

# Lecture 1

## Overview

- course mechanics
- outline & topics
- what is a linear dynamical system?
- why study linear systems?
- some examples

# Course mechanics

- all class info, lectures, homeworks, announcements on class web page:

`www.stanford.edu/class/ee263`

course requirements:

- weekly homework
- takehome midterm exam (date TBD)
- takehome final exam (date TBD)

# Prerequisites

- exposure to linear algebra (*e.g.*, Math 104)
- exposure to Laplace transform, differential equations

**not needed**, but might increase appreciation:

- control systems
- circuits & systems
- dynamics

# Major topics & outline

- linear algebra & applications
- autonomous linear dynamical systems
- linear dynamical systems with inputs & outputs
- basic quadratic control & estimation

# Linear dynamical system

*continuous-time* linear dynamical system (CT LDS) has the form

$$\frac{dx}{dt} = A(t)x(t) + B(t)u(t), \quad y(t) = C(t)x(t) + D(t)u(t)$$

where:

- $t \in \mathbf{R}$  denotes *time*
- $x(t) \in \mathbf{R}^n$  is the *state* (vector)
- $u(t) \in \mathbf{R}^m$  is the *input* or *control*
- $y(t) \in \mathbf{R}^p$  is the *output*

- $A(t) \in \mathbf{R}^{n \times n}$  is the *dynamics matrix*
- $B(t) \in \mathbf{R}^{n \times m}$  is the *input matrix*
- $C(t) \in \mathbf{R}^{p \times n}$  is the *output or sensor matrix*
- $D(t) \in \mathbf{R}^{p \times m}$  is the *feedthrough matrix*

for lighter appearance, equations are often written

$$\dot{x} = Ax + Bu, \quad y = Cx + Du$$

- CT LDS is a first order vector *differential equation*
- also called *state equations*, or ‘*m*-input, *n*-state, *p*-output’ LDS

## Some LDS terminology

- most linear systems encountered are *time-invariant*:  $A$ ,  $B$ ,  $C$ ,  $D$  are constant, *i.e.*, don't depend on  $t$
- when there is no input  $u$  (hence, no  $B$  or  $D$ ) system is called *autonomous*
- very often there is no feedthrough, *i.e.*,  $D = 0$
- when  $u(t)$  and  $y(t)$  are scalar, system is called *single-input, single-output* (SISO); when input & output signal dimensions are more than one, MIMO

# Discrete-time linear dynamical system

*discrete-time* linear dynamical system (DT LDS) has the form

$$x(t+1) = A(t)x(t) + B(t)u(t), \quad y(t) = C(t)x(t) + D(t)u(t)$$

where

- $t \in \mathbf{Z} = \{0, \pm 1, \pm 2, \dots\}$
- (vector) signals  $x, u, y$  are *sequences*

DT LDS is a first-order vector *recursion*



# Why study linear systems?

applications arise in **many** areas, *e.g.*

- automatic control systems
- signal processing
- communications
- economics, finance
- circuit analysis, simulation, design
- mechanical and civil engineering
- aeronautics
- navigation, guidance

# Usefulness of LDS

- depends on availability of **computing power**, which is large & increasing exponentially
- used for
  - analysis & design
  - implementation, embedded in real-time systems
- like DSP, was a specialized topic & technology 30 years ago

# Origins and history

- parts of LDS theory can be traced to 19th century
- builds on classical circuits & systems (1920s on) (transfer functions . . . ) but with more emphasis on linear algebra
- first engineering application: aerospace, 1960s
- transitioned from specialized topic to ubiquitous in 1980s (just like digital signal processing, information theory, . . . )

# Nonlinear dynamical systems

many dynamical systems are **nonlinear** (a fascinating topic) so why study **linear** systems?

- most techniques for nonlinear systems are based on linear methods
- methods for linear systems often work unreasonably well, in practice, for nonlinear systems
- if you don't understand linear dynamical systems you certainly can't understand nonlinear dynamical systems

## Examples (ideas only, no details)

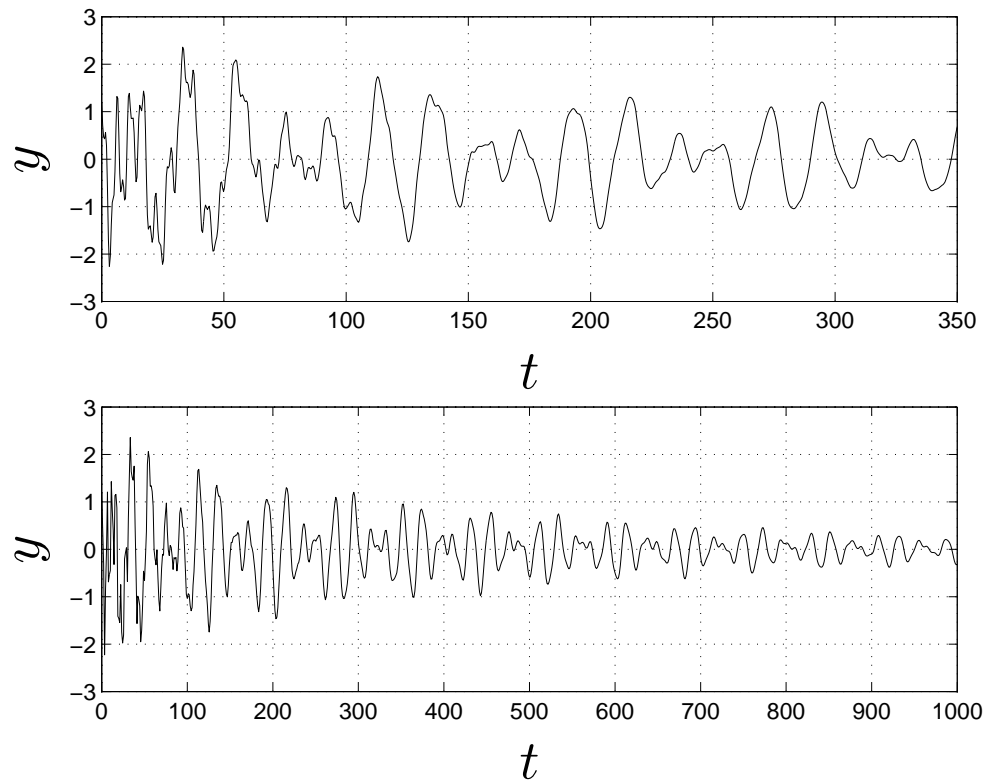
- let's consider a specific system

$$\dot{x} = Ax, \quad y = Cx$$

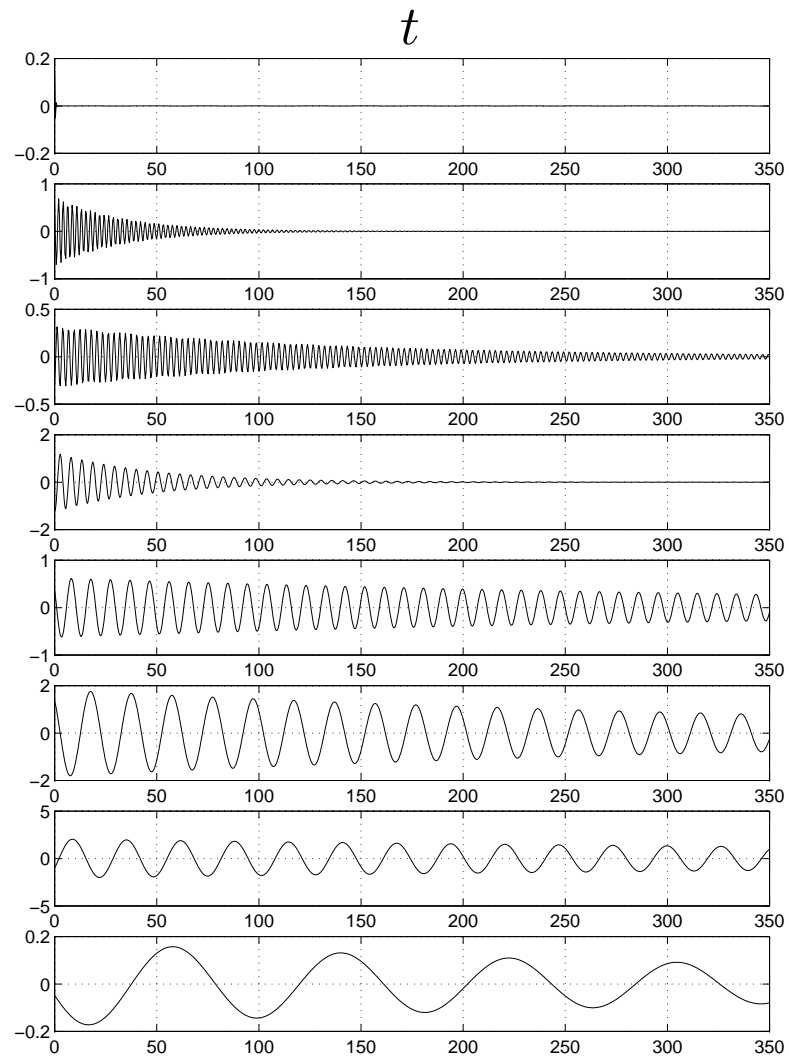
with  $x(t) \in \mathbf{R}^{16}$ ,  $y(t) \in \mathbf{R}$  (a '16-state single-output system')

- model of a lightly damped mechanical system, but it doesn't matter

typical output:



- output waveform is very complicated; looks almost random and unpredictable
- we'll see that such a solution can be decomposed into much simpler (modal) components



(idea probably familiar from ‘poles’)

# Input design

add two inputs, two outputs to system:

$$\dot{x} = Ax + Bu, \quad y = Cx, \quad x(0) = 0$$

where  $B \in \mathbf{R}^{16 \times 2}$ ,  $C \in \mathbf{R}^{2 \times 16}$  (same  $A$  as before)

**problem:** find appropriate  $u : \mathbf{R}_+ \rightarrow \mathbf{R}^2$  so that  $y(t) \rightarrow y_{\text{des}} = (1, -2)$

**simple approach:** consider static conditions ( $u, x, y$  constant):

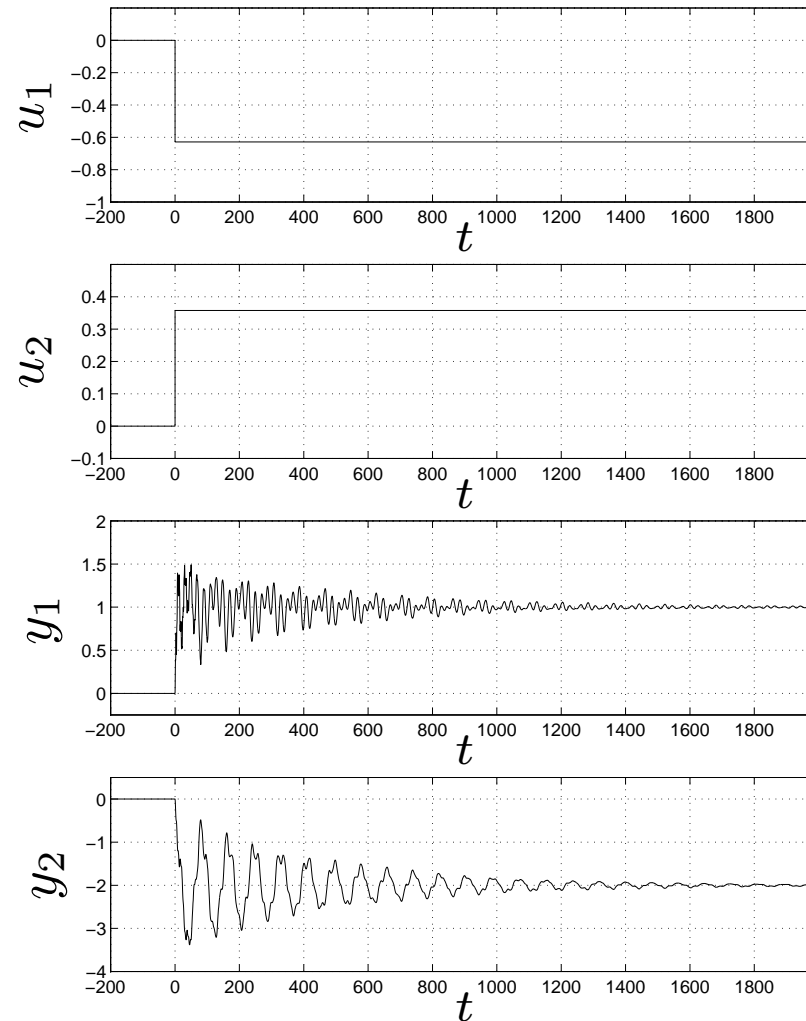
$$\dot{x} = 0 = Ax + Bu_{\text{static}}, \quad y = y_{\text{des}} = Cx$$

solve for  $u$  to get:

$$u_{\text{static}} = (-CA^{-1}B)^{-1} y_{\text{des}} = \begin{bmatrix} -0.63 \\ 0.36 \end{bmatrix}$$

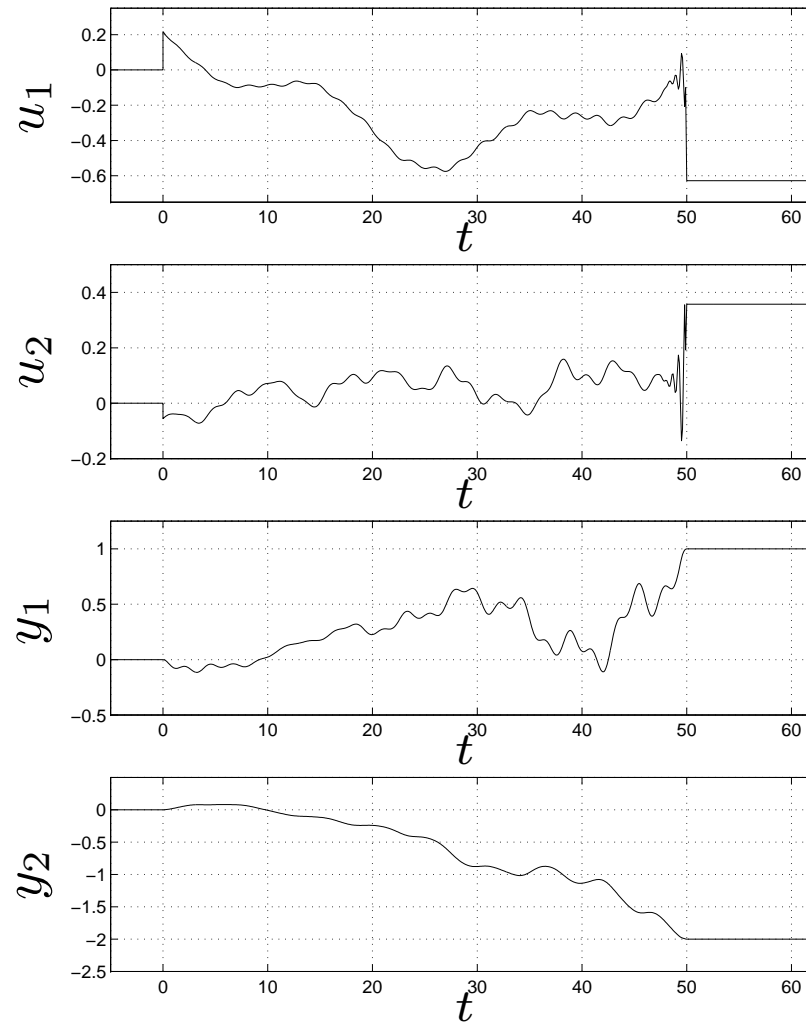


let's apply  $u = u_{\text{static}}$  and just wait for things to settle:



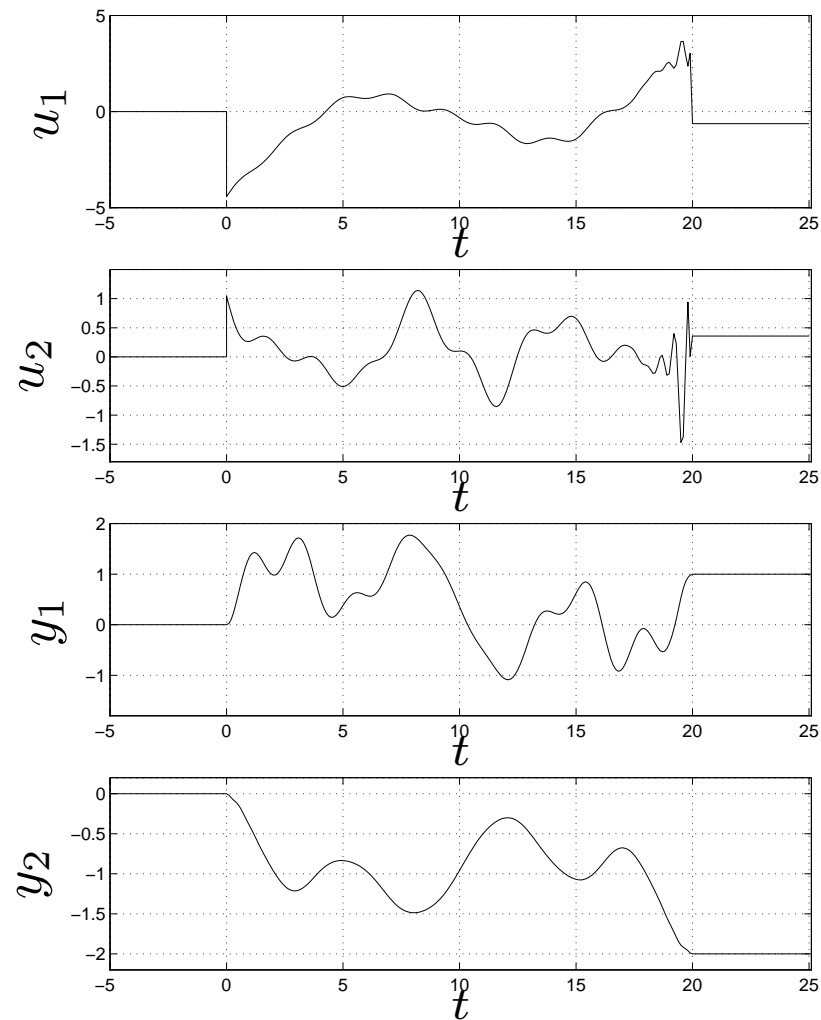
... takes about 1500 sec for  $y(t)$  to converge to  $y_{\text{des}}$

using very clever input waveforms (EE263) we can do much better, *e.g.*



. . . here  $y$  converges *exactly* in 50 sec

in fact by using larger inputs we do still better, *e.g.*

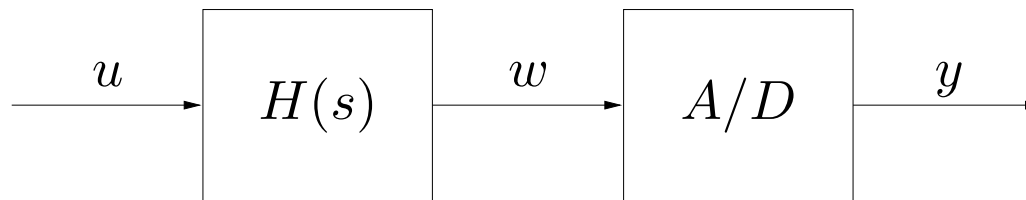


. . . here we have (exact) convergence in 20 sec

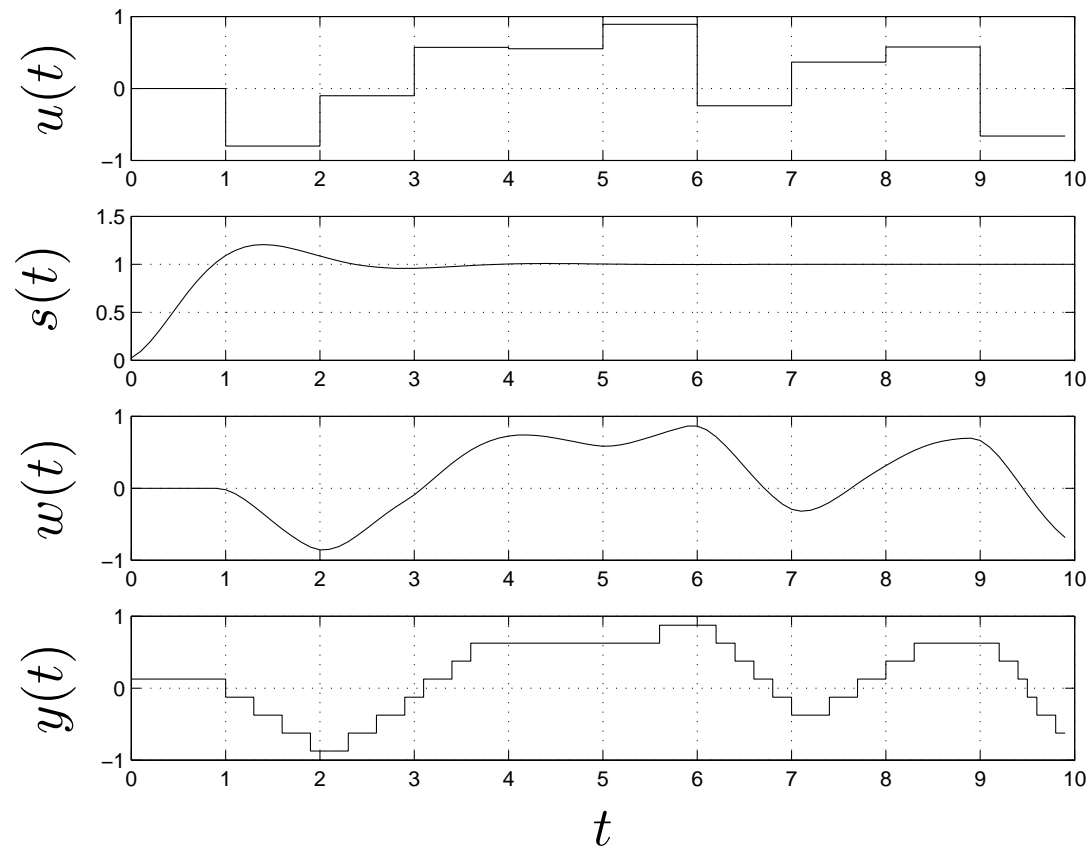
in this course we'll study

- how to synthesize or design such inputs
- the tradeoff between size of  $u$  and convergence time

# Estimation / filtering



- signal  $u$  is piecewise constant (period 1 sec)
- filtered by 2nd-order system  $H(s)$ , step response  $s(t)$
- A/D runs at 10Hz, with 3-bit quantizer



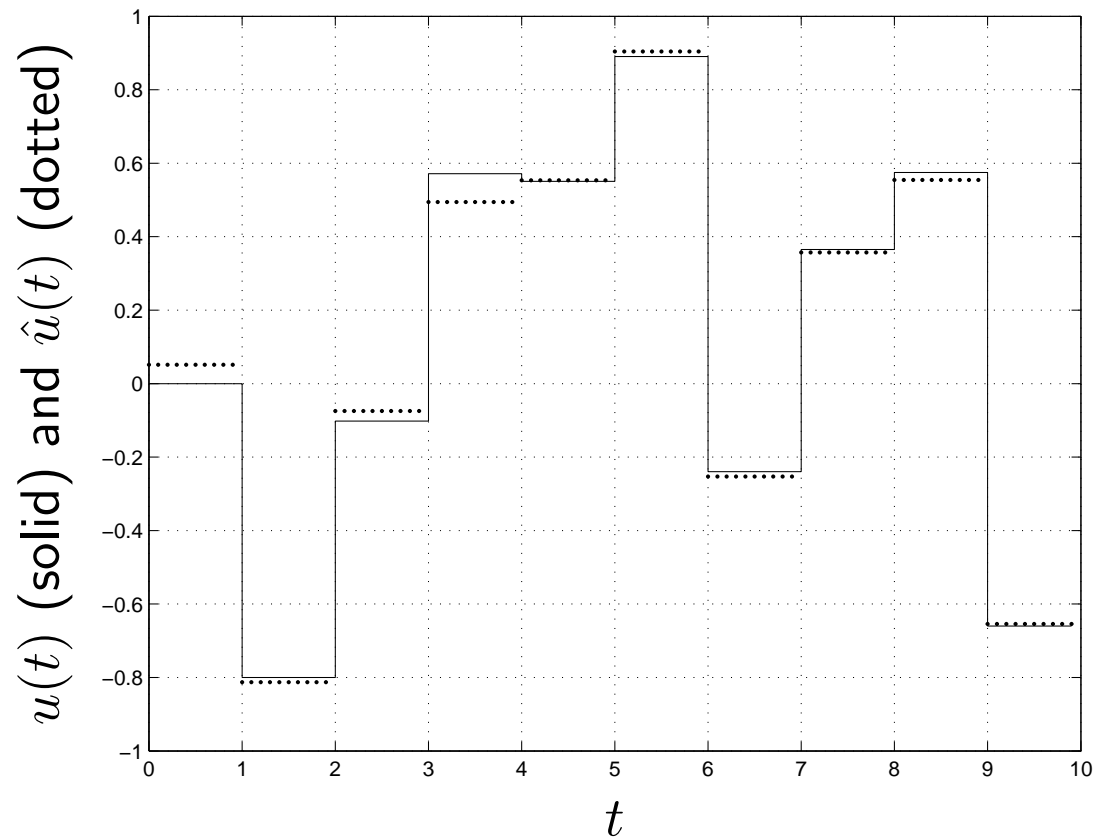
**problem:** estimate original signal  $u$ , given quantized, filtered signal  $y$

simple approach:

- ignore quantization
- design equalizer  $G(s)$  for  $H(s)$  (*i.e.*,  $GH \approx 1$ )
- approximate  $u$  as  $G(s)y$

. . . yields terrible results

formulate as *estimation problem* (EE263) . . .



RMS error 0.03, well **below** quantization error (!)