

# A statisticians view of the surge in warming debate

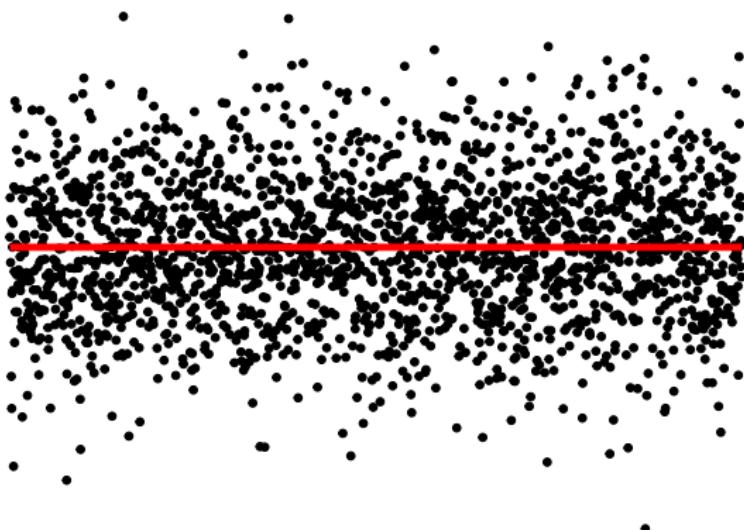
Rebecca Killick(r.killick@lancs.ac.uk)

Joint work with Claudie Beaulieu (Uni California Santa Cruz)

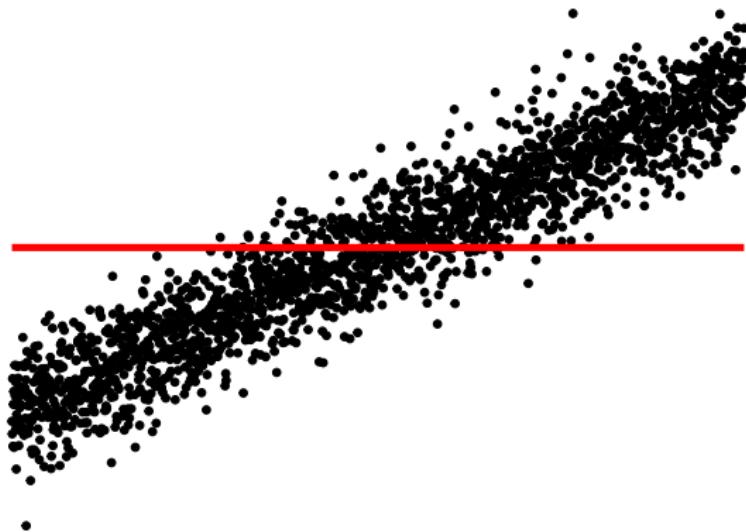
Parts with: Adelicia Johnson, Colin Gallagher, Robert Lund, Xueheng Shi

Queen Mary Nov 2025

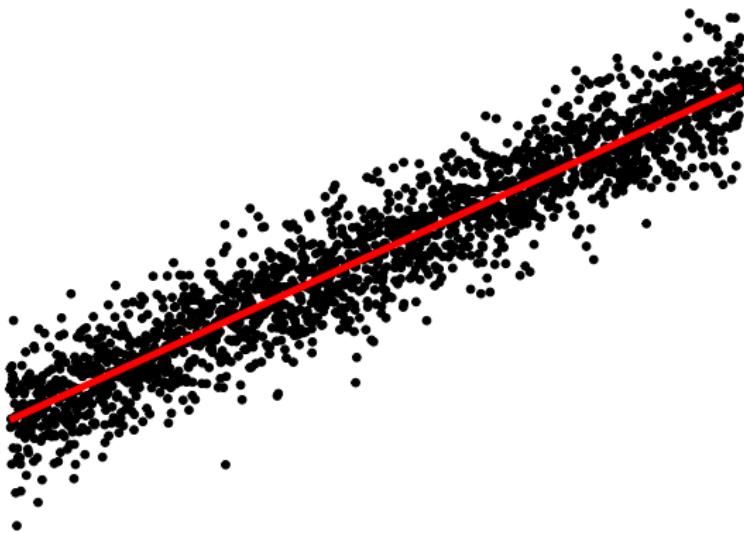
# Model development



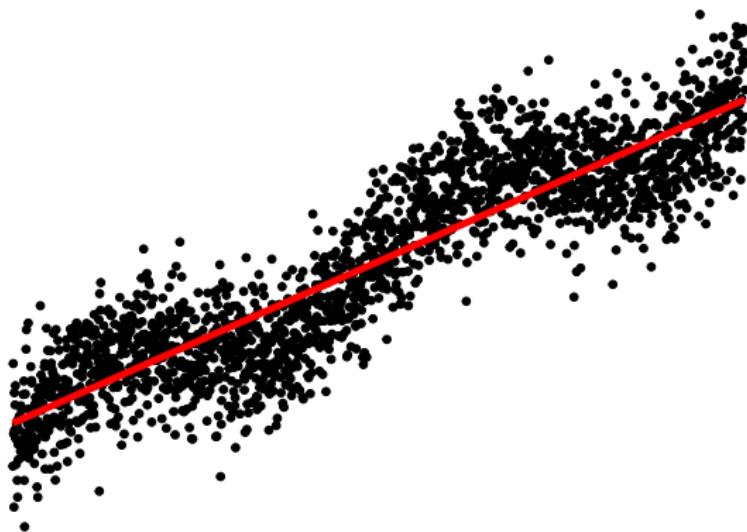
# Model development



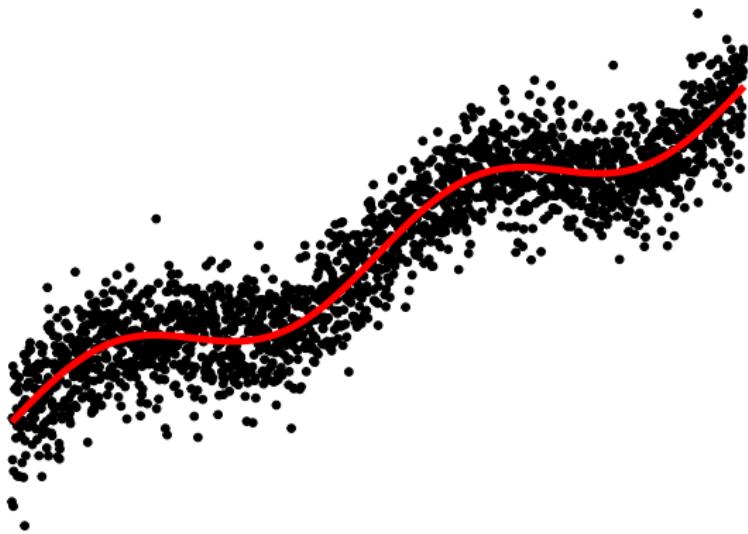
# Model development



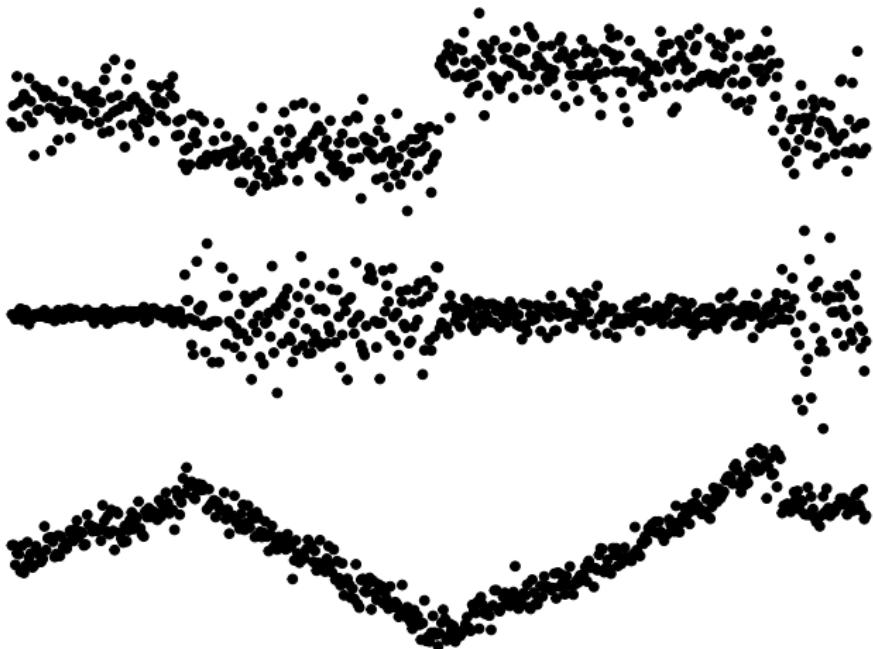
# Model development



# Model development



# Model development



# Outline

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- What is the warming debate in climate science?
- What are changepoints?
- The story of a statistical solution
- Some frustrations

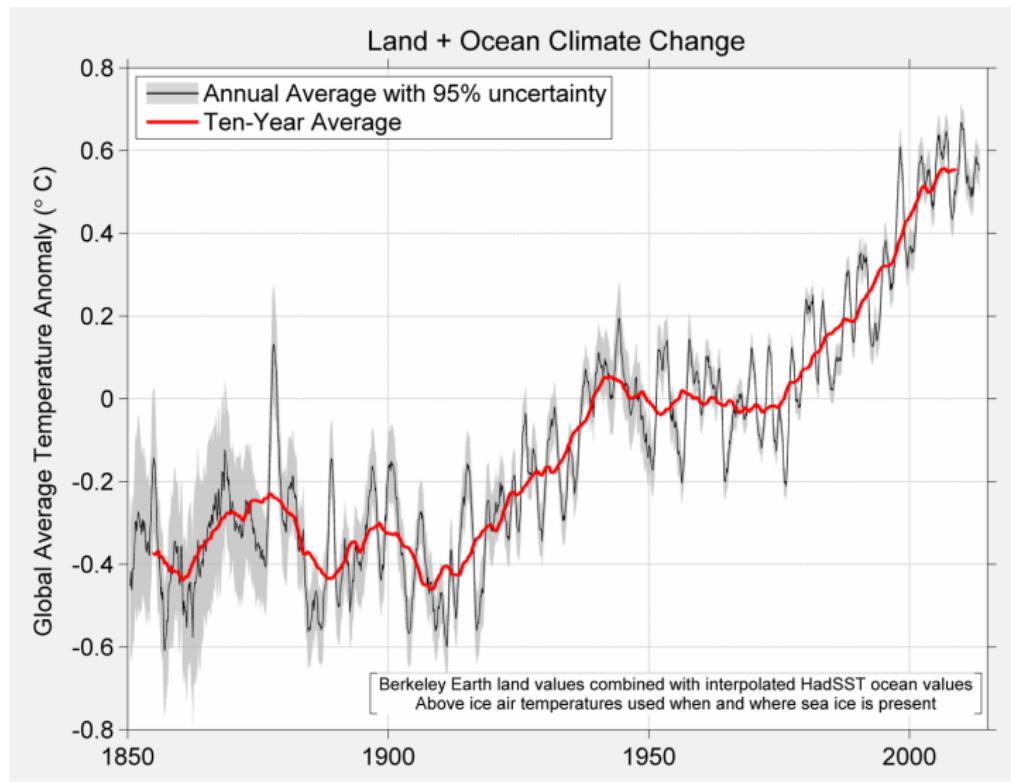
# Where my interest began

School of  
Mathematical Sciences

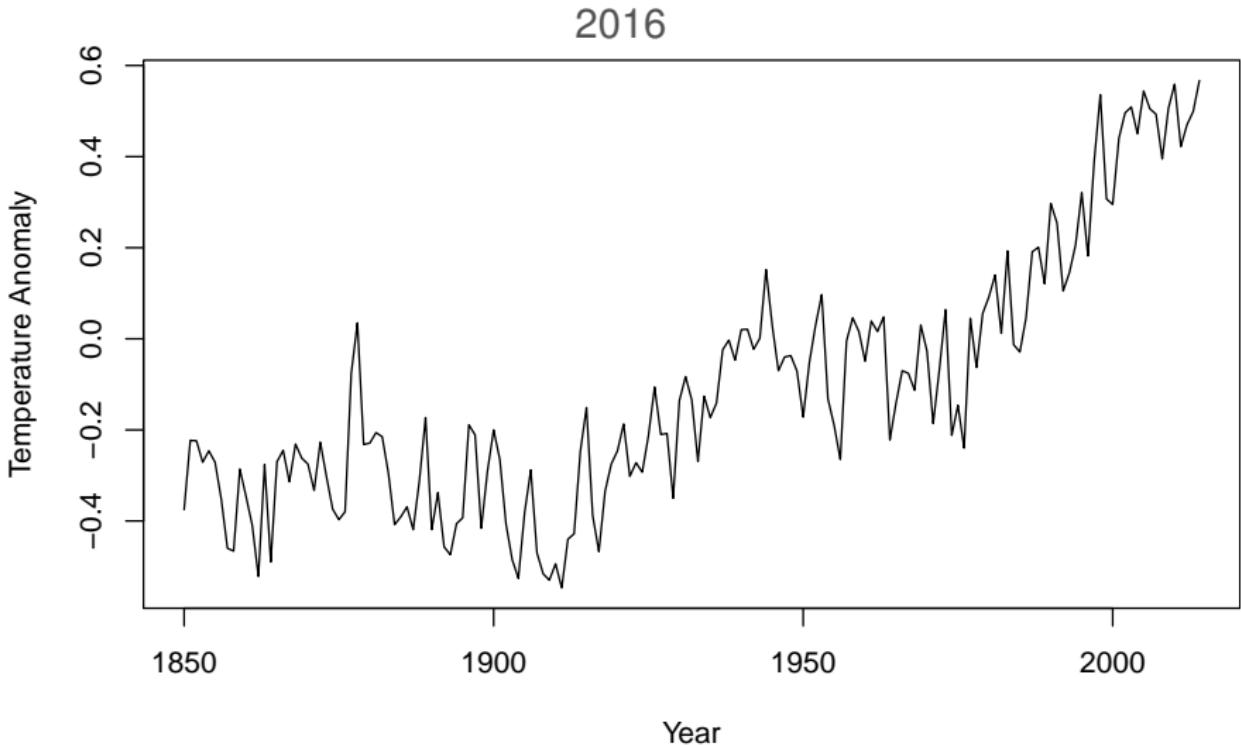
Lancaster  
University



# Global Mean Temp 2016



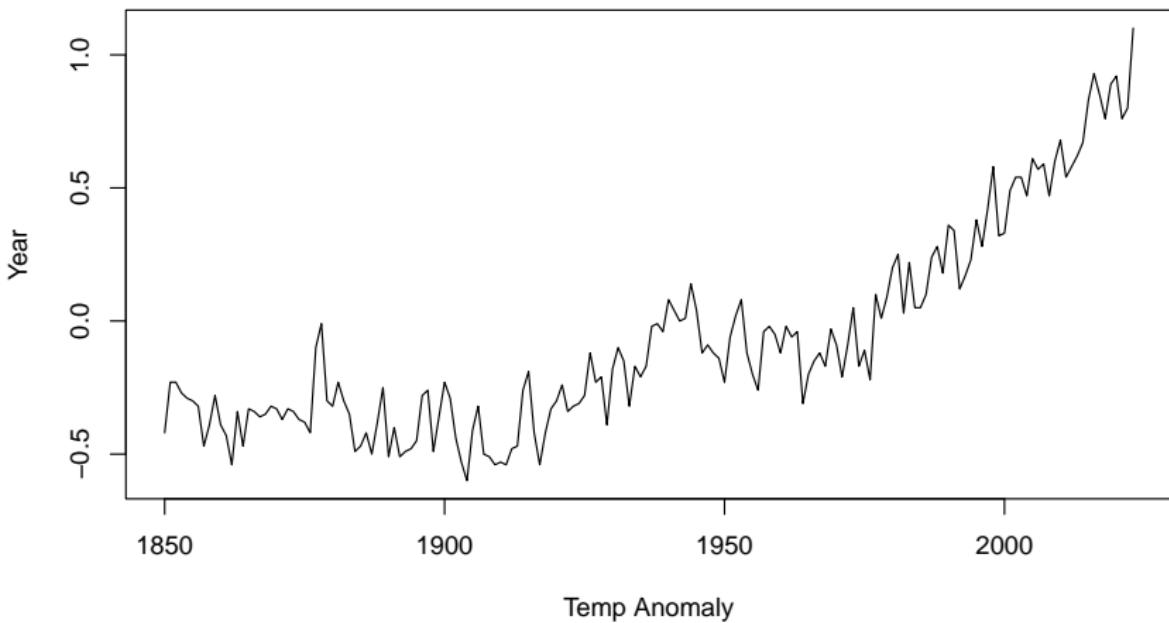
# Global Mean Temp



# Global Mean Temp



Today





# Is climate change speeding up? Here's what the science says.

This year's record temperatures have some scientists concerned that the pace of warming may be accelerating. But not everyone agrees.

By [Chris Mooney](#) and [Shannon Osaka](#)

December 26, 2023 at 6:30 a.m. EST

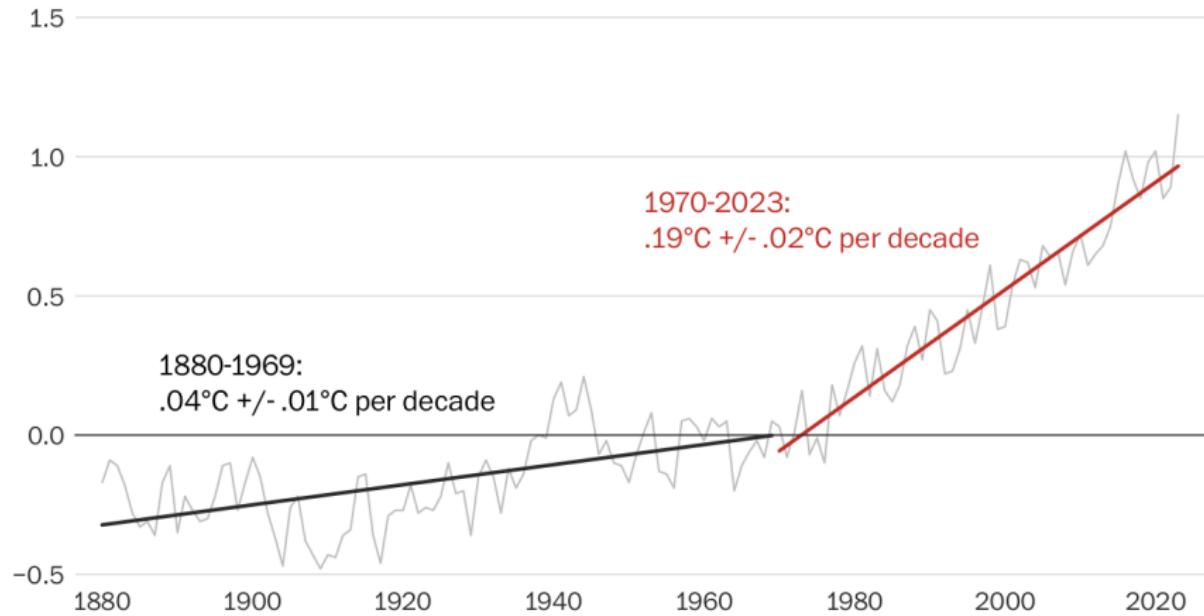
For the past several years, a small group of scientists has warned that sometime early this century, the rate of global warming — which has remained largely steady for decades — might accelerate. Temperatures could rise higher, faster. The drumbeat of weather disasters may become more insistent.

# Where is Statistics?



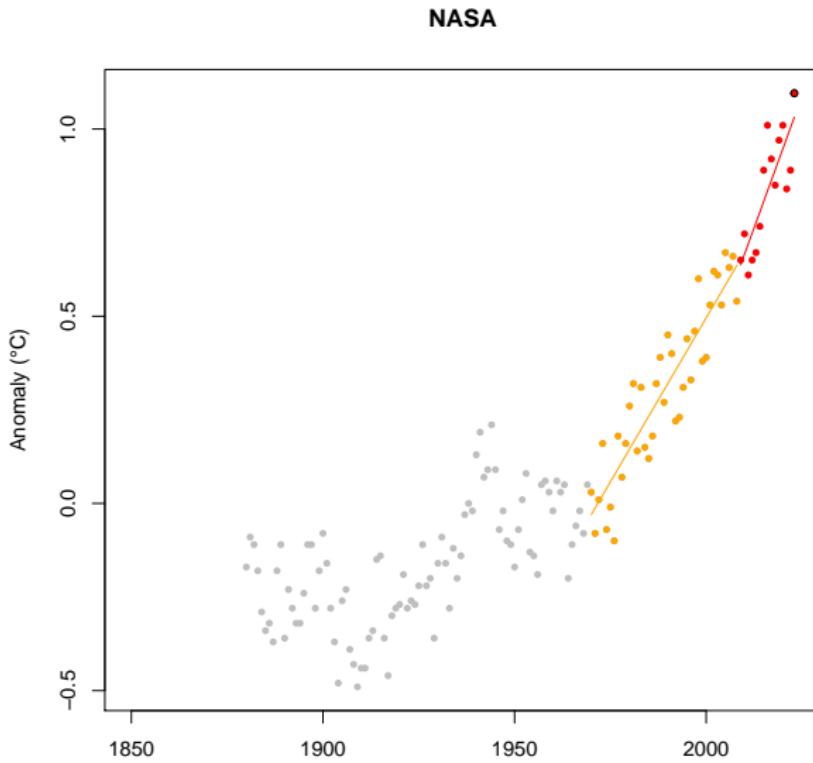
## The increased rate of global warming

Values are relative to the 1951-1980 global mean temperature, in degrees Celsius



Note: 2023 is an estimate based on values from January through November.

# Where is Statistics? NYT

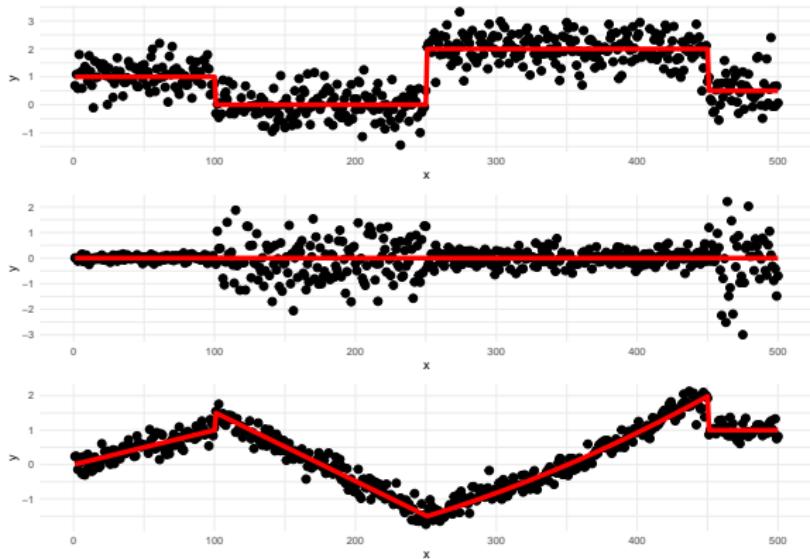


# What are changepoints?



For data  $y_1, \dots, y_n$ , if a changepoint exists at  $\tau$ , then  $y_1, \dots, y_\tau$  differ from  $y_{\tau+1}, \dots, y_n$  in some way.

There are many different types of change.

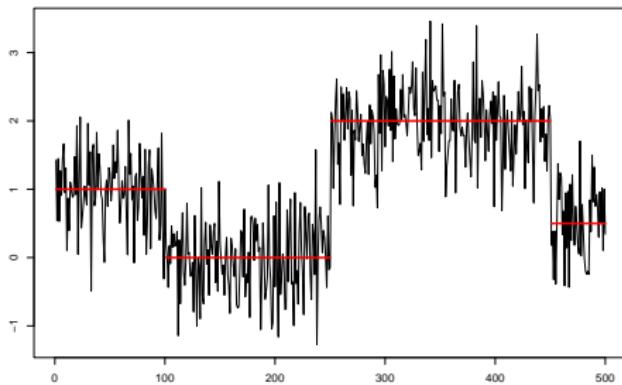


# Change in mean

Assume we have time-series data where

$$Y_t | \theta_t \sim N(\theta_t, 1),$$

but where the means,  $\theta_t$ , are piecewise constant through time.



We want to infer the number and position of the points at which the mean changes. One approach:

## Likelihood Ratio Test

To detect a single changepoint we can use the likelihood ratio test statistic:

$$LR = \max_{\tau} \{\ell(y_{1:\tau}) + \ell(y_{\tau+1:n}) - \ell(y_{1:n})\}.$$

We infer a changepoint if  $LR > \beta$  for some (suitably chosen)  $\beta$ . If we infer a changepoint its position is estimated as

$$\tau = \arg \max \{\ell(y_{1:\tau}) + \ell(y_{\tau+1:n}) - \ell(y_{1:n})\}.$$

This can test can be repeatedly applied to new segments to find multiple changepoints.

# Likelihood Ratio Tests

Define  $m$  to be the number of changepoints, with positions  $\tau = (\tau_0, \tau_1, \dots, \tau_{m+1})$  where  $\tau_0 = 0$  and  $\tau_{m+1} = n$ .

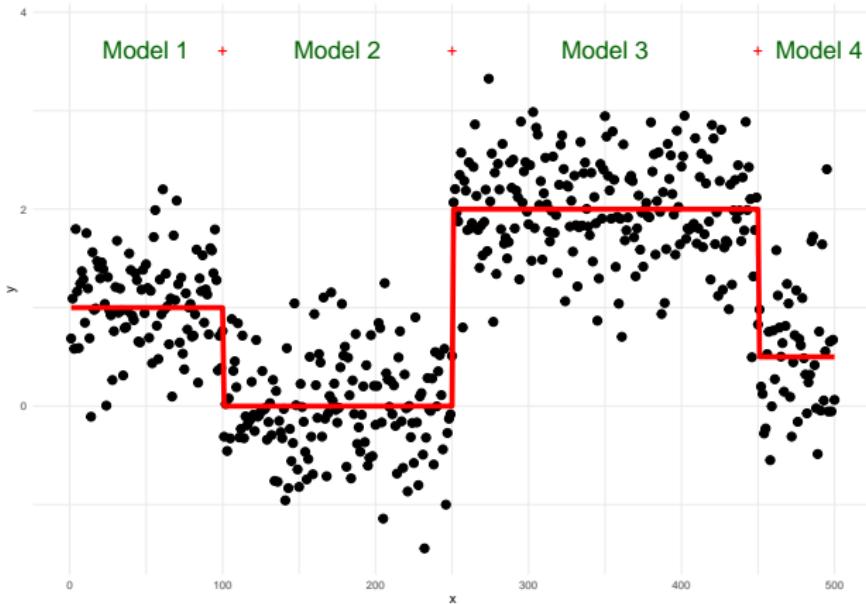
Then one application of the Likelihood ratio test can be viewed as

$$\min_{m \in \{0,1\}, \tau} \left\{ \sum_{i=1}^{m+1} [-\ell(y_{\tau_{i-1}:\tau_i})] + \beta m \right\}$$

Repeated application is thus aiming to minimise

$$\min_{m, \tau} \left\{ \sum_{i=1}^{m+1} [-\ell(y_{\tau_{i-1}:\tau_i})] + \beta m \right\}$$

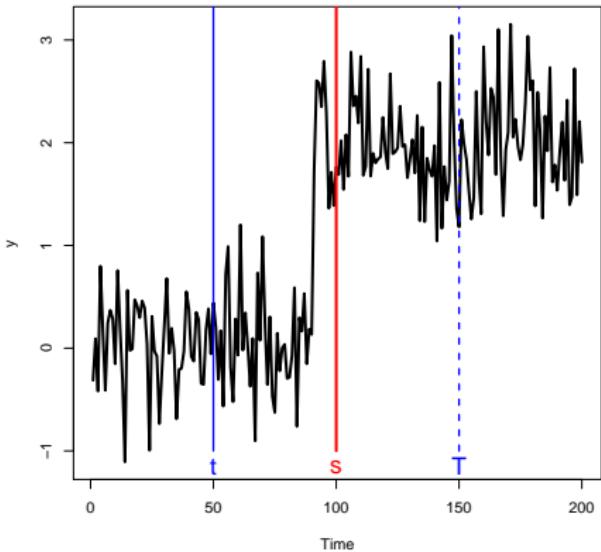
# Problem



- How many changes?
- Where are the changes?  $2^{n-1}$  possible solutions!

# PELT in a nutshell

- Dynamic programming allows us to only worry about the location of the *last* change.
- Pruning means that as we go through the data we are smart about which locations are potential last change locations.



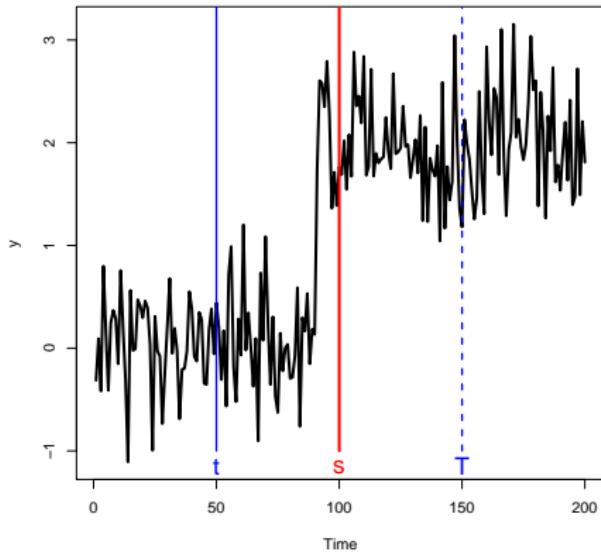
# PELT: Pruning

Let  $0 < t < s < T$ , if

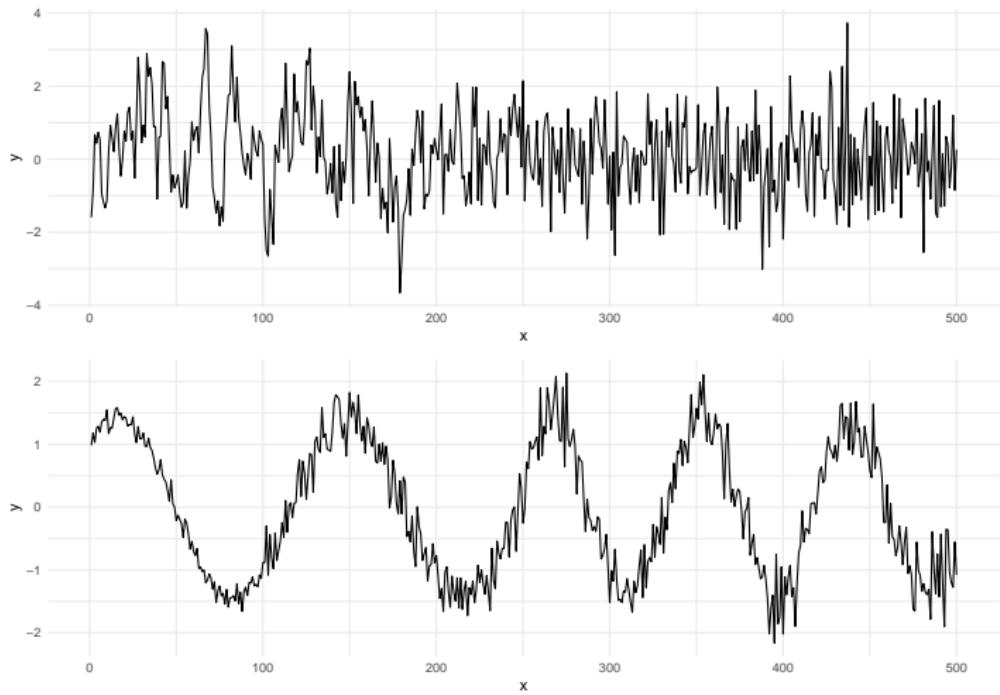
$$F(t) + \mathcal{C}(y_{(t+1):s}) < F(s)$$

then at any future time  $T > s$ ,  $t$  can never be the optimal last changepoint prior to  $T$ .

We can prove that, under certain regularity conditions, the expected computational complexity will be  $\mathcal{O}(n)$ .

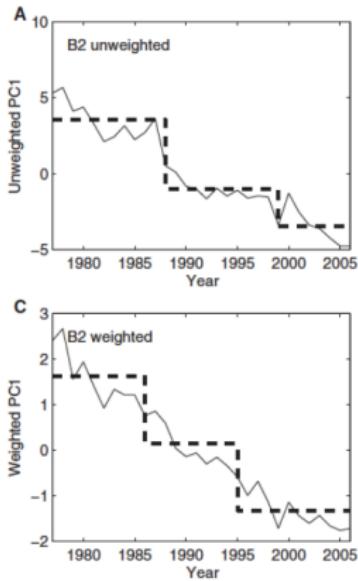


# More complicated changes

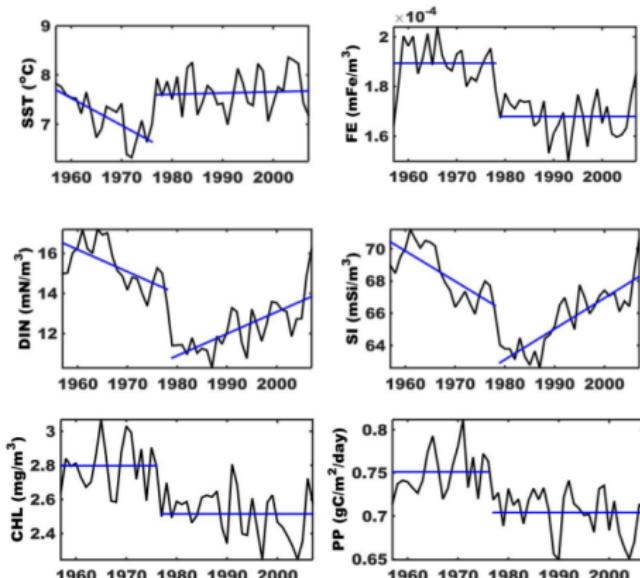


# Tooling not enough

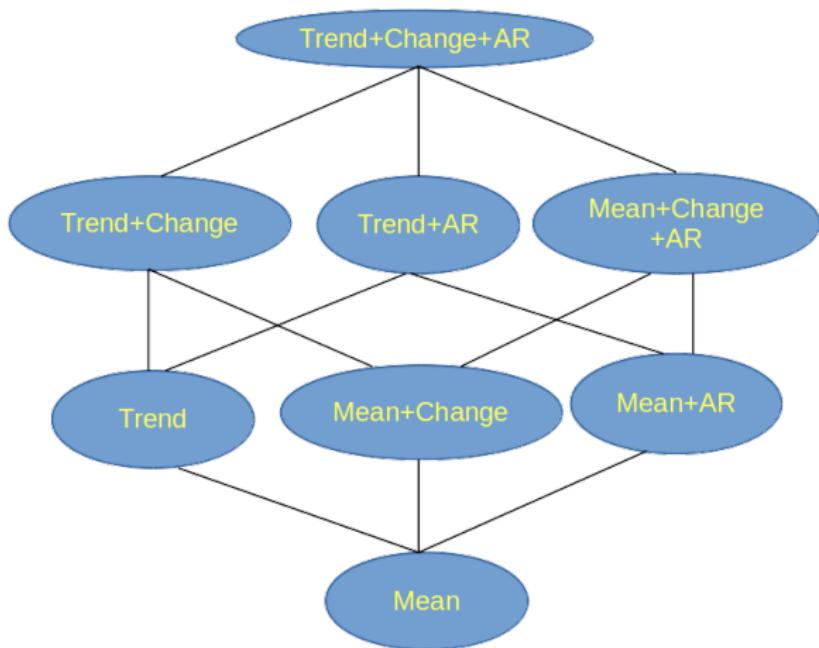
- From a publication in Marine Ecology (not the only one)
- Used the Rodionov (2004) method.
- Very popular but cannot deal with trend or autocorrelation.



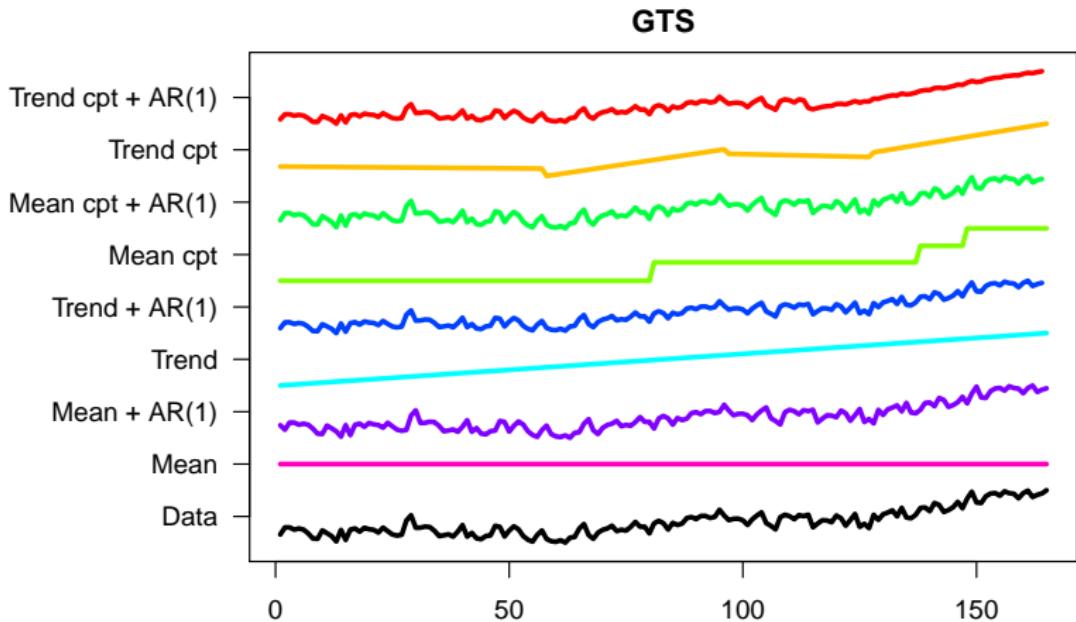
- potentially hundreds or thousands of series
- no time to consider the format of change for each
- need to include both the potential for trends and also red noise (autocorrelation).



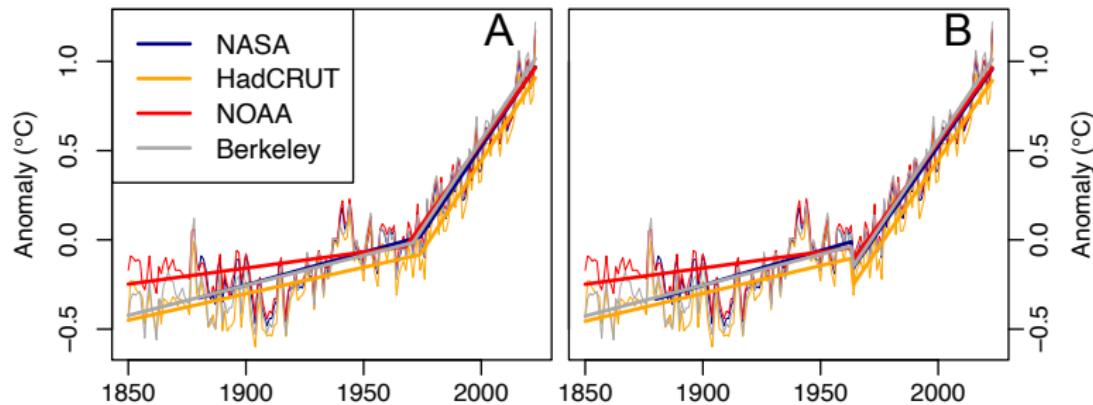
EnvCpt: select the most parsimonious but accurate model for the data. Simple to extend with other types of models.



# GMST All Fits



# Join-pin Models



Fearnhead, Maidstone, Letchford, JCGS (2019)

$$Y_t = \theta_i + \frac{\theta_{i+1} - \theta_i}{\tau_{i+1} - \tau_i} (t - \tau_i) + Z_t$$

where  $Z_t \sim N(0, \sigma^2)$ .

Implemented in the `cpop` R package.

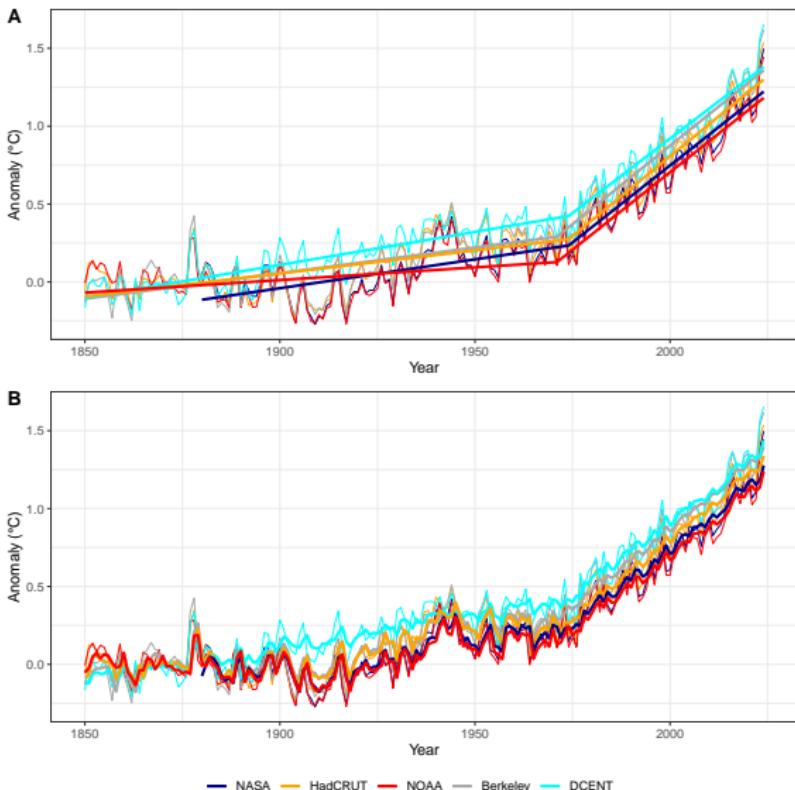
$$Y_t = \theta_i + \frac{\theta_{i+1} - \theta_i}{\tau_{i+1} - \tau_i} (t - \tau_i) + Z_t$$

but now  $Z_t$  is AR( $p$ ),  $Z_t = \phi_1 Z_{t-1} + \phi_2 Z_{t-2} + \dots + \phi_p Z_{t-p} + \epsilon_t$ .

## Challenge:

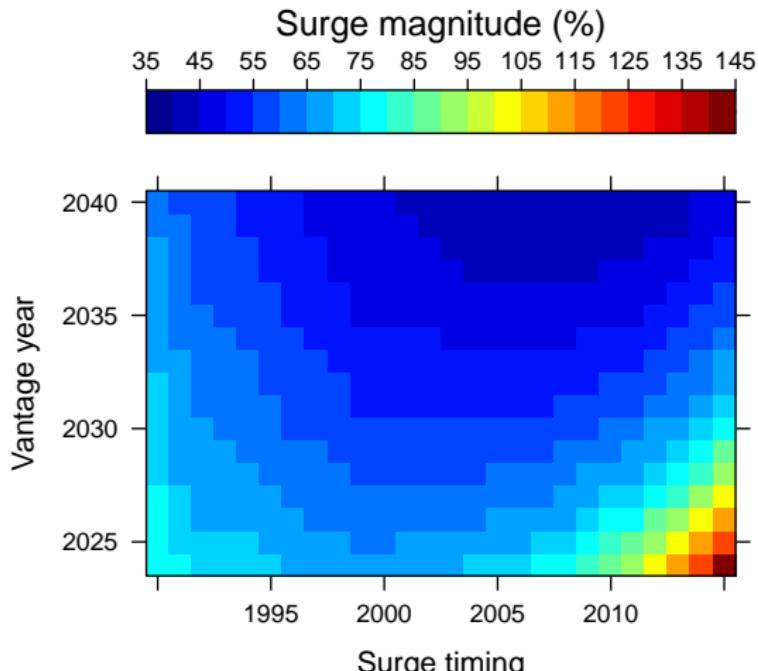
- Definition of join-pin
- Fitting AR parameter across segments:
  - Fixed we use EM algorithm
  - Varying we can embed in PELT

# Join-pin Models



# So what?

What would we need to see (in the statistically preferred model).





## Global warming is NOT surging, scientists say - despite record-breaking temperatures

- Scientists find no evidence for a 'surge' in global temperatures
- READ MORE: 2023 was officially the hottest year on RECORD

By JONATHAN CHADWICK FOR MAILONLINE

PUBLISHED: 14:01, 15 October 2024 | UPDATED: 07:52, 16 October 2024



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From the UK's hottest day to the hottest year on record globally, there's no doubt some worrying temperature records have been broken in recent years.

Many people think the rate of **global warming** has dramatically accelerated or 'surged' over the past 15 years – and is a cause of more extreme weather.

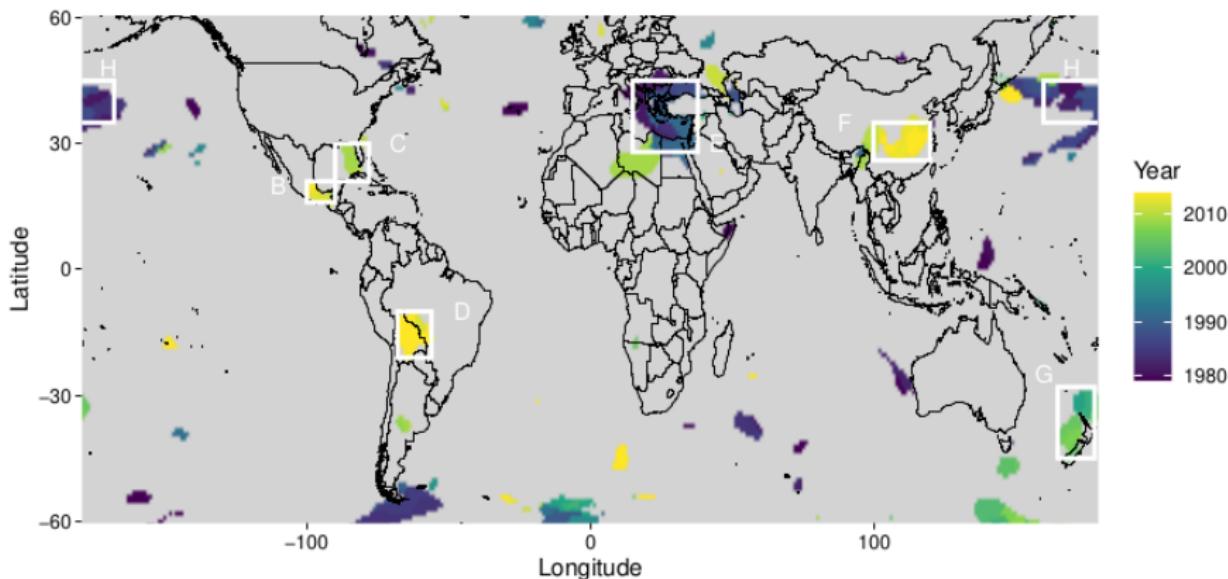
But a new study says there is not any statistical evidence for this so-called 'surge' or 'leap'.

Stoney-McBoney • 2mo ago •  
This is that article your uncle is going to bring up next Thanksgiving as to why "all this global warming stuff is liberal nonsense".  
Upvote 30 Downvote Reply Award Share ...

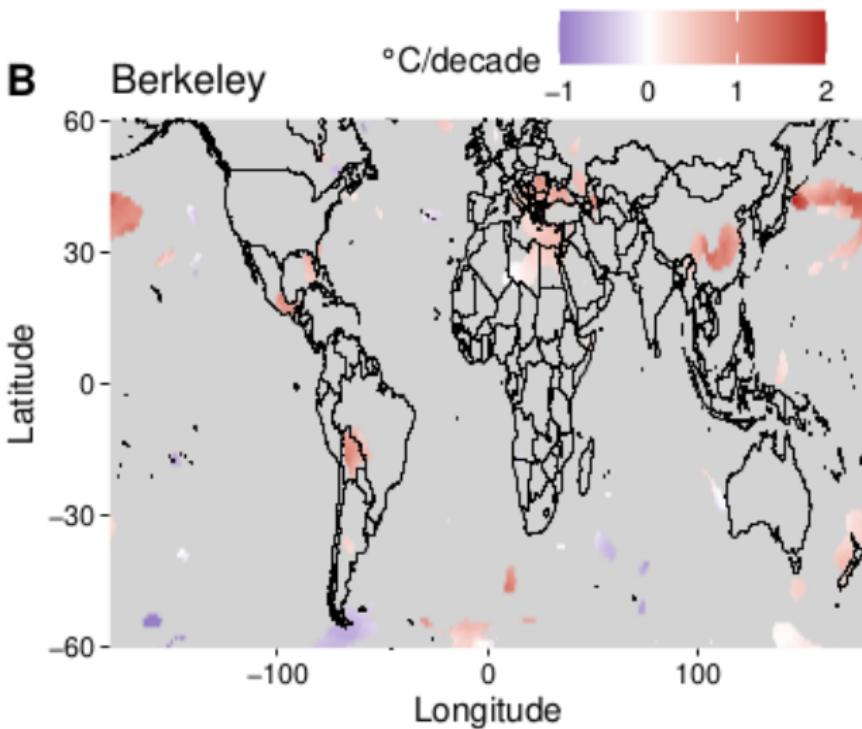
Siphilius • 2mo ago •  
Well, if there's no discernible proof that we've attributed to an increase in climate change, as this paper suggests, he would be correct wouldn't he?  
Upvote 36 Downvote Reply Award Share ...

Aacron • 2mo ago •  
That's not what the article says, so no he wouldn't be.  
Upvote 24 Downvote Reply Award Share ...

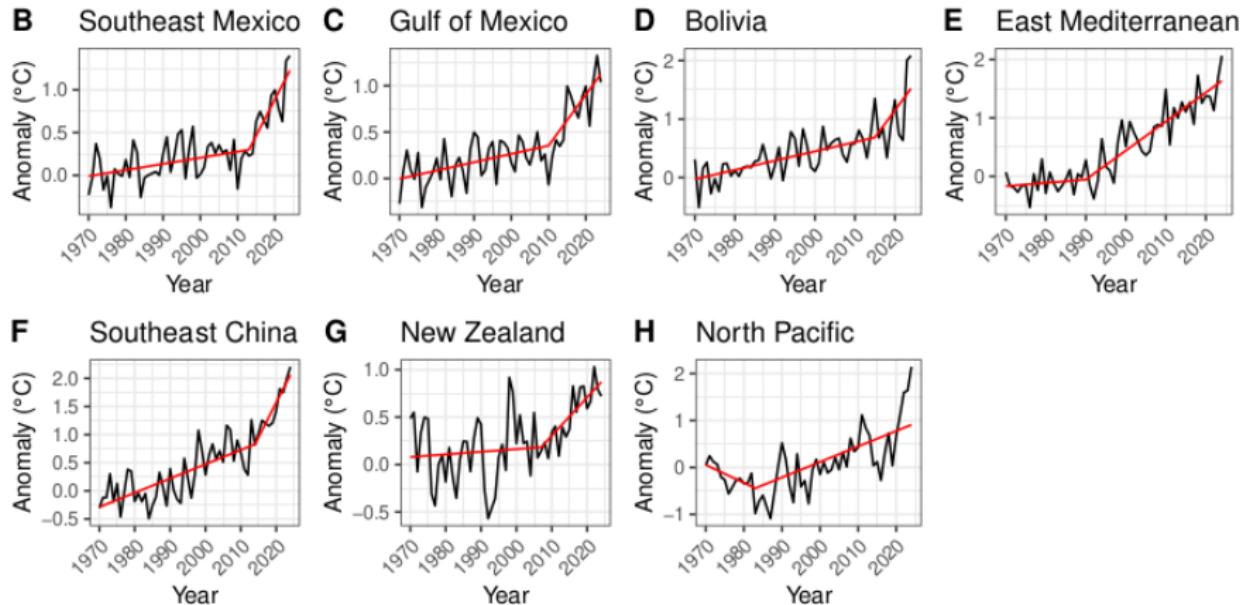
# Extension to gridded



# Extension to gridded



# Extension to gridded



- Detecting and documenting changepoints improves analyses . . .
  - . . . and can help answer pertinent questions in different domains
  - Extending to more complex model structures is interesting statistically
  - Extending to monthly/grid point is interesting climatically
- 
- I enjoy working with different disciplines . . .
  - . . . as often it sparks my next research challenge.

# Papers discussed

Preprints of all available at: [www.lancs.ac.uk/~killick/pub.html](http://www.lancs.ac.uk/~killick/pub.html)

PELT: <https://doi.org/10.1080/01621459.2012.737745>

Model Choice: <https://doi.org/10.1175/JCLI-D-17-0863.1> &  
<https://doi.org/10.1002/qre.2712>

LMvsCpts: <https://doi.org/10.1007/s11222-017-9731-0> &  
<https://doi.org/10.1002/env.2568>

Warming Surge: <https://arxiv.org/abs/2403.03388>

Regional Surge:

<https://www.researchsquare.com/article/rs-7731926/v1>

# Climate Dynamics

