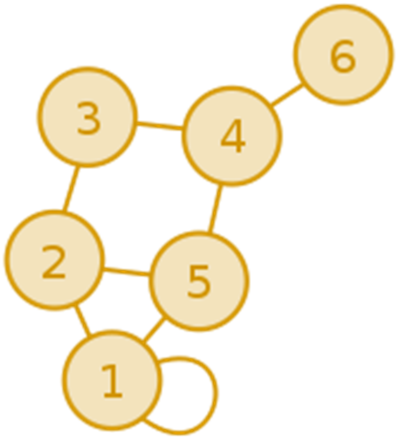
Adjacency matrix

Adjacency list

Incidence matrix

Adjacency matrix

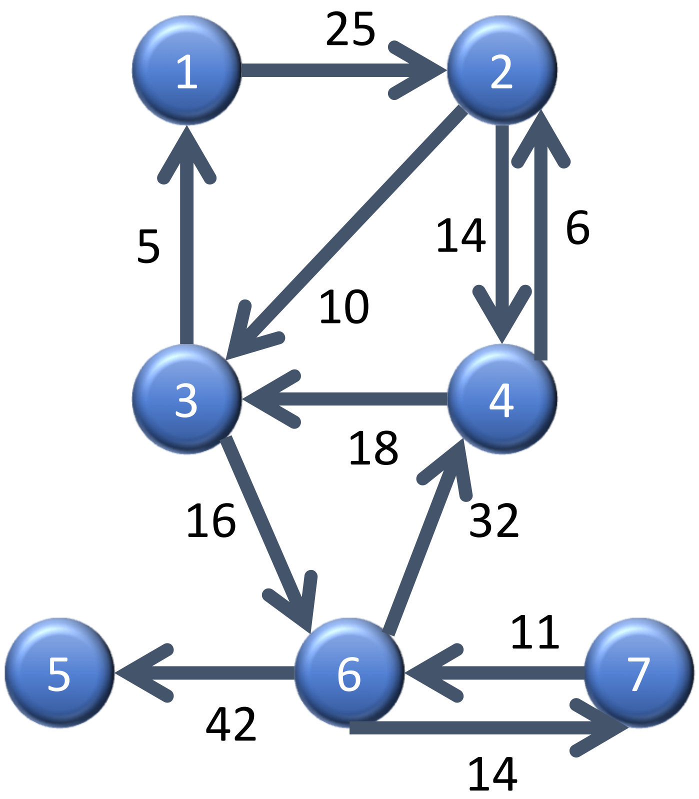
- Let $G = (V, E)$ be a graph with $|V| = n$. Suppose that the vertices of G are listed in arbitrary order as $v_1, v_2, \dots, v_n$. The **adjacency matrix** A (or A_G) of G, with respect to this listing of the vertices, is the $n \times n$ zero-one matrix with 1 as its $(i, j)^{th}$ entry when v_i and v_j are adjacent, and 0 otherwise. In other words, for an adjacency matrix $A = [a_{ij}]$,
- $a_{ij} = 1$ *if* $\{v_i, v_j\}$ is an edge of G,
- $a_{ij} = 0$ otherwise.



$$\begin{pmatrix} 2 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

https://graphonline.ru/en/create_graph_by_matrix

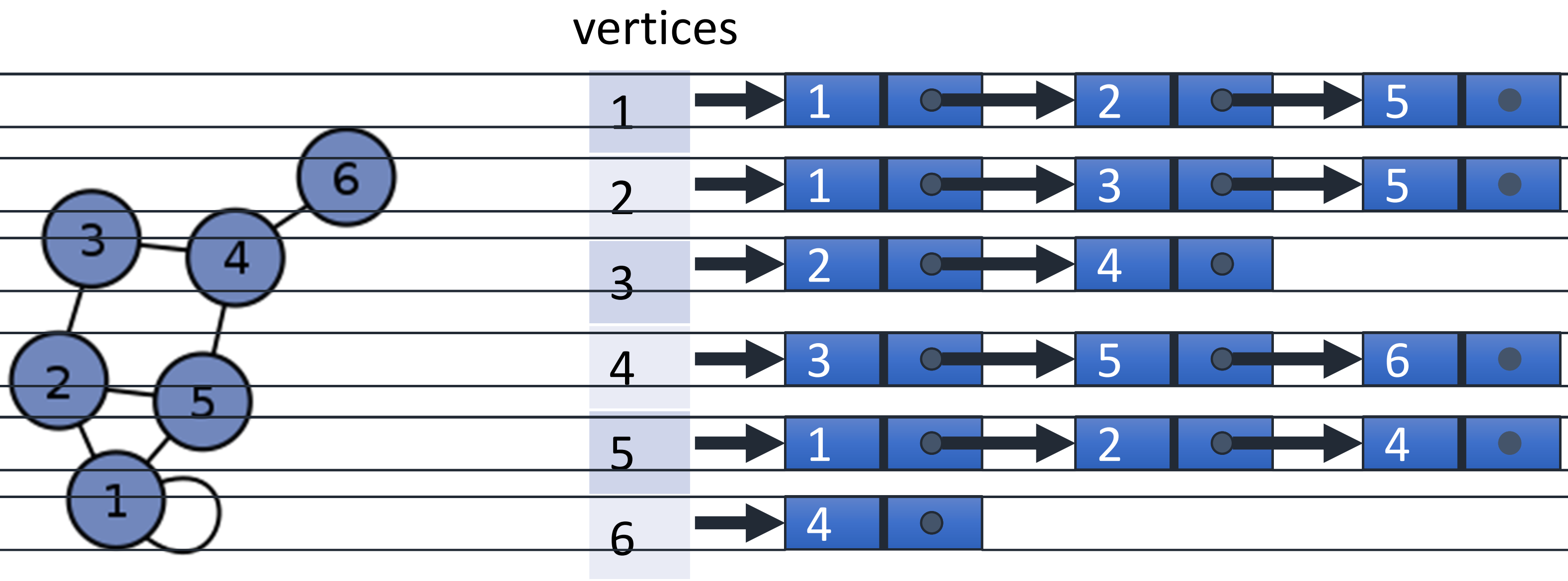
Adjacent matrix for a weighted digraph



$$\begin{pmatrix} 0 & 25 & \infty & \infty & \infty & \infty & \infty \\ \infty & 0 & 10 & 14 & \infty & \infty & \infty \\ 5 & \infty & 0 & \infty & \infty & 16 & \infty \\ \infty & 6 & 18 & 0 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & 0 & \infty & \infty \\ \infty & \infty & \infty & 32 & 42 & 0 & 14 \\ \infty & \infty & \infty & \infty & \infty & 11 & 0 \end{pmatrix}$$

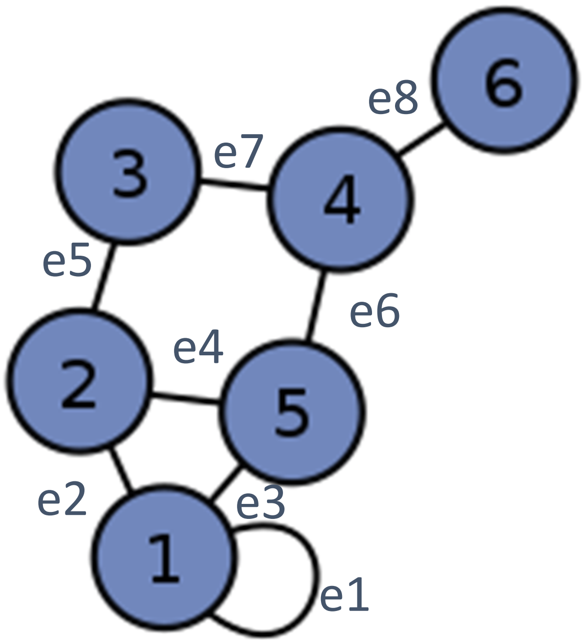
Adjacent list

- A collection of unordered lists used to represent a finite graph.
- Each list describes the set of neighbors of a vertex in the graph



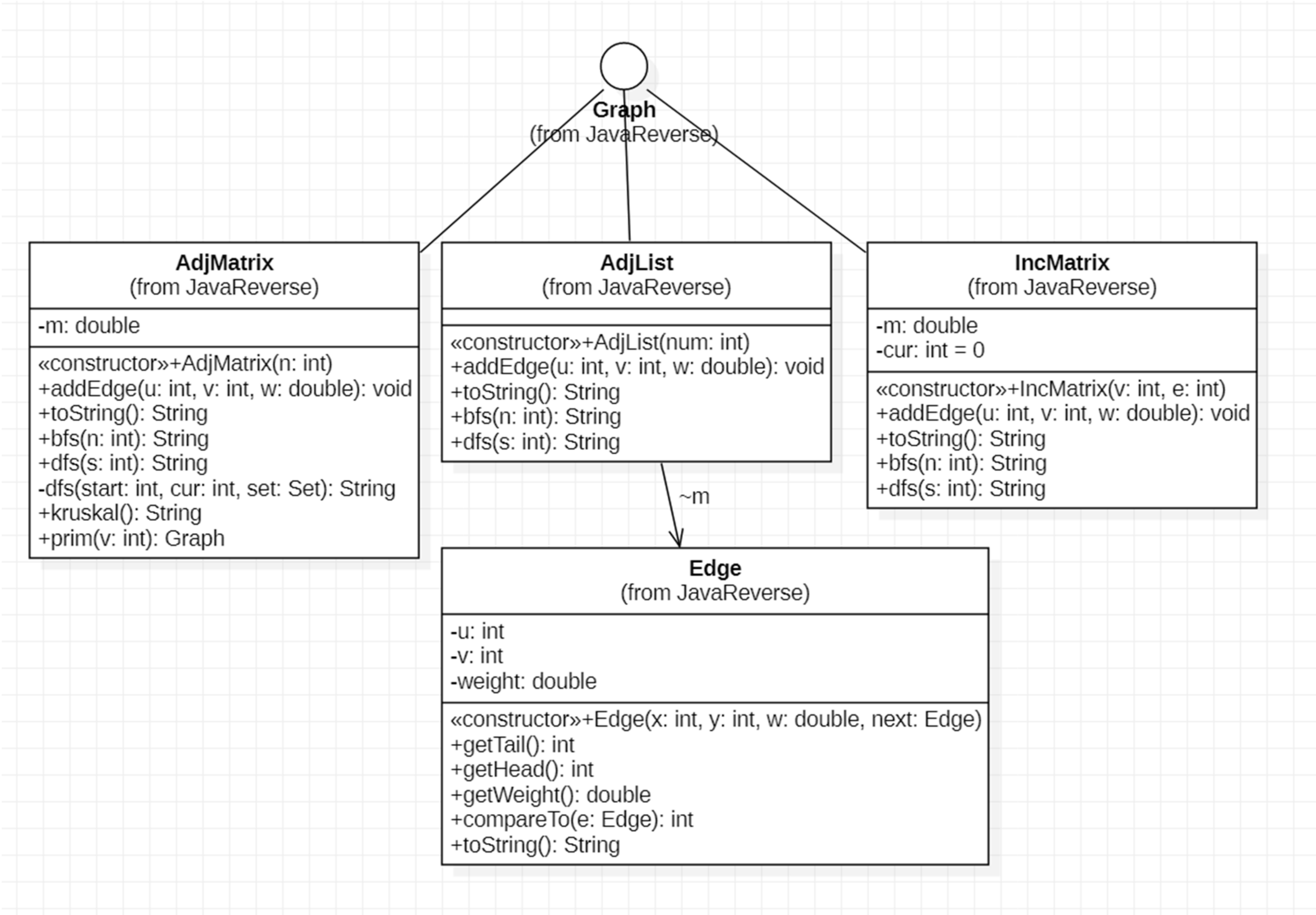
Incidence matrix

- listing of the vertices and edges is the $n \times m$ zero-one matrix with 1 as its $(i, j)^{th}$ entry when edge is incident with v_i, and 0 otherwise. In other words, for an incidence matrix $M = [m_{ij}]$,
  - $m_{ij} = 1$ if edge e_j is incident with v_i
  - $m_{ij} = 0$ otherwise.



$$
\begin{array}{cc}
\text{vertex} & \begin{array}{cccccccc} e1 & e2 & e3 & e4 & e5 & e6 & e7 & e8\text{: edge} \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} &
\begin{bmatrix}
1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\
0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\end{array}
$$

Class diagram



Thanks