Mathcounts / AMC 8

Name _____

(1) _____ More than 20 and less than 50 people went to a concert. The price of each ticket was the same whole number of dollars, and the total cost was \$377. How many dollars did each individual ticket cost?

(2) _____ The sum of each pair of positive integers x, y that satisfy $3x + 4y = 50$ is computed. What is the largest such sum?

(3) _____ The numbers 3, 5 and 7 are important in many aspects of Japanese life. Most positive integers can be expressed as a sum of only 3's, 5's and 7's. For example, 15 can be expressed as $5 + 5 + 5$ or $3 + 5 + 7$. What is the greatest even integer that cannot be expressed as a sum of 3's, 5's and 7's?

(4) _____ How many possible products can be made from two or more of the numbers 2, 3, 4, 5?

(5) _____ In professional football it is possible to score 6 points (a touchdown), 3 points (a field goal), or 2 points (a safety). If a touchdown is scored it is possible to score an additional point (the point after). What is the largest total score that cannot be achieved in football.

(6) _____ What is the sum of all positive odd multiples of 3 that are less than 100?

(7) _____ How many two-digit numbers less than 50 have an odd number of factors?

(8) _____ What is the sum of the reciprocals of the three smallest prime numbers expressed as a ~~mixed number~~ common fraction?

(9) _____ Michael wants to mail a package using exact postage. If he chooses to use only 23-cent stamps, he needs 9 more stamps than if he uses only 32-cent stamps. How many dollars will it cost Michael to mail the package? Express your answer to the nearest cent.

(10) _____ A solar system contains four planets and a sun as shown. Planet A completes an orbit around the sun in 20 months; planet B completes an orbit in 36 months; planet C completes an orbit in 40 months; and planet D completes an orbit in 48 months. If the four planets lie along a straight line in May of the year 2000, what is the next year that they will again lie along the same line? Assume that the orbits are circular and that a year has exactly 12 months.

(11) _____ For how many prime numbers between 10 and 100 is the last digit also a prime number?

(12) _____ A positive integer has all single-digit prime factors. None of its prime factors occurs more than twice in the prime factorization, but the number is not a perfect square. What is the greatest possible number with these properties?

(13) _____ How many positive three-digit prime numbers less than 200 have two digits the same?

(14) _____ What is the remainder when 5^{30} is divided by 7?

(15) _____ Two joggers run around a one-mile track in the same direction. One jogger does a mile in 6 minutes, while the second jogger takes 9 minutes. Given that they start at the same place on the track and at the same time, what is the smallest number of minutes that will elapse before they are again together at their starting place?

(16) _____ Ioana has three ropes whose lengths are 39 inches, 52 inches and 65 inches. She wants to cut the ropes into equal length pieces for magic tricks. No rope is to be wasted. What is the greatest number of inches possible in the length of each piece?

(17) _____ Mohan is selling cookies at the economics fair. As he decides how to package the cookies, he finds that when he bags them in groups of 4, he has 3 left over. When he bags them in groups of 5, he has 2 left over. When he bags them in groups of 7, he has 4 left over. What is the least number of cookies that Mohan could have?

(18) _____ Express the sum of $0.\overline{31}$ and $0.\overline{8}$ as a common fraction.

(19) _____ The Goldbach Conjecture, which has not been proven, states that every even integer bigger than 2 is expressible as the sum of two prime numbers. In how many ways can the number 36 be expressed as the sum of two primes?

(20) _____ What is the positive difference between the two greatest prime factors of 2730?

(21) _____ What is the smallest positive integer greater than 1 such that the division by each of 4, 5, 6, 9, and 10 gives a remainder of 1?

(22) _____ How many positive integers t will make the expression $\frac{19}{t} + \frac{5}{t}$ have an integral value?

(23) _____ How many positive integers less than 400 are divisible by both 7 and 11?

(24) _____ Pierre Fermat, a French mathematician who lived from 1601-1665, theorized that every prime number of the form $4n+1$, for $n \geq 1$, is the sum of two squares in one and only one way. For instance, 13 can be written in the form $4(3)+1$ and is equal to $9+4$. Two squares can be added to form the largest prime number less than 100 that can be written in the form $4n+1$. What is the positive difference between these two squares?

(25) _____ The product of two whole numbers is 1000. If neither of the numbers is a multiple of 10, what is their sum?

(26) _____ What positive, two-digit number, when doubled and added to two, gives a sum that is the original number with its digits reversed?