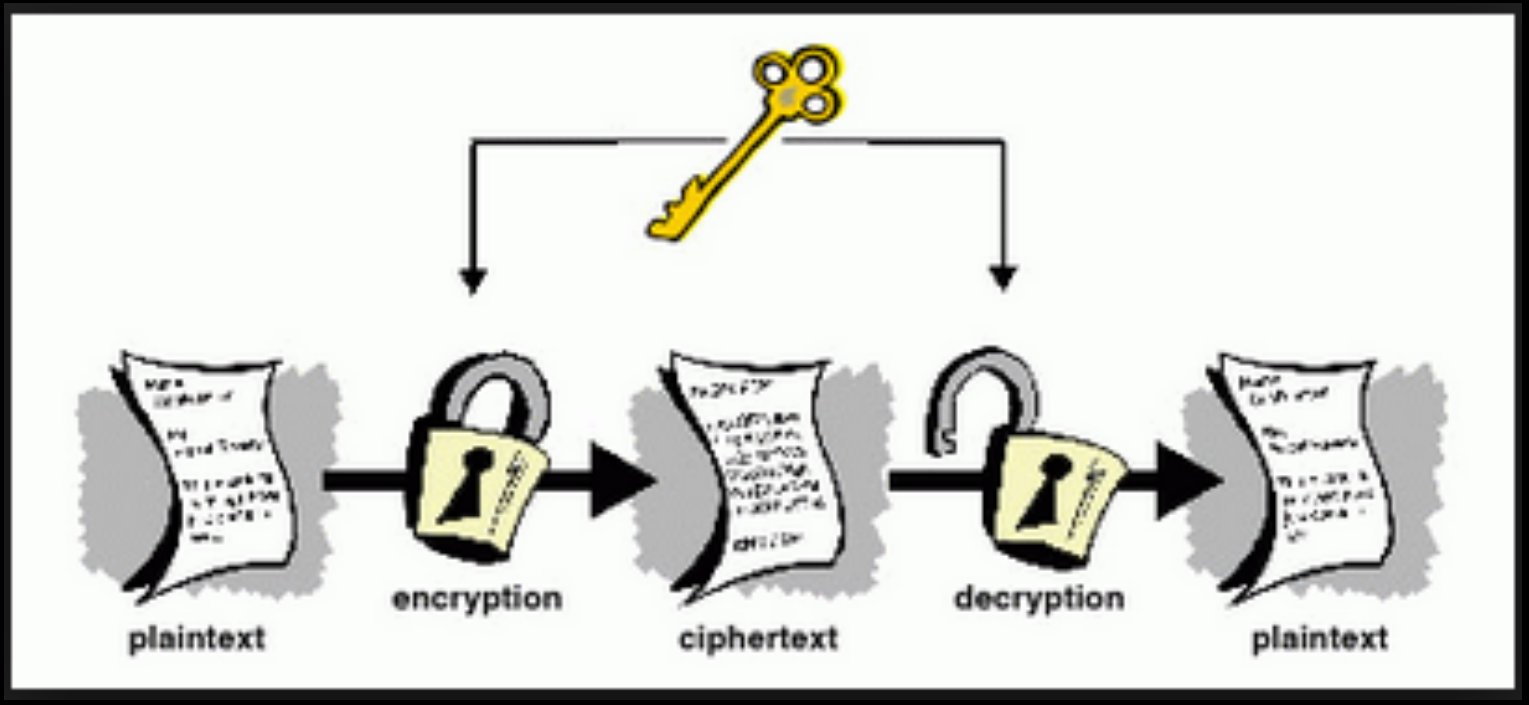
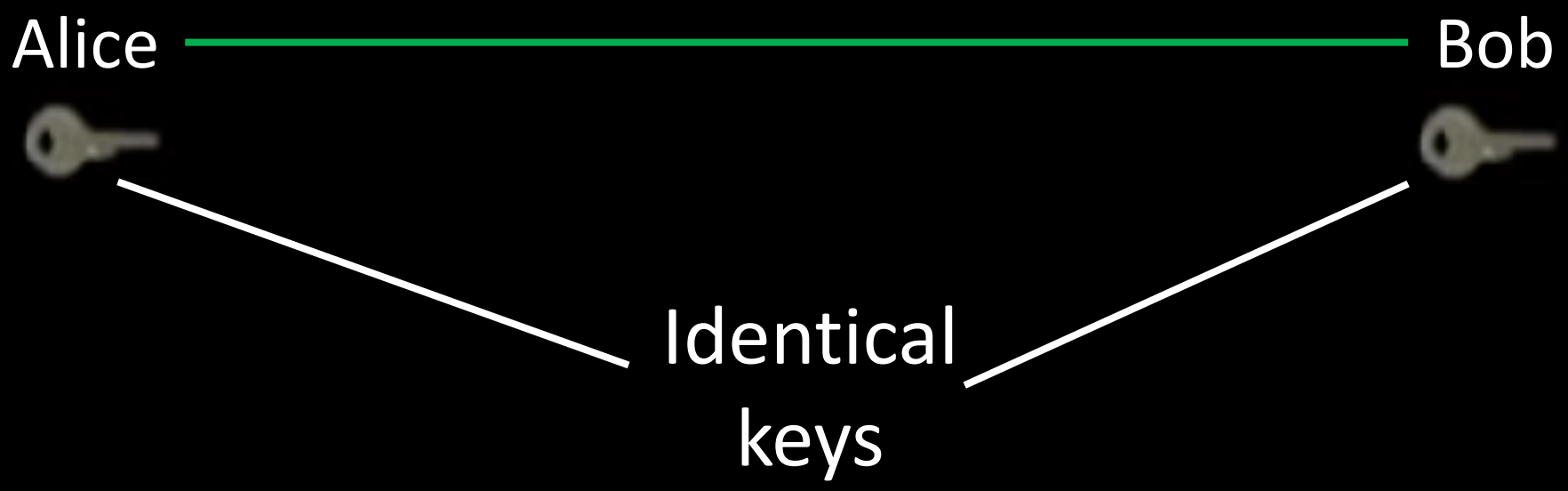
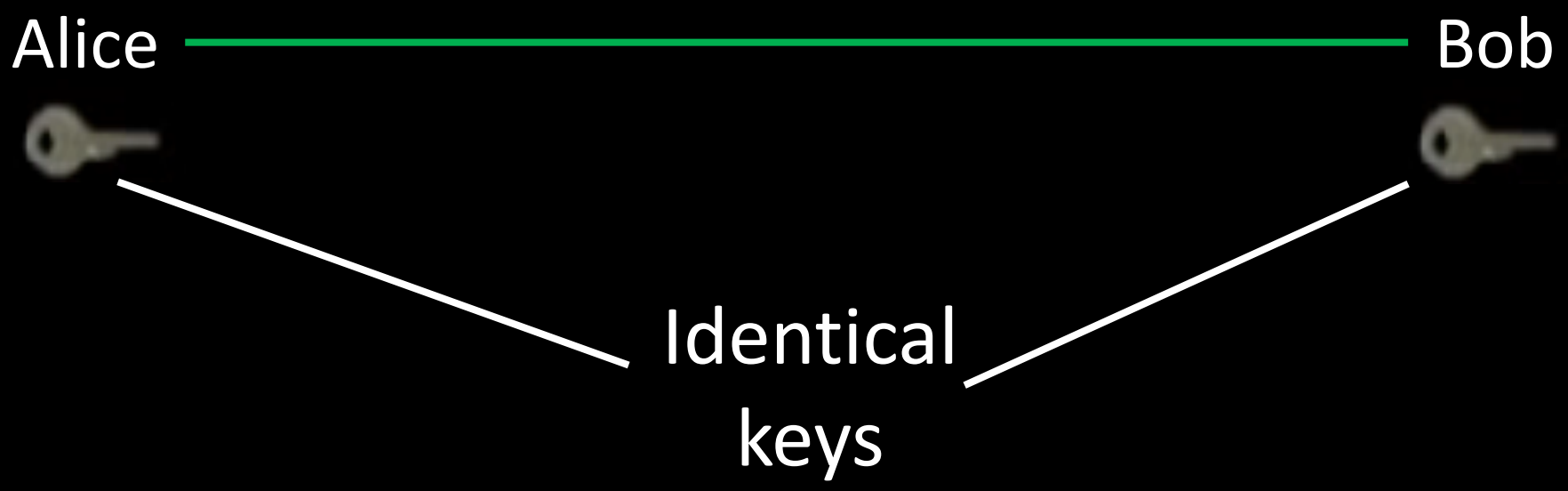
How secure is RSA: Mathematical Approach

By: Rana Khalil
Supervised by: Dr. Monica Nevins

Symmetric Key Cryptography




Alice
Bob
Identical
keys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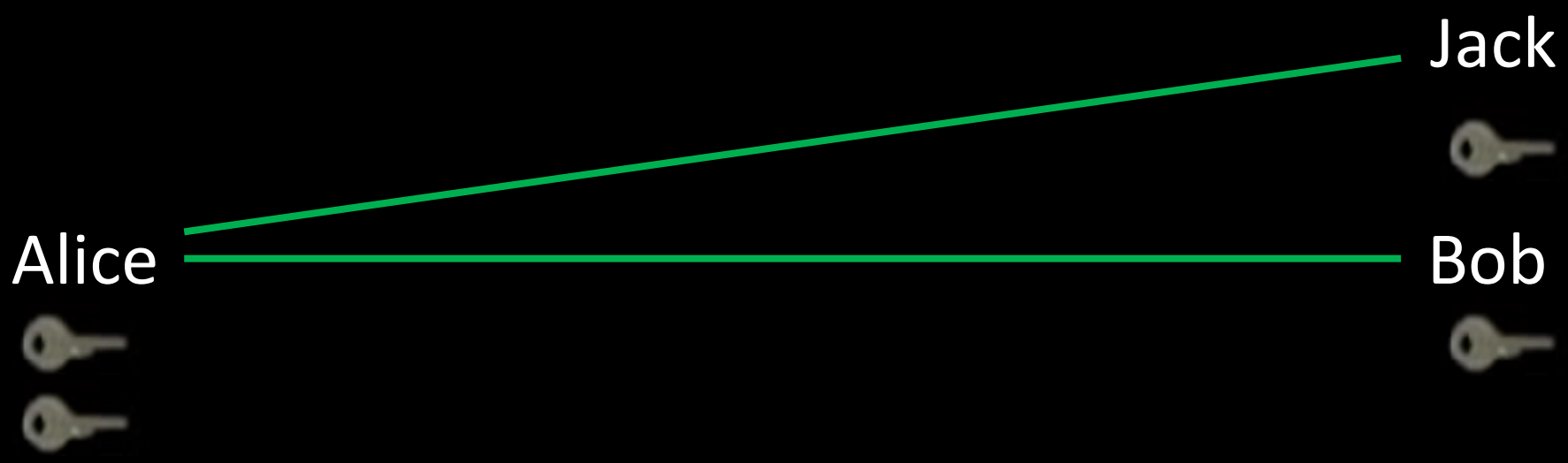
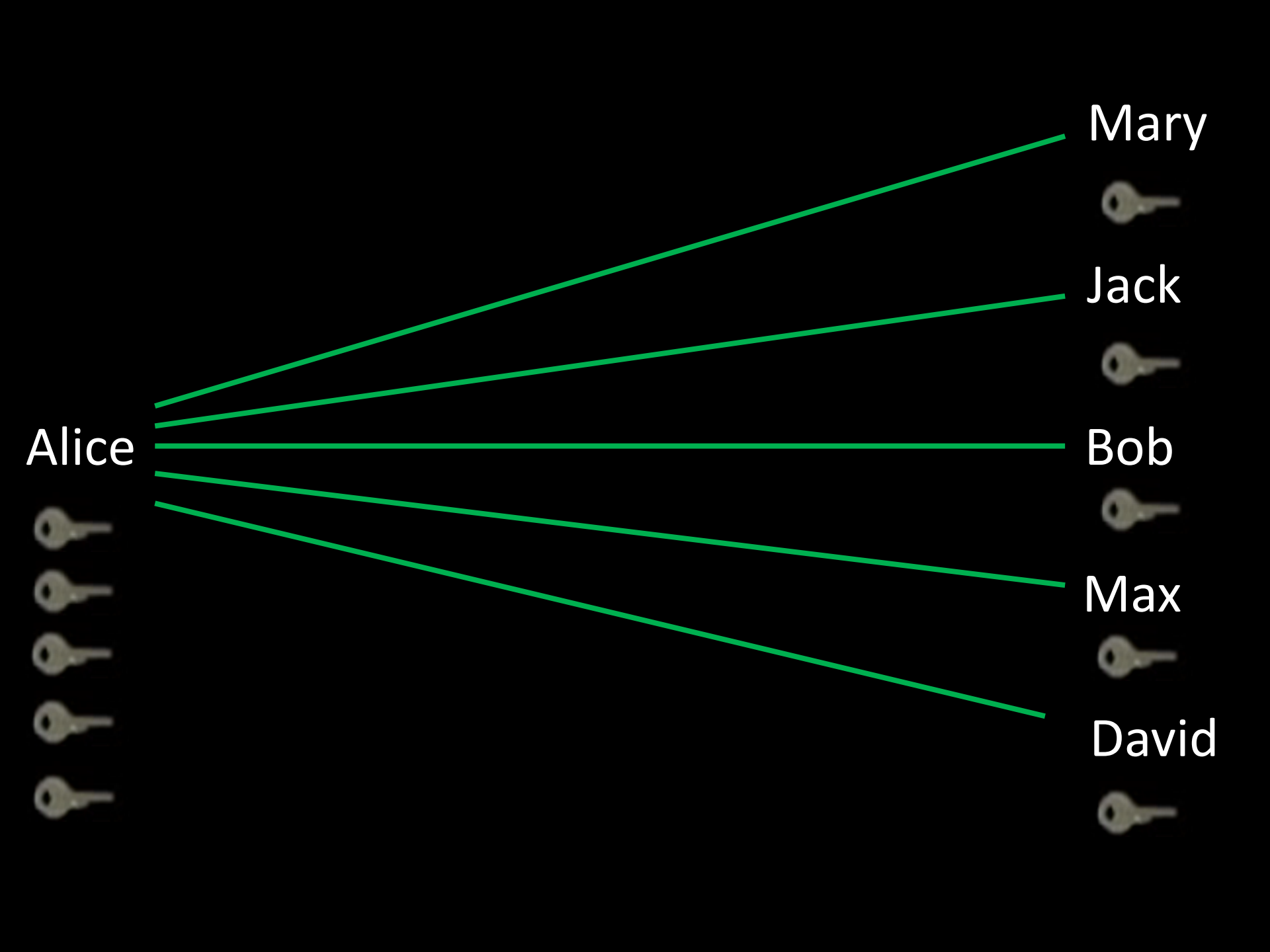

Jack
Alice
Bob


Mary
Jack
Alice
Bob
Max
David

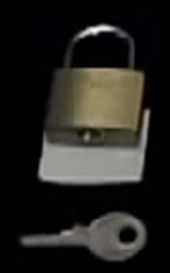
Asymmetric Key Cryptography

Alice

Bob

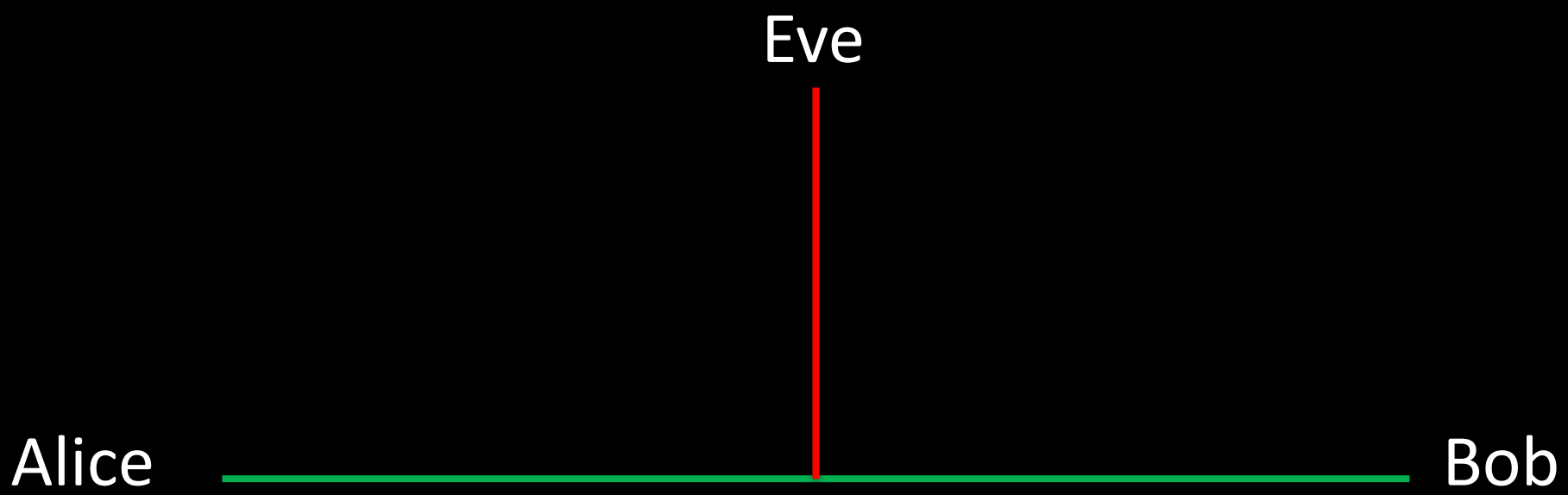
Alice Bob

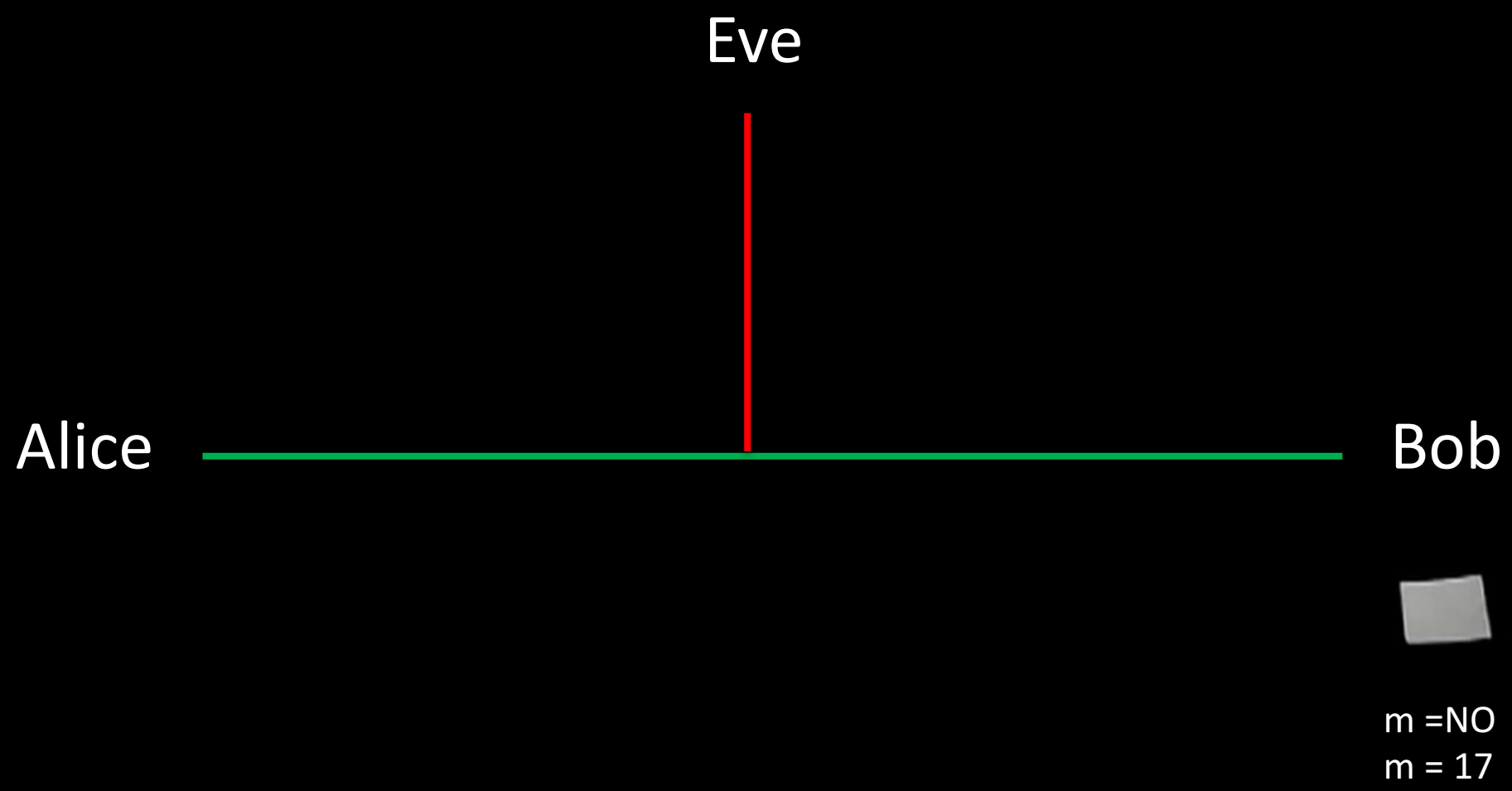
Alice Bob



Alice Bob

RSA




Eve
Alice
Bob
m =NO
m = 17

Eve

Alice

Bob

p1 = 3
p2 = 11
N = 3* 11 = 33

m =NO
m = 17

Eve

Alice

Bob

p1 = 3
p2 = 11
N = 3* 11 = 33

$\Phi(n) = (p1 – 1) (p2 -1)$
$= 2 * 10 = 20$

m =NO
m = 17

Eve

Alice

Bob

p1 = 3
p2 = 11
N = 3* 11 = 33

$\Phi(n) = (p1 – 1) (p2 -2)$
$= 2 * 10 = 20$

e = 3

m =NO
m = 17

Eve

Alice

Bob

p1 = 3
p2 = 11
N = 3* 11 = 33
Φ(n) = (p1 – 1) (p2 -2)
= 2 * 10 = 20
e = 3

$$ed \equiv 1 \; mod \; \Phi(n)$$

$$3d = 1 \bmod 20$$

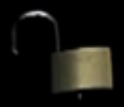
$$d = 7$$

m =NO
m = 17

Eve

N = 33
e = 3

Alice



N = 33
e = 3

Bob

m =NO
m = 17

Eve

N = 33
e = 3

Alice

Bob

N = 33
e = 3

m =NO
m = 17

$$m^e \; mod \; n \equiv c$$

$$17^3 mod \; 33 \equiv 29$$

Eve

n = 33
e = 3
c = 29

Alice

Bob

c =29

m = 17

$$m \equiv c^d \bmod N$$

$$m \equiv 29^7 \bmod 33$$

m = 17

EQUAL!

How Hard is factoring?

Let N be 1024 digits long

- In order to find a factor you need to search

$$\sqrt{N} \sim 512 \text{ digits}$$

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- $\mathbf{10^{512}}$ numbers

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- National Security Agency (NSA) – Access to classified information
- Lenstra et al.
  - Prime factor in common
  - Among 4.7 million 1024-bit RSA moduli collected, more than 12500 have a single prime factor in common!

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  - Prime factor in common
  - Among 4.7 million 1024-bit RSA moduli collected, more than 12500 have a single prime factor in common!
  - Probability is $\frac{1}{2^{459}} \approx 0$!