Number Theory 3A1

Name _____

(1) _____ A whole number, N, is chosen so that $\frac{N}{3}$ is strictly between 7.5 and 8. What is the value of N ?

(2) _____ What fraction of the one-digit positive integers is prime? Express your answer as a common fraction.

(3) _____ What is the greatest three-digit multiple of 19?

(4) _____ What is the positive difference between the greatest and least prime factors of 2000?

(5) _____ The game of Ibish is played in rounds. In the first round, you earn 0, 10 or 11 points; in the second round, you earn 0, 10, 11 or 12 points; in the third round, you earn 0, 10, 11, 12 or 13 points, and so on, including the next greatest integer in the possible point values. What is the fewest number of rounds after which your total score can have a 9 in the units digit?

(6) _____ What is the median of all values defined by the expression $2^x - 1$, where x is a prime number between 0 and 20?

(7) _____ What is the greatest three-digit multiple of 33 that can be written using three different digits?

(8) _____ A 16-page booklet is made from a stack of four sheets of paper that is folded in half and then joined along the common fold. The 16 pages are then numbered from front to back, starting with page 1. What are the other three page numbers on the same sheet of paper as page 5?

(9) _____ Four consecutive positive integers have a product of 840. What is the largest of the four integers?

(10) _____ What is the units digit of the product of $3^{35} \times 7^{35}$?

(11) _____ What is the median of the composite integers that are greater than 20 and less than 35?

(12) _____ What is the least natural number that has exactly three factors?

(13) _____ Use the clues below to determine the value of n. The sum of the digits of n is 11, and $50 < n < 100$. When n is divided by 2, the result is a prime integer. What is the value of n?

(14) _____ What is the number of factors of the product 126×216?

(15) _____ What is the sum of the reciprocals of the natural-number factors of 6?

(16) _____ To determine whether a number N is prime, we must test for divisibility by every prime less than or equal to the square root of N. How many primes must we test to determine whether 2003 is prime?

(17) _____ A *composite number* is a number that has two or more prime factors. The number 87 can be expressed as the sum of two composite numbers in many ways. What is the minimum positive difference between two such numbers?

(18) _____ What is the least three-digit positive integer that has 2, 5 and 7 as factors?

(19) _____ How many positive integer factors of $2^2 \times 3^2 \times 5$ are multiples of 12?

(20) _____ What is the greatest three-digit number which is a multiple of 13?

(21) _____ For how many positive integers n will $\frac{60}{n}$ also be an integer?

(22) _____ What positive two-digit integer is exactly twice the sum of its digits?

(23) _____ What is the smallest positive integer n such that $2n$ is a perfect square and $3n$ is a perfect cube?

(24) _____ What is the least common multiple of 6, 8, and 10?

(25) _____ What is the sum of the prime numbers between 30 and 50?

(26) _____ What is the probability that a randomly selected positive integer less than or equal to 100 is divisible by 7? Express your answer as a common fraction.

(27) _____ What is the sum of the proper divisors of 256?

(28) _____ How many natural-number factors does N have if $N = 2^3 \cdot 3^2 \cdot 5^1$?

(29) _____ A relatively prime day is one where the month number (e.g. January = 1, February = 2, etc.) and date of the month $(1, 2, \ldots, 31)$ have no common factor other than 1. Which month has the fewest relatively prime days?

(30) _____ What is the least whole number that is divisible by 7, but leaves a remainder of 1 when divided by any integer 2 through 6?