1.1 Maxwell Equations

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1 Maxwell's equations

$$\nabla \times \vec{H} - \frac{\partial}{\partial t}\vec{D} = \vec{j} \tag{1}$$

$$\nabla \times \vec{E} + \frac{\partial}{\partial t}\vec{B} = 0 \tag{2}$$

$$\nabla \cdot \vec{D} = \rho \tag{3}$$

$$\nabla \cdot \vec{B} = 0 \tag{4}$$

1.1 Continuity equation

$\nabla \cdot (1)$ and using (div curl = 0) : $-\frac{\partial}{\partial t}(\nabla \cdot \vec{D}) = \nabla \cdot \vec{j} \rightarrow$

$$\boxed{\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{j} = 0}$$

1.2 Material equations

To determine $\vec{E}, \vec{H}, \vec{D}, \vec{B}$ from $\rho, \vec{j}$, we need *material equations*.

(Example) Time-harmonic, slow, isotropic case :

$$\vec{j} = \sigma \vec{E} \tag{5}$$

$$\vec{D} = \epsilon \vec{E} \tag{6}$$

$$\vec{B} = \mu \vec{H} \tag{7}$$

1.3 Boundary conditions

(1) Normal to the boundary surface :

$$\vec{n}_{12} \cdot (\vec{B}^{(1)} - \vec{B}^{(2)}) = 0 \tag{8}$$

$$\vec{n}_{12} \cdot (\vec{D}^{(1)} - \vec{D}^{(2)}) = \rho \tag{9}$$

(2) Parallel to the boundary surface :

$$\vec{n}_{12} \times (\vec{E}^{(1)} - \vec{E}^{(2)}) = 0 \tag{10}$$

$$\vec{n}_{12} \times (\vec{H}^{(1)} - \vec{H}^{(2)}) = \vec{j} \tag{11}$$