Mathcounts / AMC 8 (Week 7)

(1) _____ What is the arithmetic mean of all of the positive two-digit integers with the property that the integer is equal to the sum of its first digit plus its second digit plus the product of its two digits?

(2) _____ How many two-digit prime numbers have a units digit of 7?

(3) _____ Diane wants to arrange her collection of 96 bottle caps into a rectangular grid rather than keep them in the line of 96 bottle caps she has now. How many distinct, rectangular-grid arrangements could she use instead of the 1 by 96 grid of bottle caps she has now? One such arrangement is 3 by 32, which is considered the same as 32 by 3.

(4) _____ The difference between two prime numbers is 17. What is their sum?

(5) _____ What is the only integer value of n for which $\frac{n+1}{13-n}$ is a positive prime number?

(6) _____ One gear turns $33\frac{1}{3}$ times in a minute. Another gear turns 45 times in a minute. Initially, a mark on each gear is pointing due north. After how many seconds will the two gears next have both their marks pointing due north?

(7) _____ A segment has endpoints P(1, 1) and Q(x, y). The coordinates of the midpoint of segment PQ are positive integers with a product of 36. What is the maximum possible value of x?

(8) _____ The letters of the alphabet are given numeric values based on the two conditions below.

- • Only the numeric values of -2, -1, 0, 1 and 2 are used.
- • Starting with A and going through Z, a numeric value is assigned to each letter according to the following pattern:

$$1, 2, 1, 0, -1, -2, -1, 0, 1, 2, 1, 0, -1, -2, -1, 0, \ldots$$

Two complete cycles of the pattern are shown above. The letter A has a value of 1, B has a value of 2, F has a value of -2 and Z has a value of 2. What is the sum of the numeric values of the letters in the word "numeric"?

8 ~~(9)~~ _____ How many integers between 100 and 300 have both 11 and 8 as factors?

9 ~~(10)~~ _____ What is the digit in the tens place when 7^{2005} is expressed in decimal notation?

10 ~~(11)~~ _____ Zach has three bags and a bunch of pencils to be placed into the bags. He is told to place the greatest number of pencils possible into each of the three bags while also keeping the number of pencils in each bag the same. What is the greatest number of pencils he could have left over?

11 ~~(12)~~ _____ What is the sum of all the distinct positive two-digit factors of 144?

12 ~~(13)~~ _____ A number is called perfect if the sum of its divisors, except itself, is equal to the original number. What is the sum of the two perfect numbers between 2 and 30?

13 (14) _____ If $x + 5 < 8$ and x is a prime number, what is the value of x?

14 ~~(15)~~ _____ How many perfect squares less than 1000 have a ones digit of 2, 3 or 4?

15 ~~(16)~~ _____ The positive difference of the cube of an integer and the square of the same integer is 100. What is the integer?

16 ~~(17)~~ _____ What is the least positive multiple of 72 that has exactly 16 positive factors?

17 ~~(18)~~ _____ The members of a band are arranged in a rectangular formation. When they are arranged in 8 rows, there are 2 positions unoccupied in the formation. When they are arranged in 9 rows, there are 3 positions unoccupied. How many members are in the band if the membership is between 100 and 200?

18 ~~(19)~~ _____ How many positive whole-number divisors does 196 have?

19 ~~(20)~~ _____ We know the following to be true:

- 1. Z and K are integers with $500 < Z < 1000$ and $K > 1$;
- 2. $Z = K \times K^2$.

What is the value of K for which Z is a perfect square?

20 ~~(21)~~ _____ The number 24 can be made by multiplying together four prime numbers: 2, 2, 2 and 3. How many primes must be multiplied to make 2400?