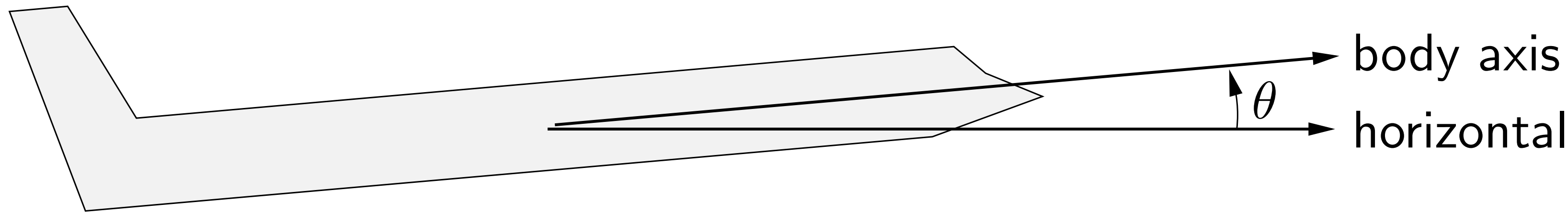
Lecture 14
Example: Aircraft dynamics

- longitudinal aircraft dynamics
- wind gust & control inputs
- linearized dynamics
- steady-state analysis
- eigenvalues & modes
- impulse matrices

Longitudinal aircraft dynamics



variables are (small) deviations from operating point or *trim conditions*

state (components):

- u: velocity of aircraft along body axis
- v: velocity of aircraft perpendicular to body axis (down is positive)
- θ: angle between body axis and horizontal (up is positive)
- $q = \dot{\theta}$: angular velocity of aircraft (pitch rate)

Inputs

disturbance inputs:

- u_w: velocity of wind along body axis
- v_w: velocity of wind perpendicular to body axis

control or actuator inputs:

- δ_e: elevator angle ($\delta_e > 0$ is down)
- δ_t: thrust

Linearized dynamics

for 747, level flight, 40000 ft, 774 ft/sec,

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} -.003 & .039 & 0 & -.322 \\ -.065 & -.319 & 7.74 & 0 \\ .020 & -.101 & -.429 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} u - u_w \\ v - v_w \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} .01 & 1 \\ -.18 & -.04 \\ -1.16 & .598 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_e \\ \delta_t \end{bmatrix}$$

- units: ft, sec, crad $(= 0.01\text{rad} \approx 0.57^\circ)$
- matrix coefficients are called *stability derivatives*

outputs of interest:

- aircraft speed u (deviation from trim)
- climb rate $\dot{h} = -v + 7.74\theta$

Steady-state analysis

DC gain from $(u_w, v_w, \delta_e, \delta_t)$ to $(u, \dot{h})$:

$$H(0) = -CA^{-1}B + D = \begin{bmatrix} 1 & 0 & 27.2 & -15.0 \\ 0 & -1 & -1.34 & 24.9 \end{bmatrix}$$

gives steady-state change in speed & climb rate due to wind, elevator & thrust changes

solve for control variables in terms of wind velocities, desired speed & climb rate

$$\begin{bmatrix} \delta_e \\ \delta_t \end{bmatrix} = \begin{bmatrix} .0379 & .0229 \\ .0020 & .0413 \end{bmatrix} \begin{bmatrix} u - u_w \\ \dot{h} + v_w \end{bmatrix}$$

- level flight, increase in speed is obtained mostly by increasing elevator ($i.e.$, downwards)

- constant speed, increase in climb rate is obtained by increasing thrust and increasing elevator ($i.e.$, downwards)

(thrust on 747 gives strong pitch up torque)

Eigenvalues and modes

eigenvalues are

$$-0.3750 \pm 0.8818i, \quad -0.0005 \pm 0.0674i$$

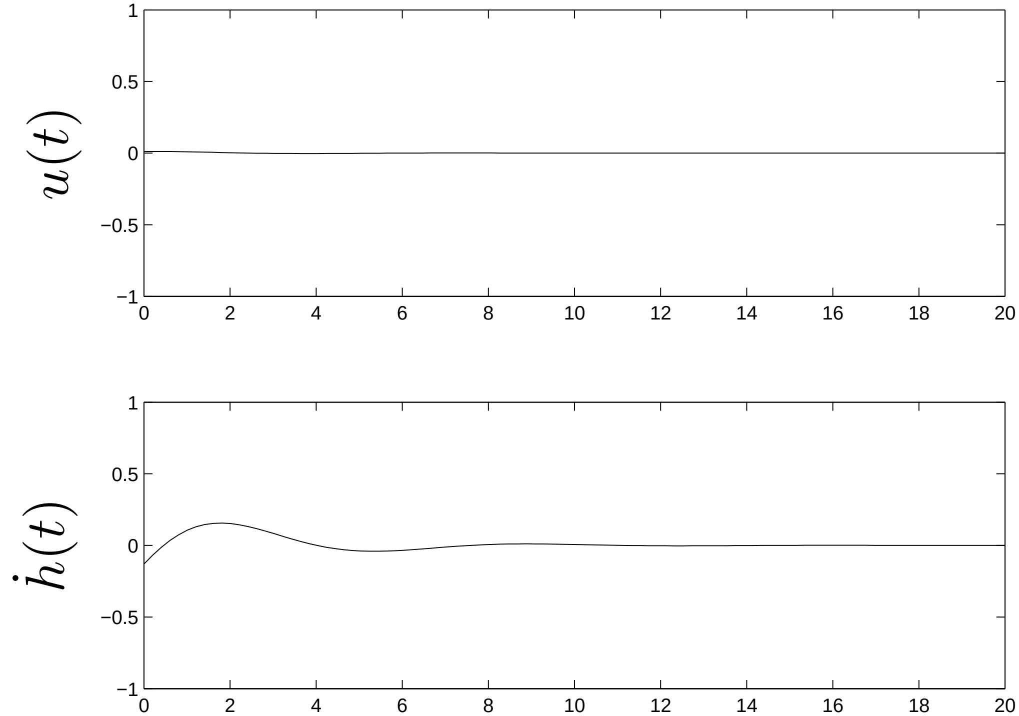
- two complex modes, called *short-period* and *phugoid*, respectively
- system is stable (but lightly damped)
- hence step responses converge (eventually) to DC gain matrix

eigenvectors are

$$
\begin{aligned}
x_{\text{short}} &= \begin{bmatrix} 0.0005 \\ -0.5433 \\ -0.0899 \\ -0.0283 \end{bmatrix} \pm i \begin{bmatrix} 0.0135 \\ 0.8235 \\ -0.0677 \\ 0.1140 \end{bmatrix}, \\
x_{\text{phug}} &= \begin{bmatrix} -0.7510 \\ -0.0962 \\ -0.0111 \\ 0.1225 \end{bmatrix} \pm i \begin{bmatrix} 0.6130 \\ 0.0941 \\ 0.0082 \\ 0.1637 \end{bmatrix}
\end{aligned}
$$

Short-period mode

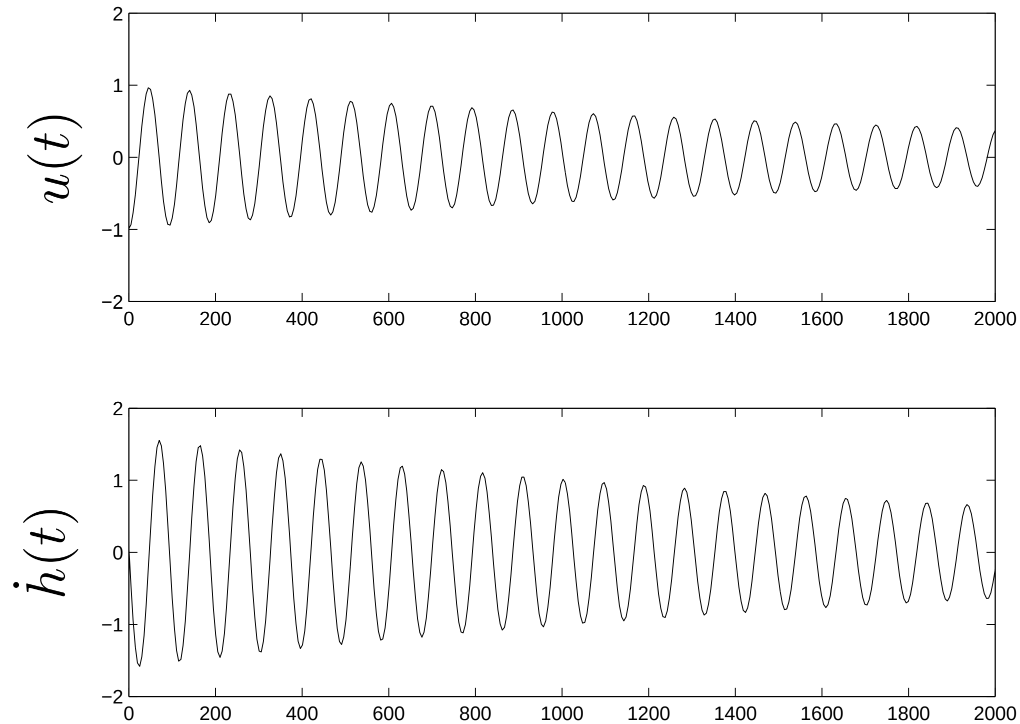
$y(t) = Ce^{tA}(\Re x_{\text{short}})$ (pure short-period mode motion)



- only small effect on speed u
- period ≈ 7 sec, decays in ≈ 10 sec

Phugoid mode

$y(t) = Ce^{tA}(\Re x_{\mathrm{phug}})$ (pure phugoid mode motion)

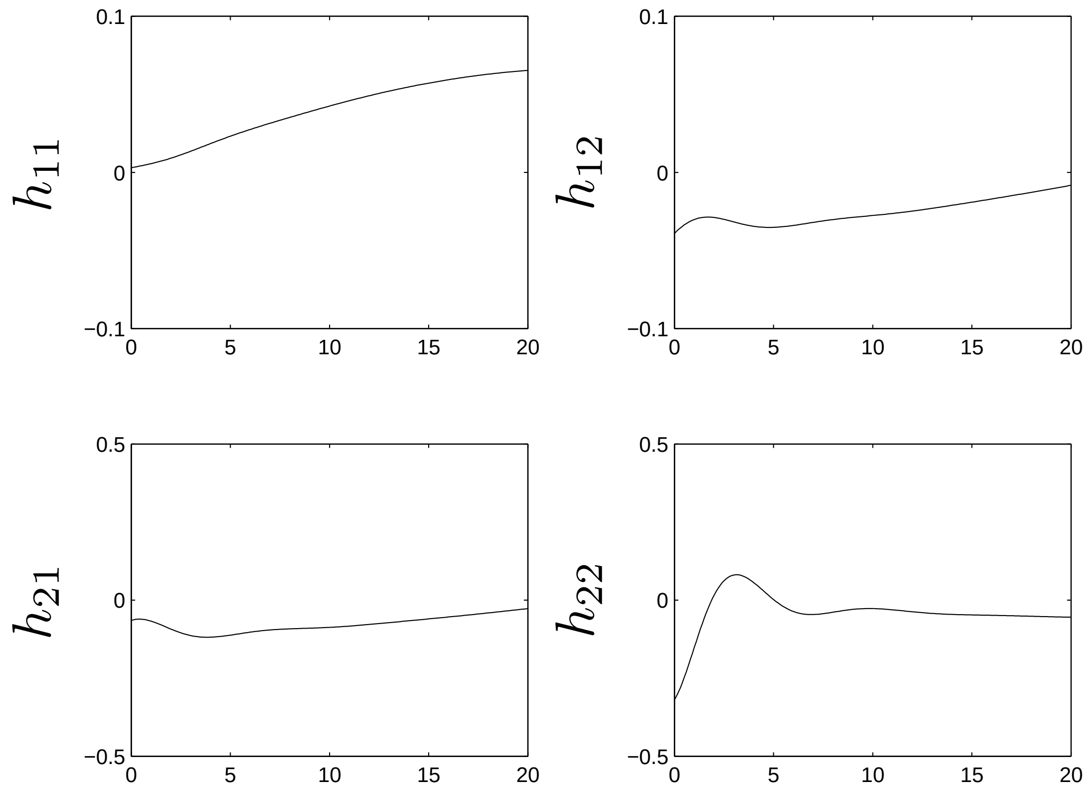


- affects both speed and climb rate
- period ≈ 100 sec; decays in ≈ 5000 sec

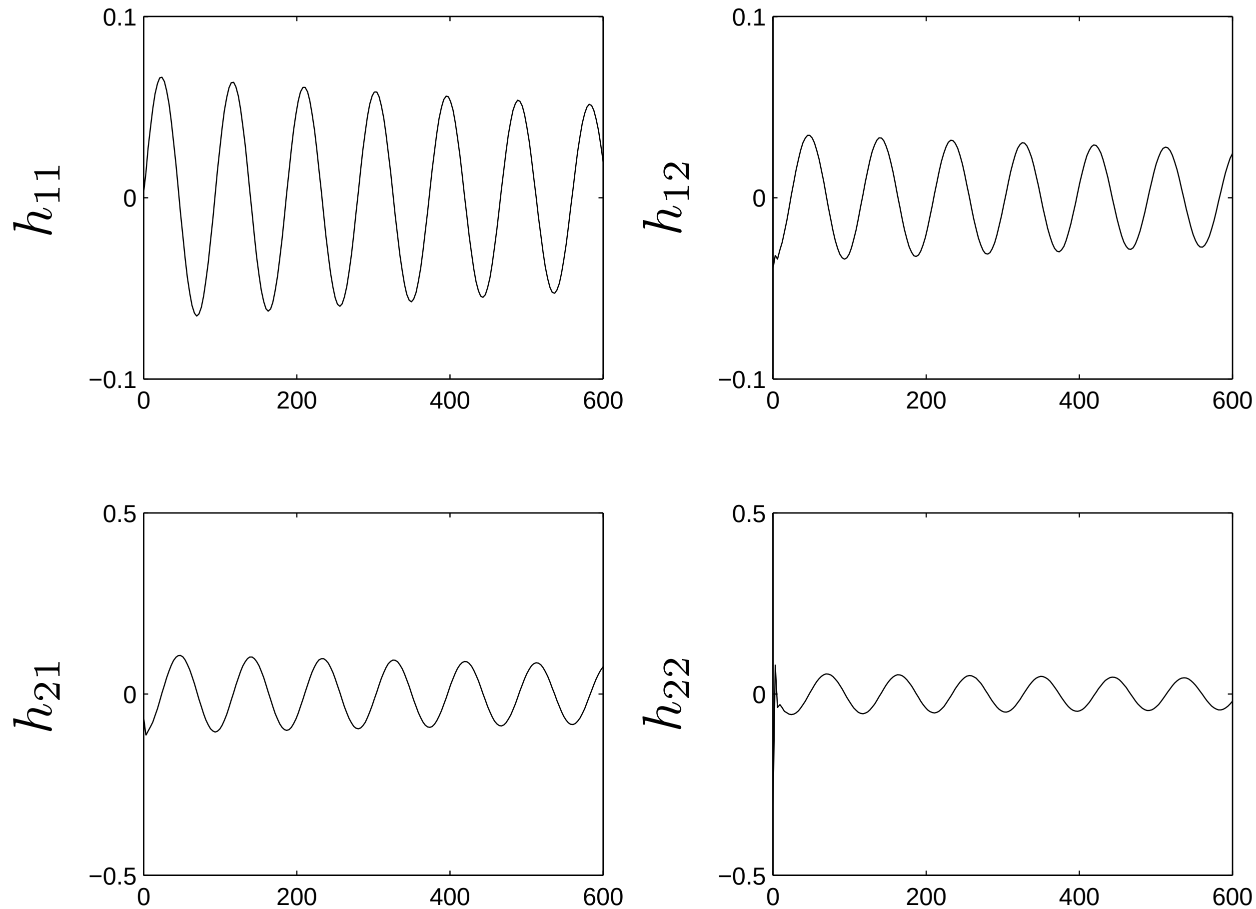
Dynamic response to wind gusts

impulse response matrix from (u_w, v_w) to $(u, \dot{h})$ (gives response to short wind bursts)

over time period $[0, 20]$:



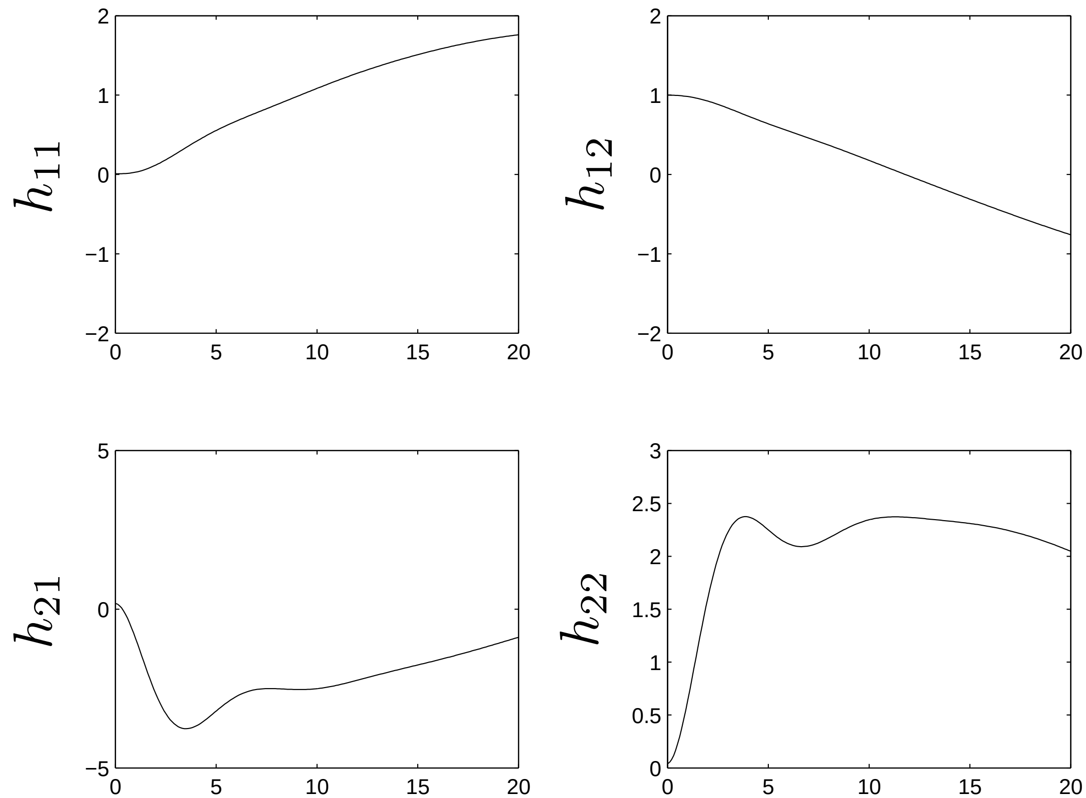
over time period $[0, 600]$:



Dynamic response to actuators

impulse response matrix from (δ_e, δ_t) to $(u, \dot{h})$

over time period $[0, 20]$:



over time period $[0, 600]$: