Mathcounts / AMC 8

1. _____ What is the smallest integer greater than 2 that will have a remainder of 2 when divided by any member of the set $\{3, 4, 5, 6, 8\}$?

2. _____ The total area of four squares, each with whole-number side measurements, is 23 square inches. In inches, what is the positive difference between the perimeter of the largest square and the perimeter of the smallest square?

3. _____ John, Joe and James go fishing. At the end of the day, John comes to collect his third of the fish. However, there is one too many fish to make equal thirds, so John throws it out, takes his third and leaves. Joe comes to get his fish without realizing John has already taken his third. He notices there is one too many fish to make equal thirds so he throws one fish out, takes his third and leaves. James notice that there is one too many fish to make equal thirds so he throws one out, takes his fish and leaves. Assuming no fish are divided into pieces, what is the minimum possible number of fish before John threw out the first fish?

4. _____ What is the tens digit of the product of the first six prime numbers?

5. _____ What is the sum of the three distinct prime factors of 47,432?

6. _____ Cards are numbered from 1 to 100. One card is removed and the values on the other 99 are added. The resulting sum is a multiple of 77. What number was on the card that was removed?

7. _____ What is the sum of the last two digits of this portion of the Fibonacci Factorial Series: $1! + 1! + 2! + 3! + 5! + 8! + 13! + 21! + 34! + 55! + 89!$?

8. _____ What is the remainder when the sum of the first 100 positive integers is divided by 9?

9. _____ What is the base two representation of the sum of the binary numbers 1011_2 and 111_2?

10. _____ The number 839 can be written as $19q + r$ where q and r are positive integers. What is the greatest possible value of $q - r$?

11. _____ What is the 453rd digit to the right of the decimal point in the decimal expansion of $\frac{6}{13}$?

12. _____ What is the sum of the tens digit and the units digit in the decimal representation of 9^{2004}?

13. _____ What is the largest perfect square factor of 1512?

14. _____ What is the number of positive factors of 648?

15. _____ What is the remainder when 10! is divided by 2^7? (Reminder: If n is a positive integer, then $n!$ stands for the product $1 \cdot 2 \cdot 3 \cdots (n-1) \cdot n$.)

16. _____ How many odd perfect square factors does $2^4 \times 3^6 \times 5^{10} \times 7^9$ have?

17. _____ Let m and n denote the greatest and least positive three-digit multiples of 7, respectively. What is the value of $m + n$?

18. _____ In any month with five Mondays, a school declares the fifth Monday a Marvelous Monday. School begins on Monday, September 4. What is the date of the first Marvelous Monday after school starts?

19. _____ The natural number n has exactly two natural-number factors. How many natural number factors does n^5 have?

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20. _____ A magic square is an array of numbers in which the sum of the numbers in each row, in each column, and along the two main diagonals are equal. The numbers in the magic square shown are not written in base 10. For what base will this be a magic square?
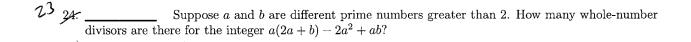
13	1	11
3	10	12
4	14	2

21. _____ What is the units digit of $(133^{13})^3$?

22. _____ If $2^x \cdot 9^y$ is equal to the four-digit number $2x9y$ where x is the hundreds digit and y is the units digit, what is the product of x and y?

23. _____ The whole numbers are written consecutively in the pattern below. In which row (A, B, C, D or E) will the number 500 be written?

Row E: 1

Row D: 2

Row C: 3

Row B: 4

Row A: 5, 6, 7

Row B: 8

Row C: 9

Row D: 10

Row E: 11, 12, 13

Row D: 14

Row C: 15

Row B: 16

Row A: 17,18,19

24. _____ Suppose a and b are different prime numbers greater than 2. How many whole-number divisors are there for the integer $a(2a + b) - 2a^2 + ab$?

25. _____ How many prime positive integers are divisors of 555?

26. _____ A positive multiple of 45 less than 1000 is randomly selected. What is the probability that it is a two-digit integer? Express your answer as a common fraction.

27. _____ What is the base five product of the numbers 121_5 and 11_5?