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Аналіз криптографічних властивостей перспективних симетричних перетворень

(тема роботи)

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2013 р.

РЕФЕРАТ

Магістерська робота містить 87 сторінок, 15 рисунок, 0 таблиць, 6 додатків та 64 джерел.

У роботі представлено аналіз перспективних симетричних шифрів, що є стандартами на державному та міжнародному рівні.

Розроблено методи побудови системи нелінійних рівнянь низького степеня від багатьох невідомих, що описують криптоалгоритми MISTY1 та ГОСТ 28147-89. Представлено характеристики алгебраїчної системи рівнянь кожного шифру та їх порівняння з аналогічними системами рівнянь для криптоалгоритмів AES, Camellia та PRESENT.

Оцінено криптографічну стійкість шифрів ГОСТ 28147-89 та MISTY1 до алгебраїчного криптоаналізу. Здійснено алгебраїчну атаку на зменшені версії криптоалгоритмів використовуючи методи SAT-solver для вирішення системи нелінійних рівнянь та відновлення ключа шифрування.

СИМЕТРИЧНІ ШИФРИ, MISTY1, ГОСТ 28147-89, АЛГЕБРАЇЧНИЙ КРИПТОАНАЛІЗ.

ABSTRACT

This thesis contains 87 pages, 15 figures, 0 tables, 6 appendices and 64 references.

The work presents analysis of symmetric block ciphers that are adopted standards on country and international levels.

Methods for constructing non-linear multivariate quadratic (MQ) equations systems that define cryptoalgorithms MISTY1 and GOST 28147-89 are developed. Characteristics for each algebraic system are presented and compared to analogous systems for cryptoalgorithms AES, DES and PRESENT.

Further the strength of GOST 28147-89 and MISTY1 ciphers to algebraic cryptanalysis is researched. Algebraic attack on reduced rounds versions of the ciphers is executed using SAT-solver methods for solving non-linear equations systems and recovering the enciphering key.

SYMMETRIC CIPHERS, ALGEBRAIC CRYPTANALYSIS, MISTY1, GOST 28147-89.

CONTENTS

INTRODUCTION

Symmetric cryptographic transformations are known to be the only effective method of providing data confidentiality and integrity in all fields of communication technologies [1]. They need to be not only cryptographically strong, but also have high performance and low resources consumption to satisfy modern needs for securing information. Consequently, robust requirements on cryptographic security, lightweight implementation and performance are entrusted to such transformations.

Symmetric ciphers came through a long history of development and improvement from the most primitive schemes based on symbols substitution and disk encryptors to advanced mathematical algorithms following Kerckhoffs' principle [2]. A tremendous contribution to enciphering theory has been done by Claude Shannon's work [3] back in 1949. He introduced the fundamentals of information theory and made it possible to evaluate and mathematically prove cipher security. Ubiquitous computerization, mass deployment of pervasive devices and extensive Internet access caused cryptography to find comprehensive applications in information systems. Despite the advanced mathematics behind modern cryptoalgorithms, real operating security systems often have weaknesses due to incorrect usage or implementation errors.

Security of mobile communication systems fell far behind from what was state-of-the-art in modern cryptography. The A5/1 cipher used in GSM standard for over 10 years can be broken within seconds using a combined distributed rainbow table code book to decrypt GSM voice calls and text messages [4]. Communication over satellite phones has also been shown to be insecure after reverse engineering the proprietary ciphers GMR-1 and GMR-2. GMR-1 is a variant of A5/2 cipher (which is prohibited for implementation in mobile phones as of July 2007) and is vulnerable to a known ciphertext-only attack with an average case complexity of 2^{32} [5]. GMR-2 is an original cipher, but its session key can be recovered with 65 bytes of keystream at a moderate computational complexity [6].

In [7] an effective attack on KASUMI cipher [1)] used in 3G systems is presented. Key recovery for the full cipher was done in hours on low-end computer that used unoptimized cipher reference implementation. It is worth noting that KASUMI is based on MISTY cryptoalgorithm which however could not be broken by the same attack. Therefore even slight modifications to robust cipher may significantly impact the security of the algorithm.

The need of deep analysis and public evaluation of cryptographic algorithms before deploying them into real systems is obvious. Several ciphers are considered for becoming worldwide standards in the field of providing data confidentiality at the moment.

MISTY1 is a symmetric block cipher designed in 1995 which has been one of the selected algorithms in the European NESSIE project [8] and recommended for Japaneese government use [9]. Though no vulnerabilities have yet been found in MISTY1 cipher, some of its successors were broken (KASUMI) or provided with a theoretical attack (Camellia) which may be feasible in future with increase of computations power [10].

GOST 28147-89 is a legacy cipher that has been a subject to cryptanalysis for more than 20 years. Despite its wide usage in Ukraine and other CIS countries since the publication in 1990, the cipher has been proposed for international standardization only in 2010, but hasn't been accepted however [11]. Therefore a detailed security evaluation and properties analysis of these ciphers are essential to validate their pervasive deployment into security systems.

Chapter **??** analyses PC users working conditions and their compliance with normative documents on safety engineering and sanitation. For retrieving and evaluating the influence of possible dangerous or harmful production factors an interaction system "Human–Machine–Environment" (HME) is developed. Safety measures are developed as the result of such system analysis.

[1)]KASUMI cipher is adopted by 3GPP as A5/3 cryptographic algorithm for securing mobile communications.

1 STATE OF ART ANALYSIS AND SYNTHESIS OF SYMMETRIC CRYPTOGRAPHIC TRANSFORMATIONS

Evaluation and design rationale are essential parts of developing and deploying the considered cryptographic security system. The primary goal of cryptography is to design mathematical methods for providing security against adversary's malicious actions under any predetermined conditions.

However, as practice shows, it's unfeasible to take into account all possible attacks that may be invented in the future due to technological and scientific innovations during the development stage. Therefore, the most widespread method for analysing computationally strong cryptographic primitives is complexity evaluation of every known applicable attack.

1.1 Classification of attacks on symmetric ciphers

Attacks on symmetric ciphers are defined by the abilities and data that an adversary can operate with. Most of them refer actions that may be done to cipher entities such as plaintexts and ciphertexts, however a separate class of attacks that exploits cipher implementation rather than the algorithm itself exists.

A ciphertext-only attack leaves the adversary with a set of ciphertexts that she [1)] can use to recover the corresponding plaintext or encryption key. Such conditions are the most complex for cryptanalyst. In a known plaintext attack the adversary has a fixed set of plaintext and ciphertext pairs [12].

In chosen plaintext and chosen ciphertext attacks the adversary gets an ability to encrypt plaintexts or decrypt ciphertexts of her choice respectively. So the cipher needs to have such plaintext and ciphertext spaces that would make it infeasible for the attacker to get full dictionary of all possible plaintexts and corresponding ciphertexts [12].

Related key attacks exploits possible relations between encryption keys to break the cryptographic system [13].

[1)] Following the tradition started by Shafi Goldwasser in her Lecture Notes on Cryptography, "she" is used throughout the text as referring to a subject of unknown gender.

Side channel attacks are somewhat outside of mathematical attacks scope. They use some cipher implementation peculiarities to gain information about the encryption key observing the cryptographic system during operation. So far no mathematical countermeasures exist that would guarantee the security of the transformation against these types of attacks [14].

A cipher is considered to be insecure if any type of attack exists that allows to get some information about the key with a complexity lower than a brute force.

1.2 Block ciphers

A block cipher is a function which maps n-bit plaintext blocks to n-bit ciphertext blocks, parameterized by a k-bit key K [12]. Here n is called the blocklength. The encryption key K has to be random. In order to always provide unique and correct decryption the mapping function defined by a chosen key must be bijective.

A straightforward usage of a block cipher to encrypt separate blocks of data may have disadvantage in many applications. Consequently, several block cipher modes of operations have been developed to satisfy various security purposes.

1.2.1 Algebraic representation

A symmetric block cipher may be represented as an algebraic system [15]

$$\Sigma_A = \langle X, K, Y, E, D \rangle \quad , \tag{1.1}$$

where X is a plaintext space defined over finite alphabet Q_X,
K — a key set (usually defined by fixed length strings over Q_X),
Y — ciphertext space defined over finite alphabet Q_Y,
$E : X \times K \to Y$ — a set of enciphering rules based on parametrized maps $e_k(x) = y$ for which $k \in K$, $x \in X$, $y \in Y$.
Mappings $e_k(\cdot)$ and $d_k(\cdot)$ for any $k \in K$ are bijective and ensure satisfaction of both conditions $d_k(e_k(x)) = x$ and $e_k(d_k(y)) = y$. Plaintext and ciphertext

spaces of all widespread modern symmetric ciphers coincide ($Q_X = Q_Y$), so such ciphers are therefore endomorphic, that is $X = Y$. For cryptographically secure ciphers the sets of encrypting and decrypting rules must have random mapping properties, and $e_k(\cdot)$, $d_k(\cdot)$ must be random permutations.

1.2.2 Modes of operation

Simple encryption of data chunks block-by-block is called an electronics codebook mode (ECB). As seen from figure 1.1 each plaintext is encrypted independently. Error propagation is limited within one block, but this mode is insecure for enciphering large correlated data [12].

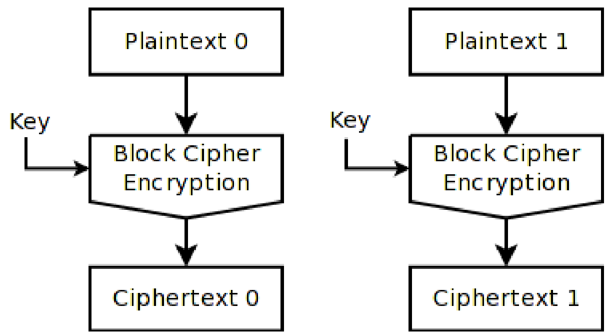

Figure 1.1 — ECB mode of operation

Cipher block chaining mode (CBC) is represented on figure 1.2. In CBC mode two identical plaintext do not encrypt to the same ciphertext as the encryption depend on the initialization vector (IV) and two previous blocks instead. This causes error propagation in ciphertext expand to two blocks, but modifications to plaintext influence all subsequent blocks and make correct decryption impossible. Such encryption mode is also more secure for enciphering correlated data [12].

Cipher feedback mode (CFB) shown on figure 1.3 turns a block cipher into self-synchronizing (see section 1.3.1) stream cipher.

Output feedback mode (OFB) is similar to CFB (figure 1.4) and differs only by a feedback connection. The main advantages of this mode is absence of error propagation (since keystream generation doesn't depend on the plaintext) and parallel processing capability.

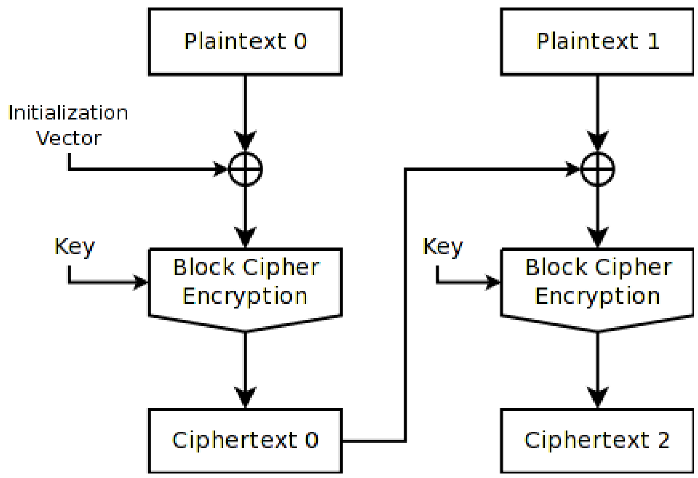

Figure 1.2 — CBC mode of operation

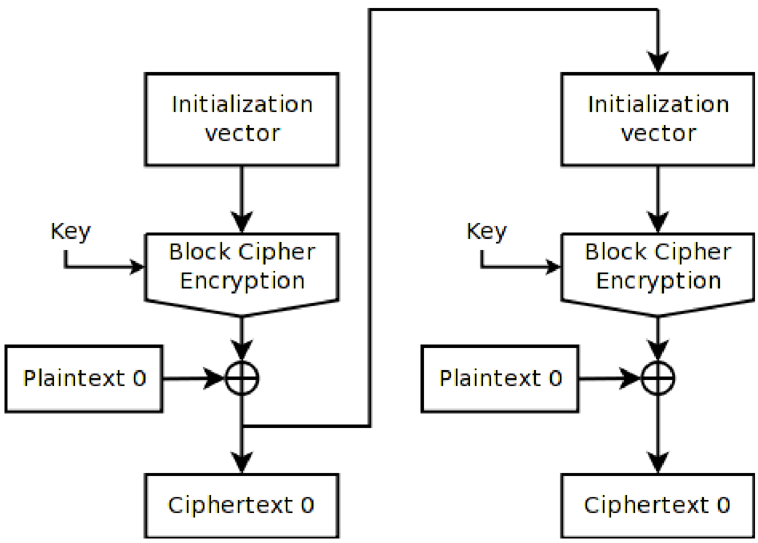

Figure 1.3 — CFB mode of operation

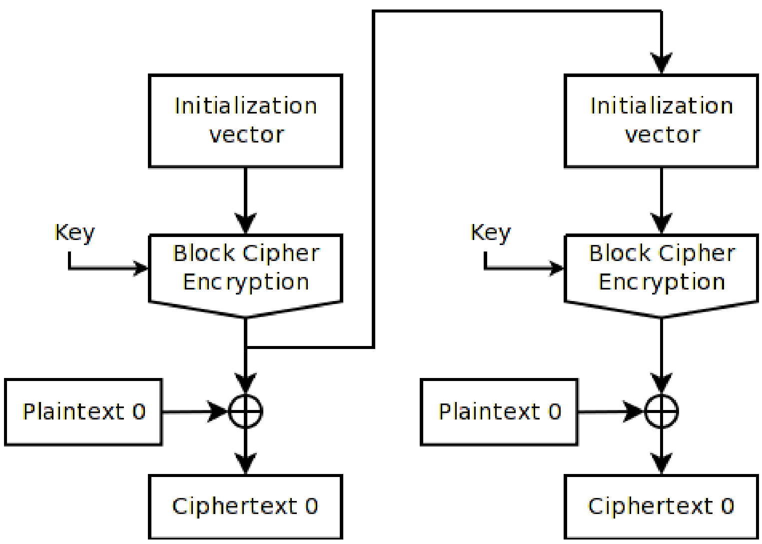

Figure 1.4 — OFB mode of operation

1.3 Stream ciphers

The distinct difference between stream ciphers and block ciphers was for the first time defined by Rainer Rueppel [16]:

"Block ciphers operate with a fixed transformation on large blocks of plaintext data; stream ciphers operate with a time-varying transformation on individual plaintext digits".

Stream ciphers gained great progress since Shannon's analysis of the Vernam cipher where he proved it to be theoretically unbreakable [3]. However such cryptosystems were complex and unprofitable to implement because of the need of secret channel to exchange key material which was the size of the message itself.

Trying to overcome disadvantages of the Vernam cryptosystem, stream ciphers inherit its idea, but use short key instead to generate a pseudo-random sequence of needed length. That is, plaintext is encrypted into ciphertext with pseudo-random sequence, called the keystream, which is produced by a finite state automaton whose initial state is determined by a secret key. Therefore stream ciphers require high structural secrecy in order to be cryptographically strong.

Stream ciphers are fast and well suited for hardware though some cryptoalgorithms designed for efficient software implementation exist. They are generally used in cases of continuous or unknown amount of data to be encrypted and strict buffering constraints.

1.3.1 Classification

Depending on the choice of how the next state of cryptosystem is generated from the current state, two types of stream ciphers are distinguished: synchronous and self-synchronizing (or asynchronous) [12].

In synchronous stream ciphers the next state of the automaton is independent of plaintext and ciphertext. Such ciphers have no error-propagation and consequently don't detect errors during decryption. This fact allows an attacker to inconspicuously alter ciphertext which will be successfully decrypted to a different plaintext. Another significance consists in the fact that encrypting and decrypting devices must constantly stay synchronized. Otherwise the decryption will fail.

Asynchronous stream ciphers are able to resume correct decryption in case transmitter and receiver fall unsynchronized. Error-propagation is limited to the state bits that depend on previously generated ciphertexts. Such ciphers are difficult for analysis because the keystream depends on input message. They are also vulnerable to playback attack: if an attacker repeats some previously recorded ciphertext, the receiver will successfully decrypt it (after synchronization) and consider the message to be valid unless time markers are used.

1.3.2 Design principles

Rainer Rueppel distinguished four approaches to stream cipher construction [17]:

a) system-theoretic approach; use fundamental design principles to create difficult and unknown problem for the cryptanalyst;

b) information-theoretic approach; try to keep the cryptanalyst in the dark about the plaintext; she will never get a unique solution;

c) complexity-theoretic approach; make the cryptosystem equivalent to some known and difficult problem (factorization, solving discrete logarithms);

d) randomized approach; generate unsolvable problem by forcing the cryptanalyst to examine lots of useless data.

Engineering and analysing of numerous stream ciphers resulted in essential design criteria [18]:

a) long period;

b) linear complexity;

c) statistical criteria (randomness, correlation, etc.);

d) confusion — every keystream bit must be a complex transformation of all the key bits;

e) diffusion — redundancies in substructures must be dissipated into long-range statistics;

f) nonlinearity criteria for Boolean functions.

However it is impossible to prove such cryptosystems are secure enough. A cipher might satisfy all criteria and still be weak to some cryptanalysis techniques.

1.3.2.1 Feedback shift registers

Any feedback shift register consists of a shift register and a feedback function [17]. The shift register itself is a sequence of bits. New pseudo-random bit is generated by shifting the sequence one bit to the right. The new input bit of the register is computed as a function of some bits already in register.

Linear feedback shift register (LFSR) is widely used in stream ciphers. Its feedback function is XOR of some bits in register (figure 1.5). The list of such bits is called a tap sequence. Such type of LFSR is called a Fibonacci configuration. A n-bit LFSR is able to produce a pseudo-random sequence of period $2^n - 1$ bits. In order to get a maximal-period linear sequence (m-sequence), the tap sequence must be formed by a primitive polynomial modulo 2. Even though using sparse polynomials leads to more efficient software implementation, dense polynomials are better for cryptographic applications. The only secret parameter of LFSR should be the initial state derived from the master key. Another type of LFSR is called a Galois configuration. It

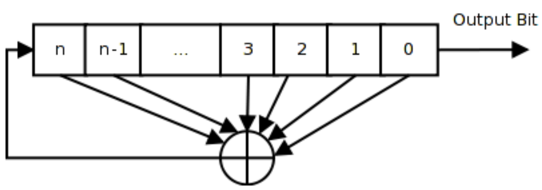

Figure 1.5 — Linear feedback shift register (Fibonacci configuration)

has the same properties, but the feedback scheme is different: each bit in the tap sequence is XORed with the output bit and replaced; the output bit then becomes the new left-most bit (figure 1.6). A sequence generated by LFSR is linear by itself and therefore useless for cryptography. It is possible to recover the LFSR structure from intercepting only $2n$ bits of the generator using Berlekamp-Massey algorithm [19].

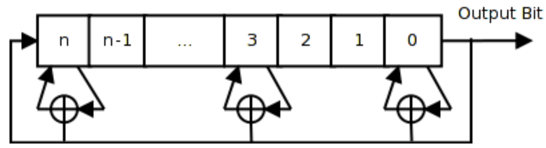

Figure 1.6 — Linear feedback shift register (Galois configuration)

Feedback with carry shift registers (FCSR) are similar to LFSRs but instead of XORing the tapping sequence bits are added to the carry register. The result reduced modulo 2 becomes the feedback bit of the register and the result divided by 2 becomes the new value of the carry register.

The carry register has to be at least $\log_2 t$, where t is the number of taps. Thus, before the carry register is filled there are some states of FCSR that never repeat. The maximum period of FCSR differs from the one of LFSR. It equals to $q - 1$, where q is the connection integer and defined as $q = 2q_1 + 2^2 q_2 + 2^4 q_4 + \cdots + 2^n q_n - 1$; q has to be a prime for which 2 is a primitive root. In fact not every initial state guarantees maximum period of the register. That means the all pseudo-random sequence generators based on FCSR will have a set of weak keys [17].

Non-linear feedback shift registers (NLFSR) use non-linear feedback function. Such stream ciphers as Grain and Trivium are based on NLFSRs. The idea both behind NLFSRs and FCSRs is to ensure high non-linearity of the output sequence. However such non-linear behavior makes the analysis of such registers almost impossible. The described registers are unpredictable — they don't guarantee maximal-period sequence, which also depends on the initial state of the register, output sequences may have biases of zeroes and ones or contain long bit series. Hereby, the advantage of these registers may at the same time lead to critical flaw. Consequently, NLFSRs and FCSRs should be used with utmost caution.

1.3.2.2 Clock control

Clock control is one of several ways to introduce high nonlinearity in pseudo-random sequence generated by linear feedback shift registers. The rate of register clocking varies either depending on several LFSRs or on certain bits of the register state [20]. As will be shown further, the combination of clock control, combination and filter generators allow to form a pseudo-random sequence satisfying all statistic requirements and yet resistant to known attacks.

1.3.2.3 Generators

The use of feedback shift registers for cryptographic applications is possible by combining several registers into a single generator.

The technique of combining outputs of several registers by a Boolean function is called *combination generator* (figure 1.7). The output sequence s_t of a combination generator composed on n LFSRs is given by

$$s_t = f(u_1, u_2, \cdots, u_n), \ \ \forall t \leq 0 \ \ , \tag{1.2}$$

where u_i denotes the sequence generated by the i-th LFSR and f is a function of n variables [21]. The output of the f function must be uniformly

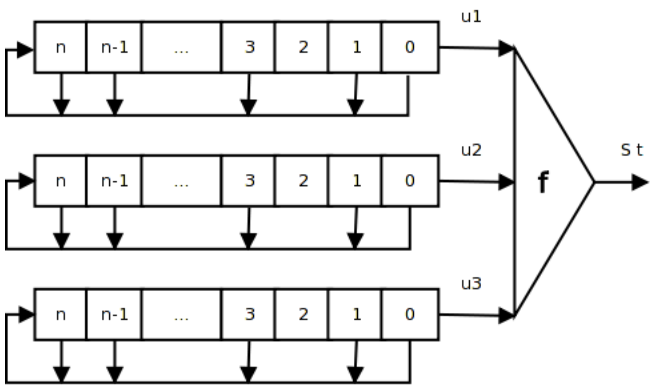

Figure 1.7 — Combination generator

distributed and balanced in order to produce pseudo-random sequences.

Linear complexity of the keystream generated by a combination generator composed of n LFSRs with primitive feedback polynomials combined by a Boolean function f equals to

$$f(L_1, L_2, \cdots, L_n) \ \ , \tag{1.3}$$

where the algebraic normal form of f is evaluated over integers and all lengths $L_1, \cdots, L_n$ are distinct and greater than 2. High linear complexity of the generator is required to ensure that Berlekamp-Massey algorithm is computationally infeasible.

Combination generators are vulnerable to correlation attacks based on recovering the initial states of all LFSRs from the knowledge of some sequence produced by the generator (known plaintext attack). In order to protect generators from this kind of attacks, the LFSR feedback polynomials should not be sparse to ensure a high correlation-immunity order of the combining function. However the correlation-immunity of a balanced Boolean function of n variables is limited with $n - 1 - deg(f)$ [21]. Tradeoffs between high algebraic degree, high nonlinearity and high correlation-immunity may be outwitted replacing the combining function by a finite state automaton with memory.

Filter generators, in distinction of combination generators, consist of single LFSR and its state is filtered by a nonlinear function (figure 1.8). The output of the function is a pseudo-random sequence formed by the generator. Just like in combination generators the filtering function must be uniformly distributed and balanced.

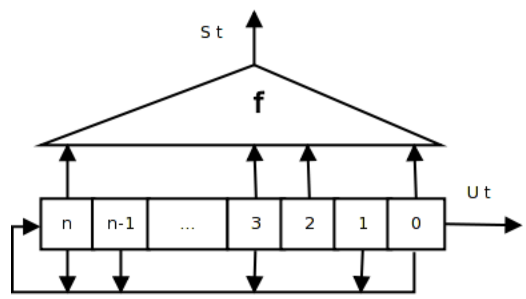

Figure 1.8 — Filter generator

Any filter generator can be represented by a corresponding combination generator consisting of n copies of the LFSR with shifted initial states when the combining function complies the filtering function.

Filter generators are vulnerable to fast correlation and generalized inversion attacks [22]. Filtering function should be highly nonlinear in order to resist the fast correlation attack. The inversion attack depends on the largest spacing between two taps of the LFSR which conflicts with the statement that LFSRs should use dense polynomials. Also the greatest common divisor of all spaces between taps should be equal to 1 or else the inversion attack could be simplified [21].

Algebraic attacks are also applicable to filter generators since a keystream bit can be represented by a function of L initial bits of the LFSR. Therefore, knowing N keystream bits allows to form an algebraic system of N equations of L variables. Usage of Gröbner bases (which may be viewed as nonlinear generalization of Gaussian elimination for linear systems) enables an attacker to lower the degree of equations until the recovery of LFSR initial state is possible by solving the algebraic system even with filtering function of high degree.

Some designs of LFSR-based generators that promise to be secure are considered further.

Alternating stop-and-go generator uses three LFSRs of different length. LFSR-1 controls clocking of the other two. If output of LFSR-1 is 0, LFSR-3 is clocked, if its output is 1, LFSR-2 is clocked. The output of the generator is the XOR of LFSR-2 and LFSR-3. A correlation attack on LFSR-1 exists, but it does not threaten the generator's security [17].

Bilateral stop-and-go generator uses two LFSRs of length n and its output bit equals to XOR of the outputs of each LFSR. The functioning of the generator is described by algorithm 1.1. So far no critical attacks on this

```
if Output of(LFSR-2 at time t − 1) == 0 and
   Output of(LFSR-2 at time t − 2) == 1
then
 | Block (LFSR-2 at time t)
end

if Output of(LFSR-1 at time t − 1) == 0 and
   Output of(LFSR-1 at time t − 2) == 1 and
   LFSR-1 clocked at time t
then
 | Block (LFSR-2 at time t)
end
```

Algorithm 1.1 — Bilateral stop-and-go generator functioning

generator have been presented. Another alternative is filter generator that consists in forming cryptographically strong pseudo-random sequence as some nonlinear function of a state of the single register [16].

The idea behind the *shrinking generator* is simple and uses two LFSRs. Both of them are clocked each time: if the output of LFSR-1 is 1, then the generator outputs bit from LFSR-2, otherwise both bits are discarded and the LFSRs are clocked again. The generator is said to be secure if no sparse polynomials are used in LFSRs, but the downside is irregular output rate. This problem can be solved by buffering though it complicates implementation.

Self-shrinking generator is similar to shrinking generator but uses pairs of bits from a single LFSR. After clocking the register twice output bits are analysed: if the first bit is 1, the output is the second bit; if the first bit is 0, bits are discarded and the register is clocked again. This generator is slower but requires less memory. However its properties are hard to analyse.

1.3.2.4 T-functions

A new building block for symmetric ciphers called T-function was introduced by Klimov and Shamir in 2003 [23]. T-function is a class of invertible mappings that mix arithmetic and boolean operations and process full machine words.

Consider a construction where each input variable has n bits, and m input variables are placed in m rows of $m \times n$ bit matrix. Than a T-function is defined by mapping

$$f : \mathbb{B}^{m\times n} \to \mathbb{B}^{k\times n} \quad , \tag{1.4}$$

where $\mathbb{B} = \{0, 1\}$ and each k-th column of the output depends only on the first k columns of the input. In general, in order to compute the k-th output bit only input bits $0, 1, \cdots, k$ must be known. Most machine instructions are T-functions: negation, addition, subtraction, multiplication, left shift, which is identical to multiplication by 2. Any combination of T-functions is also a T-function.

The name of such transformation refers to the triangular dependence of the following form [24]:

$$\begin{pmatrix} [f(x)]_0 \\ [f(x)]_1 \\ [f(x)]_2 \\ \vdots \\ [f(x)]_{n-1} \end{pmatrix} = \begin{pmatrix} f_0([x]_0) \\ f_1([x]_0, [x]_1) \\ f_2([x]_0, [x]_1, [x]_2) \\ \vdots \\ f_{n-1}([x]_0, \cdots, [x]_{n-2}, [x]_{n-1}) \end{pmatrix} \quad , \tag{1.5}$$

where $[f(x)]_k$ is the k-th output column and $[x]k-1, \cdots, [x]_0$ — first k input columns.

The primary advantage of T-functions is computation efficiency both in hardware and software implementation on modern processors. Despite of having desirable cryptographic properties, some functions revealed weaknesses to correlation, algebraic and distinguishing attacks [25] with a complexity of 2^{32}. Even though usage of T-functions is highly attractive, reasonable security of such transformations should be proved first.

1.4 Formulation of the problem

In spite of advances in mathematic methods of modern cryptography, real world security system often end up using vulnerable ciphers due to insufficient preliminary analysis. By the time the security cryptoalgorithm is properly studied, the cipher itself is already deployed in global system – switching is expensive and hard technologically.

In order to prevent such situations methods for efficient cryptographic security analysis of ciphers need to be propagated and best practices for cryptographic primitives design and evaluation provided. Most widely used cryptanalytic methods (linear and differential cryptanalysis, etc.) are based on statistical approach and require tremendous amount of data for sane evaluation. Even with exponential growth of computational power it most probably will be impossible neither in the near future nor any given time in future to apply these statistical methods to modern full-scale ciphers.

To make statistical cryptanalytic methods somewhat applicable to ciphers used in modern security systems the concept of baby-ciphers has been introduced. The concept implied proportional shrinking of all transformations of the cipher in order to get a baby-version – still similar to the original cipher but its full statistical analysis is computationally feasible. However the correspondence of such statistical evaluation to the full-scale original cipher hasn't yet been proved.

Nonetheless algebraic analysis of cryptoalgorithms and recent advances in computational algebra allow to obtain systems of multivariate equations that mathematically describe the behaviour of full-scale ciphers and require only few pairs of plaintext/ciphertext for valid analysis. Most equations systems for modern ciphers are still hard to compute, but the complexity gap for solving full-scale system of equations is times smaller than for statistical methods to analyze full-scale cipher.

Though the algebraic analysis method is promising, it is not yet widely used and has been applied to only few ciphers. In order to perform algebraic analysis of any cryptoalgorithm it first must be defined by a system of multivariate non-linear equations. Therefore to make the algebraic cryptanalysis technique commonly used by cryptologists some patterns and best practices in constructing polynomial equations for modern ciphers must be made available to the public. Not only the theoretical part is important for valid evaluation. The ready to use reference software implementation with examples is the key wide applicability of algebraic analysis methods.

In order to solve the described issues and enable aglebraic analysis efficiently applicable for modern symmetric cipher following solutions need to be provided:

a) develop guide lines for constructing system of non-linear equations for modern ciphers as well as for individual transformations commonly used in modern ciphers;

b) describe the most efficient methods and provide best practices for solving algebraic equations systems;

c) use provided methods for applying algebraic attack to actual ciphers;

d) describe needed software tools and provide reference implementation for all steps of algebraic attack for easy reproduce and application to other ciphers.

2 ALGEBRAIC ANALYSIS OF SYMMETRIC BLOCK CIPHERS

Being a fairly new technique, algebraic cryptanalysis is one of the most promissory and powerful methods for analysing cryptographic algorithms [26]. It implies modeling a cryptographic algorithm by a set of algebraic equations that form multivariate polynomial equations system over finite field. The lower degree such system has the easier it is to solve, so it is a rule of thumb in algebraic analysis to construct algebraic systems that contain only quadratic multivariate (MQ[1)]) equations.

The essence of algebraic cryptanalysis is an assumption made by Claude Shannon in [3] that binds cipher security to the difficulty of solving the corresponding algebraic equations set: "Breaking a good cipher should require as much work as solving a system of simultaneous equations in a large number of unknowns of a complex type".

The main advantage of algebraic cryptanalysis over other methods is the need for only few pairs of plaintexts and ciphertexts. Breaking Keeloq cipher is a good example of successful algebraic attack on full scale cryptoalgorithm [27]. The attack allows to recover the encryption key in $2^{14.77}$ times faster than a brute force search.

An Advanced Encryption Standard (AES) that is also widely used outside of the USA is potentially vulnerable to XSL attack (a type of algebraic cryptanalysis). The attack is claimed to significantly weaken the cipher. Even though the practical applicability of the attack hasn't yet been proven, the algebraic structure of AES that is efficiently described with algebraic equations system may be compliant to such analysis as mentioned in [28]:

"We have one criticism of AES: we don't quite trust the security. What concerns us the most about AES is its simple algebraic structure. No other block cipher we know of has such a simple algebraic representation. We have no idea whether this leads to an attack or not, but not knowing is reason enough to be skeptical about the use of AES." *(Bruce Schneier, Niels Ferguson).*

Given an equations system that completely describes the cryptographic

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[1)]It also became a widespread practice to denote the problem of solving multivariate quadratic equation systems as MQ for short.

algorithm one gets a powerful tool for researching its hidden properties. However such systems are hard to solve for full scale ciphers. Complexity of polynomial systems is usually described by the number of equations, their degree and number of variables they have [27].

It is shown in [29] that solving multivariate quadratic equations systems over $GF(2)$ (MQ) and finding satisfying solutions for boolean expressions in several variables (SAT) are NP-hard problems. But their complexity significantly drops if the system becomes overdefined, i. e. there are much more equations than unknowns [30].

2.1 Construction of algebraic equations for cryptographic primitives

Generally an algebraic attack is executed in two steps:

a) An analyzed cipher is described by multivariate equations system;

b) For given plaintext and ciphertext pairs the equation is solved for recovering the key bits.

Therefore to complete the first step, every transformation of the cipher has to be defined by polynomial equations. If all input variables are replaced by given bits and the rest of variables obtain output and intermediate values equal to those of original transformation, then the equations are correct.

Most of cryptographic algorithms consist of the following operations:

- bit permutations;
- modular addition (XOR is equivalent to modulo 2 addition);
- logical operations (AND, OR, Negation);
- substitution (S-boxes);

As will be shown further, these transformations may be completely defined by polynomial equations systems in algebraic normal form (ANF), that is expressed using operations *Exclusive Or* (XOR) and *conjunction* (logical AND).

2.1.1 Logical operations

Algebraic normal form consists of two operations: AND, XOR, so these are trivial to describe. Negation may be expressed by applying XOR with constant 1. Logical OR operation may be expressed in ANF as follows:

$$x \vee y = (x \wedge y) \oplus x \oplus y \ . \tag{2.1}$$

Correctness of the proposed equation may by verified by constructing a truth table for both expressions (figure 2.1).

x	y	$x \vee y$
T	T	T
T	F	T
F	T	T
F	F	F

(a) Logical OR in CNF

x	y	$x \veebar y \veebar (x \wedge y)$
T	T	T
T	F	T
F	T	T
F	F	F

(b) Logical OR in ANF

Figure 2.1 — Truth tables for logical OR

2.1.2 Bit permutations

Consider simple 4-bit permutation that needs to be defined by equations system (figure 2.2). After matching each bit to certain variable one could obtain the corresponding equations system.

$$\begin{cases} y_0 = x_3; \\ y_1 = x_2; \\ y_2 = x_1; \\ y_3 = x_0. \end{cases} \tag{2.2}$$

Then equations (2.2) could be transformed to implicit form to obtain the final polynomials:

$$\begin{cases} y_0 \oplus x_3 = 0; \\ y_1 \oplus x_2 = 0; \\ y_2 \oplus x_1 = 0; \\ y_3 \oplus x_0 = 0. \end{cases} \tag{2.3}$$

So the given 4-bit permutation has been defined by 4 equations in 8 variables. Such solution is simple but unfortunately introduces more variables than equations which may lead to underdefined and therefore unsolvable system. However more efficient approach exists. Since there is no multiplication in this transformation that could increase equations degree and the variables are only renamed, they may be just reordered for the following transformation to receive correct values. That way no additional equations or variables are introduced at all.

Any cyclic bit shift is just a special case of ordered permutation and is defined similarly.

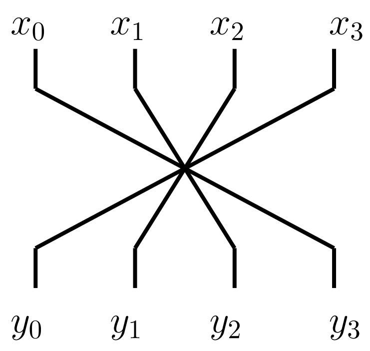


Figure 2.2 — 4-bit permutation

2.1.3 Modular addition

During the Ukrainian national public cryptographic competition the following method for defining modular addition with equations system has been proposed by the author of *Labyrinth* block cipher.

Modular addition $R = X + Y$ of two n-bit numbers is defined as

$$X = (x_0, \ldots, x_{n-1}),\ Y = (y_0, \ldots, y_{n-1}),\ R = (r_0, \ldots, r_{n-1}), \tag{2.4}$$

where i — is a bit number, so x_i represents i-th bit of number X. Then the addition on a bit level is defined as follows:

$$r_i = x_i \oplus y_i \oplus c_{i-1} \quad , \tag{2.5}$$

where c_i is a carry bit variable and is defined by (2.6).

$$c_i = r_{i+1} \oplus x_{i+1} \oplus y_{i+1} \quad . \tag{2.6}$$

The goal is to get null space equations for every addition bit without explicitly using the carry variable. It turns out that for addition bits $0 < i < (n-1)$ three implicit equations may be defined:

$$\begin{cases} (x_i \oplus r_i)(x_i \oplus c_i) = 0; \\ (y_i \oplus r_i)(y_i \oplus c_i) = 0; \\ (x_i \oplus y_i) \cdot r_i \oplus x_i y_i \oplus x_i \oplus y_i \oplus c_i = 0. \end{cases} \tag{2.7}$$

The final equation system that defines every addition bit may be extracted from (2.7) by substituting every c_i variable according to 2.6:

$$\begin{cases} x_i \oplus x_i r_i \oplus x_i r_{i+1} \oplus x_i x_{i+1} \oplus x_i y_{i+1} \oplus r_i r_{i+1} \oplus r_i x_{i+1} \oplus r_i y_{i+1} = 0; \\ y_i \oplus y_i r_i \oplus y_i r_{i+1} \oplus y_i x_{i+1} \oplus y_i y_{i+1} \oplus r_i r_{i+1} \oplus r_i x_{i+1} \oplus r_i y_{i+1} = 0; \\ x_i r_i \oplus y_i r_i \oplus x_i y_i \oplus x_i \oplus y_i \oplus r_{i+1} \oplus x_{i+1} \oplus y_{i+1} = 0. \end{cases} \tag{2.8}$$

Thereby a single addition bit is defined by three equations of degree 2 each containing 12 quadratic terms. Even though the equations in (2.8) fully describe n-bit modular addition, adding a redundant equation $r_0 = x_0 + y_0$ for the very first bit was found to give crucial increase on system solving performance.

2.1.4 S-boxes

An arbitrary S-box is fully defined by equations of degree 2 that can be obtained by finding null space equations as described in [31].

Consider the S-box $(7, 6, 0, 4, 2, 5, 1, 3)$ for example. To find corresponding null space equations the following matrix 8×22 is constructed. Each row contains the values of all possible 22 monomials for each of 8 possible inputs for variables $\{x_0, x_1, x_2, x_3\}$.

$$\begin{pmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & x_0 \\
0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & x_1 \\
0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & x_2 \\
1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & y_0 \\
1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & y_1 \\
1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & y_2 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & x_0x_1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & x_0x_2 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & x_0y_0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & x_0y_1 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & x_0y_2 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & x_1x_2 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & x_1y_0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & x_1y_1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & x_1y_2 \\
0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & x_2y_0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & x_2y_1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & x_2y_2 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & y_0y_1 \\
1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & y_0y_2 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & y_1y_2
\end{pmatrix} \tag{2.9}$$

The null space equations are then obtained by applying Gaussian elimination to the matrix.

$$\begin{pmatrix}
1&0&0&0&0&0&0&0& x_0y_0+x_1+x_2+y_0+y_1+1\\
0&1&0&0&0&0&0&0& x_0y_0+x_0+x_1+y_2+1\\
0&0&1&0&0&0&0&0& x_0y_0+x_0+y_0+1\\
0&0&0&1&0&0&0&0& x_0y_0+x_0+x_2+y_1+y_2\\
0&0&0&0&1&0&0&0& x_0y_0+x_0+x_1+x_2+y_0+y_1+y_2+1\\
0&0&0&0&0&1&0&0& x_0y_0\\
0&0&0&0&0&0&1&0& x_0y_0+x_2+y_0+y_2\\
0&0&0&0&0&0&0&1& x_0y_0+x_1+y_1+1\\
0&0&0&0&0&0&0&0& x_0x_2+x_1+y_1+1\\
0&0&0&0&0&0&0&0& x_0x_1+x_1+x_2+y_0+y_1+y_2+1\\
0&0&0&0&0&0&0&0& x_0y_1+x_0+x_2+y_0+y_2\\
0&0&0&0&0&0&0&0& x_0y_0+x_0y_2+x_1+x_2+y_0+y_1+y_2+1\\
0&0&0&0&0&0&0&0& x_1x_2+x_0+x_1+x_2+y_2+1\\
0&0&0&0&0&0&0&0& x_0y_0+x_1y_0+x_0+x_2+y_1+y_2\\
0&0&0&0&0&0&0&0& x_0y_0+x_1y_1+x_1+y_1+1\\
0&0&0&0&0&0&0&0& x_1y_2+x_1+x_2+y_0+y_1+y_2+1\\
0&0&0&0&0&0&0&0& x_0y_0+x_2y_0+x_1+x_2+y_1+1\\
0&0&0&0&0&0&0&0& x_2y_1+x_0+y_1+y_2\\
0&0&0&0&0&0&0&0& x_2y_2+x_1+y_1+1\\
0&0&0&0&0&0&0&0& y_0y_1+x_0+x_2+y_0+y_1+y_2\\
0&0&0&0&0&0&0&0& y_0y_2+x_1+x_2+y_0+y_1+1\\
0&0&0&0&0&0&0&0& y_1y_2+x_2+y_0
\end{pmatrix} \tag{2.10}$$

This yields to 14 equations that fully describe the S-box transformations.

2.1.5 Feistel network

Using the described methods of constructing algebraic equations for widely used cryptographic transformations it is straightforward to define generic Feistel network with polynomial equations system.

Consider a Feistel network on figure 2.3. All input and output bits of the Festel network are considered to be unknown and therefore are replaced with variables. Then going through each transformation those variables are

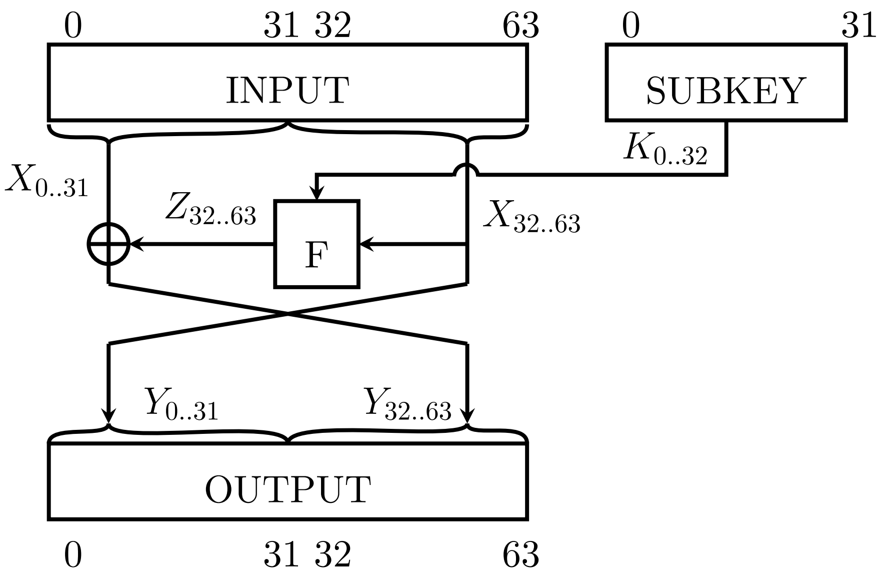


Figure 2.3 — Feistel network

used for equations construction that are further assigned to next variables for the following transformation. This example on figure 2.3 demonstrates such process for single round of Feistel network. The resulting number of equations and variables depends on transformations used in F function. For full-scale Feistel networks the internal variables for each round must be distinct. This may be achieved by prefixing variables names with current round number. However the output variables of round n are equal to input variables of adjacent round $n+1$ and therefore are common.

Some transformations (like modular addition and S-boxes) increase the degree of monomials. There are two possible ways of preventing the equations degree growth: with preprocessing or post-processing. The preprocessing implies introducing new variables before each transformation that contains multiplication. Post-processing however decreases the degree of already constructed equations until they become quadratic by applying the following operation repeatedly:

$$\{m = wxyz\} \Rightarrow \{a = wx;\ b = yz;\ m = ab\}\ . \tag{2.11}$$

Many ciphers based on Feistel networks (including the studied ones) use the key that is larger than the block size. Therefore a single known plaintext and ciphertext pair may not introduce enough information for recovering all the key bits. In this case several systems of equations should be joined together. Such systems share only the key variables (and variables for subkeys and key scheduling if any) while the rest of variables are distinct. Then the plaintext and ciphertext pairs obtained on the same key must be injected into the extended system. The number of introduced known values for plaintext/ciphertext variables must be equal to the unicity distance of the cipher.

2.2 Methods for solving algebraic equation systems

2.2.1 Gröbner basis

Finding a Gröbner basis is equivalent to solving various problems concerning polynomial systems [27]. As Gaussian elimination method solves linear equation systems, the Gröbner basis is designed to do the same for nonlinear polynomial systems.

In [32] the Gröbner basis is defined as following.

For a fixed monomial order a finite subset $G = \{g_0, \ldots, g_{m-1}\}$ of an ideal $\mathbb{I}$ is said to be a Gröbner basis if

$$\langle LT(g_0), \ldots, LT(g_{m-1})\rangle = \langle LT(I)\rangle \quad , \tag{2.12}$$

where LT denotes the leading term of a polynomial.

Notably the leading term of every polynomial from $\mathbb{I}$ is divisible by the leading term of at least one polynomial from $\mathbb{G}$.

Gröbner basis has been successfully applied for attacking several ciphers, including FLURRY and CURRY [33] and even showed to be more efficient than SAT solvers in algebraic attack on Bivium [34].

Even though in most cases Gröbner basis method is not efficient enough for attacking full-scale ciphers it is useful for exhausting more linearly independent equations by applying it to some transformations (like S-boxes).

2.2.2 SAT solvers

Another approach to solving MQ problem is SAT[2] solvers that are used for determining a set of variables that would satisfy a given boolean formula.

An efficient method of converting equation systems from the algebraic normal form (ANF) to the conjunctive normal form (CNF) was proposed in [35]. After conversion the resulting system is solved by a SAT solver of cryptanalyst's choice. SAT solvers also require less memory and make it possible to solve problems infeasible for Gröbner basis algorithms.

SAT solvers gained lots of attention lately since the problem of finding solutions satisfying a given system of boolean equations is NP-complete so developing a SAT solving algorithm running in polynomial time would help to experimentally show equality of P and NP. So proving (or disapproving) the equivalence of these two complexity classes would affect not only asymmetric ciphers based on factorization problem as was though before, but any cipher that may be efficiently described by an algebraic equations system.

2.3 Summary

In this chapter the methods for defining most widely used cryptographic primitives with system of non-linear equations are described. The techniques of obtaining equations for bit permutations, modular addition, some logical operations and S-boxes should allow to construct full-scale non-linear equations systems for most modern symmetric ciphers.

Also some most efficient methods for solving the constructed system of equations are described. The software tools for computational algebra that provide needed functionality are described in section 4. Reference implementation for defining individual transformations with non-linear equations and constructing full-scale system of equations is provided in appendices A-D. A computational power of low-end computer will not make it possible to perform an algebraic attack on full-scale cipher but will allow to research algebraic properties of individual transformations to make predictions for cipher

[2]SAT is an abbreviation denoting boolean satisfiability problem.

security.

3 ALGEBRAIC ATTACK ON GOST 28147-89 AND MISTY1

3.1 Description of GOST 28147-89 cipher

Officially adopted in 1989 the GOST 28147 cipher has been developed in former USSR and is now the encryption standard in most CIS countries. For more than 20 years of cryptanalysis no efficient attack that would significantly reduce the cipher security had been found.

GOST 28147-89 is a symmetric block cipher with a key length of 256 bits. It's represented by a Feistel network of 32 rounds. The round function consists of key addition modulo 2^{32}, substitution layer represented by eight 4-bit S-boxes, and a cyclic left shift by 11 bits (figure 3.1). S-boxes are not defined in the original standard [36]. For some time they were considered to be another secret parameter, but such approach caused more problems than gained additional security. Use of different S-boxes set caused some cipher implementations to be incompatible. Later all possible benefits of secret substitution layer were scattered by introducing a method to recover all unknown S-boxes in 2^{32} encryptions [37]. The 256-bit key is split into 8 32-bit

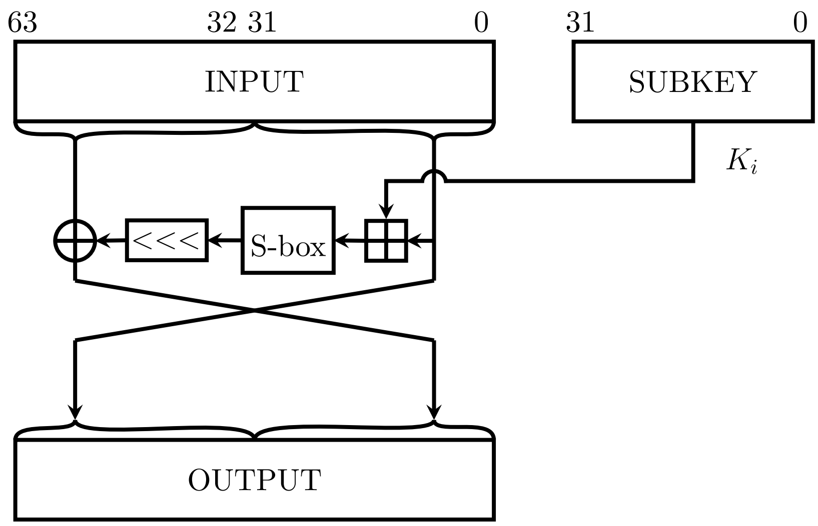


Figure 3.1 — GOST 28147-89 round function

subkeys that are used sequentially throughout the Feistel network. During the last 8 rounds subkeys are fed in reverse order. Half blocks are not switched after the last round in order to make encryption and decryption procedures similar.

However some recent works [38, 39] claim the fastest attack on GOST has complexity 2^{224} and requires 2^{32} known plaintexts.

3.2 Construction of equations system for GOST 28147-89

Using methods described in section 2.1 system of algebraic equations for GOST 28147-89 is defined as follows.

Consider variable format $X_{r,b}$, where r denotes round for which the variable is defined and b is a bit number of the block. Then defining one round of GOST cipher requires 4 sets of variables:

a) $X_{r,0\dots63}$ for input block,

b) $K_{r\%8,0\dots31}$ for a subkey (where % denotes modulo reduction),

c) $Y_{r,0\dots31}$ for the result of key addition,

d) $Z_{r,0\dots31}$ for the result of S-box substitution.

Input to a key addition modulo 2^{32} are variables $X_{r,31\dots63}$ of the right input half-block and $K_{r\%8,0\dots31}$ of the subkey. The result of key injection is assigned to variables $Y_{r,0\dots31}$. GOST 28147-89 key injection transformation can be described with 93 equations. Obtaining the equations for modular addition is described in section 2.1.3.

During this research the S-boxes used in GOST cipher implementation are those proposed in [40] and given in appendix E. Every such S-box is defined by 21 equations each containing up to 14 monomials, so the total number of equations for S-box layer is 168. Input variables $Y_{r,0\dots31}$ are passed to substitution layer which generates 32 output variables $Z_{r,0\dots31}$.

Cyclic shift is equivalent to renaming variables $Z_{r,0\dots31}$ to $Z_{r,\{21\dots31,0\dots20\}}$. Equations for XORing round function output with the left input half-block and switching the half-blocks are trivial. The resulting polynomials of a single Feistel network round are then assigned to the next round variables $X_{r+1,0\dots63}$.

Obtaining equations for modular addition described in details in section 2.1.3.

The chosen approach of constructing polynomial system for GOST 28147-89 cipher allows to define a single cipher round with 325 quadratic equations. An algebraic equation system describing full GOST 28147-89 cipher contains 10432 polynomials in 4416 variables. In case of using the subkeys in straight order (without reversing during last 8 rounds) the number of polynomials and variables in the system stays the same. That means the reversing the subkeys during last 8 rounds does not strengthen the cipher from algebraic point of view, but only complicates its implementation.

3.3 Key recovery for 6 rounds of GOST 28147-89

In order to solve the equation system some chosen plaintext and the corresponding ciphertext are injected into the system (by substituting variables of the first and the last block to their corresponding values). Since GOST 28147-89 cipher has the key length of 256 bits, having an equation system for a single pair of plaintext and ciphertext doesn't allow to recover the encryption key. The solution combines several equation systems for distinct plaintexts and ciphertexts in order to inject more information into the resulting polynomial system. Considering the 64-bit input block it is evident that a successful 256-bit key recovery should be possible using an equation system for at least 4 plaintext/ciphertext pairs.

Using a premier SAT solver [41] 6 rounds of GOST 28147-89 cipher had been broken and a complete list of the used subkeys completely recovered. After observations during the research two factors were found to influence the equation system solving complexity. The more zeroes encryption key contains, the easier the polynomial system is for solving since modular addition of zero does not generate carries. Also the chosen plaintexts should have maximum hamming distance in order to introduce enough information to the system.

Finding solution for more rounds of GOST 28147-89 polynomial system requires much larger time gap on an ordinary laptop. However the memory requirements are small so taking into account the simplicity and uniformity of the cipher structure it is rational to assume that a more powerful computer would allow to break more rounds of the cipher.

3.4 Description of MISTY1 cipher

MISTY1 is a symmetric block cipher with a 128-bit key, a 64-bit block and a variable number of rounds that inherits Feistel structure [42]. The algorithm was designed to be robust against linear and differential cryptanalysis and sustain efficiency on any platform. Therefore the operations used in the cipher do not exploit software instructions specific for certain processor architecture.

The cipher is also described in RFC 2994. It has recursive structure of nested Feistel networks – the outter level Feistel network uses round function FO which itself represents another smaller Feistel network FI (figure 3.2).

The nested Feistel round functions FO and FI are presented on figures 3.3a and 3.3b respectively.

Key schedule is performed by iterative applying of FI function to each 16-bit chunk of the key (figure 3.3c). Hereby 128 additional subkey bits are generated. Both the key and the subkey bits are used during enciphering.

The key injecting function includes conjunction and disjunction operations and is presented on figure 3.4.

S-boxes in MISTY1 are constructed algebraically. S-Box S_7 has degree 2 and s_9 is of degree 3. Equations for each S-box are defined in (3.1, 3.2).

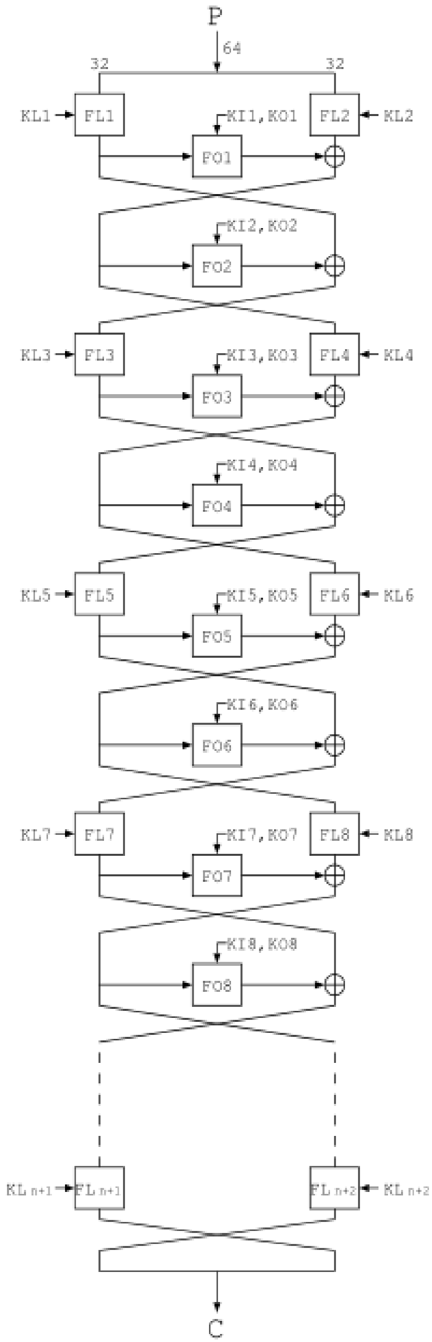
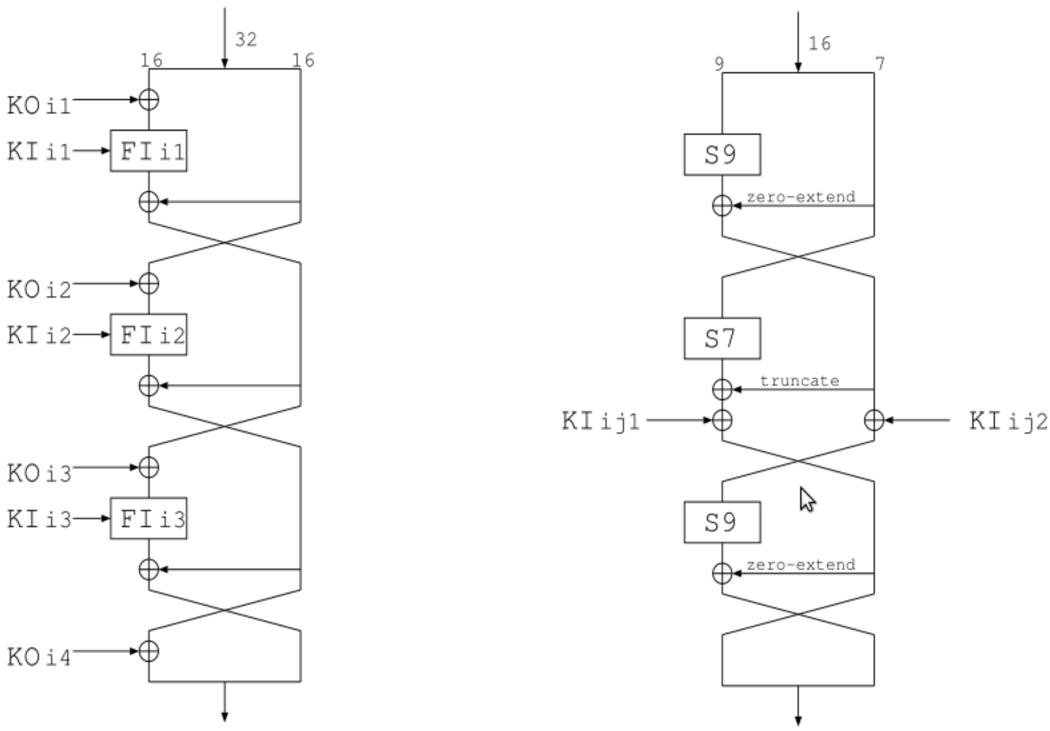
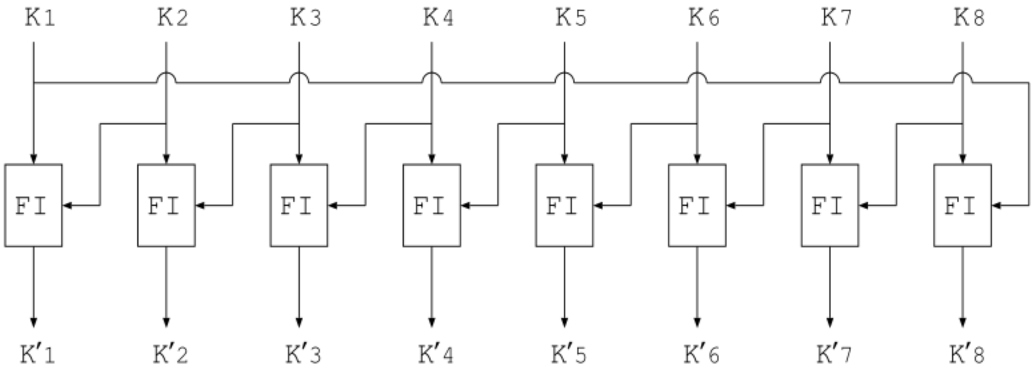


Figure 3.2 — MISTY1 cipher structure



(a) FO round function

(b) FI round function



(c) Key schedule

Figure 3.3 — MISTY1 internal functions

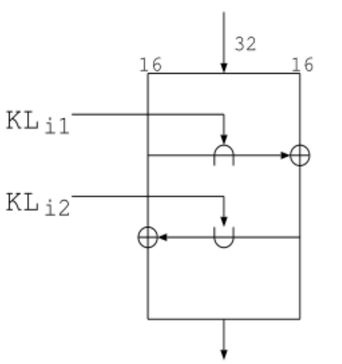


Figure 3.4 — Key injection FL function

$$\begin{aligned}
y_0 = {} & x_0 + x_1x_3 + x_0x_3x_4 + x_1x_5 + x_0x_2x_5 + x_4x_5 + \\
& + x_0x_1x_6 + x_2x_6 + x_0x_5x_6 + x_3x_5x_6 + 1 \\
y_1 = {} & x_0x_2 + x_0x_4 + x_3x_4 + x_1x_5 + x_2x_4x_5 + x_6 + \\
& + x_0x_6 + x_3x_6 + x_2x_3x_6 + x_1x_4x_6 + x_0x_5x_6 + 1 \\
y_2 = {} & x_1x_2 + x_0x_2x_3 + x_4 + x_1x_4 + x_0x_1x_4 + x_0x_5 + x_0x_4x_5 + \\
& + x_3x_4x_5 + x_1x_6 + x_3x_6 + x_0x_3x_6 + x_4x_6 + x_2x_4x_6 \\
y_3 = {} & x_0 + x_1 + x_0x_1x_2 + x_0x_3 + x_2x_4 + x_1x_4x_5 + \\
& + x_2x_6 + x_1x_3x_6 + x_0x_4x_6 + x_5x_6 + 1 \\
y_4 = {} & x_2x_3 + x_0x_4 + x_1x_3x_4 + x_5 + x_2x_5 + x_1x_2x_5 + \\
& + x_0x_3x_5 + x_1x_6 + x_1x_5x_6 + x_4x_5x_6 + 1 \\
y_5 = {} & x_0 + x_1 + x_2 + x_0x_1x_2 + x_0x_3 + x_1x_2x_3 + x_1x_4 + \\
& + x_0x_2x_4 + x_0x_5 + x_0x_1x_5 + x_3x_5 + x_0x_6 + x_2x_5x_6 \\
y_6 = {} & x_0x_1 + x_3 + x_0x_3 + x_2x_3x_4 + x_0x_5 + x_2x_5 + \\
& + x_3x_5 + x_1x_3x_5 + x_1x_6 + x_1x_2x_6 + x_0x_3x_6 + x_4x_6 + x_2x_5x_6
\end{aligned} \tag{3.1}$$

$$\begin{aligned}
y_0 = {} & x_0x_4 + x_0x_5 + x_1x_5 + x_1x_6 + x_2x_6 + x_2x_7 + \\
& + x_3x_7 + x_3x_8 + x_4x_8 + 1 \\
y_1 = {} & x_0x_2 + x_3 + x_1x_3 + x_2x_3 + x_3x_4 + x_4x_5 + x_0x_6 + \\
& + x_2x_6 + x_7 + x_0x_8 + x_3x_8 + x_5x_8 + 1 \\
y_2 = {} & x_0x_1 + x_1x_3 + x_4 + x_0x_4 + x_2x_4 + x_3x_4 + x_4x_5 + \\
& + x_0x_6 + x_5x_6 + x_1x_7 + x_3x_7 + x_8 \\
y_3 = {} & x_0 + x_1x_2 + x_2x_4 + x_5 + x_1x_5 + x_3x_5 + x_4x_5 + \\
& + x_5x_6 + x_1x_7 + x_6x_7 + x_2x_8 + x_4x_8 \\
y_4 = {} & x_1 + x_0x_3 + x_2x_3 + x_0x_5 + x_3x_5 + x_6 + x_2x_6 + \\
& + x_4x_6 + x_5x_6 + x_6x_7 + x_2x_8 + x_7x_8 \\
y_5 = {} & x_2 + x_0x_3 + x_1x_4 + x_3x_4 + x_1x_6 + x_4x_6 + x_7 + \\
& + x_3x_7 + x_5x_7 + x_6x_7 + x_0x_8 + x_7x_8 \\
y_6 = {} & x_0x_1 + x_3 + x_1x_4 + x_2x_5 + x_4x_5 + x_2x_7 + x_5x_7 + \\
& + x_8 + x_0x_8 + x_4x_8 + x_6x_8 + x_7x_8 + 1 \\
y_7 = {} & x_1 + x_0x_1 + x_1x_2 + x_2x_3 + x_0x_4 + x_5 + x_1x_6 + \\
& + x_3x_6 + x_0x_7 + x_4x_7 + x_6x_7 + x_1x_8 + 1 \\
y_8 = {} & x_0 + x_0x_1 + x_1x_2 + x_4 + x_0x_5 + x_2x_5 + x_3x_6 + \\
& + x_5x_6 + x_0x_7 + x_0x_8 + x_3x_8 + x_6x_8 + 1
\end{aligned} \tag{3.2}$$

3.5 Construction of equations system for MISTY1

MISTY1is a 64-bit Feistel network similar to GOST 28147-89and doesn't use any additional operations that are not described in 2.1, there for construction of equations is performed similarly.

There are few differences however. Even though the transformations used in MISTY1 are resembling those of GOST 28147-89 its structure is much more complicated due to nested Feistel networks. Initially equations for each function are constructed and tested for correctness so that definition of each Feistel network is self-contained. Afterwards those equations are chained into single system as usually.

Since MISTY1uses key scheduling, equations for this transformation also must be explicitly defined.

Nested structure of MISTY1 results in big number of variables so they must be named carefully to avoid confusion. The following format was used for MISTY1 variables: R<n>_<func>_<op>_<bit>, where n denotes number of current round, func stands for function to which variable belongs, op is some operation inside the function, bit is the number of bit in a block. For example variable verbn+ denotes number of current round, func stands for function to which variable belongs, op is some operation inside the function, bit is the number of bit in a block. For example variable R5_FO_KO2_15 stands for round 5, function FO, operation of injecting subkey KO_{i2}, precisely bit 15 of the subkey.

Using the described method the full-scale system of equations for MISTY1 has been constructed and shown to have 3488 equations in 3680 variables. Because of nested structure too many variables are introduced and the system becomes underdefined and therefore unsolvable. To overcome such problem it is possible to apply Gröbner basis (described in 2.2.1) to some transformation for obtaining more equations and possibly eliminating some of the variables. It has been decided to post-process equations for S-box S_7 by computing the corresponding Gröbner basis. The resulting system has 8448 equations in 3680 variables, so usage of Gröbner basis introduced another 5000 equations.

3.5.1 Key recovery for 2 rounds of MISTY1

For the attack 4 systems of equations for distinct plaintext/ciphertext pairs have been combined in order for the number of known values to reach the unicity distance.

With the use of `CryptoMiniSat` solver on a low-end computer an equations system for two MISTY1 rounds have successfully been solved and the used subkeys recovered. For the system of 4 rounds equivalent keys can be obtained for a given plaintext/ciphertext pair.

Further combinations of Gröbner basis technique with SAT-solvers and usage of powerful computers may refine the efficiency of the attack and provide more capabilities for analyzing properties of the cryptoalgorithm.

3.6 Summary

The chapter provides description of constructing systems of non-linear equations for cryptoalgorithms GOST 28147-89 and MISTY1 that are based on Feistel network and some approaches for solving the obtained equations set.

Gröbner basis method generally is not efficient enough for solving full-scale equations system but is useful for obtaining exhaustive list of linearly independent equations for given transformation. It is especially beneficial for S-box equations.

With advance of SAT-solver algorithms they become efficient enough for solving large equations sets and with the help of powerful computers it might be possible to solve system of equations for some full-scale cipher. The downside of such algorithms is their nondetermination and therefore no guarantee of average run time for given instance. This exaggerates complexity evaluation of solving given equations set and one can only predict the overall complexity of the problem based on preliminary characteristics of equations set (degree, number of equations and variables, etc.) and estimate the possibility of its solving but cannot bind these factors to some certain time complexity of the attack.

Using the proposed methods for defining symmetric ciphers with equations set, polynomial system for MISTY1 has 8448 equations in 3680 variables and the system for GOST 28147-89 has 10432 equations in 4416 variables. For comparison, the polynomial system for the PRESENT cipher that is designed for lightweight cryptography purposes contains 11067 quadratic equations in 4216 variables [43]. Even though PRESENT has very simple structure, its equations system is larger. But also PRESENT has much smaller key space and requires more space for hardware implementation. It is claimed that AES cipher can be described with an algebraic system of 8000 quadratic equations in 1600 unknowns [44] which is signivicantly smaller than the GOST 28147-89 or MISTY1 equations system. An equations set for symmetric block cipher Camellia contains 6224 equations in 3584 variables [45].

4 DESCRIPTION OF DEVELOPED METHODS FOR SOFTWARE IMPLEMENTATION

4.1 Tools and software used for computations

All computations in this thesis are performed using only free open source software. The computer algebra system used for algorithm implementations and experiments is “SAGE: Software for Algebra and Geometry Experimentation” which is provided under the terms of the GNU General Public License [46]. SAGE is described as a “free and open software that supports research and teaching in algebra, geometry, number theory, cryptography, etc.” [47].

For this thesis the most used components of SAGE were Singular computer algebra system [48], PolyBoRi C++ library for efficient reduced Gröbener basis computation [49], and `crypto` module for cryptography related routines.

4.2 Usage of implemented functionality

The implementation architecture of GOST 28147-89 cipher and its equation system generator is inspired by CTC cipher algebraic cryptanalysis description in [32].

Computation of solutions for multivariate algebraic systems of equations is performed by CryptoMiniSat [41], a SAT Race 2010 [50] winning SAT solver licensed under GNU Lesser General Public License.

Conversion of the polynomial system from ANF to CNF format and parsing the results of CryptoMiniSat computations is done by `anf2cnf.py` script [51].

4.2.1 GOST 28147-89 implementation

The `Gost` class implements GOST 28147-89 cryptographic algorithm and its multivariate quadratic equation system generator. The implementation is not efficient since it treats every bit as a boolean polynomial ring element, so it is useful for researching purposes only.

One can customize GOST 28147-89 parameters (like input block size, number of rounds, key addition mode and subkeys ordering) to get small scale cipher. A cipher instance compliant with the specification defined in the standard may be constructed as shown in listing 4.1:

Listing 4.1: Creating GOST instance

```
sage: gost = Gost(block_size=64, rounds=32, key_add='mod', key_order='frwrev')
sage: print Gost()
GOST cipher (Block Size = 64, Rounds = 32, Key Addition = mod, Key Order =
    frwrev)
```

Those parameters are default and so may be omitted. To create the small scale version of GOST 28147-89 one can specify smaller values for parameters (listing 4.2):

Listing 4.2: Small scale GOST

```
sage: gost = Gost(block_size=8, rounds=3, key_add='mod', key_order='frw')
sage: print gost.ring
Boolean PolynomialRing in K00, K01, K02, K03, Y00, Y01, Y02, Y03, Z00, Z01,
    Z02, Z03, K10, K11, K12, K13, Y10, Y11, Y12, Y13, Z10, Z11, Z12, Z13, K20,
    K21, K22, K23, Y20, Y21, Y22, Y23, Z20, Z21, Z22, Z23, X00, X01, X02, X03,
    X04, X05, X06, X07, X10, X11, X12, X13, X14, X15, X16, X17, X20, X21, X22,
    X23, X24, X25, X26, X27, X30, X31, X32, X33, X34, X35, X36, X37
```

Cipher variables are defined over boolean polynomial ring. The used notation is described in section 2.1.3. In SAGE implementation left digits of the variable index identify round number and right digits specify the number of bit defined by the variable. For instance, variable `K0325` defines 25-th bit of a subkey for round 3. It is worth noting that for small block sizes the cyclic shift value is proportionally decreased from 11 bits to ceil(block_length/3), where ceil(x) stands for the smallest integer not less than x. The minimum block size is 8 bits since the cipher becomes degenerated with smaller input blocks. There are two possible modes for key addition: `mod` for a standard modular addition and `xor` for simple XORing. Also two subkey orderings are supported: `frwrev` is for the default ordering when subkeys are reversed on the last 8 rounds and `frw` for no reversing. Default S-boxes used in the cipher are those proposed in GOST R 31.11-94 [40] and given in appendix E.

Obtaining polynomial system for GOST 28147-89 cryptoalgorithm is shown in listing 4.3:

Listing 4.3: Obtaining polynomial system

```
sage: gost = Gost()
sage: gost.polynomial_system()
Polynomial Sequence with 10432 Polynomials in 4416 Variables
```

4.2.2 MISTY1 implementation

The `Misty` class implements MISTY1 cryptographic algorithm and its multivariate equations system generator. Its interface is similar to that one of GOST 28147-89. The implementation is not efficient since it treats every bit as a boolean polynomial ring element, so it is useful for researching purposes only.

A required parameter is the number of rounds for the cipher which should be multiple of 4 according to the specification. An optional argument is a prefix that will be prepended to the names of variables for the equations system. This is used when combining several systems.

An instance of MISTY1 polynomial system generator may be created as shown in listing 4.4.

Listing 4.4: Creating MISTY1 instance

```
sage: m = Misty(8)
sage: m.polynomial_system()
Polynomial Sequence with 8448 Polynomials in 3680 Variables
```

A decorator `@groebner_basis` for post-processing the equations via Gröbner basis is implemented and should decorate every function, for which equations the Gröbner basis should be computed as show on listing 4.5).

Listing 4.5: Misty Gröbner basis

```
    @groebner_basis
    def s7(self, x, r=None):
        y = [0] * len(x)
        ...
```

4.2.3 Solving polynomial systems

Obtaining an equation system for the cipher is not enough for recovering an encryption key. Information about plaintext and ciphertext has to be injected into equation system by substituting corresponding variables. Also a single equation system for the specified plaintext and ciphertext does not allow to get a solution for the key variables, so an ability to combine several equation systems with different plaintext and ciphertext pairs is needed.

Injecting variable values is implemented by adding new `inject` method to a standard `PolynomialSequence_generic` class and is shown in listing 4.6.

Listing 4.6: Injecting variable values into equation system

```
sage: gost = Gost(block_size=64, rounds=5, key_add='mod', key_order='frw')
sage: plaintext = gost.random_block()
sage: key = gost.random_key()
sage: ciphertext = gost.encrypt(plaintext, key)
sage: f = gost.polynomial_system()
sage: print f
Polynomial Sequence with 1630 Polynomials in 864 Variables
sage: f2 = f.inject(gost.gen_vars(gost.var_names['block'], 0), plaintext)
sage: f2 = f2.inject(gost.gen_vars(gost.var_names['block'], 5), ciphertext)
sage: print f2
Polynomial Sequence with 1630 Polynomials in 736 Variables
```

For combining several equation systems one should use `join_systems` function (listing 4.7) which accepts a list of polynomial systems with injected known variables and a list of cipher instances for which the systems were constructed. The function will construct a new boolean polynomial ring that would include variables from all systems and have the same variables for encryption subkeys. Consequently, the joined equation systems must contain ciphertexts obtained on the same key, otherwise combining such systems would not help to find the solution. In order to distinct variables from different equation systems one should add a `prefix` string when constructing a cipher object as shown in listing 4.7.

Listing 4.7: Combining several equation systems

```
sage: gost1 = Gost(block_size=64, rounds=3, key_add='mod', key_order='frw',
    prefix='a')
sage: gost2 = Gost(block_size=64, rounds=3, key_add='mod', key_order='frw',
    prefix='b')
sage: plaintext1 = gost1.random_block()
sage: plaintext2 = gost1.random_block()
```

```
sage: key = gost1.random_key()
sage: ciphertext1 = gost1.encrypt(plaintext1, key)
sage: ciphertext2 = gost2.encrypt(plaintext2, key)
sage: f1 = gost1.polynomial_system()
sage: f2 = gost2.polynomial_system()
sage: f1 = f1.inject(gost1.gen_vars(gost1.var_names['block'], 0), plaintext1)
sage: f1 = f1.inject(gost1.gen_vars(gost1.var_names['block'], 5), ciphertext1)
sage: f2 = f2.inject(gost2.gen_vars(gost2.var_names['block'], 0), plaintext2)
sage: f2 = f2.inject(gost2.gen_vars(gost2.var_names['block'], 5), ciphertext2)
sage: f = join_systems([f1, f2], [gost1, gost2])
sage: print f1
Polynomial Sequence with 978 Polynomials in 416 Variables
sage: print f2
Polynomial Sequence with 978 Polynomials in 416 Variables
sage: f1 == f2
False
sage: f = join_systems([f1, f2], [gost1, gost2])
sage: print f
Polynomial Sequence with 1956 Polynomials in 736 Variables
```

All functionality for solving the GOST 28147-89 and MISTY1 polynomial systems is now implemented. Two approaches are used for solving equation systems: using reduced Gröbner basis and using SAT solver.

In listing 4.8 computing the reduced Gröbner basis for polynomial system is demonstrated. Since all equations in the resulting list equal 0 it is possible to extract the values of the key bits. In this case the method allowed to recover only 91 out of 96 subkeys bits for a 3 round GOST polynomial system.

Listing 4.8: Solving equation system using reduced Gröbner basis

```
sage: ideal = f.ideal()
sage: basis = ideal.interreduced_basis()
Polynomial Sequence with 735 Polynomials in 736 Variables
sage: key = [i for i in sorted(basis) if str(i).startswith(gost1.var_names['
   key'])]
sage: print key
[K0229, K0228, K0221, K0220, K0223 + 1, K0222, K0225, K0224, K0227 + 1, K0226,
    K0131, K0130, K0001, K0000 + 1, K0003, K0002, K0005 + 1, K0004, K0007,
   K0006, K0009 + bY0009, K0008, K0230 + 1, K0231 + 1, K0126 + 1, K0127 + 1,
   K0124 + 1, K0125, K0122, K0123, K0120 + bY0009, K0121 + bY0009 + 1, K0128,
   K0129 + 1, K0012 + 1, K0013, K0010 + 1, K0011 + 1, K0016, K0017, K0014 + 1,
    K0015, K0018, K0019 + 1, K0203 + 1, K0202 + 1, K0201 + 1, K0200 + 1, K0207
    + 1, K0206, K0205 + 1, K0204 + 1, K0209 + 1, K0208 + bY0009 + 1, K0119 +
   bY0009, K0118, K0117, K0116, K0115 + 1, K0114, K0113, K0112, K0111 + 1,
   K0110 + 1, K0029 + 1, K0028, K0023 + 1, K0022, K0021 + 1, K0020, K0027,
   K0026 + 1, K0025 + 1, K0024, K0218 + 1, K0219 + 1, K0214, K0215, K0216 + 1,
    K0217, K0210, K0211, K0212 + 1, K0213 + 1, K0108, K0109 + 1, K0100 + 1,
   K0101 + 1, K0102, K0103, K0104, K0105 + 1, K0106, K0107, K0030, K0031]
```

For solving an equation system f with CryptoMiniSat solver via SAGE one needs to install CryptoMiniSat available at [41] first. Then use `anf2cnf.py` script provided at [51] for converting the polynomial system to DIMACS CNF format, call CryptoMiniSat for solving the system and parse the result back into SAGE as shown in listing 4.9. In the example the s variable will store a solution dictionary and t will be initialized by the time needed for computation.

Listing 4.9: Solving equation system using SAT solver

```
sage: solver = ANFSatSolver(f.ring())
sage: s, t = solver(f)
```

CryptoMiniSat solver is more efficient and allows to solve a 6 round GOST equation system on regular laptop with 2 GHz processor in minutes (listing 4.10). Some code is omitted for clarity and is indicated with dots. Full source code for solving 6 rounds polynomial system is presented in appendix B.

Listing 4.10: Solving 6 round GOST system using SAT solver

```
...
sage: f = join_systems(mqsystems, gosts)
sage: solver = ANFSatSolver(f.ring())
sage: print time.ctime(); s, t = solver(f); print time.ctime();
Sat May 12 22:13:00 2012
Sat May 12 22:13:41 2012
...
sage: print key == recovered_key
True
sage: print gosts[0].encrypt(inputs[0],  key) == gosts[0].encrypt(inputs[0],
    recovered_key) == outputs[0]
True
```

The MISTY1 equations system may be solved using Sage interface to available SAT-solvers (listing 4.11).

Listing 4.11: Using Sage SAT interface

```
sage: from sage.sat.boolean_polynomials import solve as sat_solve
sage: m = Misty(4)
sage: F = m.polynomial_system()
sage: print F
Polynomial Sequence with 5156 Polynomials in 2248 Variables
```

4.3 Summary

Due to multiple interfaces supported by SAGE it is possible to efficiently combine various tools for achieving optimal results. In this work the interface to Singular has been used for computing reduced Gröbner basis which managed to solve 3 round GOST equation system. CryptoMiniSat solver with `anf2cnf.py` as a conversion layer enabled 6 round equation system to be efficiently solved in minutes. There is a substantial complexity hop for computing 7 rounds, but even though the solution hasn't been found using regular computer, the memory usage is negligible. So further optimizations including parallelizing and tweaking SAT algorithm parameters should result in solving more rounds for GOST 28147 polynomial system.

Current source code of the GOST 28147-89 and MISTY1 polynomial system generator is published by the author at [52] and the thesis sources are published at [53].

CONCLUSIONS

As global shift to portable devices usage spawned demand for very efficient and also secure cryptographic primitives, efficient methods for comprehensive security evaluation of perspective ciphers are required. Current usage of insecure ciphers in various fields of information technologies proves the importance of carefull ciphers security evaluation before deploying them into real systems.

The accomplished work resulted in development of methods for defining most widely used cryptographic primitives with system of non-linear equations. The techniques of obtaining equations for bit permutations, modular addition, some logical operations and S-boxes should allow to construct full-scale non-linear equations systems for most modern symmetric ciphers. Best approaches for solving the obtained equations sets are described and allow to solve reduced round versions of analyzed ciphers, research algebraic properties of individual transformations on low-end computers. Usage of resources with more computational power will increase feasibility of full-scale cipher analysis.

As the result of the work software tools for computational algebra that provide needed functionality are described and reference implementation for defining individual transformations with non-linear equations and constructing full-scale system of equations for modern symmetric ciphers is provided.

An algebraic equation system describing GOST 28147-89 cipher and obtained using the suggested method contains 10432 polynomials in 4416 variables. MISTY1 cipher is described with 8448 equations in 3680 variables using the same method. Number of equations and variables in GOST 28147-89system is resembling that of PRESENT and MISTY1equations set is larger than that of AES (3680 variables in MISTY1 against 1600 variables in AES).

Using the described techniques it is possible to solve a 6 round GOST 28147-89 polynomial system with 4 pairs of plaintexts and ciphertexts at the moment. Thereby the reduced GOST 28147-89 algorithm using 160 out of 256 key bits is broken by an algebraic attack. Such statistics strengthens the opinion about AES vulnerability to algebraic attacks. The algebraic attack on MISTY1 is performed. The equations system for two cipher round could be solved and equivalent keys for a given plaintext/ciphertext pair could be found for up to

4 rounds. Solving these systems of equations for additional rounds requires more computation power, however finding the solution may be possible on more efficient hi-end computers. All computations have been executed on `Intel Core i5-3570` CPU at 3.40 GHz with 8 Gb RAM.

The nondetermination of SAT-solver algorithms do not allow to bind characteristics of obtained equations set (like its degree, number of equations and variables, etc.) to some certain time complexity of solving the given system. However these factors may be used to estimate the feasibility of solving the system and rationality of allocating processing time for analysis.

Also algebraic analysis in combination with other known cryptanalytic methods (linear, differential, integral, etc.) proved to be efficient enough for security evaluation of a cipher [26]. Considering this practice algebraic analysis may increase the significance of investigating baby-ciphers that are shrinked versions of original cryptoalgorithms, so such approach is a subject for future researches.

For labour protection the employee working conditions are analysed for their compliance with normative documents on safety engineering and sanitaion. Harmful and dangerous production factors are retrieved and evaluated using the built "Human–Machine–Environment" interaction system. Corresponding safety measures are developed in order to provide favourable working conditions.

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APPENDIX A
GOST 28147-89 EQUATIONS GENERATOR

```
from copy import deepcopy
from sage.crypto.mq.sbox import SBox
from sage.rings.polynomial.multi_polynomial_sequence import PolynomialSequence
from sage.rings.polynomial.multi_polynomial_sequence import
    PolynomialSequence_generic


def inject(self, vars, values):
    sub_values = dict(zip(vars, values))
    return self.subs(sub_values)
PolynomialSequence_generic.inject = inject

def join_systems(mqsystems, instances):
    var_names = flatten([i.ring.variable_names() for i in instances])
    common_vars = list(set(var_names))
    common_ring = BooleanPolynomialRing(len(common_vars), common_vars, \
                                         order='degrevlex')
    new_mqsystem = PolynomialSequence([], common_ring)
    for s in mqsystems:
        new_mqsystem.extend(list(s))
    return new_mqsystem


class Gost:
    def _varformatstr(self, name):
        l = str(max([len(str(self.nrounds)), len(str(self.block_size - 1))]))
        return name + "%0" + l + "d" + "%0" + l + "d"

    def _varstrs(self, name, round):
        s = self._varformatstr(name)
        if s.startswith(self.var_names['block']):
            return [s % (round, i) for i in range(self.block_size)]
        else:
            return [s % (round, i) for i in range(self.halfblock_size)]

    def gen_vars(self, name, round_):
        return [self.ring(e) for e in self._varstrs(name, round_)]

    def int2bits(self, num, bits):
        num = Integer(num)
        return num.digits(base=2, padto=Integer(bits))

    def bits2int(self, num_bits):
        num = ''.join([str(i) for i in reversed(num_bits)])
        return int(num, 2)

    def __init__(self, **kwargs):
        self.SBOX_SIZE = 4
        self.nrounds = kwargs.get('rounds', 32)
        if self.nrounds < 8:
            self.key_length = self.nrounds
        else:
            if self.nrounds % 8 != 0:
                raise ValueError('Number of rounds must be multiple of 8')
```

```
            self.key_length = 8
        self.block_size = kwargs.get('block_size', 64)
        if self.block_size % self.SBOX_SIZE != 0 or self.block_size < 8:
            raise ValueError('Block size must be multiple of 4
                    (due to SBox) and greater than 8 (due S-box size)')
        self.halfblock_size = self.block_size / 2
        self.key_order = kwargs.get('key_order', 'frwrev')
        if self.key_order not in ['frw', 'frwrev']:
            raise ValueError('Unsupported key ordering')
        if self.key_order is 'frwrev' and self.nrounds % 8 != 0:
            raise ValueError('frwrev key ordering is only possible
                    for nrounds to be multiple of 8')
        self.key_add = kwargs.get('key_add', 'mod')
        if self.key_add not in ['mod', 'xor']:
            raise ValueError('key_add may be set to `mod` or `xor`')

        self._init_sboxes(kwargs.get('sboxes', None))
        pre = kwargs.get('prefix', '')
        self.var_names = {'key': 'K',
                'block': pre + 'X',
                'sum': pre + 'Y',
                'sbox': pre + 'Z'}
        self.gen_ring()

    def _init_sboxes(self, sboxes):
        self._default_sboxes = [
                [4, 10, 9, 2, 13, 8, 0, 14, 6, 11, 1, 12, 7, 15, 5, 3],
                [14, 11, 4, 12, 6, 13, 15, 10, 2, 3, 8, 1, 0, 7, 5, 9],
                [5, 8, 1, 13, 10, 3, 4, 2, 14, 15, 12, 7, 6, 0, 9, 11],
                [7, 13, 10, 1, 0, 8, 9, 15, 14, 4, 6, 12, 11, 2, 5, 3],
                [6, 12, 7, 1, 5, 15, 13, 8, 4, 10, 9, 14, 0, 3, 11, 2],
                [4, 11, 10, 0, 7, 2, 1, 13, 3, 6, 8, 5, 9, 12, 15, 14],
                [13, 11, 4, 1, 3, 15, 5, 9, 0, 10, 14, 7, 6, 8, 2, 12],
                [1, 15, 13, 0, 5, 7, 10, 4, 9, 2, 3, 14, 6, 11, 8, 12]
                ]
        if not sboxes:
            sboxes = self._default_sboxes
        else:
            for s in sboxes:
                if len(s) != 2 ^ self.SBOX_SIZE:
                    raise TypeError('S-box must be 4x4 bits (0..15)')
        self.sboxes = [SBox(i, big_endian=False) for i in sboxes]

    def gen_ring(self):
        nr = self.nrounds
        bs = self.block_size
        hbs = self.halfblock_size
        var_names = []
        halfblock_vars = [self.var_names['key'], \
                          self.var_names['sum'], \
                          self.var_names['sbox']]
        for r in range(nr):
            var_names += [self._varformatstr(v) % (r, b)
                    for v in halfblock_vars for b in xrange(hbs)]
        for r in range(nr + 1):
            var_names += [self._varformatstr(self.var_names['block']) % (r, b)
                    for b in xrange(bs)]
        self.ring = BooleanPolynomialRing(len(var_names), var_names, \
                                          order='degrevlex')
        return self.ring
```

```

    def polynomial_system(self):
        hbs = self.halfblock_size
        mqsystem_parts = []

        kvars = list()
        for i in range(self.key_length):
            kvars.append(map(self.ring, \
                    self.gen_vars(self.var_names['key'], i)))

        for i in range(self.nrounds):
            xvars = map(self.ring, self.gen_vars(self.var_names['block'], i))
            yvars = map(self.ring, self.gen_vars(self.var_names['sum'], i))
            zvars = map(self.ring, self.gen_vars(self.var_names['sbox'], i))
            next_xvars = map(self.ring, \
                    self.gen_vars(self.var_names['block'], i + 1))
            polynomials = []

            if self.key_order == 'frwrev' and \
                    i >= (self.nrounds - self.key_length):
                k = self.key_length - 1 - (i % self.key_length)
            else:
                k = i % self.key_length
            polynomials += self.add_round_key(xvars[:hbs], kvars[k], yvars)
            polynomials += self.polynomials_sbox(yvars, zvars)
            zvars = self.shift(zvars)
            xored = self.xor_blocks(xvars[hbs:], zvars)
            if i < self.nrounds - 1:
                # R_{i+1} = L_i xor f(R_i)
                polynomials += [x + y for x, y in zip(xored, \
                                                        next_xvars[:hbs])]
                # L_{i+1} = R_i
                polynomials += [x + y for x, y in zip(xvars[:hbs], \
                                                        next_xvars[hbs:])]
            else:
                # No block swapping in the last round.
                # R_{i+1} = L_i
                polynomials += [x + y for x, y in zip(xvars[:hbs], \
                                                        next_xvars[:hbs])]
                # L_{i+1} = L_i xor f(R_i)
                polynomials += [x + y for x, y in zip(xored, \
                                                        next_xvars[hbs:])]
            mqsystem_parts.append(polynomials)
        return PolynomialSequence(mqsystem_parts, self.ring)

    def polynomials_sbox(self, yvars, zvars):
        polynomials = list()
        sboxes = deepcopy(self.sboxes)
        for i in range(self.halfblock_size / self.SBOX_SIZE):
            nbit = i * self.SBOX_SIZE
            current_sbox = sboxes[i % len(sboxes)]
            pols = current_sbox.polynomials()
            gens = current_sbox.ring().gens()
            new_gens = yvars[nbit:nbit + self.SBOX_SIZE] + \
                    zvars[nbit:nbit + self.SBOX_SIZE]
            sub = dict(zip(gens, new_gens))
            pols = [p.subs(sub) for p in pols]
            polynomials += pols
        return polynomials

```

```
    def __repr__(self):
        gost_id = 'GOST cipher '
        gost_id += '(Block Size = %d, Rounds = %d, '
        gost_id += 'Key Addition = %s, Key Order = %s)'
        params = (self.block_size, self.nrounds, self.key_add, self.key_order)
        return gost_id % params

    def add_round_key(self, a, b, r=[]):
        hbs = self.halfblock_size
        is_defined  = lambda vals: all([p.constant() for p in vals])
        # If all variables are defined, just compute the result instead of
        # generating polynomials.
        if is_defined(a) and is_defined(b):
            if self.key_add is 'mod':
                data = self.bits2int(a)
                subkey = self.bits2int(b)
                modulo_mask = (1 << hbs) - 1
                result = (data + subkey) & modulo_mask
                return [self.ring(i) for i in self.int2bits(result, hbs)]
            if self.key_add is 'xor':
                return [x + y for x, y, in zip(a, b)]
        else:
            if self.key_add is 'mod':
                pols = list()
                pols.append(a[0] + b[0] + r[0])
                for i in range(0, hbs - 1):
                    pols.append(a[i] + a[i] * r[i] + a[i] * r[i+1] + a[i] *
                            a[i+1] + a[i] * b[i+1] + r[i] * r[i+1] + r[i] *
                            a[i+1] + r[i] * b[i+1])
                    pols.append(b[i] + b[i] * r[i] + b[i] * r[i+1] + b[i] *
                            a[i+1] + b[i] * b[i+1] + r[i] * r[i+1] + r[i] *
                            a[i+1] + r[i] * b[i+1])
                    pols.append(a[i] * r[i] + b[i] * r[i] + a[i] * b[i] + a[i]
                            + b[i] + r[i+1] + a[i+1] + b[i+1])
                return pols
            if self.key_add is 'xor':
                return [x + y + z for x, y, z in zip(r, a, b)]

    def shift(self, halfblock):
        shift = ceil(self.halfblock_size / 3)
        halfblock = halfblock[self.halfblock_size-shift:self.halfblock_size]+\
                halfblock[0:self.halfblock_size-shift]
        return halfblock

    def substitute(self, halfblock):
        result = []
        for i in range(self.halfblock_size / self.SBOX_SIZE):
            nbit = i * self.SBOX_SIZE
            plain = halfblock[nbit:nbit + self.SBOX_SIZE]
            sub = self.sboxes[i % len(self.sboxes)](plain)
            sub = [self.ring(j) for j in sub]
            result += sub
        return result

    def xor_blocks(self, left, right):
        return [x + y for x, y in zip(left, right)]

    def feistel_round(self, block, subkey):
        hbs = self.halfblock_size
        bs = self.block_size
```

```
            n1 = block[0:hbs] # left
            n2 = block[hbs:bs]; # right
            temp = n1
            n1 = self.add_round_key(n1, subkey)
            n1 = self.substitute(n1);
            n1 = self.shift(n1);
            n2 = self.xor_blocks(n1, n2)
            n1 = temp
            return n2 + n1

        def _cast_params(self, data_, key_):
            bs = self.block_size
            data = deepcopy(data_)
            key = deepcopy(key_)
            if not isinstance(data, list):
                data = self.int2bits(data, bs)
            data = [self.ring(i) for i in data]
            if len(key) != self.key_length:
                raise TypeError('Key should be of length ' + str(self.key_length))
            # coerse key bits to ring elements
            for i in range(len(key)):
                if not isinstance(key[i], list):
                    key[i] = self.int2bits(key[i], self.halfblock_size)
                key[i] = [self.ring(j) for j in key[i]]
            return data, key

        def encrypt(self, data_, key_):
            hbs = self.halfblock_size
            bs = self.block_size
            if isinstance(data_, list):
                is_list = true;
            else:
                is_list= false;
            data, key = self._cast_params(data_, key_)

            for i in range(self.nrounds):
                if self.key_order == 'frwrev' and \
                        i >= (self.nrounds - self.key_length):
                    k = self.key_length - 1 - (i % self.key_length)
                else:
                    k = i % self.key_length
                data = self.feistel_round(data, key[k])
            data = data[hbs:bs] + data[0:hbs]
            if is_list:
                return data
            else:
                return self.bits2int(data)

        def decrypt(self, data_, key_):
            hbs = self.halfblock_size
            bs = self.block_size
            if isinstance(data_, list):
                is_list = true;
            else:
                is_list= false;
            data, key = self._cast_params(data_, key_)

            key.reverse()
            for i in range(self.nrounds):
                if self.key_order == 'frwrev' and i < self.key_length:
```

```
                k = self.key_length - 1 - (i % self.key_length)
            else:
                k = i % self.key_length
            data = self.feistel_round(data, key[k])
        data = data[hbs:bs] + data[0:hbs]
        if is_list:
            return data
        else:
            return self.bits2int(data)

    def random_key(self):
        key = [list(random_vector(int(self.halfblock_size), x=2))
                for _ in range(self.key_length)]
        key = [map(self.ring, i) for i in key]
        return key

    def random_block(self):
        return map(self.ring, list(random_vector(self.block_size, x=2)))

    def test_mqsystem(self, f_):
        if f_.ring() is not self.ring:
            raise TypeError('Tested MQ system has been generated by a '
                    'different GOST instance')
        f = deepcopy(f_)
        print 'Testing MQ system', f
        bs = self.block_size
        plaintext = self.random_block()
        key = self.random_key()
        ciphertext = self.int2bits(self.encrypt(plaintext, key), bs)
        f = f.subs(dict(zip(self.gen_vars(self.var_names['block'], \
                                            0), plaintext)))
        f = f.subs(dict(zip(self.gen_vars(self.var_names['block'], \
                                            self.nrounds), ciphertext)))
        for i in range(self.nrounds):
            k = i % self.key_length
            f = f.subs(dict(zip(self.gen_vars(self.var_names['key'], i), \
                                                                key[k])))
        s = f.ideal().interreduced_basis()
        if s == [1]:
            print 'MQ System for' + str(self) + 'is INCORRECT'
            print f
            return False
        else:
            return True


    def test_cipher(self):
        keys = [
                [0x0, 0x0, 0x0, 0x0, 0x0, 0x0, 0x0, 0x0],
                [0xFFFFFFFF, 0xFFFFFFFF, 0xFFFFFFFF, 0xFFFFFFFF, \
                    0xFFFFFFFF, 0xFFFFFFFF, 0xFFFFFFFF, 0xFFFFFFFF],
                [0x01234567, 0x89ABCDEF, 0x01234567, 0x89ABCDEF, \
                    0x01234567, 0x89ABCDEF, 0x01234567, 0x89ABCDEF],
                [0x6ebabf8d, 0x1a8cad60, 0x124744f9, 0xd400b5d8, \
                    0xa721e3fd, 0x11d0702d, 0x06fd4827, 0x476df4bf]
                ]
        plain = [
                0x0000000000000000, 0xBDBDBDBDACACACAC, 0xFFFFFFFFFFFFFFFF, \
                    0x89ABCDEF01234567, 0xC0AE942BC8A99A39
                ]
```

```
        cipher = [
                0x12610BE2A6C2FDC9, 0xA587E5D3F6DFB6F4, 0x029BFE67A9364E44, \
                    0x523FC1A6AEC71B9A, 0x780CB7CE063F59E2,
                0x9057C2CF13AAAD6D, 0xF7B085BA4771F406, 0x780416781B29BC06, \
                    0xE596A183FD645558, 0x8EC42736538740AB,
                0xF1956B1D0A1A67DE, 0x9E4C808408DCDBDC, 0x7738AF92DE8FC770, \
                    0x9C44633FCEC0A03E, 0xC23013406002E268,
                0x91DB8E1FE489FAEF, 0x547BF353604A8190, 0x92B517E3CC91B9D0, \
                    0x1F5C2195762513E2, 0xE99D1C8DFF44A74C
                ]

        def show_failed(plain, key, result, expected):
            print 'GOST FAILED'
            print 'key:\t\t', ['0x%8.8X' % subk for subk in key]
            print 'plaintext:\t', "0x%16.16X" % plain
            print 'expected:\t', "0x%16.16X" % expected
            print 'actual:\t\t', "0x%16.16X" % result

        if self.block_size == 64 and self.nrounds == 32 and \
                self.key_add == 'mod' and self.key_order == 'frwrev':
            print 'Testing ', self, 'using test vectors...'
            it = cipher.__iter__()
            for k in keys:
                for p in plain:
                    c = self.encrypt(p, k)
                    expected = it.next()
                    if c != expected:
                        show_failed(p, k, c, expected)
                        return False
        print 'Testing', self, 'by correct decryption...'
        x = randint(0, 2^self.block_size)
        key = list(random_vector(self.key_length, x = 2^self.block_size))
        c = self.encrypt(x, key)
        d = self.decrypt(c, key)
        if x != d:
            show_failed(x, key, d, x)
            return False
        return True
```

APPENDIX B
SOLVING 6 ROUNDS OF GOST 28147-89 EQUATIONS SYSTEM

```
attach gost.sage
attach anf2cnf.py

nr = 5
bs = 64
hbs = int(bs/2)
varnames = ['a', 'b', 'c', 'd']
gosts = [Gost(block_size=bs,
    rounds=nr,
    key_add='mod',
    key_order='frw',
    prefix=i) for i in varnames]

print 'constructing MQ systems...'
mqsystems = [i.polynomial_system() for i in gosts]

print 'generating plaintext/ciphertext data...'
inputs = [i.random_block() for i in gosts]
key = gosts[0].random_key()
outputs = [gosts[i].encrypt(inputs[i], key) for i in range(len(gosts))]

print  'injecting known variables...'
for i in range(len(mqsystems)):
    mqsystems[i] = mqsystems[i].inject(gosts[i].gen_vars(
            gosts[i].var_names['block'], 0), inputs[i])
    mqsystems[i] = mqsystems[i].inject(gosts[i].gen_vars(
            gosts[i].var_names['block'], nr), outputs[i])

print 'combining MQ systems...'
f = join_systems(mqsystems, gosts)

print 'solving MQ system with SAT solver...'
print time.ctime()
solver = ANFSatSolver(f.ring())
s, t = solver(f)
print 'DONE'
print time.ctime()

recovered_key = []
r = f.ring()
for i in range(len(key)):
    var_names = map(str, gosts[0].gen_vars(
            gosts[0].var_names['key'], i))
    var_names.sort()
    var_names = map(r, var_names)
    recovered_key.append([s[j] for j in var_names])

if gosts[0].int2bits(gosts[0].encrypt(
    inputs[0], recovered_key), bs) == outputs[0]:
    if key == recovered_key:
        print recovered_key
    else:
        print 'FOUND ANOTHER KEY'
        print 'actual'
```

```
            print key
            print 'found'
            print recovered_key
```

APPENDIX C
MISTY1 EQUATIONS GENERATOR

```python
#!/usr/bin/env sage
# -*- coding: utf-8 -*-

import operator

from sage.rings.polynomial.multi_polynomial_sequence import PolynomialSequence


def split(l, chunk_size):
    """Split flat list into nested lists of length `chunk_size`. If the
    `chunk_size` is not multiple of list length, the last sublist is added as
    is without padding.

    Args:
        l: List to split into chunks.
        chunk_size: Length of a single nested list.

    Returns:
        Nested list of chunks each of the length `chunk_size`.

    """
    return [l[i:i + chunk_size] for i in xrange(0, len(l), chunk_size)]


def reverse(iterable):
    """Return reversed iterable as list."""
    return list(reversed(iterable))


def vector_do(operation, a, b):
    """Perform vector operation on two lists.

    Args:
        operation: binary operation to perform (from `operator` module).
        a: first vector.
        b: second vector.

    Returns:
        Resulting vector (represented as list).

    Example:
        vector_do(operator.__xor__, [1, 1, 1], [1, 0, 1])

    """
    if operation is operator.__xor__:
        if is_constant(a) and is_constant(b):
            return map(lambda x, y: operation(x, y), a, b)
        else:
            # Process variables over Boolean Polynomial Ring correctly.
            return map(lambda x, y: operator.__add__(x, y), a, b)
    elif operation is operator.__and__:
        if is_constant(a) and is_constant(b):
            return map(lambda x, y: operation(x, y), a, b)
        else:
```

```
                # Process variables over Boolean Polynomial Ring correctly.
                return map(lambda x, y: operator.__mul__(x, y), a, b)
        elif operation is operator.__or__:
            if is_constant(a) and is_constant(b):
                return map(lambda x, y: operation(x, y), a, b)
            else:
                # Process variables over Boolean Polynomial Ring correctly.
                return map(lambda x, y: x * y + x + y, a, b)
        else:
            return map(lambda x, y: operation(x, y), a, b)


def is_constant(vals):
    """Check of all elements in list are contants, not variables."""
    return all([isinstance(i, Integer) for i in vals])

def groebner_basis(func):
    """Decorator for Groebner basis reduce of polynomial system."""
    def wrapper(*args, **kwargs):
        result = func(*args, **kwargs)
        if not is_constant(result):
            F = PolynomialSequence(result)
            return F.groebner_basis()
        else:
            return result
    return wrapper


class Misty(object):
    """Misty cipher class.

    All method assume to take bit sequences as input. Use `get_bits` method to
    convert integer to Misty bit sequence representation and `get_integer` to
    obtain the corresponding integer back.

    """

    def get_bits(self, integer, nbytes=0):
        """Convert integer to crazy Misty bit ordering. """
        bytes = reverse(integer.digits(256, padto=nbytes))
        bits = [reverse(b.digits(2, padto=8)) for b in bytes]
        return flatten(bits)

    def get_integer(self, bits):
        """Convert crazy Misty bit sequence to sane ordering. """
        bytes = reverse(split(bits, 8))
        bytes = [reverse(b) for b in bytes]
        return Integer(flatten(bytes), 2)

    def __init__(self, nrounds, prefix='', equations_key_schedule=True):
        """Create Misty cipher object.

        It's a full scale cipher as well as its polynomial system generator.

        Args:
            nrounds: Number of enciphering rounds.
            prefix: Prefix used for variables identification during polynomial
                system construction.

        """
```

```
        self.nrounds = nrounds
        self.prefix = prefix
        self.equations_key_schedule = equations_key_schedule
        self.block_size = 64
        self.halfblock_size = self.block_size // 2
        self.halfblock_size_fo = self.halfblock_size // 2
        self.fi_left_size = 9
        self.fi_right_size = 7
        self.key = None
        self.subkeys = None
        self.gen_ring()
        # Subkey type constants.
        self.KEY_KO1 = 'ko1'
        self.KEY_KO2 = 'ko2'
        self.KEY_KO3 = 'ko3'
        self.KEY_KO4 = 'ko4'
        self.KEY_KI1 = 'ki1'
        self.KEY_KI2 = 'ki2'
        self.KEY_KI3 = 'ki3'
        self.KEY_KL1 = 'kl1'
        self.KEY_KL2 = 'kl2'

    def kindex(self, subkey_type, i):
        """Index subkey according to crazy Misty indexing rule.

        Args:
            subkey_type: string, indicating subkey type.
            subkeys: list of subkey bits (each element contains 16 subkey
                bits).
            i: Misty round in range 1 <= i <= 8.

        Returns:
            16-bit subkey for corresponding index.
        """
        if i < 1:
            raise ValueError('Subkey index must start from 1. '
                             'Got {0} instead.'.format(i))

        def normalize(x):
            while x > 8:
                x = x - 8
            return x

        if subkey_type == self.KEY_KO1:
            return self.key[i - 1]
        if subkey_type == self.KEY_KO2:
            i = normalize(i + 2)
            return self.key[i - 1]
        if subkey_type == self.KEY_KO3:
            i = normalize(i + 7)
            return self.key[i - 1]
        if subkey_type == self.KEY_KO4:
            i = normalize(i + 4)
            return self.key[i - 1]
        if subkey_type == self.KEY_KI1:
            i = normalize(i + 5)
            return self.subkeys[i - 1]
        if subkey_type == self.KEY_KI2:
            i = normalize(i + 1)
            return self.subkeys[i - 1]
```

```
        if subkey_type == self.KEY_KI3:
            i = normalize(i + 3)
            return self.subkeys[i - 1]

        if subkey_type == self.KEY_KL1:
            if i % 2 != 0:
                i = normalize((i + 1) // 2)
                return self.key[i - 1]
            else:
                i = normalize((i // 2) + 2)
                return self.subkeys[i - 1]
        if subkey_type == self.KEY_KL2:
            if i % 2 != 0:
                i = normalize((i + 1) // 2 + 6)
                return self.subkeys[i - 1]
            else:
                i = normalize((i // 2) + 4)
                return self.key[i - 1]

    def fi(self, x, subkey_ki):
        """Misty FI function.

        Args:
            x: 16-bit input value.
            subkey_ki: 16-bit KI key chunk for FI function.

        Returns: 16-bit output of FI function.

        """
        ki7 = subkey_ki[0:self.fi_right_size]
        ki9 = subkey_ki[self.fi_right_size:]

        d9 = x[0:self.fi_left_size]
        d7 = x[self.fi_left_size:]

        d9 = vector_do(operator.__xor__, self.s9(d9), [0, 0] + d7)
        d7 = vector_do(operator.__xor__, self.s7(d7), d9[2:self.fi_left_size])
        d7 = vector_do(operator.__xor__, d7, ki7)
        d9 = vector_do(operator.__xor__, d9, ki9)
        d9 = vector_do(operator.__xor__, self.s9(d9), [0, 0] + d7)
        return d7 + d9

    def key_schedule(self, key):
        """Generate subkeys according to Misty key schedule algorithm.

        Args:
            key: List of 128 bits.

        Returns:
            List of 8 subkeys (each containing list of 16 bits).
        """
        key_chunks = split(key, 16)
        self.key = key_chunks

        subkeys = list()
        for k in range(len(key_chunks)):
            if k < 7:
                subkeys.append(self.fi(key_chunks[k], key_chunks[k + 1]))
            else:
                subkeys.append(self.fi(key_chunks[k], key_chunks[0]))
```

```
            self.subkeys = subkeys
            return subkeys

        def fl(self, x, i):
            """Misty key injection FL function.

            Args:
                x: 32-bit input.
                i: number of round.

            Returns:
                Resulting 32 bits after key injection.

            """
            left = x[:self.halfblock_size_fo]
            right = x[self.halfblock_size_fo:]

            kl1 = self.kindex(self.KEY_KL1, i)
            kl2 = self.kindex(self.KEY_KL2, i)

            temp = vector_do(operator.__and__, left, kl1)
            right = vector_do(operator.__xor__, right, temp)

            temp = vector_do(operator.__or__, right, kl2)
            left = vector_do(operator.__xor__, left, temp)
            return left + right

        #@groebner_basis
        def s7(self, x, r=None):
            """Substitute with Misty S7 SBox.

            Bit ordering is reversed due to Crazy Misty Spec bit ordering.
            """
            y = [0] * len(x)
            if not r:
                y[6] = x[6] ^^x[5] &x[3] ^^x[6] &x[3] &x[2] ^^x[5] &x[1] ^^x[6] &x[4] &x[1] ^^x[2] &x[1] ^^x[6] &x[5] &x[0] ^^x[4] &x[0] ^^x[6] &x[1] &x[0] ^^x[3] &x[1] &x[0] ^^1
                y[5] = x[6] &x[4] ^^x[6] &x[2] ^^x[3] &x[2] ^^x[5] &x[1] ^^x[4] &x[2] &x[1] ^^x[0] ^^x[6] &x[0] ^^x[3] &x[0] ^^x[4] &x[3] &x[0] ^^x[5] &x[2] &x[0] ^^x[6] &x[1] &x[0] ^^1
                y[4] = x[5] &x[4] ^^x[6] &x[4] &x[3] ^^x[2] ^^x[5] &x[2] ^^x[6] &x[5] &x[2] ^^x[6] &x[1] ^^x[6] &x[2] &x[1] ^^x[3] &x[2] &x[1] ^^x[5] &x[0] ^^x[3] &x[0] ^^x[6] &x[3] &x[0] ^^x[2] &x[0] ^^x[4] &x[2] &x[0]
                y[3] = x[6] ^^x[5] ^^x[6] &x[5] &x[4] ^^x[6] &x[3] ^^x[4] &x[2] ^^x[5] &x[2] &x[1] ^^x[4] &x[0] ^^x[5] &x[3] &x[0] ^^x[6] &x[2] &x[0] ^^x[1] &x[0] ^^1
                y[2] = x[4] &x[3] ^^x[6] &x[2] ^^x[5] &x[3] &x[2] ^^x[1] ^^x[4] &x[1] ^^x[5] &x[4] &x[1] ^^x[6] &x[3] &x[1] ^^x[5] &x[0] ^^x[5] &x[1] &x[0] ^^x[2] &x[1] &x[0] ^^1
                y[1] = x[6] ^^x[5] ^^x[4] ^^x[6] &x[5] &x[4] ^^x[6] &x[3] ^^x[5] &x[4] &x[3] ^^x[5] &x[2] ^^x[6] &x[4] &x[2] ^^x[6] &x[1] ^^x[6] &x[5] &x[1] ^^x[3] &x[1] ^^x[6] &x[0] ^^x[4] &x[1] &x[0]
                y[0] = x[6] &x[5] ^^x[3] ^^x[6] &x[3] ^^x[4] &x[3] &x[2] ^^x[6] &x[1] ^^x[4] &x[1] ^^x[3] &x[1] ^^x[5] &x[3] &x[1] ^^x[5] &x[0] ^^x[5] &x[4] &x[0] ^^x[6] &x[3] &x[0] ^^x[2] &x[0] ^^x[4] &x[1] &x[0]
                return y
            else:
                # Process variables over Boolean Polynomial Ring correctly.
                polynomials = [
```

```
            r[6] + x[6] + x[5] * x[3] + x[6] * x[3] * x[2] + x[5] * x[1] + x[6] * x[4] * x[1] + x[2] * x[1] + x[6] * x[5] * x[0] + x[4] * x[0] + x[6] * x[1] * x[0] + x[3] * x[1] * x[0] + 1,
            r[5] + x[6] * x[4] + x[6] * x[2] + x[3] * x[2] + x[5] * x[1] + x[4] * x[2] * x[1] + x[0] + x[6] * x[0] + x[3] * x[0] + x[4] * x[3] * x[0] + x[5] * x[2] * x[0] + x[6] * x[1] * x[0] + 1,
            r[4] + x[5] * x[4] + x[6] * x[4] * x[3] + x[2] + x[5] * x[2] + x[6] * x[5] * x[2] + x[6] * x[1] + x[6] * x[2] * x[1] + x[3] * x[2] * x[1] + x[5] * x[0] + x[3] * x[0] + x[6] * x[3] * x[0] + x[2] * x[0] + x[4] * x[2] * x[0],
            r[3] + x[6] + x[5] + x[6] * x[5] * x[4] + x[6] * x[3] + x[4] * x[2] + x[5] * x[2] * x[1] + x[4] * x[0] + x[5] * x[3] * x[0] + x[6] * x[2] * x[0] + x[1] * x[0] + 1,
            r[2] + x[4] * x[3] + x[6] * x[2] + x[5] * x[3] * x[2] + x[1] + x[4] * x[1] + x[5] * x[4] * x[1] + x[6] * x[3] * x[1] + x[5] * x[0] + x[5] * x[1] * x[0] + x[2] * x[1] * x[0] + 1,
            r[1] + x[6] + x[5] + x[4] + x[6] * x[5] * x[4] + x[6] * x[3] + x[5] * x[4] * x[3] + x[5] * x[2] + x[6] * x[4] * x[2] + x[6] * x[1] + x[6] * x[5] * x[1] + x[3] * x[1] + x[6] * x[0] + x[4] * x[1] * x[0],
            r[0] + x[6] * x[5] + x[3] + x[6] * x[3] + x[4] * x[3] * x[2] + x[6] * x[1] + x[4] * x[1] + x[3] * x[1] + x[5] * x[3] * x[1] + x[5] * x[0] + x[5] * x[4] * x[0] + x[6] * x[3] * x[0] + x[2] * x[0] + x[4] * x[1] * x[0]
            ]
            return polynomials


    #@groebner_basis
    def s9(self, x, r=None):
        """Substitute with Misty S9 SBox. """
        y = [0] * len(x)
        if not r:
            y[8] = x[8] & x[4] ^^ x[8] & x[3] ^^ x[7] & x[3] ^^ x[7] & x[2] ^^ x[6] & x[2] ^^ x[6] & x[1] ^^ x[5] & x[1] ^^ x[5] & x[0] ^^ x[4] & x[0] ^^ 1
            y[7] = x[8] & x[6] ^^ x[5] ^^ x[7] & x[5] ^^ x[6] & x[5] ^^ x[5] & x[4] ^^ x[4] & x[3] ^^ x[8] & x[2] ^^ x[6] & x[2] ^^ x[1] ^^ x[8] & x[0] ^^ x[5] & x[0] ^^ x[3] & x[0] ^^ 1
            y[6] = x[8] & x[7] ^^ x[7] & x[5] ^^ x[4] ^^ x[8] & x[4] ^^ x[6] & x[4] ^^ x[5] & x[4] ^^ x[4] & x[3] ^^ x[8] & x[2] ^^ x[3] & x[2] ^^ x[7] & x[1] ^^ x[5] & x[1] ^^ x[0]
            y[5] = x[8] ^^ x[7] & x[6] ^^ x[6] & x[4] ^^ x[3] ^^ x[7] & x[3] ^^ x[5] & x[3] ^^ x[4] & x[3] ^^ x[3] & x[2] ^^ x[7] & x[1] ^^ x[2] & x[1] ^^ x[6] & x[0] ^^ x[4] & x[0]
            y[4] = x[7] ^^ x[8] & x[5] ^^ x[6] & x[5] ^^ x[8] & x[3] ^^ x[5] & x[3] ^^ x[2] ^^ x[6] & x[2] ^^ x[4] & x[2] ^^ x[3] & x[2] ^^ x[2] & x[1] ^^ x[6] & x[0] ^^ x[1] & x[0]
            y[3] = x[6] ^^ x[8] & x[5] ^^ x[7] & x[4] ^^ x[5] & x[4] ^^ x[7] & x[2] ^^ x[4] & x[2] ^^ x[1] ^^ x[5] & x[1] ^^ x[3] & x[1] ^^ x[2] & x[1] ^^ x[8] & x[0] ^^ x[1] & x[0]
            y[2] = x[8] & x[7] ^^ x[5] ^^ x[7] & x[4] ^^ x[6] & x[3] ^^ x[4] & x[3] ^^ x[6] & x[1] ^^ x[3] & x[1] ^^ x[0] ^^ x[8] & x[0] ^^ x[4] & x[0] ^^ x[2] & x[0] ^^ x[1] & x[0] ^^ 1
            y[1] = x[7] ^^ x[8] & x[7] ^^ x[7] & x[6] ^^ x[6] & x[5] ^^ x[8] & x[4] ^^ x[3] ^^ x[7] & x[2] ^^ x[5] & x[2] ^^ x[8] & x[1] ^^ x[4] & x[1] ^^ x[2] & x[1] ^^ x[7] & x[0] ^^ 1
            y[0] = x[8] ^^ x[8] & x[7] ^^ x[7] & x[6] ^^ x[4] ^^ x[8] & x[3] ^^ x[6] & x[3] ^^ x[5] & x[2] ^^ x[3] & x[2] ^^ x[8] & x[1] ^^ x[8] & x[0] ^^ x[5] & x[0] ^^ x[2] & x[0] ^^ 1
            return y
        else:
```

```
            # Process variables over Boolean Polynomial Ring correctly.
            polynomials = [
            r[8] + x[8] * x[4] + x[8] * x[3] + x[7] * x[3] + x[7] * x[2] + x[6] * x[2] + x[6] * x[1] + x[5] * x[1] + x[5] * x[0] + x[4] * x[0] + 1,
            r[7] + x[8] * x[6] + x[5] + x[7] * x[5] + x[6] * x[5] + x[5] * x[4] + x[4] * x[3] + x[8] * x[2] + x[6] * x[2] + x[1] + x[8] * x[0] + x[5] * x[0] + x[3] * x[0] + 1,
            r[6] + x[8] * x[7] + x[7] * x[5] + x[4] + x[8] * x[4] + x[6] * x[4] + x[5] * x[4] + x[4] * x[3] + x[8] * x[2] + x[3] * x[2] + x[7] * x[1] + x[5] * x[1] + x[0],
            r[5] + x[8] + x[7] * x[6] + x[6] * x[4] + x[3] + x[7] * x[3] + x[5] * x[3] + x[4] * x[3] + x[3] * x[2] + x[7] * x[1] + x[2] * x[1] + x[6] * x[0] + x[4] * x[0],
            r[4] + x[7] + x[8] * x[5] + x[6] * x[5] + x[8] * x[3] + x[5] * x[3] + x[2] + x[6] * x[2] + x[4] * x[2] + x[3] * x[2] + x[2] * x[1] + x[6] * x[0] + x[1] * x[0],
            r[3] + x[6] + x[8] * x[5] + x[7] * x[4] + x[5] * x[4] + x[7] * x[2] + x[4] * x[2] + x[1] + x[5] * x[1] + x[3] * x[1] + x[2] * x[1] + x[8] * x[0] + x[1] * x[0],
            r[2] + x[8] * x[7] + x[5] + x[7] * x[4] + x[6] * x[3] + x[4] * x[3] + x[6] * x[1] + x[3] * x[1] + x[0] + x[8] * x[0] + x[4] * x[0] + x[2] * x[0] + x[1] * x[0] + 1,
            r[1] + x[7] + x[8] * x[7] + x[7] * x[6] + x[6] * x[5] + x[8] * x[4] + x[3] + x[7] * x[2] + x[5] * x[2] + x[8] * x[1] + x[4] * x[1] + x[2] * x[1] + x[7] * x[0] + 1,
            r[0] + x[8] + x[8] * x[7] + x[7] * x[6] + x[4] + x[8] * x[3] + x[6] * x[3] + x[5] * x[2] + x[3] * x[2] + x[8] * x[1] + x[8] * x[0] + x[5] * x[0] + x[2] * x[0] + 1
            ]
            return polynomials

    def fo(self, x, i):
        """Misty FO function.

        Second level nested Feistel network.

        Args:
            x: 32-bit input list.
            i: number of rounds.

        Returns:
            Resulting bits list.
        """


        left = x[0:self.halfblock_size_fo]
        right = x[self.halfblock_size_fo:]

        ki1 = self.kindex(self.KEY_KI1, i)
        ki2 = self.kindex(self.KEY_KI2, i)
        ki3 = self.kindex(self.KEY_KI3, i)

        ko1 = self.kindex(self.KEY_KO1, i)
        ko2 = self.kindex(self.KEY_KO2, i)
        ko3 = self.kindex(self.KEY_KO3, i)
        ko4 = self.kindex(self.KEY_KO4, i)

        left = vector_do(operator.__xor__, left, ko1)
        temp = self.fi(left, ki1)
        left = vector_do(operator.__xor__, temp, right)
```

```python

        right = vector_do(operator.__xor__, right, ko2)
        temp = self.fi(right, ki2)
        right = vector_do(operator.__xor__, left, temp)

        left = vector_do(operator.__xor__, left, ko3)
        temp = self.fi(left, ki3)
        left = vector_do(operator.__xor__, temp, right)

        right = vector_do(operator.__xor__, right, ko4)

        return right + left

    def feistel_round(self, data, i):
        """Misty Feistel network single run.

        It actually performs first 2 rounds (look for Misty specs).

        Args:
            data: 64-bit input list.
            i: number of actual round (pay attention to indices according
                to Misty specification). Rounds are in range 1 <= i <= n + 2,
                where `n` is total number of rounds
        Returns:
            Resulting 64-bit list.

        """
        left = data[0:self.halfblock_size]
        right = data[self.halfblock_size:]

        # FL1
        left = self.fl(left, i)
        # FL2
        right = self.fl(right, i + 1)

        # FO1
        temp = self.fo(left, i)
        right = vector_do(operator.__xor__, right, temp)

        # FO2
        temp = self.fo(right, i + 1)
        left = vector_do(operator.__xor__, temp, left)

        return left + right

    def encipher(self, data, key):
        """Encipher plaintext with Misty cryptoalgorithm.

        Args:
            data: 64-bit input list (plaintext).
            key: 128-bit input list (key).

        Returns:
            64-bit list (ciphertext).

        """
        for i in range(1, self.nrounds + 1, 2):
            data = self.feistel_round(data, i)

        left = data[0:self.halfblock_size]
```

```
        right = data[self.halfblock_size:]
        # FL n+1
        left = self.fl(left, self.nrounds + 1)
        # FL n+2
        right = self.fl(right, self.nrounds + 2)
        return right + left

    def selftest(self):
        """Check Misty test vectors compliance."""
        plaintext = 0x0123456789ABCDEF
        key = 0x00112233445566778899AABBCCDDEEFF
        self.key_schedule(self.get_bits(key, 16))
        c = self.encipher(self.get_bits(plaintext, 8),
                          self.get_bits(key, 16))
        result = self.get_integer(c)
        expected = 0x8b1da5f56ab3d07c
        return result == expected

    def _varformatstr(self, name):
        """Prepare formatting string for variables notation.

        Args:
            name: Variable identificator string.

        Returns:
            Variable identificator string appended with format specificators
            that contains round number and block bit number.
            Format: R<round number>_<var id>_<bit number>

        """
        l = str(len(str(self.block_size - 1)))
        return "R%s_" + name + "_%0" + l + "d"

    def _varstrs(self, name, nbits, round='', start_from=0):
        """Construct strings with variables names.

        Args:
            name: variable string identificator.
            nbits: number of variables set of the same type.
            round: number of round for which variables are defined. If not
                specified, no round prefix is prepended to the string.

        Returns:
            List of strings with variables names.

        """
        round = str(round)
        s = self._varformatstr(name)
        if not round:
            # Exclude round prefix.
            s = s[s.find('_') + 1:]
            var_names = [s % (i) for i in range(start_from, start_from + nbits)]
        else:
            var_names = [s % (round, i) for i in range(start_from, start_from + nbits)]
        if not name == 'K' and not name == 'KS' and not name.startswith('FIKS'):
            # Include polynomial system prefix.
            var_names = [self.prefix + var for var in var_names]
```

```
        return var_names

    def vars(self, name, nbits, round='', start_from=0):
        """Construct variables in predefined Misty ring.

        Refer to `_varstrs()` and `gen_ring()` for details.

        """
        var_names = self._varstrs(name, nbits, round=round, start_from=
    start_from)
        return [self.ring(e) for e in var_names]

    def gen_round_var_names(self, round):
        """Generate variables names set for given round number."""
        var_names = list()
        # FL
        var_names += self._varstrs('FL_KL1', 16, round)
        var_names += self._varstrs('FL_KL2', 16, round)
        var_names += self._varstrs('FL_XOR', 16, round)
        var_names += self._varstrs('FL', 32, round)
        # FI
        for i in range(1, 4):
            # FI has 3 subrounds in FO function.
            var_names += self._varstrs('FI' + str(i), 16, round)
            var_names += self._varstrs('FI' + str(i) + '_S9', 9, round)
            var_names += self._varstrs('FI' + str(i) + '_S7', 7, round)
            var_names += self._varstrs('FI' + str(i) + '_SS9', 9, round)
            var_names += self._varstrs('FI' + str(i) + '_KI2', 16, round)
        # FO
        var_names += self._varstrs('FO', 32, round)
        var_names += self._varstrs('FO_KO1', 16, round)
        var_names += self._varstrs('FO_KO2', 16, round)
        var_names += self._varstrs('FO_KO3', 16, round)


        return var_names

    def gen_ring(self):
        """Generate ring for Misty polynomial equations system.

        Construct all variables needed for describing Misty cryptoalgorithm
        with polynomial equations system and generate the corresponding
        Boolean Polynomial Ring.

        """
        var_names = list()

        # Input plaintext.
        var_names += self._varstrs('IN', 64)
        # Output ciphertext.
        var_names += self._varstrs('OUT', 64)

        # Key variables.
        var_names += self._varstrs('K', 128)
        # Subkey variables.
        var_names += self._varstrs('KS', 128)

        if self.equations_key_schedule == True:
            for i in range(8):
                var_names += self._varstrs('FIKS' + str(i), 16)
```

```
                var_names += self._varstrs('FIKS' + str(i) + '_S9', 9)
                var_names += self._varstrs('FIKS' + str(i) + '_S7', 7)
                var_names += self._varstrs('FIKS' + str(i) + '_SS9', 9)
                var_names += self._varstrs('FIKS' + str(i) + '_KI2', 16)

        for i in range(1, self.nrounds + 1):
            var_names += self.gen_round_var_names(i)
        for i in range(1, self.nrounds + 1):
            if i % 2 == 1:
                var_names += self._varstrs('FX', 32, i)
            else:
                var_names += self._varstrs('F', 64, i)
        for i in range(self.nrounds + 1, self.nrounds + 3):
            # FL
            var_names += self._varstrs('FL_KL1', 16, i)
            var_names += self._varstrs('FL_KL2', 16, i)
            var_names += self._varstrs('FL_XOR', 16, i)
            var_names += self._varstrs('FL', 32, i)

        self.ring = BooleanPolynomialRing(len(var_names), var_names, order='degrevlex')


    def polynomials_fl(self, x, i):
        """Construct polynomials for Misty FL function."""

        left = x[:self.halfblock_size_fo]
        right = x[self.halfblock_size_fo:]

        kl1 = self.kindex(self.KEY_KL1, i)
        kl2 = self.kindex(self.KEY_KL2, i)

        polynomials = list()

        ## Generate variables for given round
        vars_kl1 = self.vars('FL_KL1', 16, i)
        vars_kl2 = self.vars('FL_KL2', 16, i)
        vars_xor = self.vars('FL_XOR', 16, i)
        vars_out = self.vars('FL', 32, i)

        temp = vector_do(operator.__and__, left, kl1)
        polynomials.extend(vector_do(operator.__xor__, temp, vars_kl1))

        right = vector_do(operator.__xor__, right, vars_kl1)
        polynomials.extend(vector_do(operator.__xor__, right, vars_xor))

        # Replace `x or y` operation with equivalent `x * y ^ x + y`.
        temp = vector_do(operator.__or__, vars_xor, kl2)
        polynomials.extend(vector_do(operator.__xor__, temp, vars_kl2))

        left = vector_do(operator.__xor__, left, vars_kl2)
        polynomials.extend(vector_do(operator.__xor__, left, vars_out[0:16]))
        polynomials.extend(vector_do(operator.__xor__, vars_xor, vars_out[16:32]))

        return flatten(polynomials)

    def polynomials_fi(self, x, subkey_ki, subround, r=''):
        """Construct polynomials for Misty FI function.

```

```
        Args:
            x: list of 16 input variables.
            subkey_ki: 16-bit key chunk.
            subround: number of FI round in FO function (1..3).
            r: number of actual outer Feistel network enciphering round.

        """
        ki7 = subkey_ki[0:self.fi_right_size]
        ki9 = subkey_ki[self.fi_right_size:]

        d9 = x[0:self.fi_left_size]
        d7 = x[self.fi_left_size:]

        subround = str(subround)

        if subround in ['1', '2', '3']:
            vars_fi = self.vars('FI' + subround, 16, r)
        else:
            vars_fi = self.vars('KS', 16, start_from=int(subround[2]) * 16)

        vars_s9 = self.vars('FI' + subround + '_S9', 9, r)
        vars_s7 = self.vars('FI' + subround + '_S7', 7, r)
        vars_ss9 = self.vars('FI' + subround + '_SS9', 9, r)
        vars_ki2 = self.vars('FI' + subround + '_KI2', 9, r)

        polynomials = list()
        pad = [self.ring(0)] * 2

        polynomials.extend(self.s9(d9, vars_s9))
        d9 = vector_do(operator.__xor__, vars_s9, pad + d7) # add to pols

        polynomials.extend(self.s7(d7, vars_s7))
        d7 = vector_do(operator.__xor__, vars_s7, d9[2:self.fi_left_size]) # add to pols
        d7 = vector_do(operator.__xor__, d7, ki7)

        d9 = vector_do(operator.__xor__, d9, ki9)
        polynomials.extend(vector_do(operator.__xor__, d9, vars_ki2))

        polynomials.extend(self.s9(vars_ki2, vars_ss9))
        d9 = vector_do(operator.__xor__, vars_ss9, pad + d7) # add to pols

        polynomials.extend(vector_do(operator.__xor__, vars_fi[0:7], d7))
        polynomials.extend(vector_do(operator.__xor__, vars_fi[7:16], d9))
        return polynomials

    def polynomials_fo(self, x, i):
        """Construct polynomials for Misty FI function."""

        left = x[0:self.halfblock_size_fo]
        right = x[self.halfblock_size_fo:]

        ki1 = self.kindex(self.KEY_KI1, i)
        ki2 = self.kindex(self.KEY_KI2, i)
        ki3 = self.kindex(self.KEY_KI3, i)

        ko1 = self.kindex(self.KEY_KO1, i)
        ko2 = self.kindex(self.KEY_KO2, i)
        ko3 = self.kindex(self.KEY_KO3, i)
        ko4 = self.kindex(self.KEY_KO4, i)
```

```

        vars_fo = self.vars('FO', 32, i)
        vars_ko1 = self.vars('FO_KO1', 16, i)
        vars_ko2 = self.vars('FO_KO2', 16, i)
        vars_ko3 = self.vars('FO_KO3', 16, i)
        vars_fi1 = self.vars('FI1', 16, i)
        vars_fi2 = self.vars('FI2', 16, i)
        vars_fi3 = self.vars('FI3', 16, i)

        polynomials = list()

        left = vector_do(operator.__xor__, left, ko1)
        polynomials.extend(vector_do(operator.__xor__, left, vars_ko1))
        polynomials.extend(self.polynomials_fi(vars_ko1, ki1, 1, i))  # FI1 variables introduced.
        left = vector_do(operator.__xor__, vars_fi1, right)

        right = vector_do(operator.__xor__, right, ko2)
        polynomials.extend(vector_do(operator.__xor__, right, vars_ko2))
        polynomials.extend(self.polynomials_fi(vars_ko2, ki2, 2, i))  # FI2 variables introduced.
        right = vector_do(operator.__xor__, vars_fi2, left)

        left = vector_do(operator.__xor__, left, ko3)
        polynomials.extend(vector_do(operator.__xor__, left, vars_ko3))
        polynomials.extend(self.polynomials_fi(vars_ko3, ki3, 3, i))  # FI3 variables introduced.
        left = vector_do(operator.__xor__, vars_fi3, right)

        right = vector_do(operator.__xor__, right, ko4)
        polynomials.extend(vector_do(operator.__xor__, right, vars_fo[0:16]))
        polynomials.extend(vector_do(operator.__xor__, left, vars_fo[16:32]))

        return polynomials

    def polynomials_round(self, data, i):
        """Construct polynomials for Misty Feistel single run."""
        left = data[0:self.halfblock_size]
        right = data[self.halfblock_size:]

        vars_fl1 = self.vars('FL', 32, i)
        vars_fl2 = self.vars('FL', 32, i + 1)
        vars_fo1 = self.vars('FO', 32, i)
        vars_fo2 = self.vars('FO', 32, i + 1)
        vars_fx = self.vars('FX', 32, i)
        vars_f = self.vars('F', 64, i + 1)

        polynomials = list()

        polynomials.extend(self.polynomials_fl(left, i))  # FL1 fariables introduced.
        polynomials.extend(self.polynomials_fl(right, i + 1))  # FL2 fariables introduced.

        polynomials.extend(self.polynomials_fo(vars_fl1, i))  # FO1 variables introduced.
        right = vector_do(operator.__xor__, vars_fo1, vars_fl2)
        polynomials.extend(vector_do(operator.__xor__, right, vars_fx))

```

```
            polynomials.extend(self.polynomials_fo(vars_fx, i + 1))  # FO2 variables introduced.
            left = vector_do(operator.__xor__, vars_fo2, vars_fl1)
            polynomials.extend(vector_do(operator.__xor__, left, vars_f[0:32]))
            polynomials.extend(vector_do(operator.__xor__, vars_fx, vars_f[32:64]))

        return polynomials

    def polynomials_key_schedule(self):
        """Construct polynomials for Misty key scheduling."""
        self.key = split(self.vars('K', 128), 16)

        polynomials = list()
        for k in range(len(self.key)):
            subround = 'KS' + str(k)
            if k < 7:
                polynomials.extend(self.polynomials_fi(self.key[k], self.key[k + 1], subround))
            else:
                polynomials.extend(self.polynomials_fi(self.key[k], self.key[0], subround))
        self.subkeys = split(self.vars('KS', 128), 16)
        return polynomials

    def polynomial_system(self):
        """Construct polynomials system for Misty cipher."""
        polynomials = list()

        plain = self.vars('IN', 64)
        if self.equations_key_schedule is True:
            polynomials.extend(self.polynomials_key_schedule())
        else:
            self.key = split(self.vars('K', 128), 16)
            self.subkeys = split(self.vars('KS', 128), 16)

        polynomials.extend(self.polynomials_round(plain, 1))  # R2_F variables introduced.
        for i in range(3, self.nrounds + 1, 2):
            vars_f_prev = self.vars('F', 64, i - 1)
            polynomials.extend(self.polynomials_round(vars_f_prev, i))

        vars_f = self.vars('F', 64, self.nrounds)
        vars_fl1 = self.vars('FL', 32, self.nrounds + 1)
        vars_fl2 = self.vars('FL', 32, self.nrounds + 2)
        vars_out = self.vars('OUT', 64)

        left = vars_f[0:self.halfblock_size]
        right = vars_f[self.halfblock_size:]

        polynomials.extend(self.polynomials_fl(left,  self.nrounds + 1))
        polynomials.extend(self.polynomials_fl(right,  self.nrounds + 2))
        polynomials.extend(vector_do(operator.__xor__, vars_fl1, vars_out[32:64]))
        polynomials.extend(vector_do(operator.__xor__, vars_fl2, vars_out[0:32]))
        return PolynomialSequence(polynomials)
```

APPENDIX D
SOLVING 2 ROUNDS OF MISTY1 EQUATIONS SYSTEM

```
from sage.rings.polynomial.multi_polynomial_sequence import PolynomialSequence
from sage.rings.polynomial.multi_polynomial_sequence import PolynomialSequence_generic

from sage.sat.boolean_polynomials import solve as sat_solve

load('misty.sage')

def inject_vars(F, vars, values):
    """Inject vars values into polynomial system. """
    sub_values = dict(zip(vars, values))
    return F.subs(sub_values)


def get_vars(solution, vars):
    """Obtain variable values from solution dict. """
    var_names = map(str, vars)
    solution = dict(zip(map(str, solution.keys()), solution.values()))
    values = list()
    for i in var_names:
        values.append(solution.get(i))
    return values


def join_systems(mqsystems):
    """Join polynomial systems into one.

    Key variables aren't prefixed since they must be the same through all
    systems. Joined systems with different keys injected are incorrect.

    """
    var_names = flatten([system.ring().variable_names() for system in mqsystems])
    common_vars = list(set(var_names))
    common_ring = BooleanPolynomialRing(len(common_vars), common_vars, order='degrevlex')
    new_mqsystem = PolynomialSequence([], common_ring)
    for s in mqsystems:
        new_mqsystem.extend(list(s))
    return new_mqsystem

def solve_single_system():
    m = Misty(2)
    plaintext = [1] * 64
    key = [1] * 128
    m.key_schedule(key)
    ciphertext = m.encipher(plaintext, key)

    print 'constructing polynomials...'
    polynomials = m.polynomial_system()
    print 'constructing equations system...'
    F = PolynomialSequence(polynomials)
    print 'injecting variables...'
    F = inject_vars(F, m.vars('IN', 64), plaintext)
```

```
    F = inject_vars(F, m.vars('OUT', 64), ciphertext)
    print 'solving system...'
    result = sat_solve(F)
    print 'Done.'
    key = get_vars(result[0], m.vars('K', 128))
    print key


def solve_joined_systems():
    nrounds = 2
    instances = [Misty(nrounds, prefix) for prefix in ['a', 'b']]

    print 'constructing equation systems...'
    eqsystems = [i.polynomial_system() for i in instances]

    inputs = [[0] * 64, [1] * 64]
    key = [1] * 128
    outputs = list()
    for i, m in enumerate(instances):
        m.key_schedule(key)
        outputs.append(m.encipher(inputs[i], key))
    print 'INPUTS:'
    for i in inputs:
        print hex(instances[0].get_integer(i))
    print 'KEY:'
    print hex(instances[0].get_integer(key))
    print 'OUTPUTS:'
    for i in outputs:
        print hex(instances[0].get_integer(i))

    print '\ninjecting known variables...'
    for i, m in enumerate(instances):
        eqsystems[i] = inject_vars(eqsystems[i], m.vars('IN', 64), inputs[i])
        eqsystems[i] = inject_vars(eqsystems[i], m.vars('OUT', 64), outputs[i
   ])

    for i in eqsystems:
        print i.__str__()

    print '\ncombining equation systems...'
    eqsystem = join_systems(eqsystems)
    print eqsystem.__str__()

    print '\nsolving equation system...'
    solution = sat_solve(eqsystem)[0]
    print solution
```

APPENDIX E
S-BOXES FOR GOST 28147-89 IMPLEMENTATION

$$S_1 = \{4, 10, 9, 2, 13, 8, 0, 14, 6, 11, 1, 12, 7, 15, 5, 3\}$$
$$S_2 = \{14, 11, 4, 12, 6, 13, 15, 10, 2, 3, 8, 1, 0, 7, 5, 9\}$$
$$S_3 = \{5, 8, 1, 13, 10, 3, 4, 2, 14, 15, 12, 7, 6, 0, 9, 11\}$$
$$S_4 = \{7, 13, 10, 1, 0, 8, 9, 15, 14, 4, 6, 12, 11, 2, 5, 3\}$$
$$S_5 = \{6, 12, 7, 1, 5, 15, 13, 8, 4, 10, 9, 14, 0, 3, 11, 2\}$$
$$S_6 = \{4, 11, 10, 0, 7, 2, 1, 13, 3, 6, 8, 5, 9, 12, 15, 14\}$$
$$S_7 = \{13, 11, 4, 1, 3, 15, 5, 9, 0, 10, 14, 7, 6, 8, 2, 12\}$$
$$S_8 = \{1, 15, 13, 0, 5, 7, 10, 4, 9, 2, 3, 14, 6, 11, 8, 12\}$$

APPENDIX F
LIST OF PUBLICATIONS

1. Oliynykov R. V., Kiyanchuk R. I. Perspective Symmetric Block Cipher optimized for Hardware Implementation // 6-th International Conference "Dependable Systems, Services & Technologies (DESSERT'12)". 2012.
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4. Kiyanchuk R. I. Differential analysis of S-functions [In Russian] // Scientific youth researching for European integration / Kharkiv university of banking. Kharkiv, 2012. Electronic resource on CD-ROM.
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6. Kiyanchuk R. I. Comparative analysis of IDEA-like Block Symmetric Ciphers [In Ukrainian] // International Conference "Computer Engineering" / Kharkiv National University of Radio Electronics. Vol. 5. Kharkiv, 2011. April. P. 225 – 227.
7. Oliynykov R. V., Kiyanchuk R. I. Perspective Symmetric Block Cipher Optimized for Hardware Implementation [In Russian] // "Telecommunication Systems and Technologies" / Kharkiv National University of Radio Electronics. Vol. II. Kharkiv, Ukraine, 2011. October. P. 321 – 330.
8. Oliynykov R. V., Kiyanchuk R. I. Usage of T-functions in Symmetric Cryptographic Transformations [In Russian] // "Perspectives of Information and transport-customs technologies in customs affairs, external economic industry and organizations management" / Kharkiv National University of Radio Electronics. Dnipropetrovs'k, 2011. December. P. 213 – 215. Section 2.
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11. Kiyanchuk R. I. Algebraic cryptanalysis of GOST 28147-89 Radioelectronics and youth in XXI century / Kharkiv National University of Radio and Electronics. Kharkiv, 2013. P. 119 – 120.