Mathcounts / AMC 8

Name _____

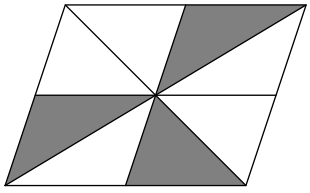
(1) _____ Angle A of parallelogram $ABCD$ measures 135°. Find the number of degrees in the difference between angle A and the smallest of the other three angles of the parallelogram.

(2) _____ A man has a 10 m $\times$ 10 m square garden. In the center is a 2 m $\times$ 2 m square patch which he cannot use. He divides his usable space into four congruent rectangular patches. What is the number of meters in the perimeter of each rectangle?

(3) _____ How many $\frac{1}{2}$-inch cubes are needed to make 1 cubic foot?

(4) _____ Find the number of square meters in the area of a regular hexagon inscribed in a circle of diameter 12 meters.

(5) _____ In the parallelogram shown, the midpoints of opposite sides are connected with line segments. Likewise, the opposite vertices are connected. What is the probability that a point randomly selected inside the parallelogram will lie inside one of the shaded regions? Express your answer as a common fraction.



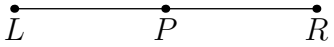
(6) _____ Micah is building a corral for his pet buffalo. He equally spaces and consecutively numbers the posts as he pounds them around a circle. The seventh and seventeenth posts lie on the same diameter. How many posts are there?

(7) _____ One side of a square A is a diagonal of square B. What is the ratio of the area of square B to the area of square A? Express your answer as a common fraction.

(8) _____ A caterer wants to cut a circular pie into congruent wedges so that each person gets the same percent of the pie. If each person is to receive 12.5% of the pie, how many degrees should form the central angle of the wedge?

(9) _____ The radius of a right circular cylinder is decreased by 20% and its height is increased by 25%. What is the absolute value of the percent change in the volume of the cylinder?

(10) _____ Point P is the midpoint of segment LR and $LR = 8$ cm. If L is rotated clockwise around point P until equilateral triangle LPR can be formed, through how many degrees is L rotated?



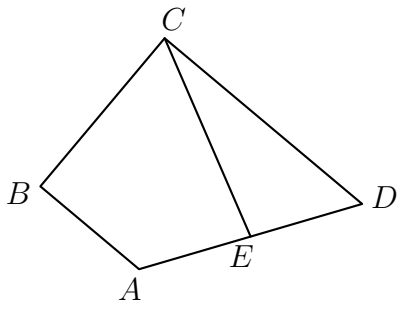
(11) _____ Given that it takes four miles of fence to enclose a square 640-acre field, how many acres are in a square field enclosed by two miles of fence?

(12) _____ A triangle is made of wood sticks of lengths 8, 15 and 17 inches joined end-to-end. Pieces of the same integral length are cut from each of the sticks so that the three remaining pieces can no longer form a triangle. How many inches are in the length of the smallest piece that can be cut from each of the three sticks to make this happen?

(13) _____ Two concentric circles are drawn such that the inner circle coveres 81% of the area of the outer circle. Given that the radius of the outer circle is ten units, how many units are in the radius of the inner circle?

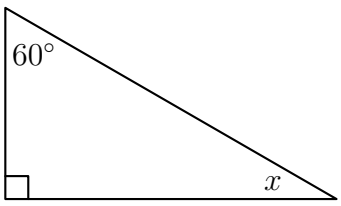
(14) _____ The radius of one of two concentric circles is 4 cm long, and the other is 9 cm long. How many centimeters are in the circumference of a circle which is concentric to the given circles and whose radius is equal to the average of the other two radii? Express your answer in terms of π.

(15) _____ In the diagram, $AB \perp BC$ and $BC \perp CD$. $AB = 8''$, $BC = 12''$, and $CD = 16''$. E is the midpoint of AD. How many square inches are in the area of $ABCD$?



(16) _____ A popcorn company wants to create a circular cylindrical container with diameter 10 inches and volume 1256 cubic inches. How many inches should the height of the container be? Express your answer to the nearest inch.

(17) _____ How many degrees are in the sum of the complement and supplement of x?



(18) _____ The area of a square is 49 square inches. Find the number of inches in the length of a diagonal. Express your answer in simplest radical form.

(19) _____ What is the common name for an equiangular quadrilateral?

(20) _____ Given a cube with volume 40 cubic centimeters, find the number of centimeters in the length of an edge.

(21) _____ A square is inscribed in a circle. The area of the square is 64 square inches. How many square inches are in the area of the circle? Express your answer in terms of π.

(22) _____ The lengths of the sides of an $8x8$ cm^2 square are increased by 2 cm each. The area of the square has been increased by what percent? Express your answer to the nearest whole percent.

(23) _____ Each dimension of a parallelogram is increased to four times its original size to form a similar parallelogram. If the new parallelogram has an area of 880 square units, what is the number of square units in the area of the original parallelogram?

(24) _____ By what number is the number of cubic inches in the volume of a sphere multiplied when the number of inches in the radius is doubled?

(25) _____ An expensive perfume is packaged in a box that has an octagonal base with area of ten square inches. The box is a prism. Given that the height of the box is four inches, what is the number of cubic inches in its volume?

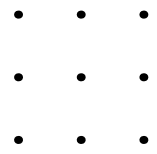
(26) _____ What is the number of centimeters in the length of a longer side of a rectangle which has a perimeter of 64 centimeters and an area of 192 square centimeters?

(27) _____ Regular pentagon $ABCDE$ is inscribed in circle O. What is the number of degrees in the measure of $\angle OCE$?

(28) _____ Point B is the midpoint of $\overline{PQ}$. $\overline{PQ}$ is eight centimeters longer than $\overline{PB}$. What is the number of centimeters in the length of $\overline{QB}$?

(29) _____ What is the length of a diameter of a circle which has an area of 36π square units?

(30) _____ How many non congruent quadrilaterals can be formed on this square dot grid if each vertex must coincide with a dot?



Answer Sheet

Number	Answer	Problem ID
1	90	4BB22
2	20 meters	A1421
3	13824 cubes	B2041
4	$54\sqrt{3}$ square meters	03041
5	3/8	20DD
6	20 posts	00441
7	1/2	D2B11
8	45	0CB22
9	20 percent	52041
10	120 degrees	0B1B
11	160 acres	23041
12	6	C33D
13	9 units	52B21
14	13π	05531
15	144	3BB22
16	16	5DC5
17	210	BAB22
18	$7\sqrt{2}$	BCB22
19	Rectangle	5D011
20	$\sqrt[3]{30}$ cm	3D1B
21	32π	B0DD
22	56	2BB22
23	55	B2C5
24	8	C2021
25	40	C43D
26	24 cm	B2B11
27	18	0DC5
28	8	543D
29	12	053D
30	16 quadrilaterals	BC4B

Solutions

(1) **90 ID: [4BB22]**
No solution is available at this time.

(2) **20 meters ID: [A1421]**
No solution is available at this time.

(3) **13824 cubes ID: [B2041]**
No solution is available at this time.

(4) $54\sqrt{3}$ **square meters ID: [03041]**
No solution is available at this time.

(5) **3/8 ID: [20DD]**
No solution is available at this time.

(6) **20 posts ID: [00441]**
No solution is available at this time.

(7) **1/2 ID: [D2B11]**
No solution is available at this time.

(8) **45 ID: [0CB22]**
No solution is available at this time.

(9) **20 percent ID: [52041]**
Let the original radius and height be r and h respectively, so the original volume is $\pi r^2 h$.
The new radius and height are $\frac{4}{5}r$ and $\frac{5}{4}h$ respectively, so the new volume is $\pi\left(\frac{4}{5}r\right)^2 \frac{5}{4} = \frac{4}{5}\pi r^2 h$, which is 20% less than the original volume.
Hence the desired percent change is $\boxed{20}$ percent.

(10) **120 degrees ID: [0B1B]**
No solution is available at this time.

(11) **160 acres ID: [23041]**
No solution is available at this time.

(12) **6 ID: [C33D]**
Our current triangle lengths are 8, 15, and 17. Let us say that x is the length of the piece that we cut from each of the three sticks. Then, our lengths will be $8-x$, $15-x$, and $17-x$. These lengths will no longer form a triangle when the two shorter lengths added together is shorter than or equal to the longest length. In other words, $(8-x)+(15-x) \leq (17-x)$. Then, we have $23-2x \leq 17-x$, so $6 \leq x$. Therefore, the length of the smallest piece that can be cut from each of the three sticks is $\boxed{6}$ inches.

(13) **9 units ID: [52B21]**
No solution is available at this time.

(14) 13π **ID: [05531]**
No solution is available at this time.

(15) **144 ID: [3BB22]**
No solution is available at this time.

(16) **16 ID: [5DC5]**
No solution is available at this time.

(17) **210 ID: [BAB22]**
No solution is available at this time.

(18) $7\sqrt{2}$ **ID: [BCB22]**
No solution is available at this time.

(19) **Rectangle ID: [5D011]**
No solution is available at this time.

(20) $\sqrt[3]{30}$ **cm ID: [3D1B]**
No solution is available at this time.

(21) 32π **ID: [B0DD]**
No solution is available at this time.

(22) **56 ID: [2BB22]**
No solution is available at this time.

(23) **55 ID: [B2C5]**
No solution is available at this time.

(24) **8 ID: [C2021]**
No solution is available at this time.

(25) **40 ID: [C43D]**
No solution is available at this time.

(26) **24 cm ID: [B2B11]**
No solution is available at this time.

(27) **18 ID: [0DC5]**
No solution is available at this time.

(28) **8 ID: [543D]**
No solution is available at this time.

(29) **12 ID: [053D]**
No solution is available at this time.

(30) **16 quadrilaterals ID: [BC4B]**

No solution is available at this time.