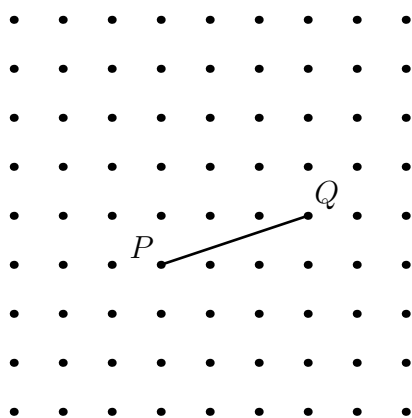


## Counting / Probability Worksheet (2A4)

- (1) \_\_\_\_\_ The digits 2, 3, 4, 7 and 8 will be put in random order to make a positive five-digit integer. What is the probability that the resulting integer will be divisible by 11? Express your answer as a common fraction.
- (2) \_\_\_\_\_ An o-Pod MP3 player stores and plays entire songs. Celeste has 10 songs stored on her o-Pod. The time length of each song is different. When the songs are ordered by length, the shortest song is only 30 seconds long and each subsequent song is 30 seconds longer than the previous song. Her favorite song is 3 minutes, 30 seconds long. The o-Pod will play all the songs in random order before repeating any song. What is the probability that she hears the first 4 minutes, 30 seconds of music - there are no pauses between songs - without hearing every second of her favorite song? Express your answer as a common fraction.
- (3) \_\_\_\_\_ In how many ways can 81 be written as the sum of three positive perfect squares if the order of the three perfect squares does not matter?
- (4) \_\_\_\_\_ Tamyra is making four cookies and has exactly four chocolate chips. If she distributes the chips randomly into the four cookies, what is the probability that there are no more than two chips in any one cookie? Express your answer as a common fraction.
- (5) \_\_\_\_\_ How many integers between 1000 and 9999 have exactly one pair of equal digits, such as 4049 or 9902, but not 4449 or 4040?
- (6) \_\_\_\_\_ What is the sum, in dollars, of the total values of every possible combination of three coins using only pennies, nickels, dimes and quarters?
- (7) \_\_\_\_\_ Six students are being grouped into three pairs to work on a science lab. How many different combinations of three pairs are possible?
- (8) \_\_\_\_\_ How many combinations of two or more consecutive positive integers have a sum of 45?

- (9) \_\_\_\_\_ The integers  $r$  and  $k$  are randomly selected, where  $-3 < r < 6$  and  $1 < k < 8$ . What is the probability that the division  $r \div k$  is an integer value? Express your answer as a common fraction.
- (10) \_\_\_\_\_ Four boys (Bs) and three girls (Gs) will be seated in a row. When a boy is next to a girl, we will call this a “meeting point.” When the seven kids are seated, there may be only one meeting point, as in BBBBGGG, or there may be as many as six meeting points, as in BGBGBGB (the arrangement Boy-Girl-Boy-Girl-Boy-Girl-Boy). Given all of the possible seating arrangements for these seven kids, what is the average number of meeting points per seating arrangement? Express your answer as a common fraction.
- (11) \_\_\_\_\_ How many positive integers divisible by 4 can be formed using the digits 1, 2, 3 and 4, each at most once for each integer?
- (12) \_\_\_\_\_ Two different prime numbers are selected from the first eight prime numbers. What is the probability that the sum of the two chosen prime numbers is greater than or equal to 16? Express your answer as a common fraction.
- (13) \_\_\_\_\_ Kevin will start with the integers 1, 2, 3 and 4 each used exactly once and written in a row in any order. Then he will find the sum of the adjacent pairs of integers in each row to make a new row, until one integer is left. For example, if he starts with 3, 2, 1, 4, and then takes sums to get 5, 3, 5, followed by 8, 8, he ends with the final sum 16. Including all of Kevin's possible starting arrangements of the integers 1, 2, 3 and 4, how many possible final sums are there?
- (14) \_\_\_\_\_ How many two-digit prime numbers less than 50 exist such that the difference of their digits is even?
- (15) \_\_\_\_\_ Suppose  $x$  and  $y$  are two distinct two-digit positive integers such that  $y$  is the reverse of  $x$ . (For example,  $x = 12$  and  $y = 21$  is one such combination.) How many different sums  $x + y$  are possible?
- (16) \_\_\_\_\_ Derek's phone number, 336 - 7624, has the property that the three-digit prefix, 336, equals the product of the last four digits,  $7 \times 6 \times 2 \times 4$ . How many seven-digit phone numbers beginning with 336 have this property?

- (17) \_\_\_\_\_ Julie baked cupcakes for her family at home and for a party at school. She iced 4 cupcakes with red frosting, 2 cupcakes with orange frosting, 2 with yellow, 2 with green, 3 with blue and the last 3 with violet frosting. Each cupcake is iced with exactly one color of frosting. Julie plans to take exactly 10 of the cupcakes to her party, and will take either all of the cupcakes of a particular color or none of the cupcakes of that color. How many different combinations of cupcakes could she take to her party?
- (18) \_\_\_\_\_ When five standard six-sided dice are rolled sequentially there are  $6^5 = 7776$  possible outcomes. For how many outcomes is the sum of the five rolled numbers exactly 27?
- (19) \_\_\_\_\_ On this 9 by 9 grid of lattice points, at how many different lattice points can a point  $R$  be placed such that it would be the third vertex of isosceles triangle  $PQR$  with  $PQ = PR$  or  $PQ = QR$ ? (The sum of the lengths of any two sides of a triangle is greater than the length of the remaining side.)



- (20) \_\_\_\_\_ How many positive four-digit odd integers can be created using only the digits 0, 1, 2, 3, 4 and 5 if repetition of digits is not allowed?
- (21) \_\_\_\_\_ Amanda has a 2-cm rod, a 5-cm rod and a 9-cm rod. She will randomly choose a fourth rod from a box containing rods of lengths 2, 4, 6, 8, 10, 12, 14, 16, 18 and 20 cm. What is the probability that Amanda's four rods will be able to form a convex quadrilateral, if the rods are to be connected endpoint to endpoint? Express your answer as a common fraction.

- (22) \_\_\_\_\_ In a school of 100 students, 90 study English, 75 study Spanish and 42 study French. Every student must study at least one of the three languages. What is the least possible number of students who could be studying all three languages?
- (23) \_\_\_\_\_ If  $1 \leq a \leq 10$  and  $1 \leq b \leq 36$ , for how many ordered pairs of integers  $(a, b)$  is  $\sqrt{a + \sqrt{b}}$  an integer?
- (24) \_\_\_\_\_ If  $x$  is an element of the set  $\{-1, 1, 2\}$  and  $y$  is an element of  $\{-2, -1, 0, 1, 2\}$ , how many distinct values of  $x^y$  are positive?
- (25) \_\_\_\_\_ Four packages are delivered to four houses, one to each house. If these packages are randomly delivered, what is the probability that exactly two of them are delivered to the correct houses? Express your answer as a common fraction.
- (26) \_\_\_\_\_ Mandvil has one standard quarter and one special quarter with heads on both sides. He selects one of these two coins at random, and without looking at it first, he flips the coin three times. If he flips a head three straight times, what is the probability that he selected the special quarter? Express your answer as a common fraction.
- (27) \_\_\_\_\_ A subset  $S$  of the set of integers from 50 to 100, inclusive, has the property that no two distinct elements of  $S$  sum to 130. What is the maximum possible number of elements in  $S$ ?
- (28) \_\_\_\_\_ Four red candies and three green candies can be combined to make many different flavors. Flavors are different if the percent red is different, so 3 red / 0 green is the same flavor as 2 red / 0 green; and likewise 4 red / 2 green is the same flavor as 2 red / 1 green. If a flavor is to be made using some or all of the seven candies, how many different flavors are possible?
- (29) \_\_\_\_\_ A box contains some green marbles and exactly four red marbles. The probability of selecting a red marble is  $x\%$ . If the number of green marbles is doubled, the probability of selecting one of the four red marbles from the box is  $(x - 15)\%$ . How many green marbles are in the box before the number of green marbles is doubled?

- (30) \_\_\_\_\_ An integer is randomly chosen from the integers 1 through 100, inclusive. What is the probability that the chosen integer is a perfect square or a perfect cube, but not both? Express your answer as a common fraction.