Sampling and Aliasing

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Abstract

This article is about sampling and aliasing problems that I have encountered while studying numerical simulation of wave optics with computer to implement computer generated holography (CGH).

1 The Nyquist-Shannon sampling theorem

1.1 Statement of the theorem

If a function $x(t)$ contains no frequencied higher than B hertz, then it can be completely determined from its ordinates at a sequence of points spaced lass than $1/(2B)$ seconds apart.

We can rewrite the above theorem with the following expressions : $f_s \geq 2B$. In other words, when we denote a sampling interval as Δx and a period of an original signal as λ, the theorem is : $\Delta x < (\lambda/2)$. Intuitively, it says that we need to sample the original signal sufficiently densely in order to represent the signal without loss of information.

1.2 Motivation

The sampling theorem is required to avoid a type of distortion called *aliasing*.

2 Aliasing

2.1 Definition

In signal processing, aliasing is the overlapping of frequency components[1]. For example, suppose we sample a high frequency(f_1) signal $x_1(t)$ with a sampling frequency f_s lower then the Nyquist frequency($f_s < f_{Nyquist}$). And denote the resulting sampled signal $x_1[n]$. The problem is, there is a lower frequency(f_2) signal $x_2(t)$ such that when we sample $x_2(t)$ with the same sampling frequency f_s, the sampled signal $x_2[n]$ becomes identical to $x_1[n]$: $x_1[n] = x_2[n]$.

In other words, when we sample a signal sparsely, then high-frequency information is lost and thus appears as a lower-frequency signal.

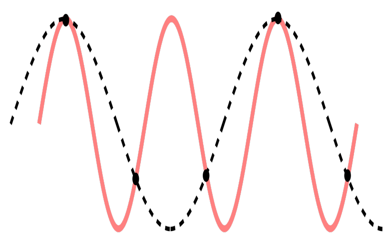

Figure 1: Aliasing. red line represents original signal and dotted line represents sampled signal. You can see the original signal is sampled as a lower frequency signal.

2.2 Mathematical description

References

[1] Nyquist–Shannon sampling theorem. In Wikipedia, The Free Encyclopedia.