Personal Notes

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Preface

This page contains notes I've taken over time for several different subjects of interest. Currently these subjects include

- Classical Mechanics
- Electrodynamics
- Circuit Analysis
- Quantum Mechanics
- Statistical Mechanics

Feel free to use whatever you find helpful.

Part I

Classical Mechanics

1 Newtonian Mechanics

Classical Mechanics can be thought of as the branch of physics that focuses on studying the mechanical properties of physical systems using classical laws. By *mechanical*, we mean we're focused on analyzing how objects behave in response to *forces*.

By *classical*, we mean we're limiting ourselves to objects that are neither too big nor too small, and aren't moving too fast. Roughly speaking, this means the size of objects are between the size of a human cell to the size of a galaxy cluster, and that those objects aren't moving anywhere near close to the speed of light.

The oldest and most intuitive formulation of classical mechanics is **Newton's Laws**. The idea is that objects move because of forces that act on them. Understanding mechanics in Newton's formulation is about understanding the underlying forces, as well as derived quantities like momentum, torque, and energy.

1.1 Point Particles

In nature, an object is made of matter. It can be composed of many different molecules arranged in intricate and complicated ways. Further, each molecule is itself made of atoms, and each atom is itself made up of subatomic particles. Trying to model the motion of an object would be extremely cumbersome if we insisted on modeling the dynamics of each subatomic particle.

Instead, it's convenient to make abstractions. The most convenient abstraction to make is that we can describe the global behavior of an object as if it were a point object with no width. It can't spin or deform. It's one indivisible thing. We call these **point particles**.

We'll think of a point particle as following some trajectory in the 3-dimensional Euclidean space $\mathbb{R}^3$. The trajectory or **position** is a time-dependent vector

$$\mathbf{x}(t) = x(t)\mathbf{e}_x + y(t)\mathbf{e}_y + z(t)\mathbf{e}_z.$$

A moving particle also has associated to it a **velocity** vector given by

$$\mathbf{v} = \dot{\mathbf{x}} = \frac{d\mathbf{x}}{dt}.$$

Perhaps the most fundamental goal of classical mechanics is to find these two vectors as a function of time. In the Newtonian formulation, if we want to find a particle's trajectory, we start with the particle's **acceleration** vector

$$\mathbf{a} = \dot{\mathbf{v}} = \ddot{\mathbf{x}} = \frac{d^2\mathbf{x}}{dt^2},$$

and match it with the **force** vector $\mathbf{F}$ via Newton's Second Law to get a second-order differential equation for $\mathbf{x}(t)$.

1.2 Newton's Laws

Newton's Laws efficiently encapsulate the fundamental physics of classical mechanics. They're stated below specifically for a point particle, or *body*, but can be extended to more complex systems as well.

1. A body remains at rest, or in motion at a constant speed in a straight line, unless acted upon by a force. That is,

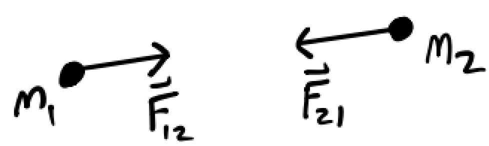
$$\mathbf{F} = \mathbf{0} \Rightarrow \mathbf{v} = const.$$

2. When a body is acted upon by a force, the time rate of change of its acceleration is proportional to the force. That is,

$$\mathbf{F} = m\mathbf{a}.$$

3. If two bodies exert forces on each other, these forces have the same magnitude but opposite directions. That is,

$$\mathbf{F}_{12} = \mathbf{F}_{21}.$$

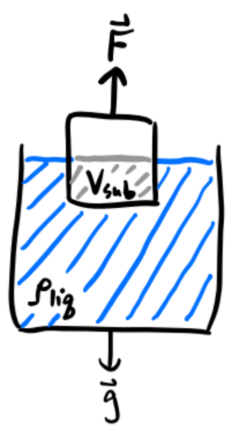


Forces are **vectors**, which means they obey the superposition principle, and can be analyzed in components. Position, velocity, and acceleration are vectors as well. The proportionality constant between $\mathbf{F}$ and $\mathbf{a}$ is called the **mass** m. Loosely speaking, the mass of an object is a measure of its *inertia* or resistance to motion.

The functional form of the forces themselves depend on the particular type of forces applied. Some common forces are:

- Gravitational Force: $\mathbf{F} = -\frac{GMm}{r^2}\mathbf{e}_r$
- Coulomb Force: $\mathbf{F} = k_e\frac{Qq}{r^2}\mathbf{e}_r$
- Harmonic Oscillator: $\mathbf{F} = -k\mathbf{x}$
- Lorentz Force: $\mathbf{F} = q\mathbf{E} + \frac{q}{c}\mathbf{v} \times \mathbf{B}$
- Thrust: $\mathbf{F} = -|\mathbf{v}_{ex}|\dot{m}\mathbf{e}_v$

- Normal Forces: $\mathbf{F} = \mathbf{N}$
- Tension Forces: $\mathbf{F} = \mathbf{T}$
- Frictional Forces: $\mathbf{F} = -\mu|\mathbf{N}|\mathbf{e}_v$
- Drag Forces: $\mathbf{F} = -f(\mathbf{v})\mathbf{e}_v \approx -a\mathbf{v} - b|\mathbf{v}|^2\mathbf{e}_v$
- Centrifugal Forces: $\mathbf{F} = m\omega \times (\mathbf{x} \times \omega)$
- Coriolis Forces: $\mathbf{F} = 2m\mathbf{v} \times \omega$
- Buoyant Forces: $\mathbf{F} = -\rho_{liq}V_{sub}\mathbf{g}$



1.3 Conservation Laws

A quantity Q is said to be **conserved** if its time derivative is zero, $\dot{Q} = 0$. That is, Q is conserved it it's constant in time.

1.3.1 Momentum

For an object moving at velocity $\mathbf{v}$, define its linear momentum $\mathbf{p}$ by

$$\mathbf{p} = m\mathbf{v}.$$

If the mass m is constant, we evidently have

$$\mathbf{F} = \dot{\mathbf{p}}.$$

If $\mathbf{F} = \mathbf{0}$, then $\mathbf{p} = const$, hence momentum is conserved if there are no forces applied. This is the conservation of momentum.

1.3.2 Angular Momentum

Define the angular momentum $\mathbf{L}$ of an object by

$$\mathbf{L} = \mathbf{x} \times \mathbf{p}.$$

Similarly, define the **torque** or **moment** $\mathbf{N}$ by

$$\mathbf{N} = \mathbf{x} \times \mathbf{F}.$$

Note both angular momentum and torque depend on the choice of coordinate system used since the position vector $\mathbf{x}$ depends on choice of origin. Now, observe that

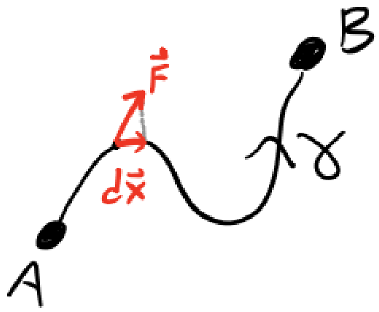
$$\dot{\mathbf{L}} = \dot{\mathbf{x}} \times \mathbf{p} + \mathbf{x} \times \dot{\mathbf{p}} = m\mathbf{v} \times \mathbf{v} + \mathbf{x} \times \mathbf{F} = \mathbf{N}.$$

Thus, $\mathbf{N} = \dot{\mathbf{L}}$. If $\mathbf{N} = \mathbf{0}$, then $\mathbf{L} = const$, hence angular momentum must be conserved if there are no torques applied. This is the conservation of angular momentum.

1.3.3 Work and Energy

Define the **work** done *on* an object as it moves along a path γ from A to B by

$$W = \int_A^B \mathbf{F} \cdot d\mathbf{x}.$$



In general, work depends on the path taken to get from A to B, hence it isn't a unique property of the system.

Observe that

$$dW = \mathbf{F} \cdot d\mathbf{x} = \mathbf{F} \cdot \mathbf{v} dt = d\left(\frac{1}{2} m \mathbf{v}^2\right).$$

Define the **kinetic energy** of the system by $T = \frac{1}{2} m \mathbf{v}^2$. Then we evidently have $dW = dT$. That is, the work done on the system to get from A to B via γ is just the change in kinetic energy between A and B,

$$W = \Delta T = T_B - T_A.$$

When the work done is independent of the path taken it's a state function of the kinetic energy. In this case, the force $\mathbf{F}$ is said to be **conservative**.

By the Helmholtz theorem, the following conditions are all equivalent:

- $\mathbf{F}$ is conservative,
- W is path-independent,
- $\nabla \times \mathbf{F} = \mathbf{0}$,
- There is a scalar potential $V = V(\mathbf{x})$ such that $\mathbf{F} = -\nabla V$.

The scalar potential V is called the **potential energy** of the system. Evidently, if $\mathbf{F}$ is conservative, we have

$$W = \int_A^B \mathbf{F} \cdot d\mathbf{x} = -\int_A^B \nabla V \cdot d\mathbf{x} = -\int_A^B dV = V_A - V_B = -\Delta V = \Delta T.$$

That is, $\Delta T + \Delta V = 0$. Define the total mechanical **energy** E of the system by

$$E = T + V.$$

Then $\Delta E = \Delta(T + V) = 0$. That is, energy is conserved when the forces on the system are conservative. This is the conservation of energy.

Energy isn't generally conserved if the forces aren't conservative. Examples of non-conservative forces include any force that's a function of velocity. These include dissipative forces like friction or drag, as well as magnetic forces.

1.4 Examples

The primary goal of mechanics is to understand how systems evolve with time. To understand a particle's given trajectory in Newtonian Mechanics, we need to

- Write down all the forces acting on the particle,
- Use $\mathbf{F} = m\mathbf{a}$ to set up the equations of motion,
- Solve the equations of motion for the trajectory $\mathbf{x}(t)$, either analytically or (usually) numerically.

Here are some examples.

1.4.1 Example: Projectile motion

Suppose a cannon is launched from the origin at an angle θ above the ground with initial velocity $\mathbf{v}_0$.