Counting / Probability Worksheet (2A4)

(1) ________ The digits 2, 3, 4, 7 and 8 will be put in random order to make a positive five-digit integer. What is the probability that the resulting integer will be divisible by 11? Express your answer as a common fraction.

(2) ________ An o-Pod MP3 player stores and plays entire songs. Celeste has 10 songs stored on her o-Pod. The time length of each song is different. When the songs are ordered by length, the shortest song is only 30 seconds long and each subsequent song is 30 seconds longer than the previous song. Her favorite song is 3 minutes, 30 seconds long. The o-Pod will play all the songs in random order before repeating any song. What is the probability that she hears the first 4 minutes, 30 seconds of music - there are no pauses between songs - without hearing every second of her favorite song? Express your answer as a common fraction.

(3) ________ In how many ways can 81 be written as the sum of three positive perfect squares if the order of the three perfect squares does not matter?

(4) ________ Tamyra is making four cookies and has exactly four chocolate chips. If she distributes the chips randomly into the four cookies, what is the probability that there are no more than two chips in any one cookie? Express your answer as a common fraction.

(5) ________ How many integers between 1000 and 9999 have exactly one pair of equal digits, such as 4049 or 9902, but not 4449 or 4040?

(6) ________ What is the sum, in dollars, of the total values of every possible combination of three coins using only pennies, nickels, dimes and quarters?

(7) ________ Six students are being grouped into three pairs to work on a science lab. How many different combinations of three pairs are possible?

(8) ________ How many combinations of two or more consecutive positive integers have a sum of 45?

(9) ________ The integers r and k are randomly selected, where $-3 < r < 6$ and $1 < k < 8$. What is the probability that the division $r \div k$ is an integer value? Express your answer as a common fraction.

(10) ________ Four boys (Bs) and three girls (Gs) will be seated in a row. When a boy is next to a girl, we will call this a "meeting point." When the seven kids are seated, there may be only one meeting point, as in BBBBGGG, or there may be as many as six meeting points, as in BGBGBGB (the arrangement Boy-Girl-Boy-Girl-Boy-Girl-Boy). Given all of the possible seating arrangements for these seven kids, what is the average number of meeting points per seating arrangement? Express your answer as a common fraction.

(11) ________ How many positive integers divisible by 4 can be formed using the digits 1, 2, 3 and 4, each at most once for each integer?

(12) ________ Two different prime numbers are selected from the first eight prime numbers. What is the probability that the sum of the two chosen prime numbers is greater than or equal to 16? Express your answer as a common fraction.

(13) ________ Kevin will start with the integers 1, 2, 3 and 4 each used exactly once and written in a row in any order. Then he will find the sum of the adjacent pairs of integers in each row to make a new row, until one integer is left. For example, if he starts with 3, 2, 1, 4, and then takes sums to get 5, 3, 5, followed by 8, 8, he ends with the final sum 16. Including all of Kevin's possible starting arrangements of the integers 1, 2, 3 and 4, how many possible final sums are there?

(14) ________ How many two-digit prime numbers less than 50 exist such that the difference of their digits is even?

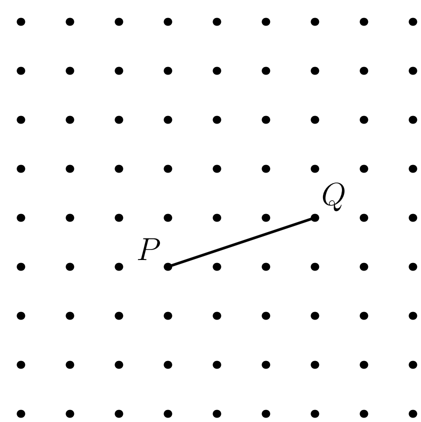
(15) ________ Suppose x and y are two distinct two-digit positive integers such that y is the reverse of x. (For example, $x = 12$ and $y = 21$ is one such combination.) How many different sums $x + y$ are possible?

(16) ________ Derek's phone number, 336 - 7624, has the property that the three-digit prefix, 336, equals the product of the last four digits, $7 \times 6 \times 2 \times 4$. How many seven-digit phone numbers beginning with 336 have this property?

(17) ________ Julie baked cupcakes for her family at home and for a party at school. She iced 4 cupcakes with red frosting, 2 cupcakes with orange frosting, 2 with yellow, 2 with green, 3 with blue and the last 3 with violet frosting. Each cupcake is iced with exactly one color of frosting. Julie plans to take exactly 10 of the cupcakes to her party, and will take either all of the cupcakes of a particular color or none of the cupcakes of that color. How many different combinations of cupcakes could she take to her party?

(18) ________ When five standard six-sided dice are rolled sequentially there are $6^5 = 7776$ possible outcomes. For how many outcomes is the sum of the five rolled numbers exactly 27?

(19) ________ On this 9 by 9 grid of lattice points, at how many different lattice points can a point R be placed such that it would be the third vertex of isosceles triangle PQR with $PQ = PR$ or $PQ = QR$? (The sum of the lengths of any two sides of a triangle is greater than the length of the remaining side.)



(20) ________ How many positive four-digit odd integers can be created using only the digits 0, 1, 2, 3, 4 and 5 if repetition of digits is not allowed?

(21) ________ Amanda has a 2-cm rod, a 5-cm rod and a 9-cm rod. She will randomly choose a fourth rod from a box containing rods of lengths 2, 4, 6, 8, 10, 12, 14, 16, 18 and 20 cm. What is the probability that Amanda's four rods will be able to form a convex quadrilateral, if the rods are to be connected endpoint to endpoint? Express your answer as a common fraction.

(22) ________ In a school of 100 students, 90 study English, 75 study Spanish and 42 study French. Every student must study at least one of the three languages. What is the least possible number of students who could be studying all three languages?

(23) ________ If $1 \le a \le 10$ and $1 \le b \le 36$, for how many ordered pairs of integers (a, b) is $\sqrt{a+\sqrt{b}}$ an integer?

(24) ________ If x is an element of the set $\{-1, 1, 2\}$ and y is an element of $\{-2, -1, 0, 1, 2\}$, how many distinct values of x^y are positive?

(25) ________ Four packages are delivered to four houses, one to each house. If these packages are randomly delivered, what is the probability that exactly two of them are delivered to the correct houses? Express your answer as a common fraction.

(26) ________ Mandvil has one standard quarter and one special quarter with heads on both sides. He selects one of these two coins at random, and without looking at it first, he flips the coin three times. If he flips a head three straight times, what is the probability that he selected the special quarter? Express your answer as a common fraction.

(27) ________ A subset S of the set of integers from 50 to 100, inclusive, has the property that no two distinct elements of S sum to 130. What is the maximum possible number of elements in S?

(28) ________ Four red candies and three green candies can be combined to make many different flavors. Flavors are different if the percent red is different, so 3 red / 0 green is the same flavor as 2 red / 0 green; and likewise 4 red / 2 green is the same flavor as 2 red / 1 green. If a flavor is to be made using some or all of the seven candies, how many different flavors are possible?

(29) ________ A box contains some green marbles and exactly four red marbles. The probability of selecting a red marble is $x\%$. If the number of green marbles is doubled, the probability of selecting one of the four red marbles from the box is $(x-15)\%$. How many green marbles are in the box before the number of green marbles is doubled?

(30) ________ An integer is randomly chosen from the integers 1 through 100, inclusive. What is the probability that the chosen integer is a perfect square or a perfect cube, but not both? Express your answer as a common fraction.

Answer Sheet

Number	Answer	Problem ID
1	1/10	05D5
2	79/90	C15C
3	3 ways	C5D1
4	51/64	A5D1
5	3888	C2A1
6	6.15 dollars	A1A1
7	15	0D212
8	5 combos	45D1
9	1/4	5CB1
10	24/7 mtg pts	11A1
11	16 integers	34C
12	19/28	4011
13	5 sums	315C
14	6 numbers	3A51
15	15 sums	BA51
16	84 phone numbers	44C
17	5 combinations	54D5
18	35 outcomes	34D5
19	11 points	435
20	144 integers	05D1
21	3/5	1502
22	7 students	25D1
23	10 ordered pairs	C5212
24	5 values	A402
25	1/4	0B51
26	8/9	1DD1
27	36 elements	4B322
28	11 flavors	341
29	6 green marbles	44D5
30	1/10	BBB1

Solutions

(1) **1/10 ID: [05D5]**

If the resulting integer is divisible by 11 then the sum of the first, third, and fifth digits has the same remainder when divided by 11 as the sum of the second and fourth digits. This only occurs when the first, third, and fifth digits are 2, 3, and 7 (in some order) and the second and fourth digits are 4 and 8 (in some order).

There are $\binom{5}{2}$ total ways to partition these five digits into a group of 3 and a group of 2. From above, only one of these partitions will result in five-digit integers that are divisible by 11. Therefore, our answer is $\boxed{1/10}$.

(2) **79/90 ID: [C15C]**

We will calculate the probability of her hearing every second of her favorite song and then subtract that from 1 to get the probability that we're looking for. There are a total of 10! ways in which the 10 songs can be ordered. If her favorite song is the first song, she obviously hears the whole thing, and then there are 9! ways to order the other songs. If the first song is the 30 second song, then she will hear the entirety of her favorite song if and only if it is played as the second song, after which there are 8! ways to order the other songs. Finally, if the first song is the 1 minute song, she will hear her favorite song if and only if it is played as the second song, after which there are 8! ways to order the other songs. If the first song is longer than a minute, or if two songs are played before her first song, she won't have time to hear all of her favorite song in the first 4 minutes, 30 seconds. So out of the 10! ways of ordering the 10 songs, there are $9! + 8! + 8!$ ways that result in her hearing the full song for a probability of $\dfrac{9! + 8! + 8!}{10!} = \dfrac{8!}{8!} \cdot \dfrac{9 + 1 + 1}{10 \cdot 9} = \dfrac{11}{90}$. But that is the probability that what we want *doesn't* happen, so we need to subtract it from 1 to get our final probability of $1 - \dfrac{11}{90} = \boxed{\dfrac{79}{90}}$

(3) **3 ways ID: [C5D1]**

Since we are partitioning 81 into sums of perfect squares, we proceed by subtracting out perfect squares and seeing which work: $81 - 64 = 17 = 16 + 1$. Further, $81 - 49 = 32 = 16 + 16$. And finally, $81 - 36 = 45 = 36 + 9$. Although there is more to check through, this sort of method should convince us that these are the only $\boxed{3}$ solutions: $1^2 + 4^2 + 8^2 = 81$, $4^2 + 4^2 + 7^2 = 81$, and $3^2 + 6^2 + 6^2 = 81$.

(4) **51/64 ID: [A5D1]**

No solution is available at this time.

(5) **3888 ID: [C2A1]**

Temporarily disregard the case in which we have a pair of zeros. Now, the first number of the pair can be placed in any one of four slots (the thousands, hundreds, tens, or units place), and the third can be placed in any one of the remaining three. However, because the numbers are the same, we've overcounted by a factor of two. Thus, there are $\frac{4 \cdot 3}{2} = 6$ ways to place a pair of digits in which the pair is not a pair of zeros. There are nine such pairs (1 through 9).

We cannot begin a number with a zero. Thus, there are three places in which the first zero may be put, and two for the second zero. Again, we've overcounted by a factor of two. So there are $\frac{3 \cdot 2}{2} = 3$ ways to place a pair of zeros.

After the pair of equal digits is chosen, there are nine choices for the third number and eight for the fourth (since they must be distinct from each other and from the digit that is repeated.)

Finally, we've overcounted, as we have not yet eliminated cases in which an integer with a repeated digit other than zero begins with a zero. There are nine pairs of digits other than zero, and then there are eight choices for the other digit (not zero, and not the digit that is repeated). Also, there are three ways in which the pair of digits can be placed in the hundreds, tens, and units digit places. Thus, we must subtract off $9 \cdot 8 \cdot 3$.

Our final answer is thus $(9 \cdot 6 + 3) \cdot 9 \cdot 8 - 9 \cdot 8 \cdot 3 = 54 \cdot 72 = \boxed{3888}$.

(6) **6.15 dollars ID: [A1A1]**

No solution is available at this time.

(7) **15 ID: [0D212]**

Given four students A, B, C, and D, there are 3 possible pairings: $AB|CD$, $AC|BD$, and $AD|BC$. With six students A, B, C, D, E, and F, there are 5 ways to choose A's partner, and then 3 ways to group of the remaining four students in pairs. In total, there are $3 \times 5 = \boxed{15}$ ways to group the students in pairs.

(8) **5 combos** **ID: [45D1]**

Let x be the smallest positive integer in each sequence.

Case 1: Two consecutive positive integers

$x + (x + 1) = 45$

$x = 22$, $x + 1 = 23$

Case 2: Three consecutive positive integers

$x + (x + 1) + (x + 2) = 45$

$x = 14$, $x + 1 = 15$, $x + 2 = 16$

Case 3: Four consecutive positive integers

$x + (x + 1) + (x + 2) + (x + 3) = 45$

Combining like terms gives $4x = 39$, which has no integer solution.

Case 4: Five consecutive positive integers

$x + (x + 1) + (x + 2) + (x + 3) + (x + 4) = 45$

$x = 7$, $x + 1 = 8$, $x + 2 = 9$, $x + 3 = 10$, $x + 4 = 11$

Case 5: Six consecutive positive integers

$x + (x + 1) + \ldots + (x + 5) = 45$

$x = 5$, $x + 1 = 6$, $\ldots$, $x + 5 = 10$

Case 6: Seven consecutive positive integers

$x + (x + 1) + \ldots + (x + 6) = 45$

This simplifies to $7x = 24$, which has no integer solution.

Case 7: Eight consecutive positive integers

$x + (x + 1) + \ldots + (x + 7) = 45$

This simplifies to $8x = 17$, which has no integer solution.

Case 8: Nine consecutive positive integers

$x + (x + 1) + \ldots + (x + 8) = 45$

$x = 1$, $x + 1 = 2$, $\ldots$, $x + 8 = 9$.

x cannot be less than 1, so we are done. Thus, there are $\boxed{5}$ such combinations.

(9) **1/4 ID: [5CB1]**

The possible values of r are represented by the set

$$R = \{-2, -1, 0, 1, 2, 3, 4, 5\}$$

and for k the set

$$K = \{2, 3, 4, 5, 6, 7\}.$$

There are thus $8 \cdot 6 = 48$ pairs of integers.

Now, we see which satisfy the divisibility requirement that $k|r$. If $r = -2$ then k can only be 2, for 1 integer. If $r = -1$, then k can be no integer. If $r = 0$, then k can be any integer, or 6 choices. If $r = 1$, then k cannot be any integer. If $r = 2$, then k can only be 2, or 1 integer. If $r = 3$ then k can only be 3, or 1 integer. If $r = 4$, then k can be 2 or 4, or 2 different integers. If $r = 5$, then $k = 5$ is the only possibility, for 1 integer. So, $1 + 6 + 1 + 1 + 2 + 1 = 12$ possibilities. So, $\frac{12}{48} = \boxed{\frac{1}{4}}$ is the probability of $r \div k$ being an integer.

(10) **24/7 mtg pts ID: [11A1]**

No solution is available at this time.

(11) **16 integers ID: [34C]**

The only one-digit integer divisible by 4 that we can construct is 4.

We can construct 3 two-digit integers divisible by 4: 12, 32, and 24.

An integer is divisible by 4 if its rightmost two digits are divisible by 4. Thus we can append either or both of the remaining two digits to any of these two-digit integers and preserve divisibility by 4. For each, there are 2 ways to choose one digit to append, and 2 ways to order the digits if we append both of them. Thus we get 4 more integers for each, or 12 total.

The full number is $12 + 3 + 1 = \boxed{16}$ integers.

(12) **19/28 ID: [4011]**

No solution is available at this time.

(13) **5 sums** **ID: [315C]**

Let's say the four integers were arranged as a, b, c, d across to start. We can then compute each row in terms of these variables:

$$\begin{array}{ccccccc} a & & b & & c & & d \\ & a+b & & b+c & & c+d & \\ & & a+2b+c & & b+2c+d & & \\ & & & a+3b+3c+d & & & \end{array}$$

As we can see from the final sum, the order of a and d, as well as the order of b and c, will not affect the final sum. So, we consider each of the six possible ways to choose two numbers from possible starting values of $1, 2, 3, 4$.

a	**d**	**b**	**c**	**a+3b+3c+d**
1	2	3	4	24
1	3	2	4	22
1	4	2	3	20
2	3	1	4	20
2	4	1	3	18
3	4	1	2	16

As we can see from the table, there are a total of $\boxed{5}$ different possible sums.

(14) **6 numbers** **ID: [3A51]**

Two odd numbers or two even numbers have an even difference. Any two digit number that ends in an even number will not be prime, so we can focus on the first case. Listing primes less that 50 that have two odd digits gives: 11, 13, 17, 19, 31, and 37. Thus, there are $\boxed{6}$ such primes.

(15) **15 sums** **ID: [BA51]**

No solution is available at this time.

(16) **84 phone numbers ID: [44C]**

We begin by factoring 336. $336 = 2^4 \cdot 3 \cdot 7$. Because we are looking for phone numbers, we want four single digits that will multiply to equal 336. Notice that 7 cannot be multiplied by anything, because $7 \cdot 2$ is 14, which is already two digits. So, one of our digits is necessarily 7. 3 can be multiplied by at most 2, and the highest power of 2 that we can have is $2^3 = 8$. Using these observations, it is fairly simple to come up with the following list of groups of digits whose product is 336 :

$$1, 6, 7, 8$$
$$2, 4, 6, 7$$
$$2, 3, 7, 8$$
$$3, 4, 4, 7$$

For the first three groups, there are $4! = 24$ possible rearrangements of the digits. For the last group, 4 is repeated twice, so we must divide by 2 to avoid overcounting, so there are $\frac{4!}{2} = 12$ possible rearrangements of the digits. Thus, there are $3 \cdot 24 + 12 = \boxed{84}$ possible phone numbers that can be constructed to have this property.

(17) **5 combinations ID: [54D5]**

If Julie includes one of the colors that cover three cupcakes, she must also include the other color that covers three cupcakes. This is because she must make ten cupcakes total, and all of the other colors cover an even number of cupcakes, so there is no way to make ten with three and some combination of even numbers. Thus, if she includes blue and violet, she has four cupcakes left to choose. There are three ways in which she can choose four cupcakes if she chooses colors that cover two (green and orange, green and yellow, or orange and yellow). Alternately, she can choose a color that covers four (red). Finally, if she doesn't include any colors that cover three cupcakes, she must choose all of the other cupcakes in order to make ten. Thus, Julie has $\boxed{5}$ different combinations of cupcakes.

(18) **35 outcomes   ID: [34D5]**

There are three ways for the sum of the five rolled numbers to equal 27: we can have two 6s and three 5s; three 6s, one 5, and one 4; or four 6s and one 3.

In the first case, we rolled the numbers 6, 6, 5, 5, 5 in some order. There are $\dfrac{5!}{2!3!} = 10$ distinct ways that we could have rolled these numbers.

In the second case, we rolled the numbers 6, 6, 6, 5, 4 in some order. There are $\dfrac{5!}{3!1!1!} = 20$ distinct ways that we could have rolled these numbers.

In the third case, we rolled the numbers 6, 6, 6, 6, 3 in some order. There are $\dfrac{5!}{4!1!} = 5$ distinct ways that we could have rolled these numbers.

Adding all these outcomes, we see that there are $10 + 20 + 5 = \boxed{35}$ possible outcomes.

(19) **11 points   ID: [435]**

The length of PQ is $\sqrt{3^2 + 1^2} = \sqrt{10}$. It is seen that no other pair of positive integers a, b is such that $a^2 + b^2 = 10$. So all lattice points which are $\sqrt{10}$ away from a given lattice point are 3 units away in one direction and 1 unit away in an orthogonal direction.

There are 4 ways to choose the direction in which to move 3 units, and then 2 ways to choose the direction to move 1 unit. So there should be 8 valid points for each of P and Q, or 16 in total. But not all of these work. We are counting P as one of Q's points and Q as one of P's points, but these are not valid placements for R. If we move 3 units right from Q, we are off the grid, so the 2 points produced from that move are invalid. Finally, if we move 3 units left and 1 unit down from P, the resulting triangle will be degenerate. However, it is easy to check that all other points produce valid, unique triangles. So the answer is

$$16 - 2 - 2 - 1 = \boxed{11}$$

(20) **144 integers   ID: [05D1]**

No solution is available at this time.

(21) **3/5 ID: [1502]**

The rods will be able to form a convex quadrilateral if and only if every set of three sides adds up to more than the fourth side. (To see why the "only if" part of this statement is true, consider a quadrilateral $ABCD$. The path going from A to B, B to C, then C to D is longer than the direct path AD, since the shortest distance between two points is a straight segment. So $AB + BC + CD > AD$ for any quadrilateral. This holds for the sides other than AD, as well.) Suppose Amanda picks a rod of length x out of the box. Then we can make a convex quadrilateral if and only if all the following are true:

$$2 + 5 + 9 > x$$
$$2 + 5 + x > 9$$
$$2 + 9 + x > 5$$
$$5 + 9 + x > 2$$

The last two inequalities are always true for positive x. The first inequality gives $x < 16$ and the second inequality gives $x > 2$. So out of the 10 rods Amanda can choose, 4, 6, 8, 10, 12, and 14 will work. So the probability is $\frac{6}{10} = \boxed{\frac{3}{5}}$.

(22) **7 students ID: [25D1]**

Let $x =$ the number of students studying both English and Spanish. $y =$ the number of students studying both English and French. $z =$ the number of students studying both Spanish and French. $a =$ the number of students studying all three languages. From given, we know $90 + 75 + 42 - x - y - z + a = 100 \Rightarrow x + y + z - a = 107$ from the Inclusion-Exclusion principle. Also we know the number of students studying English/Spanish and French/Spanish can not exceed the total number of students studying Spanish. So we have $x + z - a \leq 75$. Similarly, $x + y - a \leq 90$ and $y + z - a \leq 42$. Combine the three inequalities we have $2(x + y + z) - 3a \leq 207$. Combining with our result above we have $2(x + y + z - a) - a \leq 207 \Rightarrow 2 \times 107 - a \leq 207 \Rightarrow a \geq 7$. Note that this situation is also obtainable, with 90 students taking English, 75 taking Spanish, 42 taking French, 65 taking English and Spanish, 32 taking French and English, 17 taking Spanish and French, and $\boxed{7}$ taking all three: 90+75+42-65-32-17+7=100

(23) **10 ordered pairs ID: [C5212]**
If $\sqrt{a+\sqrt{b}}$ is an integer, then its square $a+\sqrt{b}$ is also an integer. Therefore, $\sqrt{b}$ is an integer. In other words, b must be a perfect square. If we define $c=\sqrt{b}$, then the problem has asked us to find the number of ordered pairs (a,c) for which $1 \leq a \leq 10$, $1 \leq c \leq 6$, and $a+c$ is a perfect square. We check the 6 possibilities for c separately. If $c=1$, then a is 3 or 8. If $c=2$, then a is 2 or 7. If $c=3$, then a is 1 or 6. If $c=4$, then $a=5$, and if $c=5$, then $a=4$. Finally, if $c=6$, then a is either 10 or 3. Altogether, there are $2+2+2+1+1+2=\boxed{10}$ ordered pairs (a,c) satisfying the given conditions.

(24) **5 values ID: [A402]**
Raising -1 and 1 to integer powers will only result in answers of 1 or -1. Raising 2 to each of the powers in $\{-2,-1,0,1,2\}$ gives $\frac{1}{4}$, $\frac{1}{2}$, 1, 2, and 4. Thus, there are $\boxed{5}$ distinct *positive* values of x^y.

(25) **1/4 ID: [0B51]**
Since there are 4 houses and 4 packages, we can choose $\binom{4}{2}=6$ pairs of houses to be the pair that will receive the correct package. In that case, the other two houses must have one another's package. The probability of this occuring for any arrangement is $\frac{1}{4}\cdot\frac{1}{3}\cdot\frac{1}{2}$, as the first fraction represents the probability of a given house getting the correct package, and the second fraction the subsequent probability that the other given house gets the correct package, and the final fraction the probability that the last two houses have each other's packages. So, the probability is $6\cdot\frac{1}{2\cdot3\cdot4}=\boxed{\frac{1}{4}}$.

(26) **8/9 ID: [1DD1]**
First, let's find the total probability of flipping 3 straight heads. He has a $\frac{1}{2}$ chance of choosing the fair quarter, after which he has a $\frac{1}{2^3}=\frac{1}{8}$ chance of getting 3 heads, for a $\frac{1}{16}$ probability in this case. If he gets the special quarter, with probability $\frac{1}{2}$, he is guaranteed to get three heads. So the total probability is $\frac{1}{16}+\frac{1}{2}=\frac{9}{16}$. So, we already know that there were 3 heads, so the realm of possible outcomes is restricted to this probability of $\frac{9}{16}$. And the special quarter was used $\frac{1}{2}$ so we find the probability, given 3 heads, that we used the special quarter to be the ratio $\frac{\frac{1}{2}}{\frac{9}{16}}=\boxed{\frac{8}{9}}$.

(27) **36 elements ID: [4B322]**
From each of the pairs $\{50, 80\}$, $\{51, 79\}$, $\{52, 78\}$,..., $\{64, 66\}$, we may take at most one member. Since there are 15 such pairs, we must exclude a minimum of 15 numbers from the set S. This shows that no more than $51 - 15 = 36$ elements could be in S. The set $\{65, 66, 67, \ldots, 100\}$ satisfies the requirements and has $\boxed{36}$ elements.

(28) **11 flavors ID: [341]**
Denote the ratio by $x : y$, where x is the number of red candies and y is the number of green. We can have 0, 1, 2, 3, or 4 red candies and 0, 1, 2, or 3 green candies. Thus, there are $5 \cdot 4 = 20$ potential ratios. However, a $0 : 0$ ratio is not allowed (there would be no candy!), so we subtract one for a total of 19 ratios possible. Now we must subtract the ratios we've over-counted. In particular, $0 : 1$ is the same as $0 : 2$ and $0 : 3$, and $1 : 0$ is the same as $2 : 0$, $3 : 0$, and $4 : 0$. Also, $1 : 1$ is the same as $2 : 2$ and $3 : 3$, and $2 : 1$ is the same as $4 : 2$. Thus, we have over-counted by 8 ratios, so our final answer is $19 - 8 = \boxed{11}$.

(29) **6 green marbles ID: [44D5]**
Let g be the initial number of green marbles. We are given that $4/(g + 4) = x/100$ and $4/(2g + 4) = (x - 15)/100$. Subtracting the second equation from the first, we obtain

$$\frac{4}{g+4} - \frac{4}{2g+4} = \frac{15}{100}.$$

After clearing fractions and some manipulation, we have

$$3g^2 - 22g + 24 = (g - 6)(3g - 4) = 0.$$

It follows that $g = 6$ or $g = 4/3$. Since we have to have an integer amount of marbles, our answer is $\boxed{6}$.

(30) **1/10 ID: [BBB1]**
A number is a perfect square and a perfect cube if and only if it is a perfect sixth power. Note that $10^2 = 100$ and $4^3 < 100 < 5^3$, while $2^6 < 100 < 3^6 = 9^3$. Hence, there are 10 squares and 4 cubes between 1 and 100, inclusive. However, there are also 2 sixth powers, so when we add $10 + 4$ to count the number of squares and cubes, we count these sixth powers twice. However, we don't want to count these sixth powers at all, so we must subtract them twice. This gives us a total of $10 + 4 - 2 \cdot 2 = 10$ different numbers that are perfect squares or perfect cubes, but not both. Thus, our probability is $\frac{10}{100} = \boxed{\frac{1}{10}}$.