ST 501 Final Project - Fall 2018

Bayesian Inference and Analysis

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Being submitted to fulfill the course requirements of
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Honor Pledge

I have neither given nor received unauthorized aid on this assignment.

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Solutions

1 Analysis of Log Returns on Google Stock

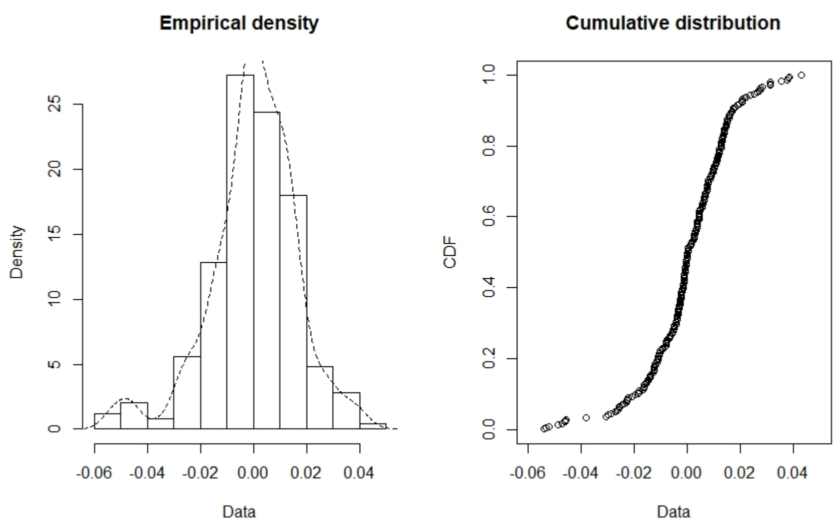
1.1 Sample Quantile and Sample Estimate

The 0.05 sample quantile and the sample estimate are respectively,

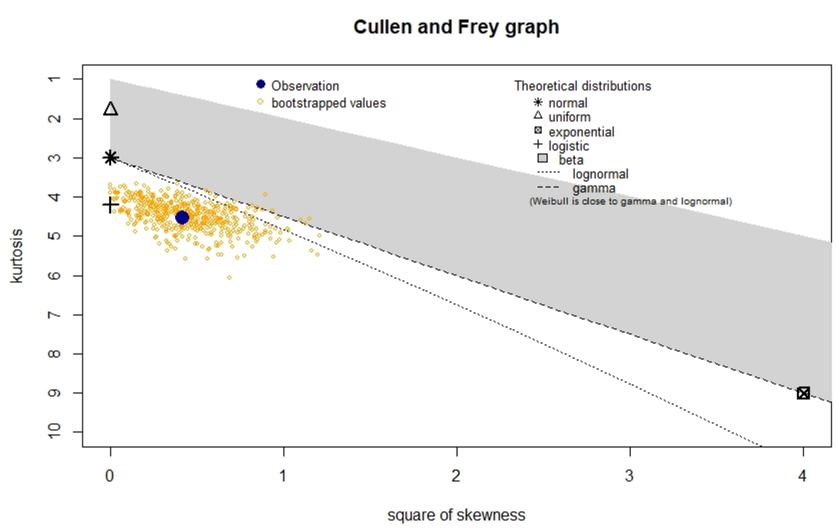
$$Q_{0.05} = -0.02692224$$
$$E[R|R < Q_{0.05}] = -0.04221229$$

1.2 Histogram and deciding the distribution for R_t

The histogram for the R_t of Google stock is as follows,



Using the *decdist* function, we obtained the following plot,



1.3 Usage of $fitdist$ and $ad.test$ functions

The summary of all the tests run to find a better fit to the data is in the table below. Please refer to the R-code file for reference.

Distribution	P-Value
T-Distribution (df=3)	**0.5939**
Logistic Distribution	0.3967
Logistic Distribution on Differentiated $R_t s$	0.9585
Logistic Distribution on $(R_t + (R_t)^2)$	0.502
Gamma Distribution on $(R_t)^2$	0.3041
Beta Distribution on $(R_t)^2$	0.3019
Cauchy Distribution	0.06303

1.4 To prove for any constant c, $E[R|R < c] = \int_{-\infty}^{c} \frac{r f_R(r)}{F_R(c)} dr, F_R(c) = \int_{-\infty}^{c} f_R(r) dr$

The data has a more suitable fit with **T-distribution** with 3 degrees of freedom, hence we will be proceeding with that.
From 1.1 we have the sample estimate to be,

$$E[R|R < Q_{0.05}] = -0.04221229$$

And now by evaluating the complex integral, we have the output to be,

$$E[R|R < c] = \int_{-\infty}^{c} \frac{r f_R(r)}{F_R(c)} dr = -0.0374903$$

Also, c has been calculated with value as below:-

$$Q_{0.05} = -0.02454694$$

Hence proved.

2 Analysis of Log Returns on Amazon Stock

2.1 Sample Quantile and Sample Estimate

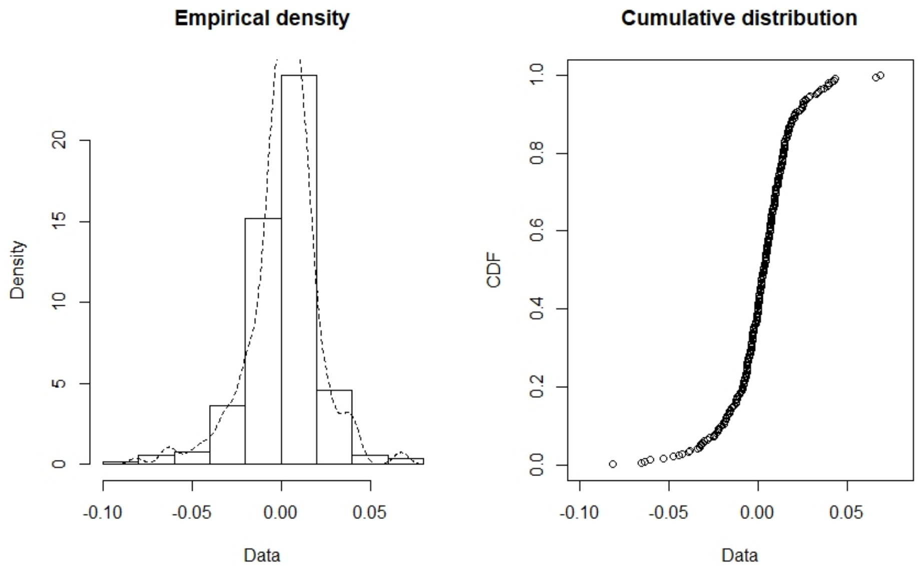
The 0.05 sample quantile and the sample estimate are respectively,

$$Q_{0.05} = -0.03228566$$

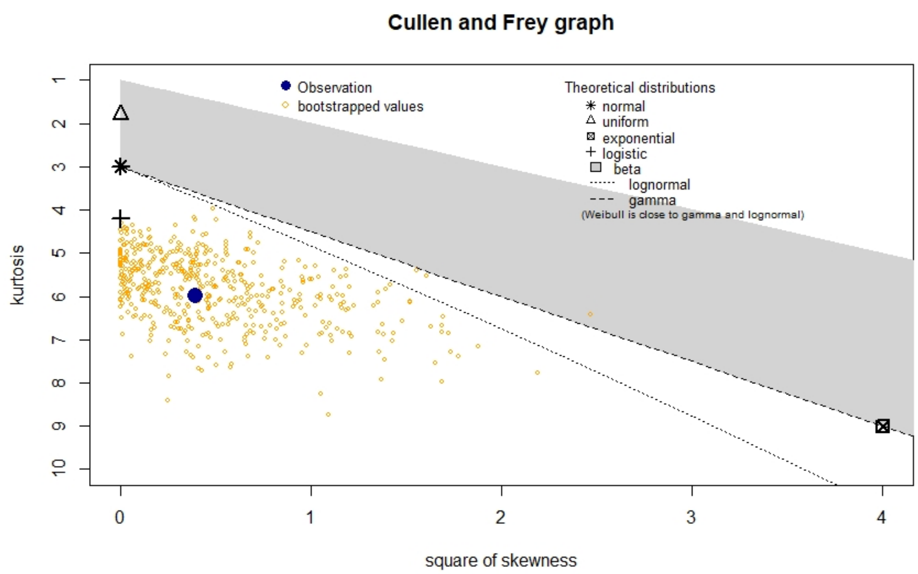
$$E[R|R < Q_{0.05}] = -0.04896195$$

2.2 Histogram and deciding the distribution for R_t

The histogram for the R_t of Google stock is as follows,



Using the *decdist* function, we obtained the following plot,



2.3 Usage of $fitdist$ and $ad.test$ functions

The summary of all the tests run to find a better fit to the data is in the table below. Please refer to the R-code file for reference.

Distribution	P-Value
T-Distribution (df=3)	**0.7502**
Logistic Distribution	0.2098
Logistic Distribution on Differentiated $R_t s$	0.3929

2.4 To prove for any constant c, $E[R|R<c] = \int_{-\infty}^{c} \frac{rf_R(r)}{F_R(c)}dr, F_R(c) = \int_{-\infty}^{c} f_R(r)dr$

The data has a more suitable fit with **T-distribution** with 3 degrees of freedom, hence we will be proceeding with that.
From 2.1 we have the sample estimate to be,

$$E[R|R<Q_{0.05}] = -0.04896195$$

And now by evaluating the complex integral, we have the output to be,

$$E[R|R<c] = \int_{-\infty}^{c} \frac{rf_R(r)}{F_R(c)}dr = -0.04736586$$

Also, c has been calculated with value as below:-

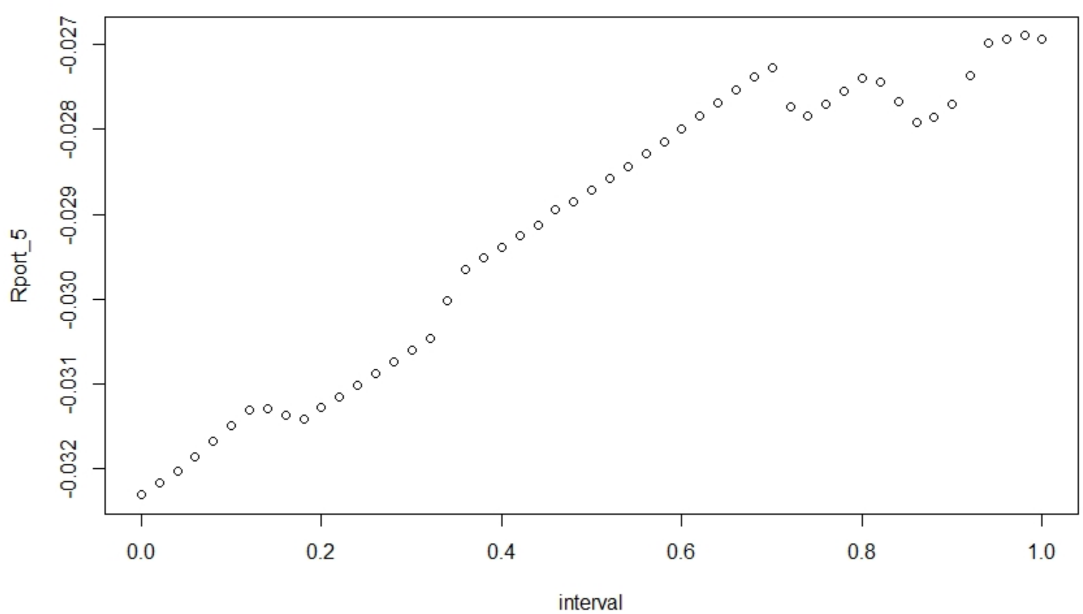
$$Q_{0.05} = -0.02635557$$

Hence proved.

3 Determining Portfolio Weights and Analysis

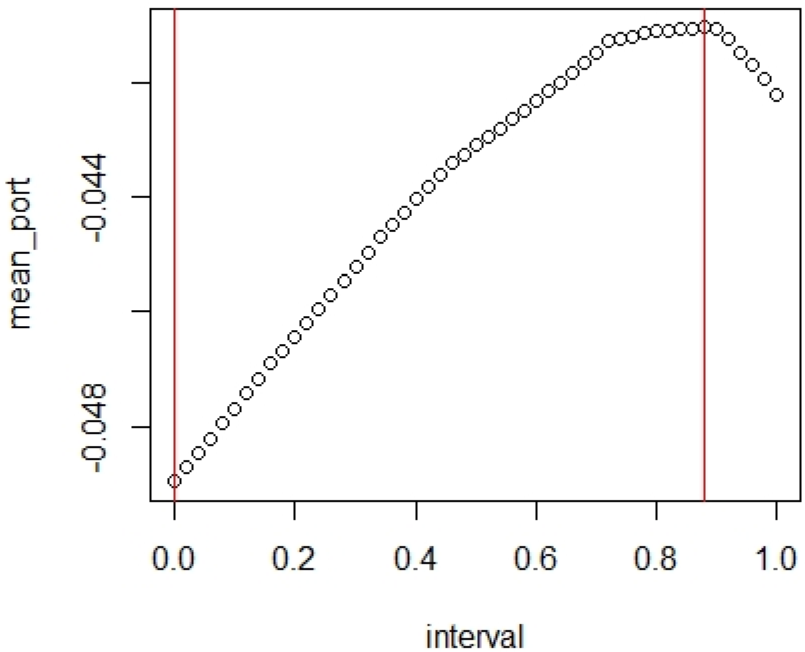
3.1 Sample quantile of the portfolio weight w,$Q_{0.05}(w)$

The 0.05 sample quantile of the portfolio weight w,$Q_{0.05}(w)$, plotted over the range of [0,1] on 50 intervals is as follows,



3.2 Sample estimate $ES(w) = E[R(w)|R(w)] < Q_{0.05}(w)]$

The plot of $ES(w)$ as a function of w on a grid of equally spaced 50 values in [0,1] is as follows,



3.3 Minima and Maxima of $ES(w)$

$ES(w)$ is minimum at $w = 0$
$ES(w)$ is maximum at $w = 0.88$