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Micro-macro changepoint inference for periodic data sequences

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RSS, Sept 2025

Outline

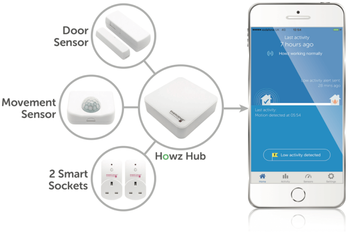
- Motivation
- What are changepoints?
- Micro changepoint model
    - Bayesian Implementation
    - Frequentist Implementation
- Macro changepoint model
- Simulations
- Application

Motivation

At home monitoring

The Howz Smart System:

- Discreet passive sensors.
- Measure activity in the home.
- Learn about your daily routine.



Objective:

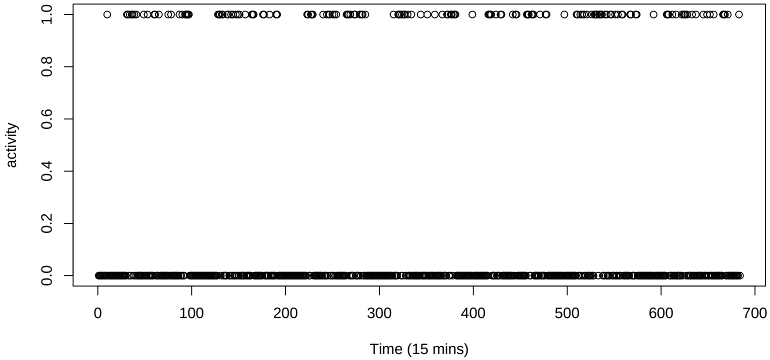
- Identify abnormal behaviour:
- Indicate a decline in health or well-being in an older person.

Data:

- Binary yes/no activity in 15 mins

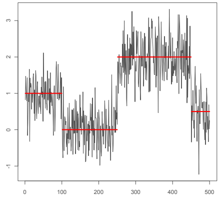
Howz

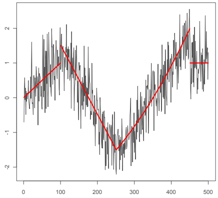
- How can we study changes in periodic and cyclical behaviours?

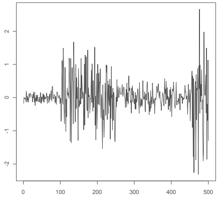


What are changepoints?

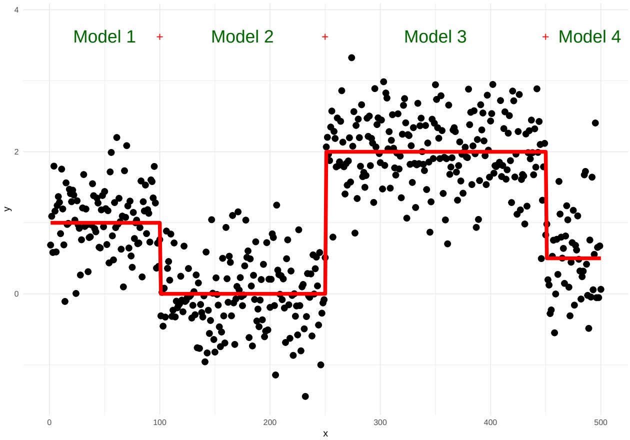
For data $y_1, \ldots, y_n$, a changepoint is a location τ where the statistical properties of $y_1, \ldots, y_\tau$ are different from $y_{\tau+1}, \ldots, y_n$ in some way. Traditional changepoints include: mean, regression, variance.





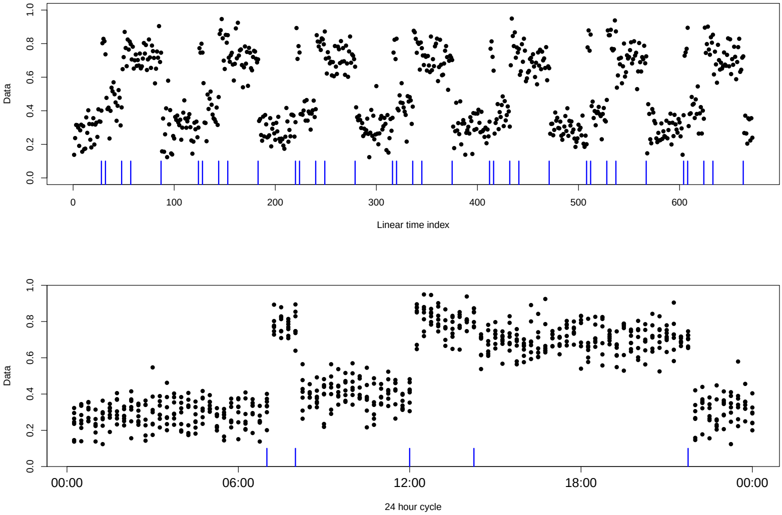


Problem



- How many changes?
- Where are the changes? 2^{n-1} possible solutions!

Periodic Changepoints



Our Contribution

Create method for periodic and global changepoint detection

- Wide range of data structures (assume likelihood available)
- Present frequentist method for local periodic changes
- Use PELT to extend to global changes in periodic

Create layered visualizations for ease of inference.

We denote changepoints as micro (periodic) and macro (global).

Periodic level changepoint detection

Thinking

Challenge: How to encorporate circular nature

Soution: Reframe $t = cN + i$, $c \in \mathbb{N}_0$ and $i \in \{0, 1, \ldots, N-1\}$

Challenge: How to detect using (c, i) instead of t?

Challenge: There is no single changepoint setting anymore!

Segment Neighbourhood

Challenge: No single changepoint setting to build upon.
Standard:

$$c_{0,t}^{m} = \min_{\tau_1,\ldots,\tau_m} \sum_{j=1}^{m+1} \mathcal{C}(x_{(\tau_{j-1}+1):\tau_j})$$
$$= \min_{\tau_{m-1}} [c_{0,\tau_{m-1}}^{m-1} + \mathcal{C}(x_{(\tau_{m-1}+1):t})].$$

Circular:

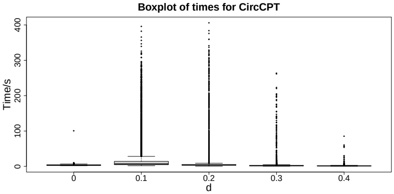
$$c_{0,t}^{2} = \min_{\tau_1,\tau_2} [\mathcal{C}(x_{(\tau_1+1):\tau_2}) + \mathcal{C}(x_{(\tau_2+1):\tau_1})]$$
$$c_{0,t}^{m} = \min_{\tau_{m-1}} [c_{0,\tau_{m-1}}^{m-1} + \mathcal{C}(x_{(\tau_{m-1}+1):t})] \qquad m = 3,\ldots M$$

Frequentist

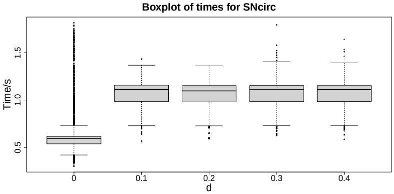
Correct detection (1-α and power rates)

d \ τ_1	4	8	16	24	32	48
0.0	1.000	**1.000**	1.000	1.000	**1.000**	**1.000**
0.1	0.000	0.004	0.042	0.102	0.123	0.156
0.2	0.177	0.467	0.550	**0.540**	**0.519**	**0.570**
0.3	0.842	**0.846**	**0.828**	**0.856**	**0.866**	**0.873**
0.4	**0.973**	**0.975**	**0.971**	**0.972**	**0.966**	**0.976**

Simulation Results

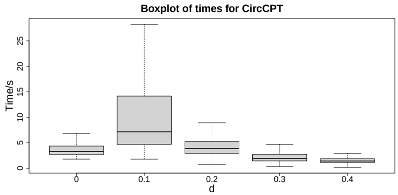


Bayesian

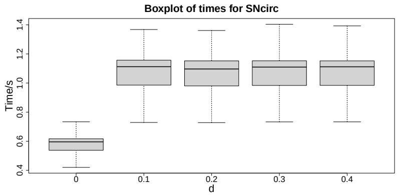


Frequentist

Simulation Results



Bayesian

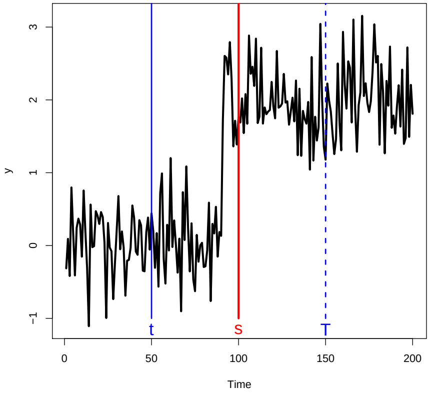


Frequentist

Macro Level Changepoint Detection

PELT in a nutshell

- Dynamic programming allows us to only worry about the location of the *last* change.
- Pruning means that as we go through the data we are smart about which locations are potential last change locations.



Macro Level Changepoints

- Treat the micro-level detection as a "fit" for a segment ...
- ... thus we can use traditional, linear time algorithms.

We use PELT which optimizes:

$$\sum_{i=1}^{q+1}[\mathcal{C}(y_{(\alpha_{i-1}+1):\alpha_i})] + \beta q. \tag{1}$$

where q is the number of macro level changepoints and $\mathcal{C}(\cdot)$ is the likelihood:

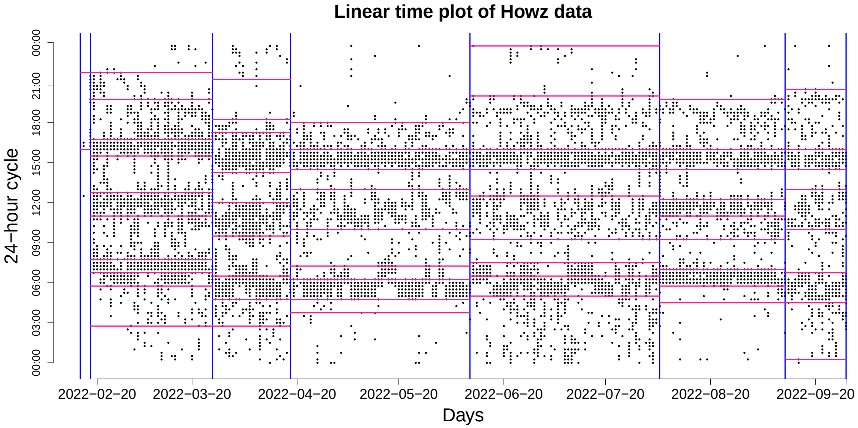
$$\mathcal{C}(y_{(\alpha_{i-1}+1):\alpha_i}) = -2\sum_{j=1}^{\hat{m}_i}\sum_{t\in B^*_{i,j}} \log f(y_t|\hat{\mu}_{i,j}, \hat{\sigma}^2_{i,j})$$

Simulations

See pre-print for simulation study.

Application

Application Results:Howz



Discussion

Overview:

- A new approach for identifying within-period changepoint events for time series.

  Taylor, S. A. C., Killick, R., Burr, J. and Rogerson, L. (2021)

  'Changepoint Detection within Periodic Binary Time Series'. JRSS:Series C.

- Bayesian pooling of evidence across multiple periods by applying a circular perspective to time.

- Introduced the micro-macro level changepoint approach

  Ushakova, A., Taylor, S. A. C., Killick, R. (2023)

  'Micro-Macro Changepoint Inference for Periodic Data Sequences'. JCGS.

- Bayesian and Frequentist viewpoints for a wide set of data types

  Li, O., Killick, R. (2025+)

  'Detecting changes in periodic data'. Submitted

WBS performance