Counting / Probability Worksheet 4A3

(1) _____ There are ten teams in a school district competition. Each team plays each other team twice. What is the total number of games played in the competition?

(2) _____ A square and both of its diagonals are drawn. Given that two different vertices of the square are selected at random, determine the probability that the first can be reflected over one of the diagonals of the square so it coincides with the other selected point. Express your answer as a common fraction.

(3) _____ A rectangular floor measuring 10 feet by 12 feet is tiled with one-foot square tiles. Through how many tiles would the diagonal of this rectangle pass?

(4) _____ Find a number between 100 and 999 such that

1. the sum of the digits is 12,
2. the hundreds digit equals the sum of the tens and units digit, and
3. one greater than the tens digit equals the hundreds digit.

(5) _____ In the land of Noom, all nouns are 4-letter words with consonants a the beginning and end and a repeated vowel (a, e, i, o, or u) in the middle. How many such words are possible?

(6) _____ An algebra course at Carver Community College can be scheduled in various ways. One possibility is for the course to meet every day from 10:00 am to 11:00 am. Another possibility is to schedule the course to meet from 10:00 am to 11:15 am each of four days. A third possibility is to schedule the course to meet from 10:00 am to 11:40 am each of three days. The class may meet on any of the five days of the school week. In how many different ways could the course be scheduled?

(7) _____ Nine digit ID codes may not begin with a zero or end with a zero. Express in scientific notation the number of possible ID codes available.

(8) _____ The faces of a regular tetrahedron are numbered 1-4, the faces of another tetrahedron are lettered A-D, and the faces of a third tetrahedron are colored red, blue, yellow and green. If these tetrahedrons are rolled, what is the probability that the bottom faces are 1, A and red, respectively? Express your answer as a common fraction.

(9) _____ In how many ways can two number cubes, each with faces numbered 1-6, be rolled so that the sum of the numbers on the top faces is divisible by 4?

(10) _____ Joel was ready to give his speech when he dropped his seven note cards and they scattered randomly on the floor. Not paying attention to the order of the cards, he quickly picked them up. What is the probability that the cards are in the correct order? Express your answer as a common fraction.

(11) _____ How many unique sets of three prime numbers exist for which the sum of the members of the set is 44?

(12) _____ If the digits can be used more than once, how many positive even three-digit integers can be created using the digits 2, 3, 4, 5, 7 and 9?

(13) _____ The pattern of Pascal's triangle is illustrated in the diagram shown. What is the fourth element in Row 15 of Pascal's triangle?

Row 0 : 1

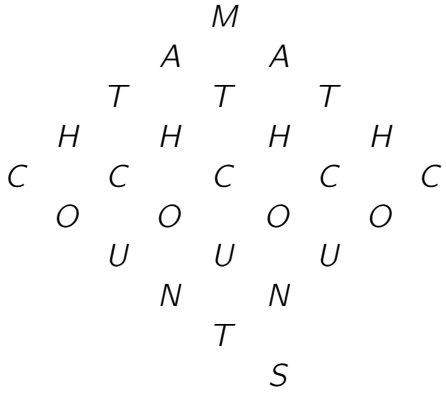
Row 1 : 1 1

Row 2 : 1 2 1

Row 3 : 1 3 3 1

Row 4 : 1 4 6 4 1

(14) _____ From any letter in the diagram, a move can only be made to a letter diagonally adjacent and below. In how many different ways can a path that spells $MATHCOUNTS$ be taken?



(15) _____ A pair of two distinct points is selected at random from the set P. What is the probability that the length of the segment formed by joining the chosen points is an integer? Express your answer as a common fraction.

$$P = \{(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)\}$$

(16) _____ A magic square is an array of numbers in which the sum of the numbers in each row, column and diagonal is the same. In a 5×5 magic square which uses the integers $1 - 25$, the numbers are arranged so that 4, 6 and 13 are in the same row. What is the sum of the other two numbers in that row?

(17) _____ A four-digit number is chosen at random from all four-digit numbers. Express as a common fraction the probability that the number is divisible by 2, 3, 4, and 5.

(18) _____ How many distinct triangles can be formed by connecting three different vertices of a cube?

(19) _____ Every camper at camp EKO is required to take exactly two of the three crafts classes offered. One summer, 47 campers took basket weaving, 59 took cabinet making, and 34 took pottery. How many campers attended camp EKO that summer?

(20) _____ The faces of a tetrahedron are numbered 1, 2, 3 and 4. Two such tetrahedra are tossed, and the numbers on the bottom faces are added. What is the most likely sum?

(21) _____ Chen's math teacher gives homework four of every five school days. Her science teacher gives homework three out of every four school days. What is the probability that on a particular school day Chen does not have homework in either subject? Express your answer as a common fraction.

(22) _____ A fair coin and a 6-sided number cube with numbers 1 through 6 on its faces are simultaneously tossed. What is the probability that a "head" and an even number appear, respectively, on the top faces of the coin and the cube?

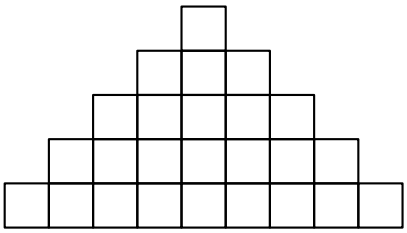
(23) _____ A penny, a nickel, a dime and a quarter are tossed simultaneously. What is the probability that at least two heads show and one of the heads showing is the dime? Express your answer as a common fraction.

(24) _____ a is randomly selected from the set $2, 3, 4, 5$, replaced, and then b is randomly selected from the same set. What is the probability that the fraction $\frac{a}{b}$ is already in reduced form?

(25) _____ The Bell System first planned in the 1940's for ten-digit telephone numbers, with the first three digits forming the area code. In the plan, the first digit could be a number 2 through 9, the second digit could be either 0 or 1, and the third digit could be any number other than 0. How many different area codes are possible under this plan?

(26) _____ What is the largest possible four-digit multiple of six, three of whose digits are 2, 3, and 5?

(27) _____ Square tiles are stacked as shown, each layer having two more tiles than the one above it. How many total square tiles will the figure contain when the bottom layer has 19 tiles?



(28) _____ Arwin needs 100 lbs of bird seed, but the seed is only sold in bags containing 16, 17, 23, 24, 39 or 40 lbs. How many bags will he need to buy in order to purchase exactly 100 lbs?

(29) _____ In how many distinct ways can five children be seated around a circular merry-go-round which has five identical seats?

(30) _____ Each letter d, n and a in the addition below represents a different digit. What is the sum $d + n + a$?

$$\begin{array}{rr} & dna \\ + & dan \\ \hline & and \end{array}$$

Answer Sheet

Number	Answer	Problem ID
1	90	4C3D
2	$\frac{1}{3}$	D2AA
3	20	A4DD
4	651	C0CB
5	2205 words	A05B
6	16	C2AA
7	8.1×10^8	54AB
8	$\frac{1}{64}$	C4DD
9	9	03DD
10	$\frac{1}{5040}$	C0AA
11	4 sets	23B21
12	72	A3AA
13	455	ADDD
14	70	BDDD
15	1/2	A1DD
16	42	B1DD
17	1/60	14041
18	56	B5DD
19	70 campers	42A11
20	5	50AA
21	$\frac{1}{20}$	21AA
22	1/4	50CB
23	7/16	35311
24	$\frac{5}{8}$	5B011
25	144	D1111
26	8532	CB111
27	100	1C111
28	6	C2DD
29	24	51DD
30	18	5ADD

Solutions

(1) **90** **ID: [4C3D]**
No solution is available at this time.

(2) $\frac{1}{3}$ **ID: [D2AA]**
No solution is available at this time.

(3) **20** **ID: [A4DD]**
No solution is available at this time.

(4) **651** **ID: [C0CB]**
No solution is available at this time.

(5) **2205 words** **ID: [A05B]**
No solution is available at this time.

(6) **16** **ID: [C2AA]**
No solution is available at this time.

(7) 8.1×10^8 **ID: [54AB]**
No solution is available at this time.

(8) $\frac{1}{64}$ **ID: [C4DD]**
No solution is available at this time.

(9) **9** **ID: [03DD]**
No solution is available at this time.

(10) $\frac{1}{5040}$ **ID: [C0AA]**
No solution is available at this time.

(11) **4 sets ID: [23B21]**
No solution is available at this time.

(12) **72 ID: [A3AA]**
No solution is available at this time.

(13) **455 ID: [ADDD]**
In Pascal's triangle, the k^{th} element in the row n has the value $\binom{n}{k-1}$. Row 15 starts with $\binom{15}{0}$, $\binom{15}{1}$, $\binom{15}{2}$, $\binom{15}{3}$, so the fourth element is

$$\binom{15}{3} = \frac{15!}{3!(15-3)!} = \frac{15 \cdot 14 \cdot 13}{3 \cdot 2 \cdot 1} = 5 \cdot 7 \cdot 13 = \boxed{455}.$$

(14) **70 ID: [BDDD]**
No solution is available at this time.

(15) **1/2 ID: [A1DD]**
No solution is available at this time.

(16) **42 ID: [B1DD]**
No solution is available at this time.

(17) **1/60 ID: [14041]**
No solution is available at this time.

(18) **56 ID: [B5DD]**
No solution is available at this time.

(19) **70 campers ID: [42A11]**
No solution is available at this time.

(20) **5 ID: [50AA]**
No solution is available at this time.

(21) $\frac{1}{20}$ **ID: [21AA]**
No solution is available at this time.

(22) 1/4 **ID: [50CB]**
No solution is available at this time.

(23) **7/16** **ID: [35311]**
No solution is available at this time.

(24) $\frac{5}{8}$ **ID: [5B011]**
No solution is available at this time.

(25) **144** **ID: [D1111]**
There are eight possible numbers that the first digit could be, two that the second digit could be, and nine that the third digit could be. Thus, there are $8 \cdot 2 \cdot 9 = \boxed{144}$ possible different area codes.

(26) **8532** **ID: [CB111]**
No solution is available at this time.

(27) **100** **ID: [1C111]**
No solution is available at this time.

(28) **6** **ID: [C2DD]**
No solution is available at this time.

(29) **24** **ID: [51DD]**
No solution is available at this time.

(30) **18** **ID: [5ADD]**
No solution is available at this time.