Mathcounts / AMC 8 (Week 11)

(1) _____ A jar contains two red marbles, three green marbles, ten white marbles and no other marbles. Two marbles are randomly drawn from this jar without replacement. What is the probability that these two marbles drawn will both be red? Express your answer as a common fraction.

(2) _____ On a given day, the probability of rain is 60%. What are the odds against rain on that day? Express your answer as ~~$a:b$ in simplest form.~~ a common fraction.

(3) _____ Five balls are numbered 1 through 5 and placed in a bowl. Josh will randomly choose a ball from the bowl, look at its number and then put it back into the bowl. Then Josh will again randomly choose a ball from the bowl and look at its number. What is the probability that the product of the two numbers will be even and greater than 10? Express your answer as a common fraction.

(4) _____ A fair, twenty-faced die has 19 of its faces numbered from 1 through 19 and has one blank face. Another fair, twenty-faced die has 19 of its faces numbered from 1 through 8 and 10 through 20 and has one blank face. When the two dice are rolled, what is the probability that the sum of the two numbers facing up will be 24? Express your answer as a common fraction.

(5) _____ A customer ordered 15 pieces of gourmet chocolate. The order can be packaged in small boxes that contain 1, 2 or 4 pieces of chocolate. Any box that is used must be full. How many different combinations of boxes can be used for the customer's 15 chocolate pieces? One such combination to be included is to use seven 2-piece boxes and one 1-piece box.

(6) _____ A play has two different male roles, two different female roles and two different roles that can be either gender. Only a man can be assigned to a male role, and only a woman can be assigned to a female role. If five men and six women audition, in how many ways can the six roles be assigned?

(7) _____ In a school of 250 students, everyone takes one English class and one history class each year. Today, 15 total students were absent from their English class and ten total students were absent from their history class. Five of the students were absent from both classes. If a student is chosen at random from this school, what is the probability that s/he was not absent from either class? Express your answer as a percent.

(8) _____ Four couples are at a party. Four people of the eight are randomly selected to win a prize. No person can win more than one prize. What is the probability that both members of at least one couple win a prize? Express your answer as a common fraction.

(9) _____ Ten distinct points are identified on the circumference of a circle. How many different convex quadrilaterals can be formed if each vertex must be one of these 10 points?

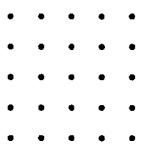
(10) _____ In an algebra class, half of the students are boys. One-third of the students are wearing glasses. Half the boys are wearing glasses. What fraction of the girls is wearing glasses? Express your answer as a common fraction.

(11) _____ Suelyn counts up from 1 to 9, and then immediately counts down again to 1, and then back up to 9, and so on, alternately counting up and down

$$(1, 2, 3, 4, 5, 6, 7, 8, 9, 8, 7, 6, 5, 4, 3, 2, 1, 2, 3, 4, \ldots).$$

What is the 1000^{th} integer in her list?

(12) _____ Set R is a set of rectangles such that (1) only the grid points shown here are used as vertices, (2) all sides are vertical or horizontal and (3) no two rectangles in the set are congruent. If R contains the maximum possible number of rectangles given these conditions, what fraction of the rectangles in set R are squares? Express your answer as a common fraction.



(13) _____ How many non-congruent triangles are there with sides of integer length having at least one side of length five units and having no side longer than five units?

(14) _____ The n^{th} triangular number is the number $1+2+3+4+\ldots+n$. Thus, the third triangular number is $1+2+3=6$. What is the sum of the fourth and fifth triangular numbers?

(15) _____ During a party, a total of 78 handshakes occurred. If each person shook hands once with each of the other people, how many people were at the party?

(16) _____ How many ordered pairs (x, y) satisfy BOTH conditions below?

- Condition I: $x = 1$ or $y = 0$ or $y = 2$
- Condition II: $x = 0$ or $x = 2$ or $y = 1$

(17) _____ A reference book lists a set of annual calendars. For any given year, there is a calendar in the set that corresponds to it. How many annual calendars must be included in the set in order to have a corresponding calendar for every possible year?

(18) _____ John, Kevin, Larry, Mary and Naomi all volunteered to do some math tutoring. If their teacher randomly chooses two of the five students, what is the probability of selecting the two girls? Express your answer as a common fraction.

(19) _____ Erika, who is 14 years old, flips a fair coin whose sides are labeled 10 and 20, and then she adds the number on the top of the flipped coin to the number she rolls on a standard die. What is the probability that the sum equals her age in years? Express your answer as a common fraction.

(20) _____ Chun is playing a game in which he can score 0, 2, 6, or 10 points in each round. After four rounds the sum of his scores is 16. How many different scoring sequences could have produced this sum? (One such sequence to include is 0, 6, 10, 0.)

(21) _____ Nathan will roll two six-sided dice. What is the probability that he will roll a number less than three on the first die and a number greater than three on the second die? Express your answer as a common fraction.

(22) _____ Thad has an unlimited supply of 3-cent and 4-cent stamps. If he has to put exactly 37 cents of postage on a letter, how many different combinations of 3-cent and/or 4-cent stamps could Thad use?

(23) _____ Two different integers are randomly chosen from the set

$$\{-5, -8, 7, 4, -2\}.$$

What is the probability that their product is negative? Express your answer as a common fraction.