

Name: \_\_\_\_\_

<b>Division</b> <b>M</b>	<b>Mathematical Olympiads</b> <b>FEBRUARY 10, 2010</b> <i>for Elementary and Middle Schools</i>	<b>Contest</b> <b>4</b>
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**4A** Time: 3 minutes

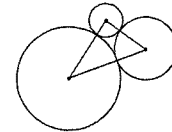
Suppose  $52 \times 50 \times N = 40 \times 13 \times 35$ .  
Find the whole number  $N$ .

**4B** Time: 5 minutes

Two consecutive positive integers are each less than 100. One integer is divisible by 17 and the other integer is divisible by 21. Find the greater of the two integers.

**4C** Time: 5 minutes

Three circles are externally tangent as shown. Their areas are  $9\pi$ ,  $25\pi$ , and  $100\pi$  sq cm. A triangle is formed by connecting the centers of the three circles. Find the perimeter of the triangle, in cm.

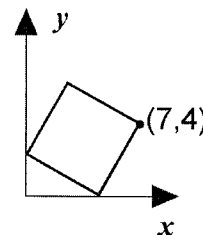


**4D** Time: 6 minutes

The four-digit whole number  $3\blacksquare 11$  is exactly divisible by 13.  
Find the missing digit  $\blacksquare$ .

**4E** Time: 7 minutes

A square is positioned in quadrant I on graph paper so that two vertices lie on the axes, while a third vertex lies at the point  $(7,4)$ . Find the area of the square.



**5A** Time: 3 minutes

For any two numbers  $a$  and  $b$ , define the value of  $a \star b$  as  $a + 3 \times b$ . For example,  $4 \star 5$  means  $4 + 3 \times 5 = 19$ . If  $2 \star 6$  and  $N \star 4$  represent the same number, what is the value of  $N$ ?

**5B** Time: 5 minutes

Express the product as a fraction in simplest terms.

$$\frac{1}{3} \times \frac{2}{4} \times \frac{3}{5} \times \frac{4}{6} \times \frac{5}{7} \times \frac{6}{8} \times \frac{7}{9} \times \frac{8}{10}$$

**5C** Time: 5 minutes

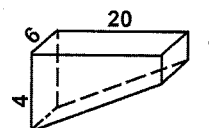
The sum of the integers from  $-10$  through  $N$ , inclusive, equals 50. Find  $N$ .

**5D** Time: 5 minutes

The Pumas lost 7 of their first 9 games. By winning 75% of their remaining games, they ended with victories in exactly  $\frac{2}{3}$  of all their games. In all, how many games did they win?

**5E** Time: 7 minutes

The rectangular top of an in-ground swimming pool is 20 m by 6 m. The pool is 4 m deep at one end and 1 m deep at the other. How many cubic meters of water can the pool hold?



Not drawn to scale.  
All measures in meters.

**1A** Time: 3 minutes

25 digits are shown.

Find the sum of the digits.

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2 2 2 2
2 2 6 2
2 6 6 2
6 6 6 2
4 4 4 4
4 4 4 4 4

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**1B** Time: 4 minutes

How many quarters (worth 25 cents each) must be added to 12 nickels (worth 5 cents each), so that the average value of a coin in the new enlarged collection is 10 cents?

**1C** Time: 5 minutes

How many different sums can be obtained by adding two different integers chosen from the set below?

$$\{-12, -11, -10, \dots, +6, +7, +8\}$$

**1D** Time: 5 minutes

561 is the product of 3 different prime numbers. How many factors of 561 are not prime?

**1E** Time: 7 minutes

In rectangle  $ABCD$ ,  $P$  is the midpoint of side  $\overline{BC}$  and  $Q$  is the midpoint of  $\overline{CD}$ . The area of  $\triangle APQ$  is what fractional part of the area of rectangle  $ABCD$ ?

