Counting / Probability Worksheet 3A2

Name ____________________

(1) ________ The number n is randomly selected from the set $\{1, 2, \ldots, 10\}$, with each number being equally likely. What is the probability that $2n - 4 > n$? Express your answer as a common fraction.

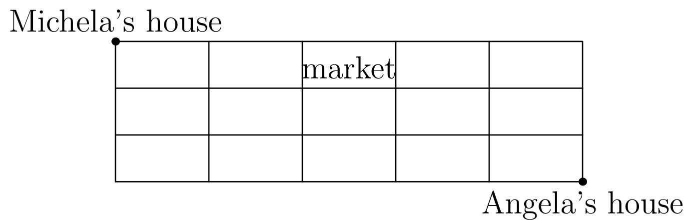
(2) ________ P and Q are whole numbers such that $0 < P < 10$ and $0 < Q < 10$. How many common fractions $\frac{P}{Q}$ exist if $\frac{1}{2} < \frac{P}{Q} < 1$?

(3) ________ The digits from 1 to 6 are arranged to form a six-digit multiple of 5. What is the probability that the number is greater than 500,000? Express your answer as a common fraction.

(4) ________ Ms. Albertson is randomly selecting the order in which her 25 students will each present a report next week. Five students will present each day, Monday through Friday. What is the probability that the shortest student will present his report on Thursday? Express your answer as a common fraction.

(5) ________ What is the positive difference between the probability of a fair coin landing heads up exactly 2 times out of 3 flips and the probability of a fair coin landing heads up 3 times out of 3 flips? Express your answer as a common fraction.

(6) __________ Michela wants to go from her house to Angela's house, but she needs to stop at the market on the way. She may only travel to the right and down. What is the number of distinct paths that Michela can take?



(7) __________ Between 3:00 p.m. and 4:00 p.m., for what fractional part of the hour does the digit "2" appear on a 12-hour digital clock that shows hour and minutes? Express your answer as a common fraction.

(8) __________ Yuliya has a piece of meat which measures 5" x 6" x 8". In order to make a stew, she would like to cut pieces which measure 2" x 3" x 4". What is the maximum number of such pieces she can cut from this piece of meat?

(9) __________ Diane is writing a book with ten chapters, which have 17, 23, 14, 26, 21, 32, 36, 19, 24 and 30 pages, respectively. The first chapter begins on page 1, and each subsequent chapter must also begin on an odd numbered page, with a blank page between chapters if needed. What is the page number of the last page of the last chapter?

(10) __________ What is the units digit of $1! + 3! + 5! + 7! + 9! + 11!$?

(11) __________ How many distinct positive integers can be represented as the difference of two numbers in the set $\{1, 3, 5, 7, 9, 11, 13\}$?

(12) __________ A graphic art designer's annual salary is a whole number of dollars between \$62,400 and \$62,600. If the hundreds, tens and units digits in her salary are all different and in descending order, how many possibilities exist for her salary?

(13) ________ If two distinct numbers are selected at random from the first seven prime numbers, what is the probability that their sum is an even number? Express your answer as a common fraction.

(14) ________ The number 121 is a palindrome, because it reads the same backwards as forward. How many integer palindromes are between 100 and 500?

(15) ________ Ilian will randomly choose a value for p from the integers 0 through 13, inclusive, and will make $\frac{p}{13}$ the coordinate of point A. The coordinate of point B is $\frac{2}{3}$. Points A and B will be on the same number line. What is the probability that point A and point B will be less than $\frac{1}{5}$ of a unit from each other? Express your answer as a common fraction.

(16) ________ A digital, 12-hour clock shows hours and minutes. During what fraction of the day will the clock show the digit 1 in its display? Express your answer as a common fraction.

(17) ________ The first 20 numbers of an arrangement are shown below. What would be the value of the 40^{th} number if the arrangement were continued?

- Row 1: 2, 2
- Row 2: 4, 4, 4, 4
- Row 3: 6, 6, 6, 6, 6, 6
- Row 4: 8, 8, 8, 8, 8, 8, 8, 8

(18) ________ What is the probability of getting an even number when a fair six-sided die is rolled? Express your answer as a common fraction.

(19) ________ The product of the digits of 3214 is 24. How many distinct four-digit positive integers are such that the product of their digits equals 12?

(20) _________ Mr. Cortes is trying to determine where each student in his class should sit. He knows that he wants Abraham, Brian, Cynthia, Delia, Eugene, and Faye to sit in the six chairs in the first row. Using the following criteria, determine who sits next to Delia.

- Eugene sits next to Cynthia but not Faye.
- Abraham's nickname, ABE, appears in the seating chart if you only use the first letters of each person's name.
- Faye and Delia sit at each end.
- There are two seats between Delia and Eugene.

(21) _________ Two numbers are chosen at random, with replacement, from the set $\{1, 2, 3, 4\}$. The two numbers are used as the numerator and denominator of a fraction. What is the probability that the fraction represents a whole number? Express your answer as a common fraction.

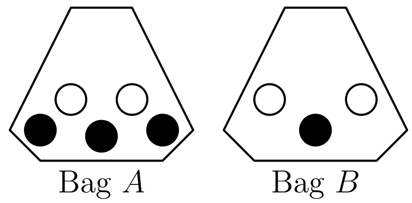
(22) _________ Chris sleeps from 10:30 p.m. to 6:30 a.m. At a random time during the night, he awakens and looks at his clock. What is the probability that it is before midnight? Express your answer as a common fraction.

(23) _________ Camy made a list of every possible distinct five-digit positive even integer that can be formed using each of the digits 1, 3, 4, 5 and 9 exactly once in each integer. What is the sum of the integers on Camy's list?

(24) _________ What is the greatest possible number of points of intersection for eight distinct lines in a plane?

(25) _________ What fraction of the eleven letters in the word "MISSISSIPPI" are I's? Express your answer as a common fraction.

(26) __________ Two bags of marbles are pictured below. One marble is randomly selected from Bag A and placed into Bag B. One marble is then randomly selected from Bag B. What is the probability that the marble selected from Bag B is black? Express your answer as a common fraction.



(27) __________ How many different four-digit numbers can be obtained by using any four of the digits 2, 3, 4, 4, and 4?

(28) __________ A printer is used to print out the squares of the first 15 positive integers. How many digits are printed by the printer?

(29) __________ How many diagonals does a regular seven-sided polygon contain?

(30) __________ If digits may not be repeated, how many positive three-digit integers can be written using the digits 1, 2, 3 and 4?

Answer Sheet

Number	Answer	Problem ID
1	3/5	13C2
2	13	55D3
3	1/5	3DB3
4	1/5	5203
5	$\frac{1}{4}$	0DB3
6	30	2DC3
7	1/4	4303
8	10	ADD3
9	246	05D3
10	7	D3C2
11	6 integers	0B42
12	16	B4D3
13	5/7	B3C3
14	40 palindromes	C041
15	5/14	5141
16	1/2	BBAC
17	12	4D03
18	1/2	0D03
19	36 integers	B141
20	Abraham	5403
21	1/2	DBAC
22	3/16	C4D3
23	1199976	2241
24	28 points	A422
25	4/11	C203
26	2/5	144C
27	20	C3BC
28	33 digits	4422
29	14 diagonals	0141
30	24	A403

Solutions

(1) **3/5 ID: [13C2]**
No solution is available at this time.

(2) **13 ID: [55D3]**
We can begin by restating the inequalities. $\frac{1}{2} < \frac{P}{Q}$ is the same as $Q < 2P$, and $\frac{P}{Q} < 1$ is the same as $P < Q$. Thus, $P < Q < 2P$. If $P = 1$, there are no possible solutions for Q. If $P = 2$, $Q = 3$ is the only possible solution. Similarly, if $P = 3$, Q could be 4 or 5; if $P = 4$, then $Q = 5$, 6, or 7; if $P = 5$, then $Q = 6$, 7, 8, or 9; if $P = 6$, $Q = 7$, 8, or 9; if $P = 7$, $Q = 8$ or 9; if $P = 8$, $Q = 9$. There are sixteen such fractions.

However, we have over-counted some fractions. $\frac{2}{3} = \frac{4}{6} = \frac{6}{9}$, and $\frac{3}{4} = \frac{6}{8}$.

Thus, there are $\boxed{13}$ possible distinct common fractions.

(3) **1/5 ID: [3DB3]**
Of the digits 1 to 6, 5 must be the units digit since our number is a multiple of 5. We have five digits remaining, 1, 2, 3, 4, and 6, for five digit places. The number is greater than 500,000 if and only if the hundred thousands digit is 6. The probability that the hundred thousands digit (which can be 1, 2, 3, 4, or 6) is 6 is $\boxed{1/5}$.

(4) **1/5 ID: [5203]**
No solution is available at this time.

(5) $\frac{1}{4}$ **ID: [0DB3]**
The probability that a fair coin lands heads up exactly 2 times out of 3 flips is $p_1 = \binom{3}{2}(1/2)^2(1/2) = 3/8$. The probability that a fair coin lands heads up 3 times out of 3 flips is $p_2 = (1/2)^3 = 1/8$. Finally, we have $p_1 - p_2 = 2/8 = \boxed{1/4}$.

(6) **30 ID: [2DC3]**
No solution is available at this time.

(7) **1/4 ID: [4303]**

The digit 2 appears for 10 minutes between 3 : 20 and 3 : 30. It also appears for a minute at each of 3 : 02, 3 : 12, 3 : 32, 3 : 42, and 3 : 52. So it appears for a total of $10 + 1 + 1 + 1 + 1 + 1 = 15$ minutes, or $\frac{15}{60} = \boxed{\frac{1}{4}}$ of the hour.

(8) 10 **ID: [ADD3]**

No solution is available at this time.

(9) **246 ID: [05D3]**

If a chapter has an even number of pages, it will start on an odd-numbered page and end on an even-numbered page, and then the next chapter can begin on the next page. But a chapter with an odd number of pages will end on an odd-numbered page, and the even numbered page following it must be skipped. There are 4 chapters with an odd number of pages, so we must add 4 to the number of used pages to get the answer,

$$4 + 17 + 23 + 14 + 26 + 21 + 32 + 36 + 19 + 24 + 30 = \boxed{246}$$

(10) **7 ID: [D3C2]**

We observe that for all $n > 5$, $n!$ has a units digit of 0, because 5! has a factor of 5 and 2, which become a factor of 10. So, the terms in the sum, 5!, 7!, 9!, and 11! all have a 0 for the units digit. And, $1! + 3! = 1 + 6 = \boxed{7}$ is the units digit of the sum.

(11) **6 integers ID: [0B42]**

Since all of the integers are odd, the differences between any pair of them is always even. So, $13 - 1 = 12$ is the largest even integer that could be one of the differences. The smallest positive (even) integer that can be a difference is 2. So, the integers include 2, 4, 6, 8, 10 and 12, for a total of $\boxed{6}$ integers.

(12) **16 ID: [B4D3]**

The hundreds digit can be either a 4 or a 5 (it can't be a 6 because then the salary would be greater than $\$62,600$). For either potential hundreds digit, possible tens and units digit combinations are as follows:

10
21, 20
32, 31, 30

Also, for a hundreds digit of 5, there are four more possible combinations: 43, 42, 41, and 40. Thus, there are $2 \cdot 6 + 4 = \boxed{16}$ possible salaries.

(13) **5/7 ID: [B3C3]**

The only way for the sum to not be even is if one of the primes chosen is 2. There are six pairs where one of the primes is 2, and there are $\binom{7}{2} = 21$ total possible pairs, so the probability that the sum is NOT even is $\frac{6}{21} = \frac{2}{7}$. Therefore, the probability that the sum IS even is $1 - \frac{2}{7} = \boxed{\frac{5}{7}}$.

(14) **40 palindromes ID: [C041]**

The hundreds digit can be any one of 1, 2, 3 or 4. Whatever the hundreds digit is, that fixes what the units digit can be. Then, there are 10 choices for the middle (tens) digit. So, we can construct $4 \cdot 10 = \boxed{40}$ palindromes by choosing the digits.

(15) **5/14 ID: [5141]**

There are 14 different choices for p. We want to determine which choices of p will allow $\left|\frac{p}{13} - \frac{2}{3}\right| < \frac{1}{5}$. So, $\frac{p}{13} < \frac{2}{3} + \frac{1}{5} = \frac{13}{15}$, or $p < \frac{169}{15} = 11\frac{4}{15}$. Likewise, $\frac{p}{13} > \frac{2}{3} - \frac{1}{5} = \frac{7}{15}$, or $p < \frac{91}{15} = 6\frac{1}{15}$. So, $7 \leq p \leq 11$ giving 5 successful choices of p. So, the probability is $\boxed{\frac{5}{14}}$.

(16) **1/2 ID: [BBAC]**

No solution is available at this time.

(17) **12 ID: [4D03]**

Since we are told there are 20 numbers in the first 4 Rows, we want to find the 20th number starting in Row 5. Since there are 10 numbers in Row 5, and there are 12 numbers in Row 6, the 20th number if we start counting in Row 5 is located at the 10th spot of Row 6, which is of course a $\boxed{12}$.

(18) **1/2 ID: [0D03]**

No solution is available at this time.

(19) **36 integers ID: [B141]**

We first have to figure out the different groups of 4 one-digit numbers whose product is 12. We obviously can't use 12 as one of the numbers, nor can we use 9, 8, or 7 (none divides 12). We can use 6, in which case one of the other numbers is 2 and the other two are 1s. So, we can have the number 6211, or any number we can form by reordering these digits. There are 4! ways to order these four numbers, but we have to divide by 2! because the two 1s are the same, so 4! counts each possible number twice. That gives us $4!/2! = 12$ numbers that consist of 6, 2, and two 1s.

Next, we note that we can't have a 5, so we think about 4. If we have a 4, then the other three numbers are 3, 1, 1. Just as there are 12 ways to order the digits in 6211, there are 12 ways to order the digits in 4311. Finally, we check if there are any ways to get a product of 12 with digits that are 3 or less. There's only one such group, the digits in 3221. As with 6211 and 4311, there are 12 distinct ways to order the digits in 3221.

Combining our three cases, we have $12 + 12 + 12 = \boxed{36}$ possible integers.

(20) **Abraham ID: [5403]**

No solution is available at this time.

(21) **1/2 ID: [DBAC]**

No solution is available at this time.

(22) **3/16 ID: [C4D3]**

Chris sleeps for 8 hours, including 1.5 hours before midnight. So the answer is

$$\frac{1.5}{8} = \boxed{\frac{3}{16}}$$

(23) **1199976 ID: [2241]**

Since Camy's numbers are even, they all must end in 4. This leaves us free to pick the remaining digits without restriction. Note that once we fix one of the remaining digits, we have $3! = 6$ ways of forming the final number. Thus each of the digits $1, 3, 5, 9$ appears in the 10s, 100s, 1000s, and 10000s spots exactly 6 times. So the 1 contributes a total of $6 \cdot 1 \cdot (10 + 100 + 1000 + 10000) = 6 \cdot 11110$ to the sum, and similarly the 3, 5, 9 contribute $6 \cdot 33330$, $6 \cdot 55550$, $6 \cdot 99990$, respectively. Since the 4 appears in $4! = 24$ numbers, our total sum is

$$6 \cdot 11110 + 6 \cdot 33330 + 6 \cdot 55550 + 6 \cdot 99990 + 4 \cdot 24 = \boxed{1,199,976}$$

(24) **28 points ID: [A422]**

To maximize the number of intersection points, we assume that the lines are in general position - that is, that no line is parallel to any other line, and that no more than two lines intersect at any given point. In this case, each pair of lines will intersect at a unique point. So the total number is

$$\binom{8}{2} = \boxed{28}$$

(25) **4/11 ID: [C203]**

There are 4 I's, and 11 letters, so the fraction is $\boxed{\dfrac{4}{11}}$.

(26) **2/5 ID: [144C]**

The probability that the marble drawn is originally from bag B is $\frac{3}{4}$. Given that the marble drawn is originally from bag B, the probability that it is black is $\frac{1}{3}$. Therefore, the probability that the marble drawn from the bag is black and is originally from bag B is $\frac{3}{4} \cdot \frac{1}{3} = \frac{1}{4}$.

The probability that the marble drawn is originally from bag A is $\frac{1}{4}$. Given that the marble drawn is originally from bag A, the probability that it is black is $\frac{3}{5}$. Therefore, the probability that the marble drawn is black and is originally from bag A is $\frac{1}{4} \cdot \frac{3}{5} = \frac{3}{20}$.

Since the marble originally came either from bag A or from bag B, the probability that it is black is the sum of the probability that it is black and originally came from bag A and the probability that it is black and originally came from bag B: $\frac{1}{4} + \frac{3}{20} = \boxed{\dfrac{2}{5}}$.

(27) **20 ID: [C3BC]**

No solution is available at this time.

(28) **33 digits ID: [4422]**

1, 2, and 3 have single-digit squares. The rest of the single digit numbers have two-digit squares, until 10, whose square is 100, the smallest three-digit integer. $15^2 = 225$, which still has three digits, so all the rest have three digits. Thus the total is

$$3(1) + 6(2) + 6(3) = \boxed{33}$$

(29) **14 diagonals ID: [0141]**

A seven-sided polygon has seven vertices. There are $\binom{7}{2} = 21$ ways to connect the pairs of these 7 points. But 7 of those pairs are pairs of consecutive vertices, so they are counted as sides. So, only $21 - 7 = \boxed{14}$ of these segments are diagonals.

(30) **24 ID: [A403]**

No solution is available at this time.