Counting / Probability Worksheet (3A1)

Name ______________________

(1) __________ The license plates for motor vehicles in a state have a sequence of three letters followed by three digits. How many license plates could start with MOM or POP?

(2) __________ How many sets of five paintings can a museum curator choose from a collection of eight paintings?

(3) __________ How many diagonals does a regular hexagon have?

(4) __________ How many different integers can be expressed as the sum of three different numbers in the set $\{1, 2, 3, 4, 5, 6, 7\}$?

(5) __________ Ohio entered the Union on March 1, 1803, which can be written as 3/1/03. The date 3/1/03 is a "friendly date" because the product of the month and day is equal to value of the last two digits of the year. How many friendly dates occur during the year 2009?

(6) __________ How many different isosceles triangles have integer side lengths and perimeter of 81 units?

(7) __________ A five-digit number is called a mountain number if the first three digits increase and the last three digits decrease. For example, $35,763$ is a mountain number but $35,663$ is not. How many five-digit numbers greater than $70,000$ are mountain numbers?

(8) ________ Three standard dice are tossed. What is the probability that the sum of the numbers on the tops of the three dice is 17 or greater? Express your answer as a common fraction.

(9) ________ During the monsoon season, torrential rainstorms occur. In how many distinct ways can the letters of MONSOON be arranged?

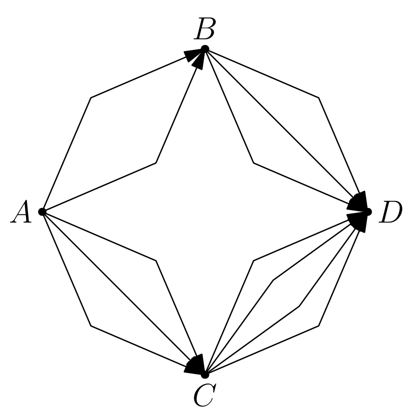
(10) ________ A basketball player makes her free throws 55% of the time. What is the probability that she will make both of her next two free throws? Express your answer as a common fraction.

(11) ________ Merina's annual salary is a whole number of dollars between $\$42,400$ and $\$42,500$. The digits in the hundreds, tens and units places are in strictly ascending order. How many distinct possibilities exist for her annual salary?

(12) ________ There are 14 red, 10 yellow, 5 green and 3 purple candies in a bag. What is the probability that a candy chosen at random is not red? Express your answer as a common fraction.

(13) ________ A school is creating a four-digit student code system. For security reasons, the code cannot start with an even number. How many codes are possible?

(14) ________ Following the arrows, how many different routes are there from A to D?



(15) __________ The perimeter of a square lot is lined with trees, and there are three yards between the centers of adjacent trees. There are eight trees on a side, and a tree is at each corner. What is the number of yards in the perimeter of the lot?

(16) __________ What percent of the integers from 1 to 100 inclusive has at least one digit that is a 7?

(17) __________ Two different natural numbers are selected from the set $\{1, 2, 3, \ldots, 6\}$. What is the probability that the greatest common factor of these two numbers is one? Express your answer as a common fraction.

(18) __________ Compute: $\frac{(3!)^6}{(3!)^3}$.

(19) __________ A drawer contains 2 brown and 3 gray socks. The socks are taken out of the drawer one at a time. What is the probability that the fourth sock removed is gray? Express your answer as a common fraction.

(20) __________ A penny, a nickel and a dime are tossed simultaneously. What is the probability that exactly two of the coins show heads when they land? Express your answer as a common fraction.

(21) __________ The probability of rain tomorrow is $\frac{3}{10}$. What is the probability that it will not rain tomorrow? Express your answer as a common fraction.

(22) __________ In a science class of 16 students, 11 students are on the basketball team, and 7 students are on the track team. What is the least number of students in this class who are on both teams?

(23) __________ Morgan has 3 hockey shirts, 2 football shirts and 7 baseball shirts in her closet. If she randomly selects one of these shirts, what is the probability that it will not be a baseball shirt? Express your answer as a common fraction.

(24) ________ In how many different ways can 30 cents be made from any combination of quarters, dimes, nickels or pennies?

(25) ________ A board game spinner is divided into three parts labeled A, B, and C. The probability of the spinner landing on A is 1/3 and on B is 5/12. What is the probability of the spinner landing on C? Express your answer as a common fraction.

(26) ________ Mr. Patrick is preparing a five-question true-false quiz for his class. He flips a coin before writing the first question. If it is heads, he writes a true statement and if it is tails, he writes a false statement. He continues this until all five statements are written. What is the probability that the correct sequence of answers is TFTFT ? Express your answer as a common fraction.

(27) ________ How many different four-digit numbers can be formed using each of the digits in 1999 exactly once?

(28) ________ A palindrome is a number that reads the same forward and backward. The year 1991 was the last palindromic year of the twentieth century. How many years between 2000 and 3000 are palindrome?

(29) ________ Eight women of different heights are at a party. Each woman decides to only participate in a handshake with women shorter than herself. How many handshakes take place?

(30) ________ The numbers 1 through 25 are written on 25 cards with one number on each card. Sara picks one of the 25 cards at random. What is the probability that the number on her card will be a multiple of 2 or 5? Express your answer as a common fraction.

Answer Sheet

Number	Answer	Problem ID
1	2000	5CA3
2	56	15B41
3	9	ACA3
4	13	ABA3
5	3 dates	02D4
6	20	2513
7	36	B213
8	1/54	C413
9	420	B0B3
10	121/400	B5B41
11	10	A413
12	9/16	5CC2
13	5000	A5B41
14	18 routes	D5C1
15	84	42AC
16	19	51B3
17	11/15	5113
18	216	D5B41
19	3/5	20B3
20	3/8	31D4
21	7/10	05A2
22	2 students	4BA3
23	5/12	D0D2
24	18	1BA3
25	1/4	0CA3
26	1/32	5513
27	4	A54C
28	10	20D4
29	0	4BC2
30	3/5	A222

Solutions

(1) **2000** **ID: [5CA3]**
Since there are 10 digits, the number of sequences of 3 digits is $10^3 = 1000$. Since we have two choices of letter sequence, the number of plate possibilities is $2(1000) = \boxed{2000}$.

(2) **56** **ID: [15B41]**
The number of sets the curator can choose is $\binom{8}{5} = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2} = \boxed{56}$.

(3) **9** **ID: [ACA3]**
No solution is available at this time.

(4) **13** **ID: [ABA3]**
No solution is available at this time.

(5) **3 dates** **ID: [02D4]**
No solution is available at this time.

(6) 20 **ID: [2513]**
No solution is available at this time.

(7) **36** **ID: [B213]**
The first three digits must be 7, 8, 9 for them to be increasing. For the last two digits, we can choose any two distinct digits less than 9, place the larger one in the tens place and the smaller one in the units place, and obtain a mountain number. There are $\frac{9}{2} = 36$ ways to choose two digits less than 9, so the answer is $\boxed{36}$.

(8) **1/54** **ID: [C413]**

For the sum to be 18, all three dice must come up 6. This happens with probability $\frac{1}{6^3} = \frac{1}{216}$.

For the sum to be 17, one die must come up 5 and the other two must be 6. There are 3 ways to choose the die that comes up as a 5, so the probability here is $\frac{3}{6^3} = \frac{3}{216}$.

Thus the total probability is

$$\frac{1}{216} + \frac{3}{216} = \frac{4}{216} = \boxed{\frac{1}{54}}$$

(9) **420** **ID: [B0B3]**

No solution is available at this time.

(10) **121/400** **ID: [B5B41]**

No solution is available at this time.

(11) 10 **ID: [A413]**

We know that the hundreds digit is 4. If we choose two distinct digits greater than 4, we can place the smaller one in the tens place and the larger one in the units place and have a valid salary. There are $\binom{5}{2} = 10$ ways to choose two such digits, so there are $\boxed{10}$ possibilities.

(12) **9/16** **ID: [5CC2]**

No solution is available at this time.

(13) **5000** **ID: [A5B41]**

There are five numbers that the code could start with (1, 3, 5, 7, or 9), and each digit of the code thereafter can be any one of ten numbers (0 through 9). Thus, there are $5 \cdot 10^3 = \boxed{5000}$ possible codes.

(14) **18 routes** **ID: [D5C1]**

We have a choice of going to D through B or through C. If we use B, there are 2 ways to get to B and then 3 ways to get from B to D, for 6 routes. If we use C, there are 3 ways to get from A to C and then 4 ways to get from C to D, for 12 routes. So there are a total of $6 + 12 = \boxed{18}$ routes.

(15) **84** **ID: [42AC]**
No solution is available at this time.

(16) **19** **ID: [51B3]**
No solution is available at this time.

(17) **11/15** **ID: [5113]**
We consider all the two-element subsets of the six-element set $\{1, 2, 3, 4, 5, 6\}$. There are $\binom{6}{2} = 15$ such subsets. And of these, only the subsets $\{2, 4\}$, $\{2, 6\}$, $\{3, 6\}$, $\{4, 6\}$ are not relatively prime. So the probability of the two-element subset's elements having greatest common factor one is $1 - \frac{4}{15} = \boxed{\frac{11}{15}}$.

(18) **216** **ID: [D5B41]**
No solution is available at this time.

(19) **3/5** **ID: [20B3]**
No solution is available at this time.

(20) **3/8** **ID: [31D4]**
No solution is available at this time.

(21) **7/10** **ID: [05A2]**
We can use complementary probability to determine that the probability of its not raining tomorrow is $1 - \frac{3}{10} = \boxed{\frac{7}{10}}$.

(22) **2 students** **ID: [4BA3]**
No solution is available at this time.

(23) **5/12 ID: [D0D2]**
There are $3+2+7=12$ shirts to choose from. A total of $2+3=5$ of these, all the hockey and football shirts, are not baseball shirts. So, the probability of not getting a baseball shirt is $\boxed{\frac{5}{12}}$.

(24) **18 ID: [1BA3]**
No solution is available at this time.

(25) **1/4 ID: [0CA3]**
No solution is available at this time.

(26) **1/32 ID: [5513]**
Since all sequences of 5 answers are equally likely, the probability of any given 5-answer sequence is simply $\frac{1}{2^5}$, since each answer is equally likely to be true or false. So, the answer evaluates to $\frac{1}{2^5} = \boxed{\frac{1}{32}}$.

(27) **4 ID: [A54C]**
No solution is available at this time.

(28) **10 ID: [20D4]**
No solution is available at this time.

(29) **0 ID: [4BC2]**
Because the women are of different heights, any handshake will take place between two people, one of whom is taller than the other. Of course, the shorter of the two will not participate in the handshake because her handshake partner is not shorter than herself. Applying this logic to all of the pairs, there can be $\boxed{0}$ handshakes.

(30) **3/5 ID: [A222]**
There are 12 even numbers and 5 multiples of 5 in the range 1 to 25. However, we have double-counted 10 and 20, which are divisible by both 2 and 5. So the number of good outcomes is $12+5-2=15$ and the probability is $\frac{15}{25} = \boxed{\frac{3}{5}}$.