

# Essential Radio Astronomy Lecture Note

Renkun Kuang

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# Contents

<b>1</b>	<b>Introduction to Radio Astronomy</b>	<b>2</b>
1.1	Related links	2
1.2	Software Preparation	2
1.2.1	AIPS	3
1.2.2	CASA	9
1.3	Introduction to Radio Astronomy	11
1.3.1	The Radio Window	11
1.3.2	The low-frequency cut-off	12
1.3.3	The high-frequency cut-off (absorption)	12
1.3.4	The high-frequency cut-off (emission)	14
1.3.5	Early Radio Astronomy	16
1.3.6	Radio Telescopes and interferometers	18
1.4	Astrophysical applications	19
1.5	LOFAR	20
1.5.1	Response of the LOFAR antenna	20
1.5.2	Power gain	20
1.5.3	Beam solid angle	22
1.5.4	Response of a reflector antenna	23
1.6	Sensitivity	24
1.7	Summary	25
<b>2</b>	<b>(Gentle) Introduction to Interferometry</b>	<b>26</b>
2.1	Adding Interferometer	26
2.2	Multiplying Interferometer	26
2.3	The Effect of Baseline Length	26
2.4	Complex Visibilities	26
2.5	Van Cittert Zernike Function	26
2.6	Visibility	26
2.7	Visibilities and Images	26
2.8	Fourier Components	26
2.9	Conclusions	26

# Chapter 1

## Introduction to Radio Astronomy

### 1.1 Related links

- <https://www.cv.nrao.edu/course/ast534/PDFnewfiles/IntroRadioastro.pdf>
- [https://www.astron.nl/eris2017/Documents/ERIS2017\\_L1\\_McKean.pdf](https://www.astron.nl/eris2017/Documents/ERIS2017_L1_McKean.pdf)  
2017 European Radio Interferometry School, a week of lectures and tutorials on how to achieve scientific results from radio interferometry.  
Topics covered:
  1. Calibration and imaging of continuum, spectral line, and polarisation data;
  2. Low-freq. (LOFAR domain), high-freq. (ALMA/IRAM domain), and VLBI-interferometry;
  3. Extracting information from the data and interpreting the results;
  4. Choosing the most suitable array and observing plan for your project.
- <http://www.radio-astronomy.org/pdf/sara-beginner-booklet.pdf>
- [http://egg.astro.cornell.edu/alfalfa/ugradteam/pdf12/radio\\_lecture\\_jess\\_uat12.pdf](http://egg.astro.cornell.edu/alfalfa/ugradteam/pdf12/radio_lecture_jess_uat12.pdf)
- <http://www.jb.man.ac.uk/tob/course/radio2.html>
- SKA Shanghai 2018

### 1.2 Software Preparation

ref: [2017 European Radio Interferometry School](#).

Examples will be drawn from m-, cm- and mm-wave instruments such as LOFAR,

JVLA, EVN, eMERLIN and ALMA.

CDs for advanced users may include tutorials on using the proposal preparation tools (the Proposal Submission Tool for the VLA, GBT, and VLBA, and the Observing Tool for ALMA) and data reduction software (CASA for the VLA and ALMA, GBTIDL for the GBT, and AIPS for VLBA). [nrao-cd](#)

### 1.2.1 AIPS

- [1996 The AIPSview Astronomy Visualization Tools, PDF](#)

AIPSview is a set of two new software tools for visual data analysis being developed by the radio astronomy group at the University of Illinois as part of the AIPS++ project. The tools provide a wide range of functionality for the display and analysis of 2D and 3D astronomical data sets. In this paper we describe how to obtain further information about AIPSview on the WWW including how to obtain executable and source code, and discuss the current functionality of AIPSview and future development plans.

- <http://www.aips.nrao.edu/index.shtml>  
[What is AIPS](#)

The Astronomical Image Processing System is a software package for calibration, data analysis, image display, plotting, and a variety of ancillary tasks on Astronomical Data. It comes from the National Radio Astronomy Observatory.

Download install.pl via ftp and the "perl install.pl -n" to install the software.

ftp host website: <ftp://ftp.aoc.nrao.edu/pub/software/aips>  
[linux下登陆FTP](#)

In terminal, using following command to login:

```
lftp ftp://ftp.aoc.nrao.edu/pub/software/aips
```

Download files:

要下载文件首先得设置用于存放下载文件的本地目录。

命令：lcd 本地目录

然后进入待下载文件的目录。根据下载文件类型的不同，命令也不同。

单个文件

命令：get file

get -c file 可以进行断点续传

批量文件

命令：mget \*.txt 批量下载目录下的txt文件

mget -c \*.txt 断点续传加上批量下载

整个目录

命令：mirror aaa/

```
lftp ftp.aoc.nrao.edu:/pub/software/aips> lcd ./
lcd ok, local cwd=/home/anything/THU/astro/software/aips
lftp ftp.aoc.nrao.edu:/pub/software/aips> mirror 31DEC19
`31DEC19.tar.gz' at 1206184 (0%) 47.0K/s eta:58m [Receiving data]
[0] 0:lftp*
```

```
===== AIPS 31DEC19 Install Wizard=====

Screen 11: FINAL REVIEW before installing!
-----
This is your last, best hope for checking the settings before
committing to the install. Please check these settings, and
make sure they are what you want:

  AIPS_ROOT (screen 3): /home/anything/THU/astro/software/aips/31DEC19
  Group (screen 4): anything
  Group Write (screen 4): YES
  Architecture (screen 5): LNX64
  Site name (screen 5): ANY
  AIPS hosts (screen 6): LOCALHOST
  Data areas (screen 7): /home/anything/THU/astro/software/aips/31DEC19/DATA/LOCALHOST_1
  Printers (screen 8): Paper type (screen 8): A4
  Tape drives (screen 9):
  Tape hosts (screen 9): 127.0.0.1
  Advanced (screen 10): (not listed here)
```

```
/home/anything/THU/astro/software/aips/31DEC19/31DEC19/LNX64/SYSTEM/ANY/MAKE.MNJ: 69: /home/anything/
THU/astro/software/aips/31DEC19/31DEC19/LNX64/SYSTEM/ANY/MAKE.MNJ: /opt/local/compilers/gcc-6/bin/gcc
: not found
MAKE.MNJ - UPDLSTDAT compiled
MAKE.MNJ - all .OLD files created
MAKE.MNJ - LAST*.DAT files created with begin date of 20190903.000000
MAKE.MNJ - Done.
MAKE.MNJ - The MNJ now uses cvs (http://www.cvshome.org) for updates;
MAKE.MNJ - looking to see if I can find a copy of it...
MAKE.MNJ - I can not seem to find it, either it is not here or it is
MAKE.MNJ - here, but it is installed in a place that is not in your
MAKE.MNJ - search path. If it is not here, you should install it and
MAKE.MNJ - run MAKE.MNJ again. Otherwise, please tell me where you
MAKE.MNJ - put it.
MAKE.MNJ - Full path to cvs: : |
```

install cvs: <http://www.cvshome.org>, [instruction](#)

[installing AIPS failed.](#)

[2019 install instruction](#)

[Installation instructions for Training Week 1](#)

# Installation instructions for Training Week 1

There are two courses in training week 1: radio astronomy and telescope systems, and radio interferometry. Both of these have a hands-on component, and software should be pre-installed on laptops beforehand. The software packages needed are IPython for the IPython notebook component of the radio astronomy course, and AIPS or CASA for radio interferometry. Either MacOS or Linux is needed (i.e. **not Windows**). On Windows laptops or PCs it should be possible to install a VirtualBox <https://www.virtualbox.org/> with Linux as the guest operating system, or alternatively have Linux as a dual boot if you are planning to use it a lot.

## 1. Radio astronomy

This course uses IPython notebooks, so IPython is needed. On Linux systems with `pip`, you may be able to get and install IPython with `pip install ipython`. (IPython does run on Windows in theory, but we have not tested the notebooks with Windows, and MacOS/Linux is needed in any case for part 2).

Probably the easiest (and on some systems the only) way to install IPython is via the installation of Jupyter, which is available on

[jupyter.readthedocs.org/en/latest/install.html](http://jupyter.readthedocs.org/en/latest/install.html)

This installs without problems provided that at least Python 2.7 is present. If you do not have Python 2.7, then the best course is to install **anaconda** - a link to installation instructions for this is given on the same webpage. Once you have anaconda, any other packages can be installed with the `conda install` command. In some cases (e.g. `jupyter`) this can be done directly: `conda install jupyter`. In other cases, all you should need to do is to google `conda [package-name]` for any package that appears to be missing.

You can test your installation by downloading the file

<http://www.jb.man.ac.uk/~njj/test.ipynb>

and running it with `jupyter notebook test.ipynb`. You should get a cell of commands with a blue line to the left. Clicking inside the cell and then selecting `cell` and then `Run cell` should produce a plot with blue dots.

## 2. Radio interferometry

### *2.1 Major packages*

There are currently two major software packages for radio interferometry, AIPS and CASA. AIPS is an older, Fortran-based system; CASA is a newer, C++/Python-based system which will eventually supersede AIPS. CASA has a fuller suite of imaging features, including much better wide-field mapping and imaging algorithms, but AIPS has some features useful for long-baseline interferometry.

AIPS is the primary software package for VLBI, e-MERLIN and LOFAR (long baselines). CASA, or CASA-based, systems are the primary system for LOFAR (other than long baselines), the JVLA, and ALMA. The syntax of the two packages is not hugely different, and there is a translation available on [https://casaguides.nrao.edu/index.php?title=aips-to-casa\\_Cheat\\_Sheet](https://casaguides.nrao.edu/index.php?title=aips-to-casa_Cheat_Sheet).

The course will be bilingual (in AIPS and CASA!) so you can choose which one to install. Both are interoperable with Python: CASA natively, and AIPS by an interface (ParselTongue, which is not covered in the first week's course).

Both are distributed as binaries and should be relatively straightforward to install on any version of Linux or any version of MacOS. Please install and test one (or both) as below if you are intending to follow the course. **Note: neither AIPS nor CASA runs on Windows.**

Both packages are available as binary installations for MacOS and Linux. AIPS can be downloaded from

<http://www.aips.nrao.edu/install.shtml>

and CASA from

[http://casa.nrao.edu/casa\\_obtaining.shtml](http://casa.nrao.edu/casa_obtaining.shtml)

In both cases, the installation should be relatively painless. For CASA, the installation comes as a unix tar file, which can be unzipped in any desired directory. If you start the executable `casapy`, which should be in the top directory, this should start CASA and produce a log window.

For AIPS, the installation is slightly more complicated, but a Perl install wizard guides you through the steps. If you start the executable `START_AIPS`, which should be in the top directory, AIPS should start up and invite you to type in an AIPS number - any number larger than 10 should be fine. If you start it with the argument `tv=local`, then a black TV window should pop up as well.

## *2.2 Difmap*

There is also a third useful package which does only some tasks in radio interferometry, but does them very well and very interactively. It is particularly useful for long-baseline interferometry. It's available from <ftp://ftp.astro.caltech.edu/pub/difmap/difmap.html>. It requires the plotting package PGPLOT, which is available from <http://www.astro.caltech.edu/~tjp/pgplot/>.

If there are any problems, I'm happy to try and help: [neal.jackson@manchester.ac.uk](mailto:neal.jackson@manchester.ac.uk) although I may need to consult the local IT support here in complicated cases. Please forward screenshots and/or as much detail of error messages as possible in these cases.



- [天文博客](#)
- [How to build AIPS on Ubuntu](#)
- [The AIPS++ Project](#)

Finally successfully installed:

```
AipsWiz: =====> We're DONE!  Let's have a nice Banana Split! <=====
AipsWiz: ***** READ CAREFULLY *****
AipsWiz: ***** READ CAREFULLY *****
AipsWiz: Services should be defined either in /etc/services
        or your YP/NIS services map (all tcp services)
This may require sudo or root privileges
AipsWiz: ***** READ CAREFULLY *****
AipsWiz: Here are the final setup instructions for running AIPS

    1.  Reference the LOGIN.SH file in your .profile file
        or perhaps your .bashrc file
        (dot it now too, via ". ./LOGIN.SH")

    2.  Check that it works:

        aips notv tpok

        (this will not start a TV or tape servers).
        Try 'print 2 + 2' for a very basic test.

    3.  Make a cron entry for the do_daily.LOCALHOST file
        that the MAKE.MNJ created, so you can run the
        AIPS 'midnight job'.  This is optional but
        strongly recommended.

AipsWiz: That's it.  You should now have the latest AIPS!  Enjoy.

# anything @ anything-ThinkPad-E470c in ~/.aips [13:40:30]
$ |
```

```
# anything @ LOCALHOST in ~/.aips [13:48:17]
$ aips notv tpok
START_AIPS: Your initial AIPS printer is the
START_AIPS: - system name , AIPS type

START_AIPS: User data area assignments:
DADEVs.PL: This program is untested under Perl version 5.026
  (Using global default file /home/anything/.aips/DA00/DADEVs.PL)
  Disk 1 (1) is /home/anything/.aips/DATA/LOCALHOST_1

Tape assignments:
  Tape 1 is REMOTE
  Tape 2 is REMOTE

START_AIPS: Assuming TPMON daemons are running or not used (you said TPOK)
Starting up 31DEC18 AIPS with normal priority
Begin the one true AIPS number 1 (release of 31DEC18) at priority = 0
AIPS 1: You are NOT assigned a TV device or server
AIPS 1: You are NOT assigned a graphics device or server
AIPS 1: Enter user ID number
?2
AIPS 1: 31DEC18 AIPS:
AIPS 1: Copyright (C) 1995-2019 Associated Universities, Inc.
AIPS 1: AIPS comes with ABSOLUTELY NO WARRANTY;
AIPS 1: for details, type HELP GNUGPL
AIPS 1: This is free software, and you are welcome to redistribute it
AIPS 1: under certain conditions; type EXPLAIN GNUGPL for details.
AIPS 1: Previous session command-line history recovered.
AIPS 1: TAB-key completions enabled, type HELP READLINE for details.
AIPS 1: Recovered POPS environment from last exit
>print 2+2
AIPS 1: 4
>|
```

## TWO VLBI TUTORIALS

From <http://www.aips.nrao.edu/index.shtml>

Two extensive tutorials on VLBI data reduction in AIPS have been prepared, complete with data sets, introductory material about AIPS, and detailed instructions. They are:

- [simple VLBA project including self-calibration](#)
- [spectral-line VLBA project plus astrometry](#)

Users new to, or rusty in, VLBI data reduction are encouraged to try these tutorials. Appendix C and Chapter 9 of the [AIPS CookBook](#) are also recommended.

### 1.2.2 CASA

[CASA-obtaining](#)

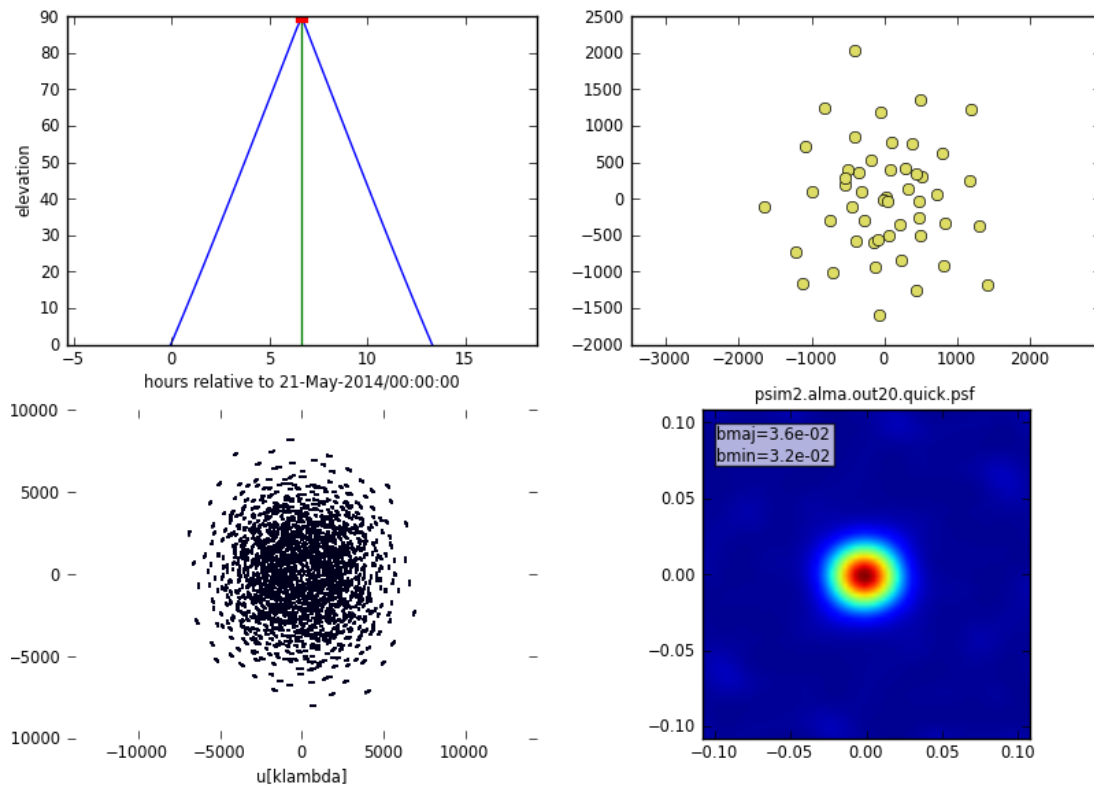
[CASA Documentation Homepage](#)

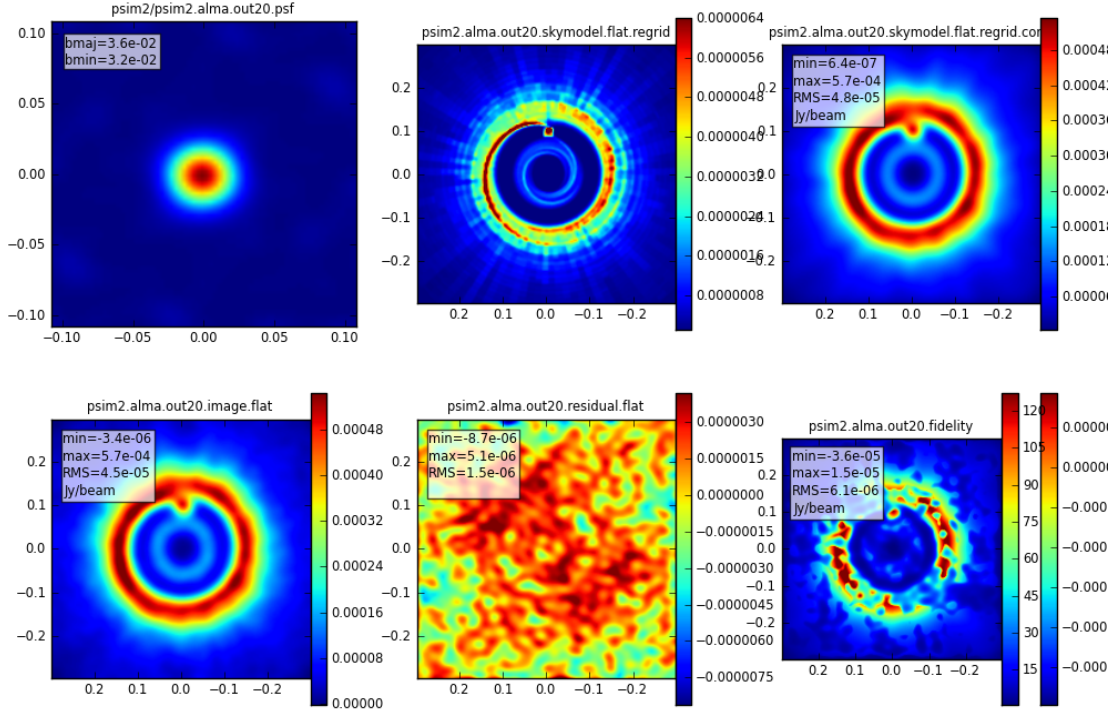
[CASA homepage](#)

[CASA Guides](#)

## Protoplanetary Disk Simulation Using CASA

[Reference](#)





## 1.3 Introduction to Radio Astronomy

The following of this note file is mainly center on [2017 European Radio Interferometry School](#)(which aims to give a general introduction to radio astronomy, focusing on the issues that you must consider for single element telescopes that make up an interferometer) materials.

### 1.3.1 The Radio Window

$$\nu \sim 10^7 Hz - 10^{12} Hz, \quad \lambda \sim 10m - 0.1mm$$

The observing window is constrained by atmospheric absorption / emission and refraction.

1. Charged particles in the ionosphere reflects radio waves back into space at  $< 10$  MHz.
2. Vibrational transitions of molecules have similar energy to infra-red photons and absorb the radiation at  $> 1$  GHz (completely by  $\sim 300$  GHz)

### 1.3.2 The low-frequency cut-off

The ionosphere consists of a plasma of charged particles (conducting layers), that has an effective refractive index of,

$$n^2 = 1 - \frac{\omega_p^2}{\omega^2} = 1 - \frac{\lambda^2}{\lambda_p^2}$$

where, the plasma frequency is defined as,

$$\nu_p[\text{Hz}] = \frac{\omega_p}{2\pi} = \left( \frac{N_e e^2}{4\pi^2 \epsilon_0 m} \right)^{1/2} = 8.97 \times 10^3 \sqrt{\frac{N_e}{[\text{cm}^{-3}]}}$$

when  $\omega < \omega_p \rightarrow n^2 < 0$ , there is no propagation, i.e. total reflection. (i.e. low-frequency cutoff)

Worked example: What is the cut-off frequency for LOFAR observations carried out when the electron density is  $N_e = 2.5 \times 10^5 \text{cm}^{-3}$  (night time) and  $N_e = 1.5 \times 10^6 \text{cm}^{-3}$  (day time)?

$$\nu_p[\text{Hz}] = 4.5 \text{ MHz (night time)}$$

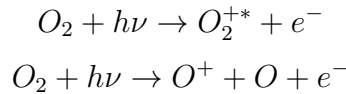
$$\nu_p[\text{Hz}] = 11 \text{ MHz (day time)}$$

(at night, the observable frequency by LOFAR can be lower)

At frequencies,

1.  $\omega < \omega_p$ :  $n^2 < 0$ , reflection ( $\nu < 10 \text{MHz}$ ),
2.  $\omega > \omega_p$ :  $n^2 > 0$ , refraction ( $10 \text{MHz} < \nu < 10 \text{GHz}$ ),
3.  $\omega \gg \omega_p$ :  $n^2 \rightarrow 1$  ( $\nu > 10 \text{GHz}$ ).

The observing conditions are dependent on the electron density, i.e. the solar conditions (space weather), since the ionisation is due to the ultra-violet radiation field from the Sun,



### 1.3.3 The high-frequency cut-off (absorption)

1. Molecules in the atmosphere can absorb the incoming radiation, but also emit radiation (via thermal emission).

2. **Mass absorption co-efficient (k)**: From atomic and molecular physics, define for various species, i,

$$k_i = \frac{\sigma n_i}{r_i \rho_0}$$

where:  $k_i$  is Mass attenuation coefficient ( $cm^2 g^{-1}$ ),  $\sigma$  Cross-section ( $cm^2$ ),  $n_i$  is Number density of particles ( $cm^{-3}$ ),  $\rho_0$  is Mass density of air ( $gcm^{-3}$ ),  $r_i$  is Mixing ratio( $= \rho_i/\rho_0$ )

3. **Optical depth ( $\tau$ )**: A measure of the absorption/ scattering (attenuation) of electromagnetic radiation in a medium (probability of an interaction),

$$\tau_i(\lambda, z_0) = \int_{z_0}^{\infty} n_i(z) \sigma dz = \int_{z_0}^{\infty} r_i(z) \rho_0(z) k_i(\lambda) dz$$

or, in terms of the **linear absorption co-efficient ( $\kappa$ )**

$$\tau_i(\lambda, z_0) = \int_{z_0}^{\infty} \kappa(\lambda, z) dz$$

where  $\kappa(\lambda, z) = k_i(\lambda) \rho_i(z)$ ,

$\kappa(\lambda, z)$ : linear absorption coefficient ( $cm^{-1}$ )

$k_i(\lambda)$ : Mass attenuation coefficient ( $cm^2 g^{-1}$ )

$\rho_i(z)$ : Mass density of species i ( $gcm^{-3}$ )

$\rho_i(z) = r_i(z) \rho_0(z)$

The attenuation of an incident ray of intensity  $I_0$ , received at altitude  $z_0$ , summed over all absorbing species is,

$$I(z_0) = I_0 \exp\left[-\sum_i \tau(\lambda, z_0)\right] = I_0 \exp[-\tau(z)]$$

Where, for convenience, **we consider all species together and define the optical depth as a function of zenith angle,  $\tau(z)$**

Worked example: What is the optical depth for sky transparencies of 0.5, 0.1 and 0.01?

Rearrange, in terms of  $\tau$ , and evaluate,  $\tau = -\ln\left(\frac{I(z_0)}{I_0}\right)$

$$\tau_{0.5} = -\ln(0.5) = 0.69$$

$$\tau_{0.1} = -\ln(0.1) = 2.3$$

$$\tau_{0.01} = -\ln(0.01) = 4.6$$

The smaller the transparency, the larger the optical depth  $\tau$

Note that the opacity changes with the path length, and so depends on the airmass  $X(z)$ , which assuming a plane parallel atmosphere,

$$\tau(z) = \tau_0 X(z), X(z) = \sec(z)$$

where  $\tau_0$ : Optical depth at Zenith,  $X(z)$ : Airmass,  $z$ : Zenith angle

The atmosphere is not completely transparent at radio wavelengths, but  $\tau(z)$  varies with frequency  $\nu$ .

Zenith opacity is the sum of several component opacities at cm  $\lambda$ .

- **Broadband (continuum) opacity:** dry air.  $\tau_z \approx 0.01$  and almost independent of  $\nu$
- **Molecular absorption:**  $O_2$  has rotational transitions that absorb radio waves and are opaque ( $\tau_z \gg 1$ ) at 52 to 60 GHz.
- **Hydrosols:** Water droplets ( $radius \leq 0.1mm$ ) suspended in clouds absorb radiation (proportional to  $\lambda^{-2}$ ).
- **Water vapor:** Emission line at  $\nu \approx 22.235$  GHz is pressure broadened to  $\Delta\nu \sim 4GHz$  width + “continuum” absorption from the “line-wings” of very strong  $H_2O$  emission at infrared wavelengths (proportional to  $\lambda^{-2}$ ).

The zenith optical depth is dependent on the path length through the material.

- Higher altitude: Move above the water vapour layer ( $> 4$  km).
- Drier locations: Move to regions with low water vapour.

### 1.3.4 The high-frequency cut-off (emission)

A partially absorbing atmosphere also emits radio noise that can de-grade ground based observations. We can define the total system noise power as an **equivalent noise temperature**

$$P = \frac{E}{\Delta t} = kT\Delta\nu$$

in terms of **spectral power**,

$$P_\nu = k T_{\text{sys}}$$

Spectral power (W Hz<sup>-1</sup>)      System temperature (Receivers; Sky, Ground; etc)  
Boltzmann constant = 1.38 x 10<sup>-23</sup> m<sup>2</sup> kg s<sup>-2</sup> K<sup>-1</sup>

where,

$$T_{\text{sys}} = T_{\text{bg}} + T_{\text{sky}} + T_{\text{spill}} + T_{\text{loss}} + T_{\text{cal}} + T_{\text{rx}}$$

Noise from Radio background (Galaxy, CMB, etc)    Noise from ground emission    Noise from injected noise    Noise from the receiver (Dominates)  
Noise from atmospheric emission    Noise from losses at receiver

The contribution from the sky opacity to the sky temperature is,

$$T_{\text{sky}} = T_{\text{atm}} [1 - \exp(-\tau_\nu)]$$

Atmospheric kinetic temperature ( $\equiv 300$  K)  
Emitted sky temperature (K)      Optical depth

Don't want  $T_{\text{sky}}$  to dominate our noise budget, need to minimise  $T_{\text{atm}}$  and  $\tau_\nu$  by observing in cold and dry locations (winter; high alt), especially at high frequencies

Worked example: Using the total opacity data for the Green Bank Telescope (West Virginia; USA; 2800 m) and  $T_{\text{atm}} = 288$  K, what is  $T_{\text{sky}}$  at  $\nu = 5$  GHz, 22 GHz and 115 GHz?

How does this compare with the typical receiver temperature,  $T_{\text{rx}} \sim 30$  K?

- At  $\nu = 5$  GHz,  $\tau_z \sim 0.007$ ,  $T_{\text{sky}} = 288[1 - \exp(-0.007)] \sim 2K$  (Good)
- At  $\nu = 22$  GHz,  $\tau_z \sim 0.15$ ,  $T_{\text{sky}} = 288[1 - \exp(-0.15)] \sim 40K$  (Bad)
- At  $\nu = 115$  GHz,  $\tau_z \sim 0.8$ ,  $T_{\text{sky}} = 288[1 - \exp(-0.8)] \sim 160K$  (Bad)

**Key concept: The partially transparent atmosphere**



allows radio waves to be detected from ground-based telescopes, but also attenuates the signal due to absorption/scattering, and also adds noise to the measured signal

### 1.3.5 Early Radio Astronomy

The first detection of radiation at radio wavelengths was not made until 1932 due to,

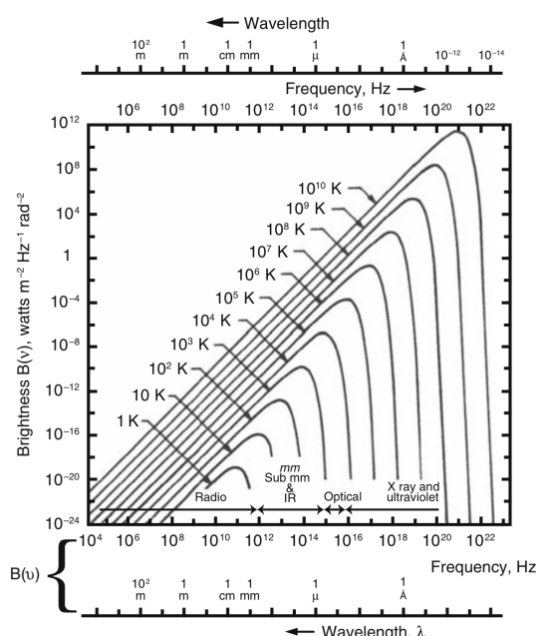
1. limitations of technology (our eyes), but then the communication era started,
2. the expectation that celestial objects would be too faint

The spectral brightness  $B_\nu$  at frequency  $\nu$  of a blackbody object (stars) is given by Planck's law.

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{\exp \frac{h\nu}{kT} - 1}$$

Spectral brightness (W m<sup>-2</sup> Hz<sup>-1</sup> sr<sup>-1</sup>)      Planck constant = 6.626 × 10<sup>-34</sup> m<sup>2</sup> kg s<sup>-1</sup>  
 Speed of light constant = 3 × 10<sup>8</sup> m s<sup>-1</sup>      Absolute temperature (K)

In the low frequency radio limit,  $h\nu/kT \ll 1$ .



Worked example: Does the low-frequency limit work for the photosphere of the Sun, which has  $T = 5800$  K? At  $\nu = 1$  GHz,

$$\frac{h\nu}{kT} = \frac{6.626 \times 10^{-34} \times 1 \times 10^9}{1.38 \times 10^{-23} \times 5800} = 8 \times 10^{-6}$$

Using this property ( $h\nu/kT \ll 1$  in the low frequency limit), we can replace the

exponential term using the Taylor expansion,

$$\exp(h\nu/kT) - 1 \approx 1 + \frac{h\nu}{kT} + \dots - 1$$

to give the Rayleigh-Jeans approximation to the Planck function at low-frequencies,

$$B_\nu(T) \approx \frac{2h\nu^3}{c^2} \frac{kT}{h\nu} = \frac{2kT}{\lambda^2}$$

**Flux-density ( $S_\nu$ ):** The power received per unit detector area in a unit bandwidth ( $\Delta\nu = 1$  Hz) at frequency  $\nu$ . The units are  $Wm^{-2}Hz^{-1}$ .

The flux-density received from a celestial source of brightness  $B_\nu(T)$  and subtending a very small angle  $\Omega \ll 1$ sr, is approximately,

$$S_\nu = B_\nu \Omega$$

---

Worked example: What is the flux-density at  $\nu = 1$  GHz of a black body with temperature  $T = 5800$  K and size  $R \approx 7 \times 10^{10}$  cm (the Sun) at about 1 parsec ( $d \approx 3 \times 10^{18}$  cm)

$$B_\nu(T) = \frac{2kT\nu^2}{c^2} = 1.78 \times 10^{-18} Wm^{-2}Hz^{-1}sr^{-1}$$

The spectral brightness is an intrinsic property of the source (independent of distance). The solid angle subtended by the source is dependent on the distance

$$\Omega = \frac{\pi R^2}{d^2} \approx 1.71 \times 10^{-15} sr$$

The flux-density is therefore,

$$S_\nu = B_\nu \Omega \approx 3 \times 10^{-33} Wm^{-2}Hz^{-1}$$

This flux density is too small for even today's telescopes to detect (easily), so the thermal emission from stars was thought to be impossible to detect at radio wavelengths, but ...

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Long distance communication developed by Marconi & Ferdinand Braun - Nobel Prize 1909

Evolution of frequency over the years

- pre-1920: <100 kHz.
- ca. 1920: shift to 1.5 MHz.

- post-1920: 10s of MHz (more voice channels, less effected by the ionosphere and thunderstorms).
- Research labs sprung up in early-1900s

Karl Jansky (1933, published) discovered a radio signal at 20.5 MHz that varied steady every 23 hours and 56 minutes (Sidereal day).

“The data give for the co-ordinates of the region from which the disturbance comes, a right ascension of 18 hours and declination -10 degrees.” He had detected the Galactic Centre.

Grote Reber (1937-39), using his own 10 m telescope, made no detection at 3300 and 910 MHz, **ruling out a Planck spectrum ( $B_\nu \propto \nu^2$ ).**

**Detection made at 150 MHz, confirming Jansky’s result and finding the spectrum must be non-thermal**

**Key concept: Radio emission from celestial objects can be measured and it can be both thermal and non-thermal in origin.**

### 1.3.6 Radio Telescopes and interferometers

- Radio telescopes are designed in a different way to optical telescopes, and the radio range is so broad (5 decades in frequency) that different telescope technologies can be used.
- The surface accuracy of a reflector is proportional to  $\lambda/16$ 
  - cm (1 GHz)  $\rightarrow$  surface accuracy of  $\sim 2$  cm
  - mm (100 GHz)  $\rightarrow$  surface accuracy of  $\sim 200 \mu m$ .
  - Optical ( $0.55 \mu m$ )  $\rightarrow$  surface accuracy of  $\sim 0.034 \mu m$ .

$$\frac{P(\delta)}{P_0} = \eta_s = \exp[-(\frac{4\pi\sigma}{\lambda})^2] \quad (1.1)$$

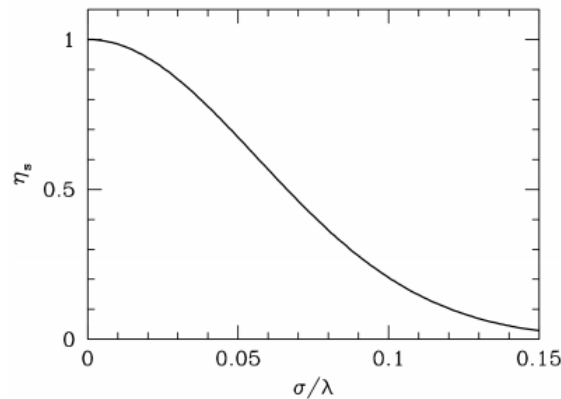


Figure 1.1: Not understand yet

### Not understand equation 1.1 and figure 1.1

- Large single-element radio telescopes can be constructed cheaply, but have limited spatial resolution,  $\theta \approx \lambda/D$

---

Worked example: What is the spatial resolution (in arcseconds) of the  $D = 300$  m Arecibo telescope, operating at  $\nu = 5$  GHz?

$\theta \sim 41$  arcsec

---

- Interferometric techniques have been developed to combine several single element telescopes into a multi-element array. Now, the resolution is limited by the distance between the elements

---

What is the spatial resolution (in arcseconds) of the Very Long Baseline Array operating at  $\nu = 5$  GHz? The longest distance between telescopes is  $D_{max} = 8611$  km.

$\sim 0.00144$  arcsec

---

**Key Concept: Radio interferometry can provide the highest angular resolution imaging possible in astronomy.**

## 1.4 Astrophysical applications

- The large radio window has allowed a wide variety of astronomical sources, thermal and non-thermal radiation mechanisms, and the propagation phe-

nomena to be studied.

1. Discrete cosmic radio sources, at first, supernova remnants and radio galaxies (1948);
2. The 21cm line of atomic hydrogen (1951);
3. Quasi Stellar Objects “Quasars” (1963);
4. The Cosmic Microwave Background (1965);
5. Inter stellar molecules and proto-planetary discs (1968);
6. Pulsars (1968);
7. Gravitational lenses (1979);
8. The Sunyaev-Zeldovich effect (1983);
9. Distance determinations using source proper motions determined from Very Long Baseline Interferometry (1993); and
10. Molecules in high-redshift galaxies (2005)

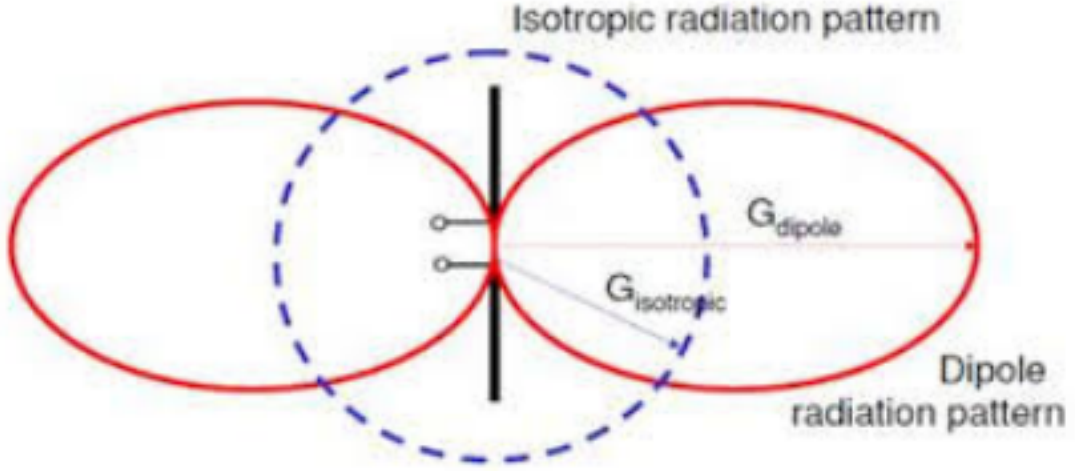
## 1.5 LOFAR

### 1.5.1 Response of the LOFAR antenna

In different frequencies, LOFAR has different Gains as a function of Zenith angle.

### 1.5.2 Power gain

$G(\theta, \phi)$  is the power transmitted per unit solid angle in direction  $(\theta, \phi)$  divided by the power transmitted per unit solid angle from an isotropic antenna with the same total power( figure [1.5.2](#)).



The power or gain are often expressed in logarithmic units of decibels (dB):

$$G(\text{dB}) = 10 \times \log_{10}(G)$$

---

Worked example: What is the maximum and half power of a normalised power pattern in decibels? Maximum power of a normalised power pattern is  $P_n = 1$

$$P_n(1) = 10 \times \log_{10}(1) = 0\text{dB}$$

Half power of a normalised power pattern is  $P_n = 0.5$

$$P_n(0.5) = 10 \times \log_{10}(0.5) = -3\text{dB}$$

---

For a lossless isotropic antenna, conservation of energy requires the directive gain averaged over all directions be,

$$\langle G \rangle = \frac{\int_{\text{sphere}} G d\Omega}{\int_{\text{sphere}} d\Omega} = 1$$

Therefore, for an isotropic lossless antenna,

$$\int_{\text{sphere}} G d\Omega = \int_{\text{sphere}} d\Omega = 4\pi$$

and

$$G = 1$$

Lossless antennas may radiate with different directional patterns, but they cannot alter the total amount of power radiated  $\rightarrow$  the gain of a lossless antenna depends only on the angular distribution of radiation from that antenna.

**Key Concept: Higher the gain, the narrower the radiation pattern (directivity)**

$$\Delta\Omega \approx \frac{4\pi}{G_{max}}$$

### 1.5.3 Beam solid angle

Beam solid angle: The beam area  $\Omega_A$  is the solid angle through which all of the power radiated by the antenna would stream if  $P(\theta, \phi)$  maintained its maximum value over  $\Omega_A$  and zero everywhere else

$$\Omega_A \equiv \int_{4\pi} P_n(\theta, \phi) d\Omega$$

The power (and temperature) received is also a function of the power pattern of the antenna. Therefore, the true antenna temperature is,

$$T_A = \frac{A_e}{2k} \int \int I_\nu(\theta, \phi) P_n(\theta, \phi) d\Omega$$

where  $P_n(\theta, \phi)$  is the power pattern normalised to unity maximum,

$$P_n = \frac{G(\theta, \Phi)}{G_{max}}$$

### 1.5.4 Response of a reflector antenna

**Paraboloidal reflectors** are useful because,

1. The effective collecting area  $A_e$  can approach the geometric area ( $= \pi D^2/4$ ).
2. Simpler than an array of dipoles.
3. Can change the feed antenna to work over a wide frequency range (e.g. for the JVLA 8 receivers on each telescope allow observations from 1–50 GHz)

For a receiving antenna, where the electric field pattern is  $f(l)$  and the electric field illuminating the aperture is  $g(u)$

$$f(l) = \int_{\text{aperture}} g(u) e^{-i2\pi l u} du$$

**Key concept: In the far-field, the electric field pattern is the Fourier transform of the electric field illuminating the aperture**

The radiated power as a function of position

$$P_n(l) = \text{sinc}^2\left(\frac{\theta D}{\lambda}\right)$$

For a one-dimensional uniformly illuminated aperture

$$\theta_{HPBW} \approx 0.89 \frac{\lambda}{D}$$

The central peak of the power pattern between the first minima is called the **main beam** (typically defined by the **half-power angular size**)

The smaller secondary peaks are called **sidelobes**.

**Main beam solid angle:** The area containing the principle response out to the first zero.

**Side-lobes:** Areas outside the principle response that are non-zero.

$$\Omega_{MB} = \int_{MB} P_n(\theta, \Phi) d\Omega$$

where  $\Omega_{MB}$  is the Main beam solid angle (sr)

**Main beam efficiency:** The fraction of the total beam solid angle inside the main beam

$$\eta_B = \frac{\Omega_{MB}}{\Omega_A}$$



## 1.6 Sensitivity

Our ability to measure a signal is dependent on the noise properties of our complete system ( $T_{sys}$ ), although this is typically dominated by the Johnson noise within the receiver. Our ability to measure a signal is dependent on the noise properties of our complete system ( $T_{sys}$ ), although this is typically dominated by the Johnson noise within the receiver.

The variations (uncertainty on some measurement) is estimated by,

1. In time interval  $\tau$  there are a minimum  $N = 2\Delta\nu\tau$  independent samples of the total noise power  $T_{sys}$ .
2. The uncertainty in the noise power (from a random gaussian distribution) is  $\approx 2^{1/2}T_{sys}$ .
3. The rms error in the average of  $N \gg 1$  independent samples is reduced by the factor  $N^{1/2}$

$$\sigma_T = \frac{2^{1/2}T_{sys}}{N^{1/2}}$$

which gives the (ideal) radiometer equation:

$$\sigma_T \approx \frac{T_{sys}}{\sqrt{\Delta\nu_{RF}\tau}}$$

where  $\sigma_T$  is rms uncertaintyK,  $T_{sys}$  is System temperature(K),  $\Delta\nu_{RF}$  is total bandwidth(Hz), and  $\tau$  is total time.

Typically  $T_{sys} \gg T_{source}$ . Need rms uncertainty in the system temperature to be as low as possible. Increase the observed bandwidth or observing for longer, or decrease the receiver temperature

The signal-to-noise ratio of our target source is:

$$\frac{S}{N} = \frac{T_{source}}{\sigma_T} = \frac{T_{source}}{T_{sys}} \sqrt{\Delta\nu_{RF}\tau}$$

It is convenient to express the rms uncertainty in terms of the system equivalent flux density (SEFD; units of Jy). Recall

$$P_\nu = kT_A = A_e \frac{S_\nu}{2}$$

$$T_A = \left(\frac{A_e}{2k}\right) S_\nu$$

where  $(\frac{A_e}{2k})$  is called the "forward gain" (K/Jy)

Define the SEFD as,

$$SEFD = \frac{2kt_{sys}}{A_e}$$

---

Worked example: What is the SEFD of a 25-m VLA antenna assuming a system temperature of 55 K and an effective area of 365 m<sup>2</sup>?

$$SEFD = \frac{2kt_{sys}}{A_e} = \frac{2 \times 1 \times 10^{-23} \times 55}{365} = 300 Jy$$

This will scale inversely with the effective area, i.e. a low SEFD suggests a more sensitive telescope

---

The SEFD is a good way to compare the sensitivity of telescopes because it takes the receiver system ( $T_{sys}$ ) and the effective area ( $A_e$ ) into account.

We can define our ideal radiometer equation to determine the sensitivity in terms of fluxdensity,

Diagram illustrating the radiometer equation in two equivalent forms:

Left form:  $\sigma_{S_\nu} = \frac{2kT_{sys}}{A_e \sqrt{\Delta\nu\tau}}$

- $\sigma_{S_\nu}$ : rms uncertainty (W m<sup>-2</sup> Hz<sup>-1</sup>)
- $T_{sys}$ : System temperature (K)
- $A_e$ : Effective area (m)
- $\Delta\nu$ : Total bandwidth (Hz)
- $\tau$ : total time (s)

Right form:  $\sigma_{S_\nu} = \frac{SEFD}{\sqrt{\Delta\nu\tau}}$

- $\sigma_{S_\nu}$ : rms uncertainty (Jy)
- $SEFD$ : System equivalent flux-density (Jy)

The two forms are separated by the word "or".

## 1.7 Summary

1. Radio astronomy covers 5 decades in frequency from ~10 MHz up to about 1 THz (ground based).
2. It is a well established area of astronomical research that allows for a large number of unique science cases to be investigated (sensitivity and resolution).
3. Keep in mind the properties of the single elements of your interferometer

# Chapter 2

## (Gentle) Introduction to Interferometry

Radio telescopes measure a voltage due to the incident EM radiation.

### 2.1 Adding Interferometer

### 2.2 Multiplying Interferometer

### 2.3 The Effect of Baseline Length

### 2.4 Complex Visibilities

### 2.5 Van Cittert Zernike Function

### 2.6 Visibility

### 2.7 Visibilities and Images

### 2.8 Fourier Components

### 2.9 Conclusions