1.4 Vector Waves

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August 2024

1 Vector waves

EM fields $E(\vec{r})$ and $H(\vec{r})$ are vector fields. When the fields propagate, they're called the *vector waves*.

1.1 General vector plane wave

The vector plane wave is a wave whose each components are plane waves :
$\vec{E} = \vec{E}(\vec{r}\cdot\vec{s} - vt)$ and $\vec{H} = \vec{H}(\vec{r}\cdot\vec{s} - vt)$. Define $u \equiv \vec{r}\cdot\vec{s} - vt$. Write $\dot{\vec{E}}$ for time derivative and $\vec{E}'$ for u derivative. Then,

$$\dot{\vec{E}} = \frac{\partial}{\partial t}\vec{E}(u) = \frac{\partial u}{\partial t}\frac{\partial}{\partial u}\vec{E}(u) = -v\vec{E}'$$

$$(\nabla \times \vec{E})_x = \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = \frac{\partial u}{\partial y}\frac{\partial E_z}{\partial u} - \frac{\partial u}{\partial z}\frac{\partial E_y}{\partial u} = s_y E_z' - s_z E_y' = (\vec{s} \times \vec{E}')_x$$

From [1.1 Maxwell Equations], with material equations, we have

$$\vec{s} \times \vec{H}' + \sqrt{\frac{\epsilon}{\mu}}\vec{E}' = 0$$

$$\vec{s} \times \vec{E}' - \sqrt{\frac{\mu}{\epsilon}}\vec{H}' = 0$$

Integrate with u ignoring integration constants :

$$\boxed{\vec{E} = -\sqrt{\frac{\mu}{\epsilon}}\vec{s} \times \vec{H}}$$

$$\boxed{\vec{H} = \sqrt{\frac{\epsilon}{\mu}}\vec{s} \times \vec{E}}$$

Note : (1) $\vec{E}$ and $\vec{H}$ are perpendicular. (2) $\vec{E}\cdot\vec{s} = \vec{H}\cdot\vec{s} = 0$. So, $\vec{s}$ is the propagation direction. (3) Considering magnitude, $\sqrt{\mu}H = \sqrt{\epsilon}E$.

For the above waves, we can calculate energy flow through unit area in unit time. Denote v be velocity of the waves and noting that $w_e = w_m$ so $w = w_e + w_m = \epsilon\vec{E}^2 = \mu\vec{H}^2 \Rightarrow \vec{S} \equiv \vec{E} \times \vec{H} = EH\vec{s} = vw\vec{s}$. So, the poynting vector $\vec{S}$ corresponds to the energy flow for the vector plane waves.