Mathcounts / AMC 8 Beginner - HW 1

Name ____________________

(1) __________ Find the smallest positive integer, N, such that the product of 135 and N is a perfect square.

(2) __________ What common fraction (that is, a fraction reduced to its lowest terms) is equivalent to $.3\overline{25}$?

(3) __________ How many positive integer values of x are there such that $\frac{36}{x+3}$ is an integer?

(4) __________ What is the sum of the positive factors of 24?

(5) __________ What is the square root of the difference between the square of 2.5 and the reciprocal of $\frac{4}{9}$?

(6) __________ How many different ways can 36 identical chairs be placed in rows if all rows have the same number of chairs, each chair is in exactly one row, and no row has more than 20 chairs or less than 3 chairs?

(7) __________ What is the smallest prime number greater than 120?

(8) __________ Express as a fraction in lowest terms: $0.\overline{1} + 0.\overline{01}$

(9) __________ Express one-half of 2^8 as a power of 2.

(10) __________ What is the greatest common factor of 518 and 294?

(11) __________ In the addition problem each letter represents a distinct digit. What is the numerical value of E?

$$\begin{array}{ccc} E & G & M \\ + & G & M \\ \hline G & M & M \end{array}$$

(12) __________ What is the sum of the prime factors of 1001?

(13) __________ Express $.\overline{28}$ as a common fraction.

(14) __________ The number 13 is prime. If you reverse the digits you also obtain a prime number, 31. What is the larger of the pair of primes that satisfies this condition and has a sum of 110?

(15) __________ Find the smallest positive integer divisible by 10, 11, and 12.

(16) __________ Find the remainder when 1992^2 is divided by 9.

(17) __________ Express the reciprocal of 0.0625 in decimal form.

(18) __________ Kim's birthday was 200 days ago. Today is Wednesday. On what day of the week did his birthday fall?

(19) __________ A school band found they could arrange themselves in rows of 6, 7, or 8 with no one left over. What is the minimum number of students in the band?

(20) __________ Express $3.\overline{7}$ as a common fraction.

Answer Sheet

Number	Answer	Problem ID
1	15	AC1B
2	161/495	DBCA
3	6	4BAB
4	60	CCAB
5	2	300B
6	6	DA0B
7	127	1D2B
8	4/33	21BB
9	2^7	4BCA
10	14	BB412
11	4	0AC41
12	31	ACD22
13	28/99	BDC41
14	73	1DAA1
15	660	1C412
16	0	4DAA1
17	16	A02B
18	Saturday	4DC51
19	168	0DAA1
20	34/9	1C1B

Solutions

(1) **15 ID: [AC1B]**
No solution is available at this time.

(2) **161/495 ID: [DBCA]**
To express the number $0.3\overline{25}$ as a fraction, we call it x and subtract it from $100x$:

$$\begin{array}{rrcl} & 100x & = & 32.5252525\ldots \\ - & x & = & 0.3252525\ldots \\ \hline & 99x & = & 32.2 \end{array}$$

This shows that $0.3\overline{25} = \frac{32.2}{99} = \frac{322}{990} = \boxed{\frac{161}{495}}$.

(Note: This last fraction is in lowest terms, because $161 = 7 \cdot 23$ and $495 = 3^2 \cdot 5 \cdot 11$.)

(3) **6 ID: [4BAB]**
No solution is available at this time.

(4) **60 ID: [CCAB]**
The prime factorization of 24 is $2^3 \cdot 3$. It follows that the sum of the divisors of 24 is equal to $(1 + 2 + 2^2 + 2^3)(1 + 3)$, as each factor of 24 is represented when the product is expanded. It follows that the sum of the factors of 24 is $(1 + 2 + 4 + 8)(1 + 3) = (15)(4)$, or $\boxed{60}$.

(5) **2 ID: [300B]**
No solution is available at this time.

(6) **6 ID: [DA0B]**
No solution is available at this time.

(7) **127 ID: [1D2B]**
No solution is available at this time.

(8) **4/33 ID: [21BB]**

We begin by realizing that $0.\overline{1} = 0.\overline{11}$, so $0.\overline{1} + 0.\overline{01} = 0.\overline{11} + 0.\overline{01} = 0.\overline{12}$. (Note that this can be done because there is no carrying involved.)

To express the number $0.\overline{12}$ as a fraction, we call it x and subtract it from $100x$:

$$\begin{array}{rrcl} & 100x & = & 12.121212\ldots \\ - & x & = & 0.121212\ldots \\ \hline & 99x & = & 12 \end{array}$$

This shows that $0.\overline{12} = \frac{12}{99}$.

But that isn't in lowest terms, since 12 and 99 share a common factor of 3. We can reduce $\frac{12}{99}$ to $\boxed{\frac{4}{33}}$, which is in lowest terms.

(9) 2^7 **ID: [4BCA]**

No solution is available at this time.

(10) **14 ID: [BB412]**

Factoring both numbers, we find that $518 = 2 \cdot 7 \cdot 37$ and $294 = 2 \cdot 3 \cdot 7^2$. Taking the lowest common powers of both, we see that the greatest common factor of the two numbers is $2 \cdot 7 = \boxed{14}$.

(11) **4 ID: [0AC41]**

We first look at the hundreds place. Since $E \neq G$, it must be that $E + 1 = G$ in order to get G in the hundreds place. Since a 1 is carried over, we have $G + G = 10 + M$. Now we look at the units place. Either $M + M = M$ or $M + M = 10 + M$. In the second case, $2M = 10 + M \quad \Rightarrow M = 10$, which is not a possible digit. So it must be that $2M = M$, which is only possible if $M = 0$. Now $2G = 10 \quad \Rightarrow G = 5$ and $E + 1 = G \quad \Rightarrow E = 4$. The numerical value of E is $\boxed{4}$. We can check that $450 + 50 = 500$, which matches the digits in the addition problem.

(12) **31 ID: [ACD22]**

No solution is available at this time.

(13) **28/99 ID: [BDC41]**
If $x = .\overline{28}$, then $100x = 28.\overline{28}$. Notice that we can eliminate the repeating decimal by subtracting $.\overline{28}$ from $28.\overline{28}$. We have $100x - x = 99x = 28$, so $x = \frac{28}{99}$. The repeating decimal can be expressed as the fraction $\boxed{\frac{28}{99}}$.

(14) **73 ID: [1DAA1]**
No solution is available at this time.

(15) **660 ID: [1C412]**
Factoring all three numbers, we find that $10 = 2 \cdot 5$, $11 = 11$, and $12 = 2^2 \cdot 3$. Taking the highest power of each, we see that the least common multiple of the three numbers is $2^2 \cdot 3 \cdot 5 \cdot 11 = 60 \cdot 11 = \boxed{660}$.

(16) **0 ID: [4DAA1]**
No solution is available at this time.

(17) **16 ID: [A02B]**
No solution is available at this time.

(18) **Saturday ID: [4DC51]**
Noting that
$$200 = 196 + 4 = 28 \cdot 7 + 4,$$
we see that Kim's birthday was 29 weeks and 4 days ago. Since today is Wednesday, Kim's birthday fell on a $\boxed{\text{Saturday}}$.

(19) **168 ID: [0DAA1]**
The problem specifies that the number of students in the band is a multiple of 6, 7, and 8. Therefore, we are looking for the least common multiple of 6, 7, and 8. Prime factorizing the three numbers and taking the maximum exponent for each prime, we find that the least common multiple is $2^3 \cdot 3 \cdot 7 = \boxed{168}$.

(20) **34/9** **ID: [1C1B]**

To express the number $3.\overline{7}$ as a fraction, we let $x = 3.\overline{7}$, so $10x = 37.\overline{7}$ and:

$$\begin{array}{rrcl} & 10x & = & 37.77777\ldots \\ - & x & = & 3.77777\ldots \\ \hline & 9x & = & 34 \end{array}$$

This shows that $3.\overline{7} = \boxed{\dfrac{34}{9}}$.