Mathcounts / AMC 8 Advanced (Week 6)

(1) _____ What is the probability that the square root of a randomly selected two-digit whole number is less than eight? Express your answer as a common fraction.

(2) _____ Express the next term in the sequence as a decimal:

$$0, 0.5, 0.\overline{6}, 0.75 \ldots$$

(3) _____ For what value of n is the four-digit number $712n$, with units digit n, divisible by 18?

(4) _____ What is the value of the least base ten number which requires six digits for its binary representation?

(5) _____ One number is chosen from the first three prime numbers, and a second number is chosen from the first three positive composite numbers. What is the probability that their sum is greater than or equal to 9? Express your answer as a common fraction.

(6) _____ Tim and Kurt are playing a game in which players are awarded either 3 points of 7 points for a correct answer. What is the greatest score that cannot be attained?

(7) _____ Brianna was having a party for 95 guests. Hot dogs are sold in package of eight; buns are sold in packages of ten. If she purchased the minimum number of packages of each to guarantee at least one hot dog and one bun for each guest, how many more buns than hot dogs did she buy?

(8) _____ *Bertrand's Postulate* states that there is at least one prime number between any counting number and its double. How many prime numbers are there between 25 and 50?

(9) _____ For what value of n is the five-digit number 7n933 divisible by 33? (Note: the underlining is meant to indicate that the number should be interpreted as a five-digit number whose ten thousands digit is 7, whose thousands digit is n, and so on).

(10) _____ The number 100 can be written as the sum of a 1-digit prime number and a 2-digit prime number. What is the product of these prime numbers?

(11) _____ The integer x has 12 factors. The numbers 12 and 15 are factors of x. What is x?

(12) _____ For what value of x is $2^3 \times 3^x = 72$?

(13) _____ Three consecutive positive prime numbers have a sum that is a multiple of 7. What is the least possible sum?

(14) _____ How many positive perfect squares less than 300 are multiples of 9?

(15) _____ Find the integer n such that $n \times 3^4 \times 7^5 = 21^6$.

(16) _____ What is the sum of all integer values of x such that $\frac{67}{2x-23}$ is an integer?

(17) _____ What is the remainder when 5^{30} is divided by 7?

(18) _____ Let A = 1, B = 2, C = 3, ..., Z = 26. The product value of a word is equal to the product of the values of its letters. For example, CAB has a product value of $3 \times 1 \times 2 = 6$. What common English word has a product value of 715?

(19) _____ A group of N students, where $N < 50$, is on a field trip. If their teacher puts them in groups of 8, the last group has 5 students. If their teacher instead puts them in groups of 6, the last group has 3 students. What is the sum of all possible values of N?

(20) _____ What perfect-square integer is closest to 273?

(21) _____ Ten days from Thursday, it will be Sunday. What day of the week will it be in 1,000,000 days from Thursday?

21 (22) _____ What is the minimum number of United States coins Samantha needs (pennies, nickels, dimes, quarters, half-dollars) to ensure she is capable of making change for any amount of money from one cent to 99 cents?

22 (23) _____ July 4, 1903, was a Thursday. On what day of the week was July 4, 1904?

23 (24) _____ The product of the base seven numbers 24_7 and 30_7 is expressed in base seven. What is the base seven sum of the digits of this product?

24 (25) _____ What is the sum of all positive integer values of n such that $\frac{n+18}{n}$ is an integer?

25 (26) _____ What is the least four-digit positive integer, with all different digits, that is divisible by each of its digits?

26 (27) _____ What is the least natural number that will have a remainder of 3 when divided by any of the numbers 4, 5, 6, 8 or 10?

27 (28) _____ When its digits are reversed, a particular positive two-digit integer is increased by 20%. What is the original number?

28 (29) _____ What is the least possible positive integer with exactly five distinct positive factors?

29 (30) _____ What is the units digit of $1! + 3! + 5! + 7! + 9! + 11!$?