School of Mathematical Sciences | Lancaster University

Generalised mixed effects models for changepoint analysis of biomedical time series

Rebecca Killick(r.killick@lancs.ac.uk)
Joint work with Mark Fiecas and Kathryn Cullen (University of Minnesota)
IBS-IR-EMR2025

Outline

- Motivation from Biomedical Time Series
- Background
  - What are changepoints?
  - How do we fit changepoint models?
- A generalised mixed effects model with changepoints
- Data Applications
- Summary

FMRI - BRIDGES

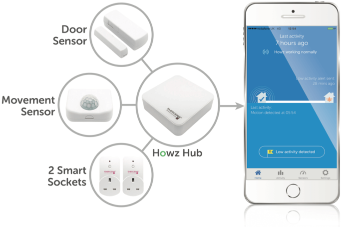
Brain Imaging Development of Girls' Emotion and Self (BRIDGES) Study

- Resting state data from 137 girls, no NSSI to severe NSSI.
- 56 spatial locations, 4 regions of interest
- 904 observations from each individual

At home monitoring

The Howz Smart System:

- Discreet passive sensors.
- Measure activity in the home.
- Learn about your daily routine.



Objective:

- Identify abnormal behaviour:
- Indicate a decline in health or well-being in an older person.

Data:

- 4 people
- Binary yes/no activity in 15 mins
- 8 weeks (56 days) of activity data

Commonalities

- Mutiple people
- Repeated observations per person
- Common structure with random variation across people
- Potential changes over time

Commonly used model = random effects model

$$\mathbb{E}(y_{i,j}) = \mu + \mu_i + \epsilon_{i,j}$$

GLMER

Generalized Linear Mixed Effects Regression (GLMER) model.

$$y_i|b \sim \mathrm{Distr}\left(\mu_i, \sigma^2\right)$$
$$g(\mu) = X\beta + Zb,$$

- X - external regressors with coefficients β
- Distr - conditional distribution
- Z - random effects with coefficients b

Standard GLMER estimators work well.

$$y_i|b \sim \mathrm{Distr}\left(0, \sigma_i^2\right)$$

We want to write the covariance in a similar random effects form.

GLMEC

For Uniform-block (UB) structure we get

$$\Sigma = \widetilde{\Sigma}_\epsilon \circ \mathbf{Id}(\mathbf{k}) + \Sigma_\mu \circ \mathbf{J}(\mathbf{k}).$$

where:

- $\widetilde{\Sigma}_\epsilon = \text{diag}(\sigma^2_{\epsilon,1}, \ldots, \sigma^2_{\epsilon,K})$
- $\mathbf{Id}(\mathbf{k}) = \text{Bdiag}(\text{Id}_{k_1}, \ldots, \text{Id}_{k_K})$, where $\text{Bdiag}(\cdot)$ constructs a block-diagonal matrix
- $\Sigma_\mu = (b_{ij})$ be a $K \times K$ covariance matrix
- $\mathbf{J}(\mathbf{k}) = (\mathbf{1}_{k_u \times k_v})$
- $\circ$ be the block Hadamard product

GLMEC

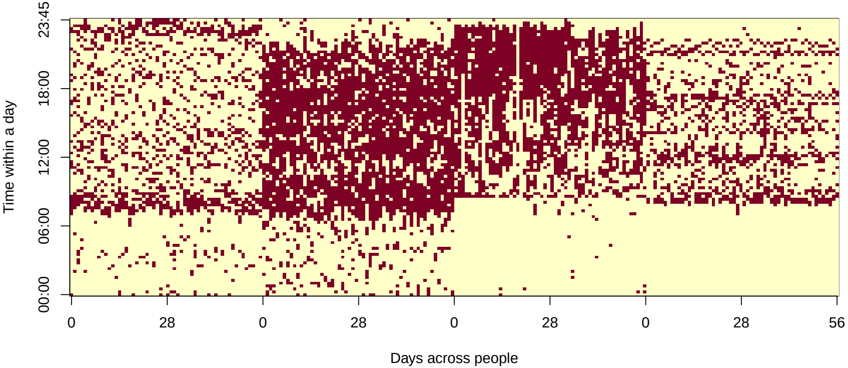
For Heterogeneous-block (HB) structure we get

$$\Sigma = \widetilde{\Sigma}_\epsilon + \Sigma_\mu \circ \mathbf{J}(\mathbf{k}).$$

- May be prohibitive for high dimensional (*n* relative to *K*)

Estimate parameters by block-wise method of moments (see ArXiV paper).

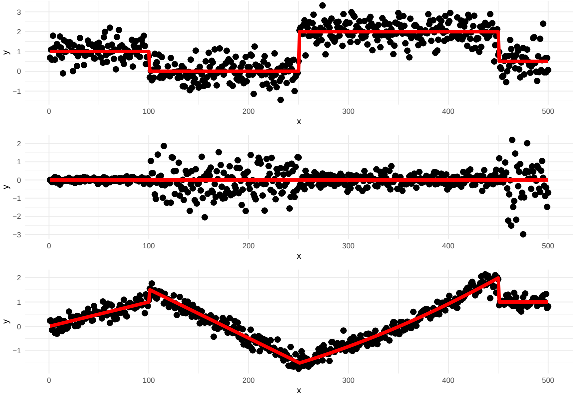
What now?


Time within a day
23:45
18:00
12:00
06:00
00:00
0
28
0
28
0
28
0
28
56
Days across people

What are changepoints?

For data $y_1, \ldots, y_n$, if a changepoint exists at τ, then $y_1, \ldots, y_\tau$ differ from $y_{\tau+1}, \ldots, y_n$ in some way.

There are many different types of change.

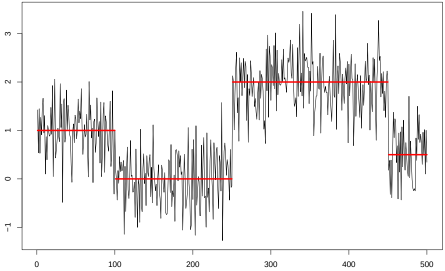


Change in mean

Assume we have time-series data where

$$Y_t|\theta_t \sim \mathrm{N}(\theta_t, 1),$$

but where the means, θ_t, are piecewise constant through time.

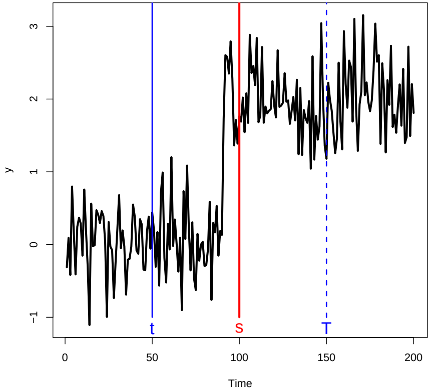


Problem

- How many changes?
- Where are the changes? 2^{n-1} possible solutions!

PELT in a nutshell

- Dynamic programming allows us to only worry about the location of the *last* change.
- Pruning means that as we go through the data we are smart about which locations are potential last change locations.

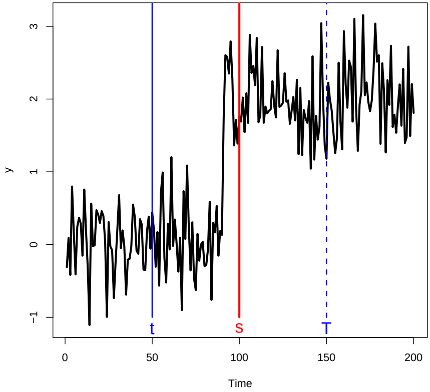


PELT: Pruning

Let $0 < t < s < T$, if

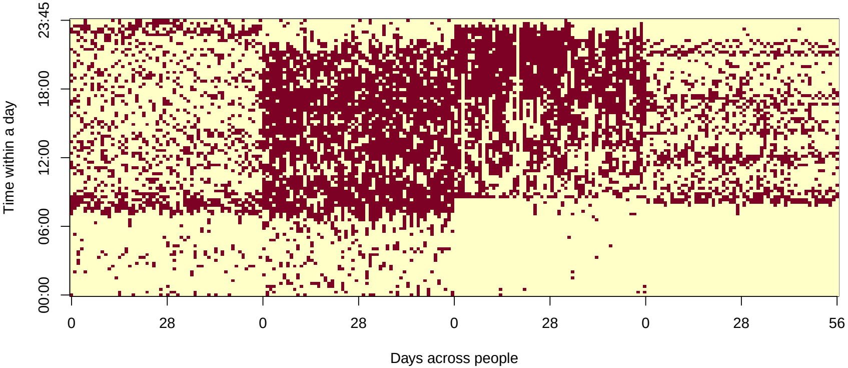
$$F(t) + \mathcal{C}(y_{(t+1):s}) < F(s)$$

then at any future time $T > s$, t can never be the optimal last changepoint prior to T.
We can prove that, under certain regularity conditions, the expected computational complexity will be $\mathcal{O}(n)$.



Final approach

- Use the GLMER or LMEC likelihood as the cost function for PELT



Practical considerations

- What can change?
- Minimum segment length - theoretical minimum? Covariance estimation is hard!
- Penalty for changepoint estimation - BIC
- Assume known group structure

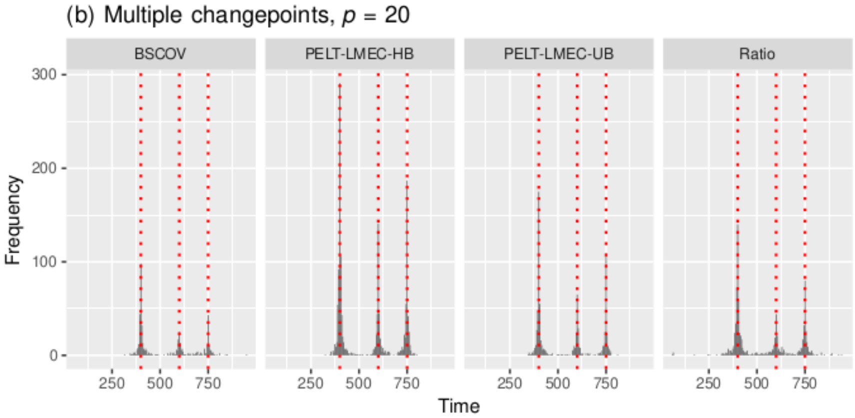
Simulations

- n=1000
- K=4 groups, 5 series per group
- Simulate covariances from Wishart distribution and manipulate

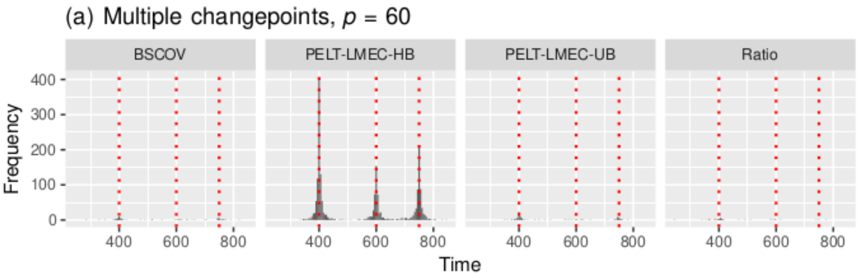
Compare to

- Ratio method (Ryan and Killick (2023)), no group structure
- Wild Sparsified Binary Segmentation (Li et al.(2023)), factor model

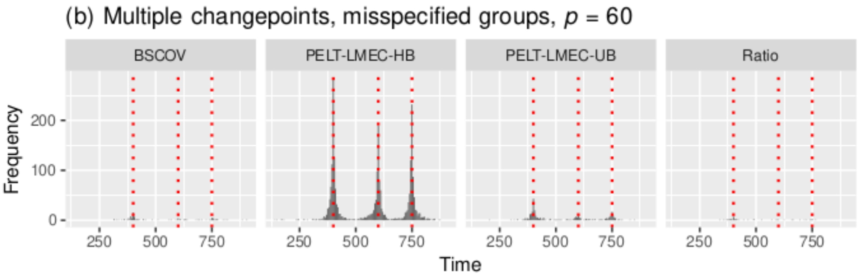
3 cpts p=20


(b) Multiple changepoints, p = 20
BSCOV
PELT-LMEC-HB
PELT-LMEC-UB
Ratio
Frequency
300
200
100
0
250
500
750
Time

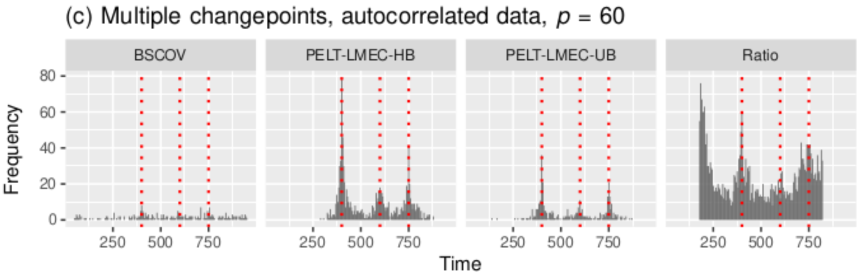
3 cpts p=60


(a) Multiple changepoints, p = 60
BSCOV
PELT-LMEC-HB
PELT-LMEC-UB
Ratio
Frequency
400
300
200
100
0
400
600
800
Time

Misspecified groups



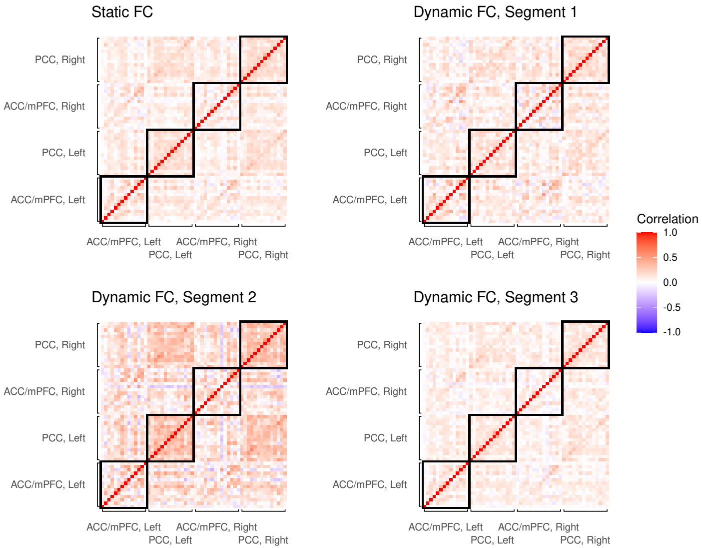
Autocorrelation



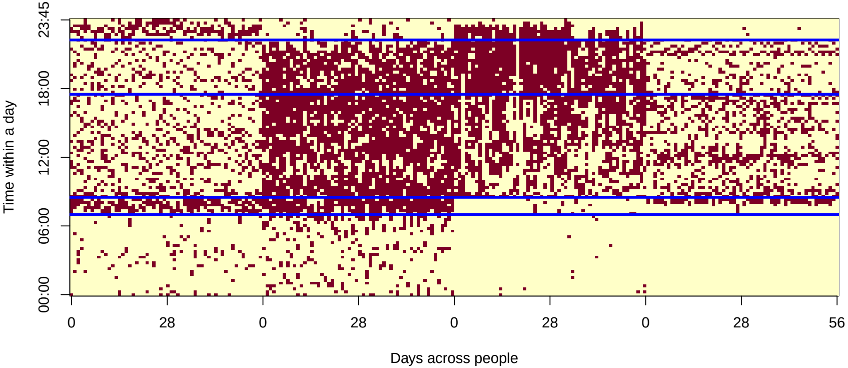
Summary of sims

- LMEC-HB best overall
- Mild misspecification doesn't affect cpt detection
- No accounting for autocorrelation is bad

FMRI


Static FC
PCC, Right
ACC/mPFC, Right
PCC, Left
ACC/mPFC, Left
ACC/mPFC, Left
ACC/mPFC, Right
PCC, Left
PCC, Right
Dynamic FC, Segment 1
PCC, Right
ACC/mPFC, Right
PCC, Left
ACC/mPFC, Left
ACC/mPFC, Left
ACC/mPFC, Right
PCC, Left
PCC, Right
Dynamic FC, Segment 2
PCC, Right
ACC/mPFC, Right
PCC, Left
ACC/mPFC, Left
ACC/mPFC, Left
ACC/mPFC, Right
PCC, Left
PCC, Right
Dynamic FC, Segment 3
PCC, Right
ACC/mPFC, Right
PCC, Left
ACC/mPFC, Left
ACC/mPFC, Left
ACC/mPFC, Right
PCC, Left
PCC, Right
Correlation
1.0
0.5
0.0
-0.5
-1.0

Howz



Time within a day
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56
Days across people

Summary

- Changepoints occur in most data
- Detecting and documenting them improves analyses
- Presented a generalised random effects model with changepoints
- Showed its utility in two very different data settings
- I enjoy working with messy real life data . . .
- . . . as often it sparks my next research challenge.