

# Number Theory 4A3

Name \_\_\_\_\_

- (1) \_\_\_\_\_ How many combinations of two positive two-digit integers have 429 as the product?
- (2) \_\_\_\_\_ What is the units digit of  $1998^{1998}$ ?
- (3) \_\_\_\_\_ How many positive even three-digit integers are divisible by 3?
- (4) \_\_\_\_\_ A positive integer less than 100 is a multiple of 3. Determine the probability that it also leaves a remainder of 1 when divided by 7. Express your answer as a common fraction.
- (5) \_\_\_\_\_ If three different odd positive integers less than 50 are randomly selected, what is the probability none of the three are prime?
- (6) \_\_\_\_\_ The decimal representation of a fraction ends in  $0.\overline{3}$ . When the decimal is changed to a common fraction and reduced to lowest terms, what is the denominator of the fraction?
- (7) \_\_\_\_\_ The digital root of a number is found by computing the sum of its digits, computing the sum of the digits of that sum, and continuing to compute the sum of the digits until a one-digit number is obtained. What is the digital root of  $2^{100}$ ?
- (8) \_\_\_\_\_ The product of the digits of a three-digit number is 63. What is the greatest possible value of the number?

- (9) \_\_\_\_\_ Prime pairs are pairs of prime numbers that differ by 2, such as 5 and 7. What is the sum of the lesser member of the first prime pair in which both members are greater than 60 and the greater member of the first prime pair in which both members are greater than 100?
- (10) \_\_\_\_\_ The number of students in Teresa's graduating class is more than 50 and fewer than 100 and is 1 less than a multiple of 3, 2 less than a multiple of 4, and 3 less than a multiple of 5. How many students are in Teresa's graduating class?
- (11) \_\_\_\_\_ When an integer is divided by 15, the remainder is 7. Find the sum of the remainders when the same integer is divided by 3 and by 5.
- (12) \_\_\_\_\_ Select two different numbers from the set  $\{4, 9, 12, 16, 18\}$ . Of all possible pairs, what is the least possible value for the sum of the greatest common factor and least common multiple of the two numbers?
- (13) \_\_\_\_\_ What is the smallest positive integer that has twelve distinct factors?
- (14) \_\_\_\_\_ Given that both  $p$  and  $p + 1$  are prime numbers, what is the least positive composite number that is not divisible by  $p$  nor  $p + 1$ ?
- (15) \_\_\_\_\_ What is the length of the longest sequence of consecutive positive composite numbers, each number less than 101?
- (16) \_\_\_\_\_ The number 3465 is divisible by 11 and by 5. Given that you change the position of two or more of these four digits, how many of the resulting numbers are divisible by 11 but not by 5?
- (17) \_\_\_\_\_ The product of a set of distinct positive integers is 84. What is the least possible sum of these integers?

- (18) \_\_\_\_\_ Find the value of  $A + B$  in the multiplication table below:

$\times$	?	?	?
?	$B$	12	20
?		21	
?	18	27	$A$

- (19) \_\_\_\_\_  $a$  is randomly selected from the set 2, 3, 4, 5, replaced, and then  $b$  is randomly selected from the same set. What is the probability that the fraction  $\frac{a}{b}$  is already in reduced form?
- (20) \_\_\_\_\_ What is the smallest positive integer greater than 5 with the property that the number of odd primes less than or equal to it equals the number of odd non-primes less than or equal to it?
- (21) \_\_\_\_\_ Find the smallest positive integer  $x$  so that the fraction below represents a fraction whose decimal equivalent terminates.
- $$\frac{1}{10 + x}$$
- (22) \_\_\_\_\_ In how many zeroes does  $20!$  end?
- (23) \_\_\_\_\_ How many positive two-digit integers are increased by exactly nine when the digits are reversed?
- (24) \_\_\_\_\_ How many positive integers are factors of  $21^{75}$ ?
- (25) \_\_\_\_\_ A proposal will make years that end in double zeroes a leap year only if the year leaves a remainder of 200 or 600 when divided by 900. Under this proposal, how many leap years will there be that end in double zeroes between 1996 and 4096?
- (26) \_\_\_\_\_ Given that  $3^n$  divides  $15!$ , what is the greatest possible integral value of  $n$ ?

- (27) \_\_\_\_\_ Natasha has more than \$1 but less than \$10 worth of dimes. When she puts her dimes in stacks of 3, she has 1 left over. When she puts them in stacks of 4, she has 1 left over. When she puts them in stacks of 5, she also has 1 left over. How many dimes does Natasha have?
- (28) \_\_\_\_\_ What is the least positive integer with exactly ten factors?
- (29) \_\_\_\_\_ When two different numbers are divided by 7, remainders of 2 and 3, respectively, are left. What is the greatest possible three-digit product of these two numbers?
- (30) \_\_\_\_\_ What is the least integer  $n \geq 2$  such that  $2^n - 1$  is a composite number?