Number Theory 5A3

(1) _____ How many positive integers less than 100 have an odd number of distinct factors?

(2) _____ A farmer has some pigs and some chickens. He finds that together they have 70 heads and 200 legs. How many pigs does he have?

(3) _____ If a three-digit number of the form $1D1$ is divided by D, the quotient is a two digit number of the form $2D$ remainder 5. What is the value of D?

(4) _____ 1993 is a:
(a) perfect square;
(b) prime number; or
(c) perfect number.

(5) _____ If the three-digit number $\underline{2d2}$ is divisible by 7, what is d?

(6) _____ What is the remainder when the product $1734 \times 5389 \times 80{,}607$ is divided by 10?

(7) _____ A and B are non-zero digits for which $\underline{A468B05}$ is divisible by 11. What is $A + B$?

(8) _____ Find the sum of the smallest and largest prime factors of 10101.

(9) _____ What is the smallest number divisible by integers 1 through 9?

(10) _____ Two factors, each with no digit greater than 5, have a product of 16,848. What is the largest possible 3-digit factor satisfying these conditions?

(11) _____ Determine the units digit of $17^{13} - 17$.