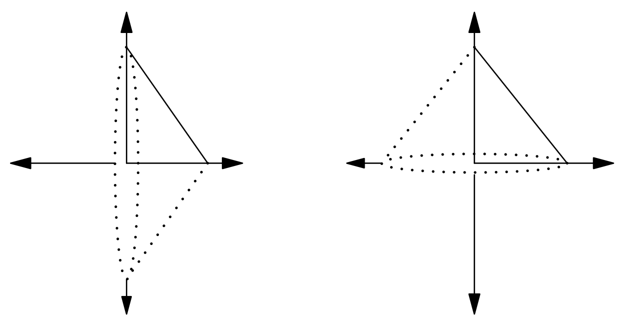
Geometry Worksheet (3A4)

(1) __________ The measures of four angles of a quadrilateral form an arithmetic sequence. The largest is 15 degrees less than twice the smallest. What is the measure, in degrees, of the largest angle?

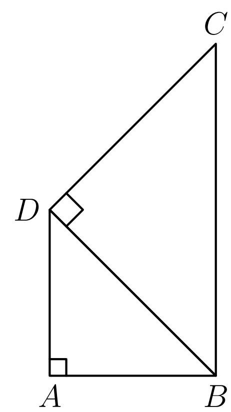
(2) __________ The triangular region having vertices $A(0,0)$, $B(3,0)$ and $C(0,4)$ is rotated about the x-axis to form a solid having volume X. The same triangular region is then rotated about the y-axis to form a solid having volume Y. What is the ratio of X to Y? Express your answer as a common fraction.



(3) __________ The point $(5,3)$ is reflected about the line $x=2$. The image point is then reflected about the line $y=2$. The resulting point is (a,b). Compute $a+b$.

(4) __________ A farmer ties his goat to the corner of a 10-foot by 15-foot rectangular shed in an otherwise empty field. The length of rope from the shed to the goat is 25 feet. Over how many square feet of the field can his goat roam? Express your answer in terms of π?

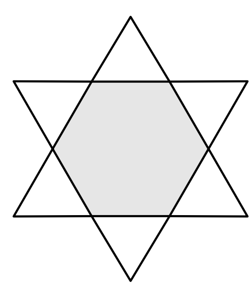
(5) __________ Each triangle in this figure is an isosceles right triangle. The length of $\overline{BC}$ is 2 units. What is the number of units in the perimeter of quadrilateral $ABCD$? Express your answer in simplest radical form.



(6) __________ A circle is inscribed in a square. What is the ratio of the area of the square *not* within the circle to the *total* area of the square? Express your answer as a common fraction in terms of π?

(7) __________ The endpoints of segment $\overline{AB}$ are $A(0, 8)$ and $B(15, 0)$. What is the shortest distance, in units, from $P(0, 0)$ to segment $\overline{AB}$? Express your answer as a common fraction.

(8) __________ Two congruent equilateral triangles intersect so the region of intersection is a regular hexagon as shown. The area of each unshaded equilateral triangle is 4m^2. How many square centimeters are in the area of the shaded hexagon?

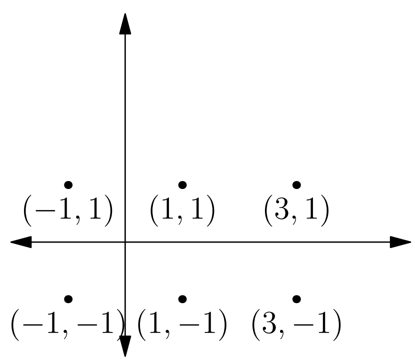


(9) ________ Each corner of a cube is sliced off, leaving a triangular face at each corner and octagonal faces for each face of the original cube. How many edges will the new polyhedron have?

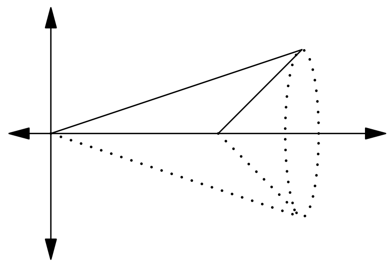
(10) ________ Two congruent cylinders each have radius 8 inches and height 3 inches. The radius of one cylinder and the height of the other are both increased by the same number of inches. The resulting volumes are equal. How many inches is the increase? Express your answer as a common fraction.

(11) ________ Find the number of cubic centimeters in the volume of the cylinder formed by rotating a square with side length 14 centimeters about its vertical line of symmetry. Express your answer in terms of π.

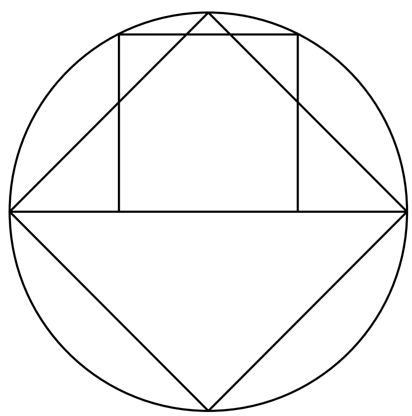
(12) ________ The graph shows six labeled points. How many distinct circles of radius 2 units are in the coordinate plane and pass through exactly two of the labeled points on this graph?



(13) ________ The vertices of a triangle are at (0, 0), (12, 0) and (18, 6). The triangle and its interior are rotated about the x-axis to form a solid figure. What is the volume, in cubic units, of this solid? Express your answer in terms of π.



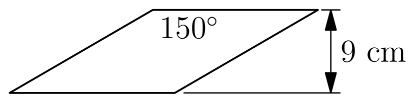
(14) ________ A square is inscribed in a semicircle, and a second square is inscribed in the whole circle, as shown below. What is the ratio of the area of the smaller square to the area of the larger square? Express your answer as a common fraction.



(15) ________ A square blanket measuring x feet by x feet was folded in half, folded in half again and finally folded in half one last time. After these three successive folds, without ever unfolding, the blanket covers an area of 8 square feet. What is the value of x?

(16) ________ What is the mean of the measures of the three exterior angles of a triangle if two of the interior angles have measures of 63 and 78 degrees?

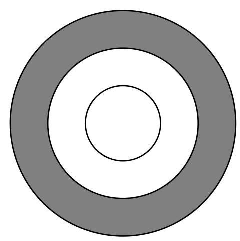
(17) ________ What is the number of centimeters in the perimeter of the rhombus shown?



(18) ________ What is the number of square inches in the surface area of a cube with space diagonal of length $3\sqrt{3}$ inches?

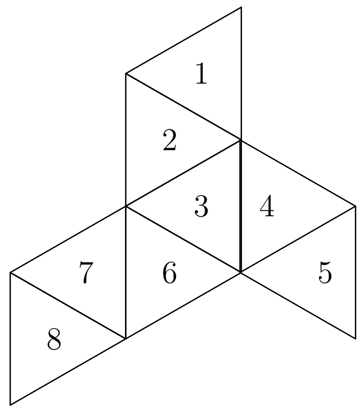
(19) ________ A triangle has vertices at $(-3, 2), (6, -2), (3, 5)$. How many square units are in the area of the triangle? Express your answer as a decimal to the nearest tenth.

(20) ________ The concentric circles are drawn as shown with radii 2, 4 and 6 units. What is the ratio of the area of the smallest circle to the area of the shaded region? Express your answer as a common fraction.



(21) ________ A line segment drawn from a vertex of a unit square to a point on the square forms two regions. The area of the smaller region is one-third of the area of the larger region. How many units are in the length of that segment? Express your answer as a common fraction in simplest radical form.

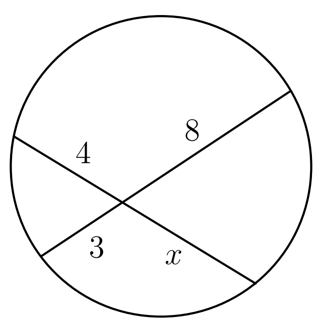
(22) ________ This net is folded into a regular octahedron. What is the sum of the numbers on the triangular faces sharing an edge with the face with a "1" on it?



(23) ________ The perimeter of a regular hexagon is 48 inches. What is the number of square inches in the positive difference between the areas of the circumscribed and the inscribed circles of the hexagon? Express your answer in terms of π.

(24) ________ The hour hand of a clock is 6 inches long and the minute hand is 8 inches long. What is the ratio of the distance in inches traveled by the tip of the hour hand to the distance in inches traveled by the tip of the minute hand from noon to 3 p.m.? Express your answer as a common fraction.

(25) ________ Two chords intersect as shown. What is the number of units in the value of x?



(26) ________ A triangle with sides $3a - 1$, $a^2 + 1$ and $a^2 + 2$ has a perimeter of 16 units. What is the number of square units in the area of the triangle?

(27) ________ If all angles are measured in degrees, the ratio of three times the measure of $\angle A$ to four times the measure of the complement of $\angle A$ to half the measure of the supplement of $\angle A$ is $3 : 14 : 4$. What is the number of degrees in the measure of the complement of $\angle A$?

(28) ________ What is the positive value of m such that the triangle bounded by the lines $y = 0, x = 12$ and $y = mx$ has an area of 1872 square units?

(29) ________ The volume of a cylinder is 60 cubic centimeters. What is the number of cubic centimeters in the volume of the sphere it circumscribes?

(30) ________ What integer is closest to the area of a triangle whose sides are 5, 6 and 7 units?

Answer Sheet

Number	Answer	Problem ID
1	115 degrees	D4B5
2	$\frac{4}{3}$	02B5
3	0	33A5
4	550π	2023
5	$4+\sqrt{2}$ units	0102
6	$\frac{4-\pi}{4}$	A1B5
7	120/17	C123
8	24	32A5
9	36 edges	ACA4
10	16/3 (inches)	22A5
11	686π cubic cm	40101
12	22 circles	A102
13	144π	35B5
14	2/5	43A5
15	8	1123
16	120 degrees	01B5
17	72	D4B01
18	54	52A5
19	25.5	02501
20	$\frac{1}{5}$	B1B5
21	$\frac{\sqrt{5}}{2}$ units	5AC2
22	14	C002
23	$\frac{16}{\pi}$	B0101
24	1/16	BCA4
25	6	13A5
26	12 square units	10B4
27	70 degrees	4102
28	26	2AC2
29	40 cubic cm	D3A5
30	15	44B5

Solutions

(1) 115 **degrees** **ID: [D4B5]**
No solution is available at this time.

(2) $\frac{4}{3}$ **ID: [02B5]**
No solution is available at this time.

(3) **0** **ID: [33A5]**
No solution is available at this time.

(4) 550π **ID: [2023]**
No solution is available at this time.

(5) $4+\sqrt{2}$ **units** **ID: [0102]**
The length of the hypotenuse of an isosceles right triangle is $\sqrt{2}$ times the length of each leg. Therefore, $BD = \frac{BC}{\sqrt{2}} = \frac{2}{\sqrt{2}} \cdot \left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{2\sqrt{2}}{2} = \sqrt{2}$ units. Applying the same rule to triangle ABD, we find that $AB = BD/\sqrt{2} = \sqrt{2}/\sqrt{2} = 1$ unit. The perimeter of quadrilateral $ABCD$ is $AB + BD + CD + DA = 1 + 2 + \sqrt{2} + 1 = \boxed{4+\sqrt{2}}$ units.

(6) $\frac{4-\pi}{4}$ **ID: [A1B5]**
No solution is available at this time.

(7) **120/17** **ID: [C123]**
No solution is available at this time.

(8) **24** **ID: [32A5]**
No solution is available at this time.

(9) **36 edges** **ID: [ACA4]**

The cube has 12 edges originally, and each cut adds 3 new edges. Since there are 8 cuts (one for each vertex), $3 \cdot 8 = 24$ edges are added. In total, the new polyhedron has $12 + 24 = \boxed{36}$ edges.

(10) **16/3 (inches)** **ID: [22A5]**

Let the increase measure x inches. The cylinder with increased radius now has volume

$$\pi(8+x)^2(3)$$

and the cylinder with increased height now has volume

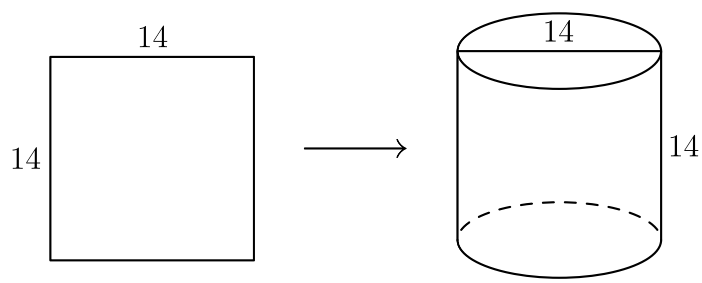
$$\pi(8^2)(3+x).$$

Setting these two quantities equal and solving yields

$$3(64+16x+x^2) = 64(3+x) \Rightarrow 3x^2 - 16x = x(3x-16) = 0$$

so $x = 0$ or $x = 16/3$. The latter is the valid solution, so the increase measures $\boxed{16/3}$ inches.

(11) 686π **cubic cm** **ID: [40101]**



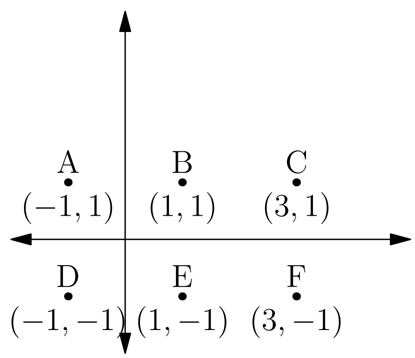
Rotating the square about its vertical line of symmetry creates a right circular cylinder with diameter 14 and height 14. Thus, the cylinder has radius $14/2 = 7$ and volume $\pi(7^2)(14) = \pi(50-1)(14) = \pi(700-14) = \boxed{686\pi}$.

(12) **22 circles** **ID: [A102]**

If two points are less than four units away, then there are exactly two circles of radius 2 containing both of them. If they are more than four units away, then no circles of radius 2 contain both of them. If the points are exactly four units apart, then there is one circle of radius 2 that contains both of them.

We would also like to know when a circle might contain three of these points. Because of this array, any three of these points (which are non-collinear) must form a right angle. If three points on a circle form a right angle, then two of them (the endpoints of the right angle) must form a diameter. This would imply that those two points are four units apart. It follows that if two points are fewer than four units apart, then none of the circles that contain both of them contain any other points on this grid.

We now label the points for clarity:



The only pairs of points which are not fewer than four units apart are the following: AF CD AC DF

The other $\binom{6}{2} - 4 = 11$ pairs must be fewer than four units apart, producing 22 circles.

The two pairs that are exactly four units apart, AC and DF, lie on a circle that also contains another one of the six points (E and B, respectively), so neither of these pairs contribute any circles.

Therefore, the total number of circles is $\boxed{22}$.

(13) 144π **ID: [35B5]**

No solution is available at this time.

(14) **2/5** **ID: [43A5]**

No solution is available at this time.

(15) **8** **ID: [1123]**

No solution is available at this time.

(16) 120 **degrees** **ID: [01B5]**

The exterior angles of a triangle sum to 360°, so the average of their measures is $360^\circ/3 = \boxed{120^\circ}$.

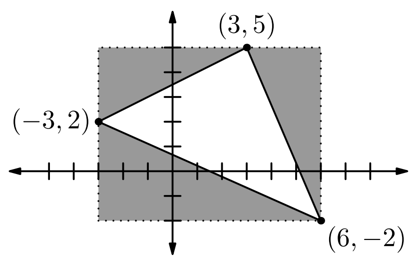
(17) **72** **ID: [D4B01]**

No solution is available at this time.

(18) **54** **ID: [52A5]**

No solution is available at this time.

(19) **25.5** **ID: [02501]**



We find the area of the given triangle by subtracting the sum of the areas of the three shaded triangles in the figure from the area of the rectangle formed by all four triangles.

The area of the rectangle is $9(7) = 63$ square units, and the sum of the areas of the shaded triangles is

$$\frac{1}{2}(6)(3) + \frac{1}{2}(3)(7) + \frac{1}{2}(4)(9) = 37.5$$

square units. The area of the fourth triangle is $63 - 37.5 = \boxed{25.5}$ square units.

(20) $\frac{1}{5}$ **ID: [B1B5]**

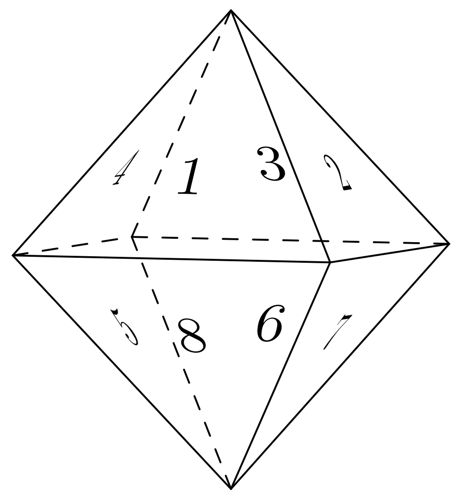
No solution is available at this time.

(21) $\frac{\sqrt{5}}{2}$ **units** **ID: [5AC2]**

No solution is available at this time.

(22) **14** **ID: [C002]**

Four faces meet at every vertex of a regular octahedron. Therefore, face 4 shares an edge with face 1, and we can picture the octahedron as having faces 1–4 forming the "upper pyramid" of the octahedron (see figure). We see that each of the faces between 1 and 4 shares an edge with only one of the faces 5, 6, 7, and 8. In the net, face 3 borders face 6 and face 4 borders face 5. Also, since four faces meet at each vertex, face 7 must border face 2. Therefore, face 1 borders face 8 in addition to faces 2 and 4. The sum of 8, 2, and 4 is $\boxed{14}$.



(23) $\frac{16}{\pi}$ **ID: [B0101]**

No solution is available at this time.

(24) **1/16** **ID: [BCA4]**

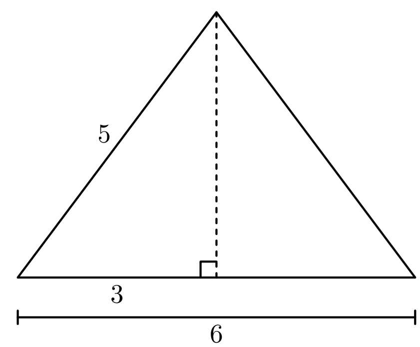
In three hours, the hour hand travels $\frac{1}{4}$ of a revolution while the minute hand travels 3 revolutions. So the ratio of the number of revolutions traveled by the hour hand to the number of revolutions traveled by the minute hand is $\frac{1}{12}$. However, the ratio of distances traveled is even smaller, because for each revolution the hour hand travels $\frac{2\pi(6 \text{ in.})}{2\pi(8 \text{ in.})} = \frac{3}{4}$ as far as the minute hand. Therefore, the ratio of the total distance traveled by the hour hand to the total distance traveled by the minute hand is $\dfrac{1}{12} \cdot \dfrac{3}{4} = \boxed{\dfrac{1}{16}}$.

(25) **6** **ID: [13A5]**

No solution is available at this time.

(26) **12 square units ID: [10B4]**

Sum $3a-1$, a^2+1, and a^2+2 to find $2a^2+3a+2=16$. Subtract 16 from both sides and factor the left-hand side to find $(2a+7)(a-2)=0 \implies a=-7/2$ or $a=2$. Discarding the negative solution, we substitute $a=2$ into $3a-1$, a^2+1, and a^2+2 to find that the side lengths of the triangle are 5, 5, and 6 units. Draw a perpendicular from the 6-unit side to the opposite vertex to divide the triangle into two congruent right triangles (see figure). The height of the triangle is $\sqrt{5^2-3^2}=4$ units, so the area of the triangle is $\frac{1}{2}(6)(4)=\boxed{12\text{ square units}}$.



(27) **70 degrees ID: [4102]**

Let x be the number of degrees in the measure of $\angle A$. Then we have

$$\frac{3x}{4(90-x)}=\frac{3}{14},$$

from the information "the ratio of three times the measure of $\angle A$ to four times the measure of the complement of $\angle A$ is $3:14$. Multiplying both sides by $\frac{2}{3}$ and clearing denominators, we find $7x=180-2x \implies 9x=180 \implies x=20$. The measure of the complement of 20 degrees is $\boxed{70}$ degrees.

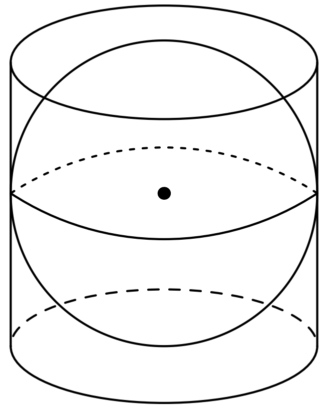
Note: The hypothesis "if all angles are measured in degrees" is not necessary. The angle is determined uniquely by the given information regardless of the units used.

(28) **26 ID: [2AC2]**

No solution is available at this time.

(29) **40 cubic cm** **ID: [D3A5]**

We begin by drawing a diagram:



Let the radius of the sphere be r. We see the radius of the cylinder is r and the height of the cylinder is $2r$. Thus, from the cylinder's volume we have

$$60 = \pi(r^2)(2r) = 2\pi r^3.$$

Dividing both sides by 2 yields $\pi r^3 = 30$. The volume of the sphere is

$$\frac{4}{3}\pi r^3 = \frac{4}{3}(30) = \boxed{40}$$

cubic centimeters. (Notice that we didn't have to solve for r!)

(30) 15 **ID: [44B5]**

No solution is available at this time.