☆ Part 2 requires use of Calculator

Mathcounts / AMC 8 (Week 9) (Part 1)

(1) _____ The point at (a, b) on a Cartesian plane is reflected over the y-axis to the point at (j, k). If $a + j = 0$ and $b + k = 0$, what is the value of b?

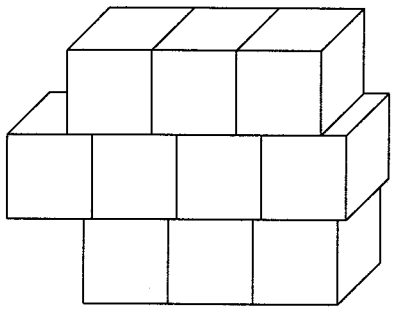
(2) _____ A smaller rectangular box has a length and width of 10 cm each and a height of 1 cm. A larger box is twice the length, three times the width, and 10 times the height of the smaller box. What is the greatest number of the smaller boxes that can fit inside one of the larger boxes?

(3) _____ Point A and line m are in the same plane, but A is not on m. How many lines containing A are parallel to m ?

(4) _____ A wheel has a circumference of 3 meters. The radius can be expressed as $\frac{A}{B\pi}$ meters, with relatively prime integers A and B. What is the value of $A + B$?

(5) _____ A parallelogram has three of its vertices at $(-1, 0)$, $(2, 4)$ and $(2, -4)$. What is the positive difference between the greatest possible perimeter and the least possible perimeter of the parallelogram?

(6) _____ Ten unit cubes are glued together as shown. How many square units are in the surface area of the resulting solid?



(7) _____ Four straight lines intersect a circular region. The lines and circle are coplanar, and two of the lines are parallel. What is the maximum number of non-overlapping regions inside the circle?

(8) _____ One right triangle has sides of length 3 cm, 4 cm, and 5 cm. Another right triangle has two sides of length 3 cm and 4 cm, but it is not congruent to the first triangle. What is the length of the third side of this triangle? ~~Express your answer in simplest radical form.~~

(9) _____ What is the ratio of the numerical value of the area, in square units, of an equilateral triangle of side length 4 units to the numerical value of its perimeter, in units? ~~Express your answer as a common fraction in simplest radical form.~~

~~(10)~~ _____ The points $B(1,1)$, $I(2,4)$ and $G(5,1)$ are plotted in the standard rectangular coordinate system to form triangle BIG. Triangle BIG is translated five units to the left and two units upward to triangle $B'I'G'$, in such a way that B' is the image of B, I' is the image of I, and G' is the image of G. What is the midpoint of segment $B'G'$? Express your answer as an ordered pair.

~~(11)~~ _____ What is the midpoint of the segment connecting $(3,4)$ and $(3.8,5.6)$ in the Cartesian plane? Express the coordinates as decimals to the nearest tenth.

10 ~~(12)~~ _____ In a convex heptagon, the degree measures of the interior angles are x, x, $x-2$, $x-2$, $x+2$, $x+2$ and $x+4$ degrees. What is the degree measure of the largest interior angle?

11 ~~(13)~~ _____ The length of a diagonal of a square is $\sqrt{2}+\sqrt{3}$ units. What is the area of the square? ~~Express your answer in simplest form as $\frac{a}{b}+\sqrt{c}$, where $\frac{a}{b}$ is a common fraction and c has no perfect square factors other than 1.~~

12 ~~(14)~~ _____ For a particular circle, a central angle of 75° will intercept an arc of length 10π feet. What is the radius of this circle?

13 ~~(15)~~ _____ What is the greatest number of interior right angles a convex octagon can have?

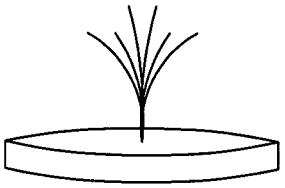
14 ~~(16)~~ _____ The sum of the lengths of all the edges of a particular cube is 24 cm. When the cube is unfolded into a flat sheet (net) of six connected squares, each sharing at least one side with another square, what is the perimeter of this resulting polygon?

15 ~~(17)~~ _____ How many non-congruent triangles are there with sides of integer length having at least one side of length five units and having no side longer than five units?

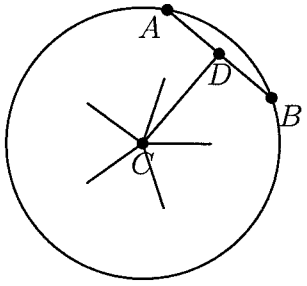
16 ~~(18)~~ _____ A bottle water company has designed a new cup for its dispenser. The cup will be a right circular cone with a three-inch radius. How tall will the cup need to be to hold 93 cubic inches of water? Express your answer to the nearest whole number.

17 ~~(19)~~ _____ In parallelogram $ABCD$, $AB = 16$ cm, $DA = 3\sqrt{2}$ cm, and sides AB and DA form a 45-degree interior angle. In isosceles trapezoid $WXYZ$ with $WX \neq YZ$, segment WX is the longer parallel side and has length 16 cm, and two interior angles each have a measure of 45 degrees. Trapezoid $WXYZ$ has the same area as parallelogram $ABCD$. What is the length in centimeters of segment YZ?

18 ~~(20)~~ _____ To be able to walk to the center C of a circular fountain, a repair crew places a 16-foot plank from A to B and then a 10-foot plank from D to C, where D is the midpoint of $\overline{AB}$. What is the area of the circular base of the fountain? ~~Express your answer in terms of π.~~



Side View



Top View

19 ~~(21)~~ _____ Either increasing the radius or the height of a cylinder by six inches will result in the same volume. The original height of the cylinder is two inches. What is the original radius in inches?

20 (~~22~~) _____ On a graph, a lattice point is an ordered pair (x, y) with integers x and y. Exactly 15 lattice points lie strictly in the interior of the triangular region with vertices $(0, 0)$, $(N, 0)$ and (N, N), where $N > 0$. What is the value of N?

Mathcounts / AMC 8 (Week 9) Part 2

2(1) _____ In the book *Holes*, Stanley must dig a cylindrical hole that measures five feet across and five feet deep. What is the volume of a cylinder with these dimensions? Express your answer to the nearest whole number.

2(2) _____ A square has an area of 1000 square centimeters. How long is its diagonal? Express your answer as a decimal to the nearest tenth.

2(3) _____ Tony will paint 16 right, cylindrical columns. The top face and bottom face of each column will be covered, so those parts will not be painted. Each column is 18 feet tall and has a diameter of 10 feet. One gallon of paint will cover 350 square feet. If paint is sold only in full gallons, how many gallons of paint must Tony buy to paint all 16 columns?

2(4) _____ A circular disk has a radius of two units. A point is marked on the edge of the disk. The disk rotates about its center, causing the point to travel a distance of 90 units. How many rotations did the disk make? Express your answer as a decimal to the nearest tenth.

2(5) _____ The coffee tin is a right circular cylinder with a radius of three inches, and it can hold exactly three pounds of coffee when it is full. Thirteen ounces of coffee fill a space of 60 cubic inches, and there are 16 ounces in one pound. How tall is the coffee tin? Express your answer as a decimal to the nearest tenth.

2(6) _____ What is the area, in sq units, of a trapezoid bounded by the lines $y = x$, $y = 10$, $y = 5$ and the y-axis? Express your answer as a decimal to the nearest tenth.

2(7) _____ What is the distance between $(3, 4)$ and $(3.8, 5.5)$ in the Cartesian plane? Express your answer as a decimal to the nearest tenth.

2(8) _____ The European equivalent of $8\frac{1}{2}$" by 11" paper is called A4 paper, and its dimensions are 0.21 meters by 0.297 meters. What is the greatest total area, in square meters, that can be covered by 21 sheets of rectangular A4 paper? Express your answer as a decimal to the nearest tenth.

2(9) _____ Keli and Mario are planning to plant rectangular gardens of the same length, side by side with fencing all around and dividing the two plots. The total amount of fencing is 100 feet. If the total area of the two plots is 336 square feet and the dimensions are integers, what is the length of the fence that divides the two plots?