

1.1 Electromagnetic Fields

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1 Maxwell's equations

$$\nabla \times \vec{H} - \frac{\partial}{\partial t} \vec{D} = \vec{j} \quad (1)$$

$$\nabla \times \vec{E} + \frac{\partial}{\partial t} \vec{B} = 0 \quad (2)$$

$$\nabla \cdot \vec{D} = \rho \quad (3)$$

$$\nabla \cdot \vec{B} = 0 \quad (4)$$

1.1 Continuity equation

$\nabla \cdot (1)$ and using $(\text{div curl} = 0) : -\frac{\partial}{\partial t}(\nabla \cdot \vec{D}) = \nabla \cdot \vec{j} \rightarrow$

$$\boxed{\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{j} = 0}$$

1.2 Material equations

To determine $\vec{E}, \vec{H}, \vec{D}, \vec{B}$ from ρ, \vec{j} , we need *material equations*.

(Example) Time-harmonic, slow, isotropic case :

$$\vec{j} = \sigma \vec{E} \quad (5)$$

$$\vec{D} = \epsilon \vec{E} \quad (6)$$

$$\vec{B} = \mu \vec{H} \quad (7)$$

1.3 Boundary conditions

(1) Normal to the boundary surface :

$$\vec{n}_{12} \cdot (\vec{B}^{(1)} - \vec{B}^{(2)}) = 0 \quad (8)$$

$$\vec{n}_{12} \cdot (\vec{D}^{(1)} - \vec{D}^{(2)}) = \rho \quad (9)$$

(2) Parallel to the boundary surface :

$$\vec{n}_{12} \times (\vec{E}^{(1)} - \vec{E}^{(2)}) = 0 \quad (10)$$

$$\vec{n}_{12} \times (\vec{H}^{(1)} - \vec{H}^{(2)}) = \vec{j} \quad (11)$$

References

- [1] Born, M., Wolf, E., Bhatia, A. B. (2019). Principles of Optics: 60th Anniversary Edition. United Kingdom: Cambridge University Press.