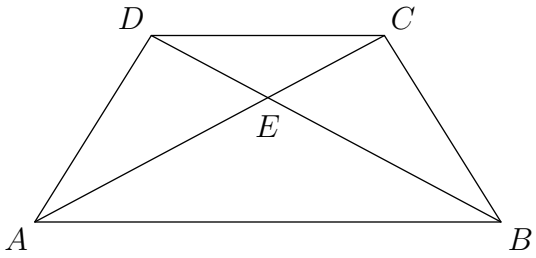
Geometry Worksheet (5A4)

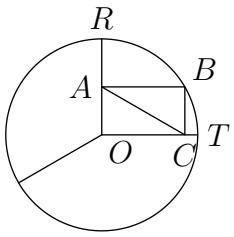
(1) _____ $ABCD$ is a trapezoid with the measure of base $\overline{AB}$ twice the measure of the base $\overline{CD}$. Point E is the point of intersection of the diagonals. The measure of diagonal $\overline{AC}$ is 11. Find the length of segment $\overline{EC}$. Express your answer as a common fraction.



(2) _____ Suppose the points A, B, C, D, E, and F are the vertices of a regular hexagon with sides of length 1 unit. What is AD?

(3) _____ A clock has struck 4 o'clock. In exactly how many minutes will the two hands first be at right angles? Express your answer as a mixed number.

(4) _____ In the figure below, O is the center of the circle and point B is on the circle, given $OR = 8$ and $CT = 2$, find the length of diagonal $\overline{AC}$ in rectangle $ABCO$.



(5) _____ Suppose the points A, B, C, D, E, and F are the vertices of a regular hexagon with sides of length 1 unit. What is AC?

(6) _____ What is the volume in cubic inches of a rectangular box that has sides of areas 48, 66, and 88 square inches?

(7) _____ Parallelogram $ABCD$ has its vertices at $A(2, -3)$, $B(4, 5)$, $C(-5, 5)$, and $D(-7, -3)$. What is the area of the parallelogram?

(8) _____ Parallelogram $ABCD$ has vertices $A(3, 3)$, $B(-3, -3)$, $C(-9, -3)$, and $D(-3, 3)$. If a point is selected at random from the region determined by the parallelogram, what is the probability that the point is not above the x-axis? Express your answer as a common fraction.

(9) _____ The sum of the number of cubic units in the volume plus the number of square units in the surface area of a cube equals six times the number of units in the sum of the lengths of its edges. What is the length of each edge of the cube?

(10) _____ A cylindrical beaker is 8 cm high and has a radius of 3 cm. How many such beakers of water will it take to fill a spherical tank of radius 6 cm?

(11) _____ Two angles of a triangle are in the ratio 7:2 and their difference is 45 degrees. Find the number of degrees in the measure of the largest angle of the triangle.

(12) _____ How many square centimeters are in the surface area of a regular octahedron which has edges of 4 cm?

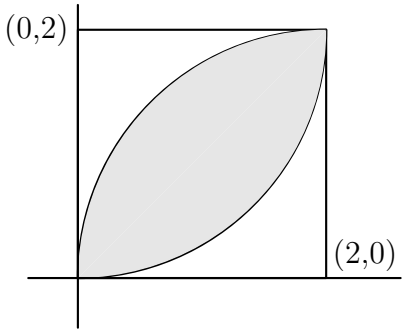
(13) _____ A regular hexagon is 4 cm on each side. How many equilateral triangles, 2 cm on each side, would be required to tile the hexagon?

(14) _____ What is the total surface area in square centimeters of a cube whose volume is 2,197 cubic centimeters?

(15) _____ If a line segment is drawn between the midpoints of 2 adjacent sides of a square to form a triangle, what is the ratio of the area of the triangle to the area of the square? Express your answer as a common fraction.

(16) _____ A square with sides of length 12 inches is circumscribed about another square as shown. What is the sum of the perimeters of the two squares? Express your answer in the form of $a + b\sqrt{c}$.

(17) _____ Circles of radius 2 with centers at (2,0) and (0,2) overlap in the shaded area as shown in the figure. Find the area in terms of π.



Answer Sheet

Number	Answer	Problem ID
1	$\frac{11}{3}$	B10D1
2	2	A14A1
3	$5\frac{5}{11}$	DB3A1
4	8	54DA1
5	$\sqrt{3}$	B14A1
6	528	24DA1
7	72	A2BB
8	$\frac{1}{2}$	C10D1
9	6	D4DA1
10	4	DB4D1
11	99	3C3A1
12	$32\sqrt{3}$	B4DA1
13	24	D2BB
14	1014 sq cm	125A1
15	1/8	C1BB
16	$48 + 24\sqrt{2}$	DC2A1
17	$2\pi - 4$	4A3A1

Solutions

(1) $\frac{11}{3}$ **ID: [B10D1]**

Since the bases of the trapezoid are $\overline{AB}$ and $\overline{CD}$, these two line segments must be parallel. Now, since $\overline{AC}$ intersects these two parallel lines, $\angle DCE$ and $\angle BAE$ are alternate interior angles and therefore must be congruent. Similarly, $\overline{DB}$ intersects the bases, so $\angle CDE$ and $\angle ABE$ are congruent. We have two pairs of congruent angles, so $\triangle DCE \sim \triangle BAE$ by the Angle-Angle Similarity Theorem.

Sides of similar triangles are proportional, so since the lengths of sides $\overline{AB}$ and $\overline{CD}$ are related in a $2:1$ proportion, we also have that $EA/EC = 2/1$, so the length of $\overline{EC}$ must be $1/3$ that of $\overline{AC}$. Since $\overline{AC}$ has length 11, $\overline{EC}$ must have length $\frac{1}{3} \cdot 11 = \boxed{\frac{11}{3}}$.

(2) **2** **ID: [A14A1]**

No solution is available at this time.

(3) $5\frac{5}{11}$ **ID: [DB3A1]**

No solution is available at this time.

(4) **8** **ID: [54DA1]**

No solution is available at this time.

(5) $\sqrt{3}$ **ID: [B14A1]**

No solution is available at this time.

(6) **528** **ID: [24DA1]**

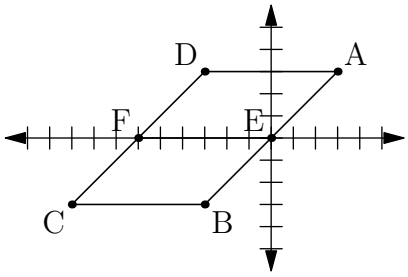
No solution is available at this time.

(7) **72** **ID: [A2BB]**

No solution is available at this time.

(8) $\frac{1}{2}$ **ID: [C10D1]**

Let us first call the point where the x-axis intersects side $\overline{AB}$ point E and where it intersects $\overline{CD}$ point F.



Now, since the x-axis is parallel to bases $\overline{AD}$ and $\overline{BC}$ of the parallelogram, $\overline{EF}$ is parallel to the two bases and splits parallelogram $ABCD$ into two smaller parallelograms $AEFD$ and $EBCF$. Since the height of each of these parallelograms is 3 and the length of their bases equals $AD = BC = 6$, both parallelograms must have the same area. Half of parallelograms $ABCD$'s area is above the x-axis and half is below, so there is a $\boxed{\frac{1}{2}}$ probability that the point selected is not above the x-axis.

(9) **6** **ID: [D4DA1]**

No solution is available at this time.

(10) **4** **ID: [DB4D1]**

We first have to remember our formulas for the volumes of 3 dimensional objects. The volume of a cylinder with radius r and height h is $r^2h\pi$ and the volume of a sphere with radius r is $\frac{4}{3}r^3\pi$. Since the cylindrical beaker has a height of 8 centimeters and a radius of 3 centimeters, that means that its volume is $3^2 \cdot 8 \cdot \pi = 72\pi$ cubic centimeters. Since the sphere has a radius of 6 centimeters, its volume is $\frac{4}{3} \cdot 6^3\pi = 288\pi$ cubic centimeters. The number of beakers of what it will take to fill the spherical tank is just the ratio of the volume of the tank to the volume of the cylinder, which is given by $\frac{288\pi}{72\pi} = \boxed{4}$.

(11) **99** **ID: [3C3A1]**

No solution is available at this time.

(12) $32\sqrt{3}$ **ID: [B4DA1]**

No solution is available at this time.

(13) **24** **ID: [D2BB]**
No solution is available at this time.

(14) **1014 sq cm** **ID: [125A1]**
No solution is available at this time.

(15) **1/8** **ID: [C1BB]**
No solution is available at this time.

(16) $48 + 24\sqrt{2}$ **ID: [DC2A1]**
No solution is available at this time.

(17) $2\pi - 4$ **ID: [4A3A1]**
No solution is available at this time.