Number Theory 3A2

Name _____

(1) _____   Find the integer n such that $n \times 3^4 \times 7^5 = 21^6$.

(2) _____   What is the probability that the square root of a randomly selected two-digit whole number is less than eight? Express your answer as a common fraction.

(3) _____   What is the least natural number that has exactly four distinct positive factors?

(4) _____   For what value of n is the five-digit number $\underline{7n933}$ divisible by 33? (Note: the underlining is meant to indicate that the number should be interpreted as a five-digit number whose ten thousands digit is 7, whose thousands digit is n, and so on).

(5) _____   What is the least four-digit positive integer, with all different digits, that is divisible by each of its digits?

(6) _____   Ten days from Thursday, it will be Sunday. What day of the week will it be in 1,000,000 days from Thursday?

(7) _____   Tim and Kurt are playing a game in which players are awarded either 3 points of 7 points for a correct answer. What is the greatest score that cannot be attained?

(8) _____   How many positive perfect squares less than 300 are multiples of 9?

(9) _____ Express the next term in the sequence as a decimal:

$$0, 0.5, 0.\overline{6}, 0.75\ldots$$

(10) _____ What perfect-square integer is closest to 273?

(11) _____ Brianna was having a party for 95 guests. Hot dogs are sold in package of eight; buns are sold in packages of ten. If she purchased the minimum number of packages of each to guarantee at least one hot dog and one bun for each guest, how many more buns than hot dogs did she buy?

(12) _____ One number is chosen from the first three prime numbers, and a second number is chosen from the first three positive composite numbers. What is the probability that their sum is greater than or equal to 9? Express your answer as a common fraction.

(13) _____ July 4, 1903, was a Thursday. On what day of the week was July 4, 1904?

(14) _____ When its digits are reversed, a particular positive two-digit integer is increased by 20%. What is the original number?

(15) _____ What is the least positive integer with exactly 10 factors?

(16) _____ The length of the year on the planet Mars is exactly 697 days. If Mars has a calendar with a 12-day week, and year 0 begins on the first day of the week, what is the next year which will begin on the first day of the week?

(17) _____ What is the minimum number of United States coins Samantha needs (pennies, nickels, dimes, quarters, half-dollars) to ensure she is capable of making change for any amount of money from one cent to 99 cents?

(18) _____ Three consecutive positive prime numbers have a sum that is a multiple of 7. What is the least possible sum?

(19) _____ The product of positive integers x, y and z equals 2004. What is the minimum possible value of the sum $x + y + z$?

(20) _____ Let A = 1, B = 2, C = 3, ..., Z = 26. The product value of a word is equal to the product of the values of its letters. For example, CAB has a product value of $3 \times 1 \times 2 = 6$. What common English word has a product value of 715?

(21) _____ What is the least natural number that will have a remainder of 3 when divided by any of the numbers 4, 5, 6, 8 or 10?

(22) _____ What is the units digit of $1! + 3! + 5! + 7! + 9! + 11!$?

(23) _____ What is the sum of all positive integer values of n such that $\frac{n+18}{n}$ is an integer?

(24) _____ The number 100 can be written as the sum of a 1-digit prime number and a 2-digit prime number. What is the product of these prime numbers?

(25) _____ *Bertrand's Postulate* states that there is at least one prime number between any counting number and its double. How many prime numbers are there between 25 and 50?

(26) _____ What is the sum of the whole-number factors of 24?

(27) _____ The integer 6 has four positive divisors: 1, 2, 3 and 6. What is the smallest positive integer with exactly five positive divisors?

(28) _____ A Chicago subway card is worth \$11. For each ride on the subway, either \$1.50 or \$1.80 is deducted from the value remaining on the card. What is the least number of cents that could be left on the card after any number of rides?

(29) _____ A group of N students, where $N < 50$, is on a field trip. If their teacher puts them in groups of 8, the last group has 5 students. If their teacher instead puts them in groups of 6, the last group has 3 students. What is the sum of all possible values of N?

(30) _____ The product of the base seven numbers 24_7 and 30_7 is expressed in base seven. What is the base seven sum of the digits of this product?