Mathcounts / AMC 8 (Week 8)

Name _____

(1) _____ The number of inches in both the length and the width of a rectangle are prime numbers. The area of the rectangle is 391 square inches. What is the number of inches in the perimeter of the rectangle?

(2) _____ Two cylindrical cans have the same volume. The height of one can is triple the height of the other. If the radius of the narrower can is 12 units, how many units are in the length of the radius of the wider can? ~~Express your answer in simplest radical form.~~

(3) _____ One angle of a triangle has a measure of 70 degrees. The other two angles have degree measures in a ratio of 5 to 6. What is the sum of the measures, in degrees, of the two largest angles?

(4) _____ The rectangular region bounded by the lines with equations $x = 1.2$, $x = 2.6$, $y = -0.2$ and $y = d$ has area 14 square units. What is the greatest possible value of d? Express your answer as a decimal to the nearest tenth.

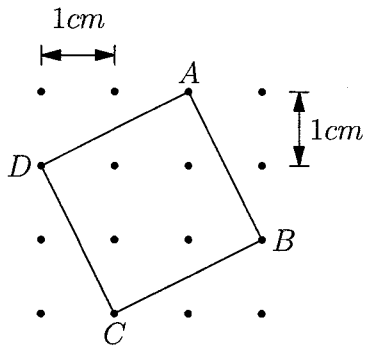
(5) _____ The coordinates of the vertices of a parallelogram are (10, 1), (7, -2), (4, 1) and (x, y). What is the sum of the distinct possible values for x ?

(6) _____ The ratio of the length to the width of a rectangle is 12 to 5 and the area of the rectangle is 540 square units. What is the number of units in the length of the rectangle?

(7) _____ Angle BCD is an exterior angle of triangle ABC and has a measure of 55 degrees. Angle BAC has a measure of 32 degrees. How many degrees are in the measure of angle ABC?

(8) _____ What is the number of square inches in the area of a rectangle having dimensions $1\frac{1}{2}$ yards by $1\frac{1}{2}$ feet?

(9) _____ The horizontally and vertically adjacent points in this square grid are 1 cm apart. How many square centimeters are in the area of square $ABCD$?



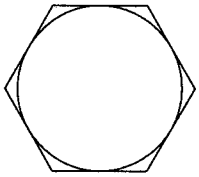
(10) _____ A 1 cm × 1 cm square is cut from each of the four corners of a square piece of cardboard with area 64 square centimeters. The sides are then folded up to make a rectangular box. What is the volume in cubic centimeters of the box?

(11) _____ A triangle with a height of 24 inches has the same area as a rectangle 12 inches by 6 inches. How many inches long is the base of the triangle that corresponds to the 24-inch height?

(12) _____ By how many degrees does the measure of an interior angle of a regular octagon exceed the measure of an interior angle of a regular hexagon?

(13) _____ What percent of the volume of a $10 \times 10 \times 10$ box can be filled with $4 \times 4 \times 4$ wooden cubes? Express your answer as a decimal to the nearest tenth.

(14) _____ How many inches are in the area of a circle inscribed in a regular hexagon with side length 12 inches? ~~Express your answer in terms of π.~~



(15) _____ Each edge of a regular octahedron is colored orange or black. If every face of the octahedron has at least one orange edge, what is the smallest possible number of orange edges?

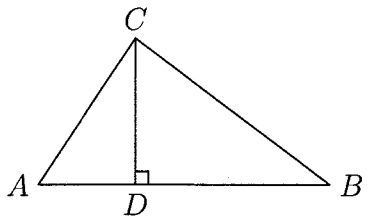
(16) _____ What is the number of square centimeters in the area of triangle DEF if angle D is a right angle, $DE = 36$ cm, and $EF = 39$ cm?

(17) _____ Mrs. Read can knit one pair of children's mittens with a ball of yarn six inches in diameter. How many pairs of identical mittens can she knit with a ball of yarn twelve inches in diameter? Assume that the balls of yarn are rolled consistently.

(18) _____ The measures of the sides of an isosceles trapezoid are in the ratio of 3:4:3:6. The perimeter of the trapezoid is 48 cm. What is the number of square centimeters in the area of the trapezoid? ~~Express your answer in simplest radical form~~.

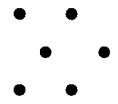
(19) _____ The measures of the three angles of a triangle are in a ratio of $4 : 5 : 6$. What is the measure in degrees of the greatest supplement of these three angles?

(20) _____ In $\triangle ABC$, D is a point on $\overline{AB}$ such that $CD = 6$ and $DB = 8$. If $\angle CAD = \angle BCD$, how many units are in the perimeter of $\triangle ACD$?



(21) _____ How many square units are in the area of the convex quadrilateral with vertices (0, 0), (3, 0), (2, 2) and (0, 3)?

(22) _____ Each point in the hexogonal lattice is one unit from its nearest neighbor. How many circles of radius one contain at least two points of the lattice?



(23) _____ The slant height of a cone is 13 cm, and the height from the vertex to the center of the base is 12 cm. What is the number of cubic centimeters in the volume of the cone? ~~Express your answer in terms of π~~.

(24) _____ A 24-foot by 72-foot rectangular dance floor is completely tiled with 1-foot by 1-foot square tiles. Two opposite corners of the dance floor are connected by a diagonal. This diagonal passes through the interior of exactly how many tiles?

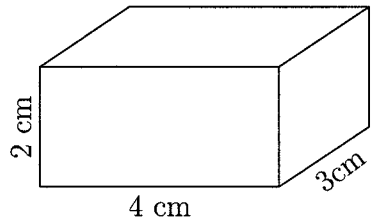
~~(25)~~ _____ Points P and R are located at (1, 3) and (7, 15) respectively. Point M is the midpoint of segment PR. Segment PR is reflected over the x-axis. What is the sum of the coordinates of the image of point M (the midpoint of the reflected segment)?

25 ~~(26)~~ _____ Exactly forty-eight non-overlapping square tiles, each 1 inch by 1 inch, fit within a rectangle. What is the least possible number of inches in the perimeter of the rectangle?

~~(27)~~ _____ The point $(4, 3)$ is reflected over the x-axis and then over the y-axis. What is the sum of the coordinates of the new point?

26 ~~(28)~~ _____ Yuliya has a piece of meat which measures 5" x 6" x 8". In order to make a stew, she would like to cut pieces which measure 2" x 3" x 4". What is the maximum number of such pieces she can cut from this piece of meat?

27 ~~(29)~~ _____ How many square centimeters are in the surface area of the rectangular solid shown?



28 ~~(30)~~ _____ A circular pizza with a radius of 6 inches is cut along radii into three wedge-shaped slices. The measures of two of the central angles are 80 degrees and 130 degrees. What is the number of square inches in the area of the largest slice? ~~Express your answer in terms of π~~.

Answer Sheet

Number	Answer	Problem ID
1	80 inches	13A2
2	~~$12\sqrt{3}$ units~~ 20.784	5422
3	130	45D3
4	9.8	A5D3
5	21	1241
6	36	D355
7	23	4503
8	972 square inches	C2C2
9	5 cm^2	4455
10	36 cubic centimeters	BDC1
11	6 inches	BAD3
12	15	5DD3
13	51.2	11001
14	~~108π~~ 339.12	34C3
15	4 edges	CA14
16	270	1D03
17	8	CCCC
18	~~$90\sqrt{2}$ centimeters~~ 127.28	35B31
19	132	2DB3
20	18	55C3
21	6 square units	A322
22	13	C3C3
23	~~100π~~ 314.20	3BAC
24	72 tiles	14A2
~~25~~	~~5~~	1B42
25 ~~26~~	28 inches	33C2
~~27~~	~~7~~	54AC
26 ~~28~~	10	ADD3
27 ~~29~~	52	D2C3
28 ~~30~~	~~15π square inches~~ 47.13	53C2

Solutions

(1) **80 inches** **ID: [13A2]**
No solution is available at this time.

(2) $12\sqrt{3}$ **units** **ID: [5422]**
Let the height of the wide can be h and the height of the narrow can be $3h$. Let the wide can have radius x units. Since the two volumes are equal, we have

$$\pi(12^2)(3h) = \pi(x^2)(h).$$

Solving yields $x = 12\sqrt{3}$ so the wide can has radius $\boxed{12\sqrt{3}}$ units.

(3) **130** **ID: [45D3]**
No solution is available at this time.

(4) 9.8 **ID: [A5D3]**
No solution is available at this time.

(5) **21** **ID: [1241]**
Let A, B, and C be the vectors for the first three points. The possible values for (x, y) are $A+B-C$, $A+C-B$, and $B+C-A$. The sum of these is $A+B+C$, and so the sum of the x-coordinates is $10+7+4=\boxed{21}$.

(6) 36 **ID: [D355]**
No solution is available at this time.

(7) **23** **ID: [4503]**
No solution is available at this time.

(8) **972 square inches** **ID: [C2C2]**
No solution is available at this time.

(9) 5 **cm^2** **ID: [4455]**
No solution is available at this time.

(10) **36 cubic centimeters** **ID: [BDC1]**
No solution is available at this time.

(11) 6 **inches** **ID: [BAD3]**
No solution is available at this time.

(12) 15 **ID: [5DD3]**
The sum of the angle measures in a polygon with n sides is $180(n-2)$ degrees. So, the sum of the octagon's angles is $180(8-2) = 1080$ degrees. The polygon is regular, so all the angles have the same measure, which means each is $\frac{1080^\circ}{8} = 135^\circ$. Similarly, the sum of the angles of a hexagon is $180(6-2) = 720$ degrees, which means each angle in a regular hexagon has measure $\frac{720^\circ}{6} = 120^\circ$.

Therefore, the desired difference is $135^\circ - 120^\circ = \boxed{15^\circ}$.

(13) **51.2** **ID: [11001]**
No solution is available at this time.

(14) 108π **ID: [34C3]**
No solution is available at this time.

(15) **4 edges** **ID: [CA14]**
No solution is available at this time.

(16) **270** **ID: [1D03]**
No solution is available at this time.

(17) **8** **ID: [CCCC]**

A ball of yarn 12 inches in diameter has twice the diameter of a ball of yarn 6 inches in diameter. Let the radius of the small ball be r and the radius of the large ball be $2r$. Then, the volume of the small ball is $\frac{4}{3}\pi r^3$ and the volume of the large ball is $\frac{4}{3}\pi(2r)^3 = 8 \cdot \frac{4}{3}\pi r^3$. Hence the large ball has 8 times the volume of the small ball, so Mrs. Read can knit $\boxed{8}$ pairs of identical mittens with the large yarn ball.

(18) $90\sqrt{2}$ **centimeters** **ID: [35B31]**

No solution is available at this time.

(19) 132 **ID: [2DB3]**

No solution is available at this time.

(20) **18** **ID: [55C3]**

No solution is available at this time.

(21) **6 square units** **ID: [A322]**

No solution is available at this time.

(22) **13** **ID: [C3C3]**

No solution is available at this time.

(23) 100π **ID: [3BAC]**

We create a right triangle with the slant height as the hypotenuse, the height from the vertex to the center of the base as one of the legs, and a radius as the other leg. By Pythagorean theorem, the radius measures $\sqrt{13^2 - 12^2} = 5$ cm. It follows that the volume of the cone is $(1/3)\pi(5^2)(12) = \boxed{100\pi}$.

(24) **72 tiles** **ID: [14A2]**

No solution is available at this time.

(25) **-5** **ID: [1B42]**

Point M has coordinates $(4, 9)$. Therefore, its image has coordinates $(4, -9)$. Thus the sum is $\boxed{-5}$.

Alternatively, the image of point M is the midpoint of the images of points P and R and thus is the midpoint of $(1, -3)$ and $(7, -15)$, which is also $(4, -9)$.

(26) **28 inches** **ID: [33C2]**

No solution is available at this time.

(27) **-7** **ID: [54AC]**

No solution is available at this time.

(28) 10 **ID: [ADD3]**

No solution is available at this time.

(29) **52** **ID: [D2C3]**

No solution is available at this time.

(30) 15π **square inches** **ID: [53C2]**

No solution is available at this time.