

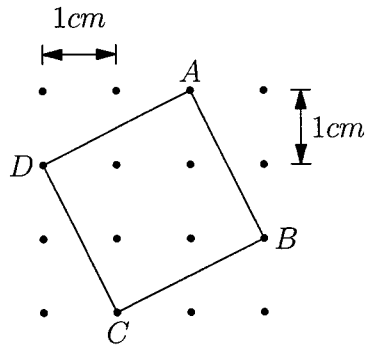
## Mathcounts / AMC 8 (Week 8)

Name \_\_\_\_\_

- (1) \_\_\_\_\_ The number of inches in both the length and the width of a rectangle are prime numbers. The area of the rectangle is 391 square inches. What is the number of inches in the perimeter of the rectangle?
- (2) \_\_\_\_\_ Two cylindrical cans have the same volume. The height of one can is triple the height of the other. If the radius of the narrower can is 12 units, how many units are in the length of the radius of the wider can? ~~Express your answer in simplest radical form.~~
- (3) \_\_\_\_\_ One angle of a triangle has a measure of 70 degrees. The other two angles have degree measures in a ratio of 5 to 6. What is the sum of the measures, in degrees, of the two largest angles?
- (4) \_\_\_\_\_ The rectangular region bounded by the lines with equations  $x = 1.2$ ,  $x = 2.6$ ,  $y = -0.2$  and  $y = d$  has area 14 square units. What is the greatest possible value of  $d$ ? Express your answer as a decimal to the nearest tenth.
- (5) \_\_\_\_\_ The coordinates of the vertices of a parallelogram are  $(10, 1)$ ,  $(7, -2)$ ,  $(4, 1)$  and  $(x, y)$ . What is the sum of the distinct possible values for  $x$ ?
- (6) \_\_\_\_\_ The ratio of the length to the width of a rectangle is 12 to 5 and the area of the rectangle is 540 square units. What is the number of units in the length of the rectangle?
- (7) \_\_\_\_\_ Angle  $BCD$  is an exterior angle of triangle  $ABC$  and has a measure of 55 degrees. Angle  $BAC$  has a measure of 32 degrees. How many degrees are in the measure of angle  $ABC$ ?

(8) \_\_\_\_\_ What is the number of square inches in the area of a rectangle having dimensions  $1\frac{1}{2}$  yards by  $1\frac{1}{2}$  feet?

(9) \_\_\_\_\_ The horizontally and vertically adjacent points in this square grid are 1 cm apart. How many square centimeters are in the area of square  $ABCD$ ?



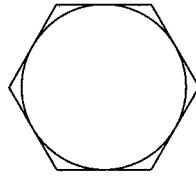
(10) \_\_\_\_\_ A  $1\text{ cm} \times 1\text{ cm}$  square is cut from each of the four corners of a square piece of cardboard with area 64 square centimeters. The sides are then folded up to make a rectangular box. What is the volume in cubic centimeters of the box?

(11) \_\_\_\_\_ A triangle with a height of 24 inches has the same area as a rectangle 12 inches by 6 inches. How many inches long is the base of the triangle that corresponds to the 24-inch height?

(12) \_\_\_\_\_ By how many degrees does the measure of an interior angle of a regular octagon exceed the measure of an interior angle of a regular hexagon?

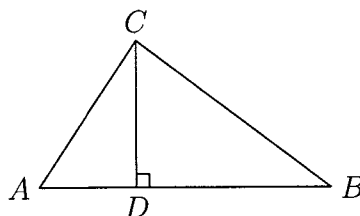
(13) \_\_\_\_\_ What percent of the volume of a  $10 \times 10 \times 10$  box can be filled with  $4 \times 4 \times 4$  wooden cubes? Express your answer as a decimal to the nearest tenth.

- (14) \_\_\_\_\_ How many inches are in the area of a circle inscribed in a regular hexagon with side length 12 inches? ~~Express your answer in terms of  $\pi$ .~~



- (15) \_\_\_\_\_ Each edge of a regular octahedron is colored orange or black. If every face of the octahedron has at least one orange edge, what is the smallest possible number of orange edges?
- (16) \_\_\_\_\_ What is the number of square centimeters in the area of triangle  $DEF$  if angle  $D$  is a right angle,  $DE = 36$  cm, and  $EF = 39$  cm?
- (17) \_\_\_\_\_ Mrs. Read can knit one pair of children's mittens with a ball of yarn six inches in diameter. How many pairs of identical mittens can she knit with a ball of yarn twelve inches in diameter? Assume that the balls of yarn are rolled consistently.
- (18) \_\_\_\_\_ The measures of the sides of an isosceles trapezoid are in the ratio of 3:4:3:6. The perimeter of the trapezoid is 48 cm. What is the number of square centimeters in the area of the trapezoid? ~~Express your answer in simplest radical form.~~
- (19) \_\_\_\_\_ The measures of the three angles of a triangle are in a ratio of 4 : 5 : 6. What is the measure in degrees of the greatest supplement of these three angles?

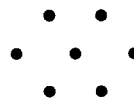
- (20) \_\_\_\_\_ In  $\triangle ABC$ ,  $D$  is a point on  $\overline{AB}$  such that  $CD = 6$  and  $DB = 8$ . If  $\angle CAD = \angle BCD$ , how many units are in the perimeter of  $\triangle ACD$ ?



- (21) \_\_\_\_\_ How many square units are in the area of the convex quadrilateral with vertices  $(0, 0)$ ,  $(3, 0)$ ,  $(2, 2)$  and  $(0, 3)$ ?

- (22) \_\_\_\_\_ Each point in the hexagonal lattice is one unit from its nearest neighbor.

How many circles of radius one contain at least two points of the lattice?



- (23) \_\_\_\_\_ The slant height of a cone is 13 cm, and the height from the vertex to the center of the base is 12 cm. What is the number of cubic centimeters in the volume of the cone? ~~Express your answer in terms of  $\pi$ .~~

- (24) \_\_\_\_\_ A 24-foot by 72-foot rectangular dance floor is completely tiled with 1-foot by 1-foot square tiles. Two opposite corners of the dance floor are connected by a diagonal. This diagonal passes through the interior of exactly how many tiles?

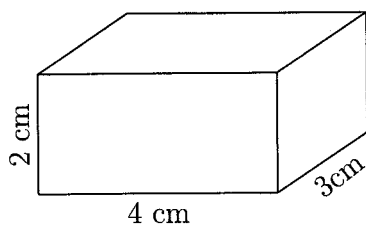
- ~~(25)~~ \_\_\_\_\_ Points  $P$  and  $R$  are located at  $(1, 3)$  and  $(7, 15)$  respectively. Point  $M$  is the midpoint of segment  $PR$ . Segment  $PR$  is reflected over the  $x$ -axis. What is the sum of the coordinates of the image of point  $M$  (the midpoint of the reflected segment)?

- 25 (26) \_\_\_\_\_ Exactly forty-eight non-overlapping square tiles, each 1 inch by 1 inch, fit within a rectangle. What is the least possible number of inches in the perimeter of the rectangle?

~~(27)~~ \_\_\_\_\_ The point  $(4, 3)$  is reflected over the  $x$ -axis and then over the  $y$ -axis. What is the sum of the coordinates of the new point?

26 (28) \_\_\_\_\_ Yuliya has a piece of meat which measures  $5'' \times 6'' \times 8''$ . In order to make a stew, she would like to cut pieces which measure  $2'' \times 3'' \times 4''$ . What is the maximum number of such pieces she can cut from this piece of meat?

27 (29) \_\_\_\_\_ How many square centimeters are in the surface area of the rectangular solid shown?



28 (30) \_\_\_\_\_ A circular pizza with a radius of 6 inches is cut along radii into three wedge-shaped slices. The measures of two of the central angles are 80 degrees and 130 degrees. What is the number of square inches in the area of the largest slice? ~~Express your answer in terms of  $\pi$ .~~