

Gravitational Lensing Theories, Questions and Applications

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Overview

GravLens

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History

Gravitational
Lensing Theories

Related Questions

Applications

History

Gravitational Lensing Theories

Related Questions

Applications

Predictions from Newtonian and GR

- ▶ First proposed by Soldner (1801) using Newtonian theory, given a deflection angle (Appendix-1 (Page 66) for derivation).

$$\alpha = \frac{2GM}{c^2 r}, 0.85 \text{ arcsec for the Sun}$$

- ▶ Einstein (1911) derived the same result using Equivalence principle and Euclidean metric
- ▶ Einstein (1915) derived the new result using General Relativity.

$$\alpha = \frac{4GM}{c^2 r}, 1.7 \text{ arcsec for the Sun}$$

Eddington's observation of the Solar Eclipse

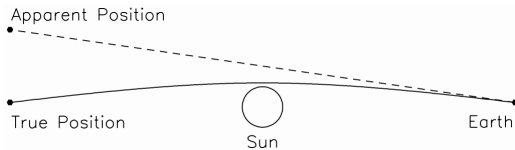
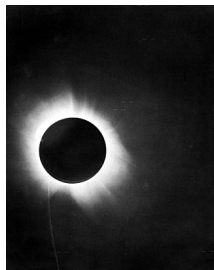
History

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- ▶ In 1919, Eddington measured a value close to GR's prediction using the data collected during an eclipse, stars with apparent position near the sun become visible.

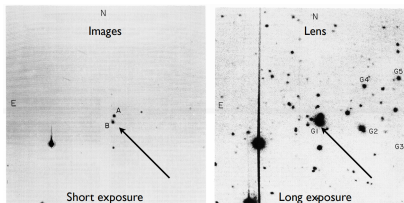


The first example for GravLens: 0957+561

- ▶ Eddington (1920): Multiple light paths \rightarrow multi images
- ▶ Walsh et al., (1979) quasar QSO 957+561 A,B found at $z \sim 1.4$, two seen images separated by $6''$

- ▶ Lens: evidence

1. Lensing galaxy at $z \sim 0.36$
2. Similar spectra
3. Ratio of optical and radio flux
4. VLBI imaging: small scale features



General Relativity and light deflection

- ▶ Einstein Field Equations:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

- ▶ Geodesic equations:

$$\frac{d^2x^\beta}{d\lambda^2} + \Gamma_{\mu\nu}^\beta \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} = 0$$

We can calculate how gravity bends light by solving geodesic equation.

- ▶ To compute the Christoffel symbols $\Gamma_{\mu\nu}^\beta$, requires solving for the metric tensor $g_{\mu\nu}$, which requires solving the curvature equations $R_{\mu\nu} = 0$,
← ten nonlinear partial differential equations.

General Relativity and light deflection

Or, the velocity of the photon from the Schwarzschild metric,

$$ds^2 = -(1 + 2\Phi)dt^2 + (1 - 2\Phi)(dx^2 + dy^2 + dz^2),$$

- ▶ and Poisson Equation, $\nabla^2\Phi = 4\pi G\rho$,
- ▶ and light interval: $g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda} = 0$
- ▶ which gives,

$$v = \frac{\sqrt{dx^2 + dy^2 + dz^2}}{dt} = \sqrt{\frac{1 + 2\Phi}{1 - 2\Phi}} \approx 1 + 2\Phi$$

The gravitational field decreases the speed of propagation

General Relativity and light deflection

$$v = \frac{\sqrt{dx^2+dy^2+dz^2}}{dt} = \sqrt{\frac{1+2\Phi}{1-2\Phi}} \approx 1 + 2\Phi \text{ (natural units)} \rightarrow$$
$$v = c \left(1 + \frac{2}{c^2}\Phi\right) \text{ (SI)}$$

- ▶ define refraction index: $n = 1 - \frac{2}{c^2}\Phi = 1 + \frac{2}{c^2}|\Phi| \geq 1$
- ▶ deflection angle: $\vec{\hat{\alpha}} = - \int \vec{\nabla}_{\perp} n dl = \frac{2}{c^2} \int \vec{\nabla}_{\perp} \Phi dl$
- ▶ which is twice of the Newtonian prediction,
- ▶ for point mass lens, $\hat{\alpha} = \frac{4GM}{bc^2}$

Thin screen approximation

- ▶ Most deflection occurs near to the lens ($|z| \sim b$)
→ treat all deflection as in the lens plane

- ▶ Projected surface density:

$$\Sigma(\vec{\xi}) = \int \rho(\vec{\xi}, z) dz$$

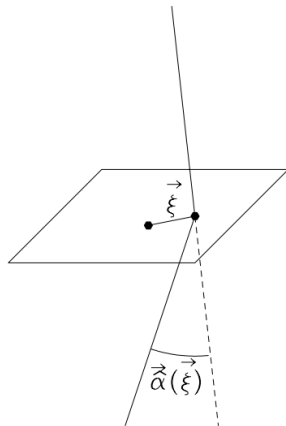
- ▶ Deflection angle:

$$\vec{\alpha}(\vec{\xi}) = \frac{4G}{c^2} \int \frac{(\vec{\xi} - \vec{\xi}') \Sigma(\vec{\xi}')}{|\vec{\xi} - \vec{\xi}'|^2} d^2 \xi'$$

- ▶ In circular symmetry cases:

$$\hat{\alpha}(\xi) = \frac{4GM(\xi)}{c^2 \xi}$$

$$M(\xi) = 2\pi \int_0^\xi \Sigma(\xi') \xi' d\xi'$$



Gravitational Lensing Theories



Case 1: Point mass Lens

$$\begin{cases} \hat{\alpha}(\xi) = \frac{4GM(\xi)}{c^2\xi} \\ \vec{\alpha}(\vec{\theta}) = \frac{D_{ds}}{D_s}\hat{\alpha}(D_d\vec{\theta}) \end{cases} \rightarrow \vec{\alpha}(\vec{\theta}) = \frac{4GM}{c^2} \frac{D_{ds}}{D_d D_s} \frac{\vec{\theta}}{|\vec{\theta}|^2}$$

and $\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta})$,

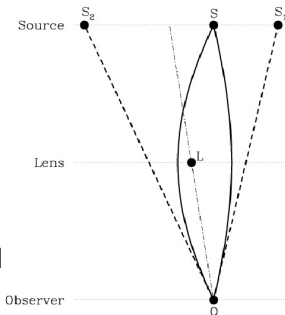
if $\beta = 0$, gives the Einstein Radius,

$$\theta_E = \sqrt{\frac{4GM}{c^2} \frac{D_{ds}}{D_d D_s}}$$

The Lens equation: $\vec{\beta} = \vec{\theta} - \theta_E^2 \frac{\vec{\theta}}{|\vec{\theta}|^2}$

- ▶ if $\beta > \theta_E \rightarrow$, weakly lensed and weakly distorted image,
- ▶ if $\beta < \theta_E \rightarrow$, strongly lensed and multi images:

$$\theta_{\pm} = \frac{1}{2} \left(\beta \pm \sqrt{\beta^2 + 4\theta_E^2} \right)$$



Case 1: Point mass Lens

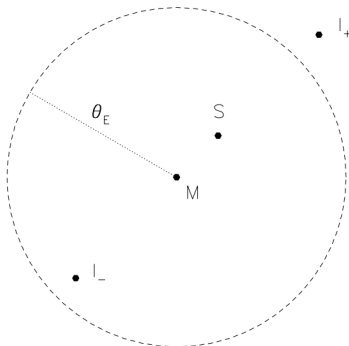
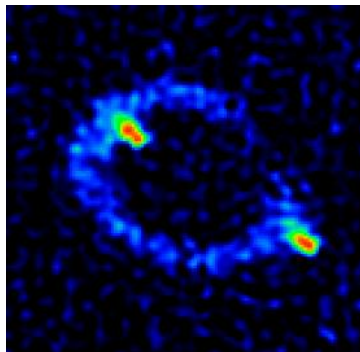
Hewitt+ 1987, First Einstein Ring discovered in Radio Band & VLA

History

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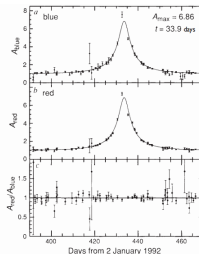
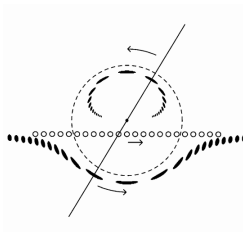
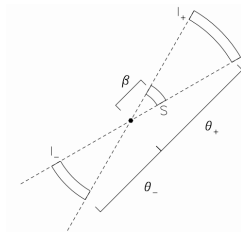
Applications



Hewitt, J., Turner, E., Schneider, D. et al. Unusual radio source MG1131+0456: a possible Einstein ring. Nature 333, 537–540 (1988) doi:10.1038/333537a0

Magnification

Gravitational lensing preserves surface brightness (**Liouville's Theorem**), but changes the apparent solid angle of the source \rightarrow *magnification* = $\frac{Area_{image}}{Area_{source}}$



Local properties of the lens mapping, described by its Jacobian matrix A

$$A \equiv \frac{\partial \vec{\beta}}{\partial \vec{\theta}} = \left(\delta_{ij} - \frac{\partial \alpha_i(\vec{\theta})}{\partial \theta_j} \right) = \left(\delta_{ij} - \frac{\partial^2 \Psi(\vec{\theta})}{\partial \theta_i \partial \theta_j} \right)$$

$$\mu = \left| \det \left(\frac{\partial \vec{\beta}}{\partial \vec{\theta}} \right) \right|^{-1} \equiv \left| \det \left(\frac{\partial \beta_i}{\partial \theta_j} \right) \right|^{-1}$$

If circularly symmetric,

$$\mu = \frac{\theta}{\beta} \frac{d\theta}{d\beta}$$

Case 1: Point mass Lens - Magnification

► Images:

$$\theta_{\pm} = \frac{1}{2} \left(\beta \pm \sqrt{\beta^2 + 4\theta_E^2} \right)$$

► Define $u = \beta\theta_E^{-1}$,

Magnification:

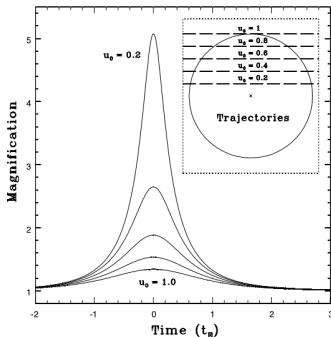
$$\begin{aligned} \mu_{\pm} &= \left[1 - \left(\frac{\theta_E}{\theta_{\pm}} \right)^4 \right]^{-1} \\ &= \frac{u^2 + 2}{2u\sqrt{u^2 + 4}} \pm \frac{1}{2} \end{aligned}$$

► Total magnification:

$$\mu = |\mu_+| + |\mu_-| = \frac{u^2 + 2}{u\sqrt{u^2 + 4}}$$

e.g. for source on the Einstein Ring:

$\beta = \theta_E, u = 1 \rightarrow \mu = 1.34 \rightarrow$ magnitude increase 0.32.



Shapiro time delay

- ▶ Passage through potential also leads to time delay

- ▶ without potential: $t_0 = \int \frac{dl}{c}$

- ▶ with potential:

$$\begin{aligned} t_1 &= \int_{src}^{obs} \frac{dl}{v} = \int_{src}^{obs} \frac{dl}{c - \frac{2}{c}|\Phi|} \\ &= \int_{src}^{obs} \frac{dl/c}{1 - \frac{2}{c^2}|\Phi|} = \int_{src}^{obs} \frac{dl}{c} \left[1 + \frac{2}{c^2}|\Phi| \right] \end{aligned}$$

- ▶ so, $\Delta t = \int_{src}^{obs} \frac{2}{c^3}|\Phi|dl \rightarrow$ Shapiro delay (1964)
- ▶ Total time delay is the sum of the extra path length from the deflection and the gravitational time delay

$$t(\vec{\theta}) = \frac{1+z_d}{c} \frac{D_d D_s}{D_{ds}} \left[\frac{1}{2}(\vec{\theta} - \vec{\beta})^2 - \psi(\vec{\theta}) \right] = t_{geom} + t_{grav}$$

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Case 2: Singular Isothermal Sphere (SIS)

- ▶ Galaxy lenses, the distributed nature of mass
- ▶ Simple model assumes that mass \rightarrow particles of ideal gas
- ▶ ideal gas:
 - ▶ equation of state: $p = \frac{\rho k T}{m}$
 - ▶ In thermal equilibrium, T is related to the 1-d velocity dispersion: $m \sigma_v^2 = k T$
 - ▶ In hydrostatic equilibrium,

$$\frac{p'}{\rho} = -\frac{GM(r)}{r^2}, M'(r) = 4\pi r^2 \rho$$
 - ▶ solve the EOS \rightarrow density profile: $\rho(r) = \frac{\sigma_v^2}{2\pi G} \frac{1}{r^2}$
 - ▶ so mass profile: $M(r) = \frac{2\sigma_v^2}{G} r$

Case 2: Singular Isothermal Sphere (SIS)

For ideal gas, rotational velocity in circular orbit:

$$\frac{v'}{\rho} = -\frac{GM(r)}{r^2}, M'(r) = 4\pi r^2 \rho$$

Surface mass density:

$$v_{rot}^2 = GM/r \rightarrow$$

$$dM = \frac{v_{rot}^2}{G} dr = 4\pi r^2 \rho(r) \rightarrow$$

$$\rho(r) = \frac{v_{rot}^2}{4\pi G} \frac{dr}{r^2} \rightarrow$$

$$\Sigma(\xi) = \frac{v_{rot}^2}{4\pi G} \int_{-\infty}^{\infty} \frac{dz}{(z + \xi)^2} = \frac{v_{rot}^2}{4G\xi} = \frac{\sigma_v^2}{2G\xi}$$

where, $M(\xi) = 2\pi \int_0^\xi \Sigma(\xi') \xi' d\xi'$, and $\hat{\alpha}(\xi) = \frac{4GM(\xi)}{c^2 \xi}$

which gives: $\hat{\alpha} = 4\pi \frac{\sigma_v^2}{c^2}$

Case 2: Singular Isothermal Sphere (SIS)

using $\begin{cases} \hat{\alpha} = 4\pi\sigma_v^2/c^2 \\ \vec{\alpha}(\vec{\theta}) = \hat{\alpha}(D_d\vec{\theta})D_{ds}/D_s \end{cases}$

and lens equation $\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta})$

we can get:

$$\alpha = \hat{\alpha} \frac{D_{ds}}{D_s} = 4\pi \frac{\sigma_v^2}{c^2} \frac{D_{ds}}{D_s} = \theta_E$$

for strong lensing, get two images as for point mass:

$$\vec{\beta} = \vec{\theta} - \theta_E \frac{\vec{\theta}}{|\vec{\theta}|}, \frac{\vec{\theta}}{|\vec{\theta}|} = \pm 1 \rightarrow$$

$$\theta_{\pm} = \beta \pm \theta_E$$

Magnification can be very large for source aligned with lens,

from $\mu = \frac{\theta}{\beta} \frac{d\theta}{d\beta} \rightarrow$:

$$\mu_{\pm} = \frac{\theta_{\pm}}{\beta} = 1 \pm \frac{\theta_E}{\beta} = \left(1 \mp \frac{\theta_E}{\theta_{\pm}}\right)^{-1}$$

Separation of the two images is typically \sim arcsec for galaxy lenses:

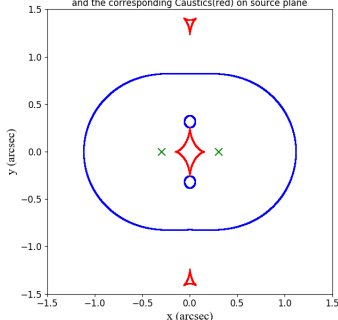
$$\theta_E = 1''.6 \left(\frac{\sigma_v}{200 \text{ km s}^{-1}} \right)^2 \left(\frac{D_{ds}}{D_s} \right)$$

- ▶ Grav Lens changes the observed brightness of the source, determined by the Jacobian matrix from the lens equation: $A \equiv \frac{\partial \vec{\beta}}{\partial \vec{\theta}} = \left(\delta_{ij} - \frac{\partial \alpha_i(\vec{\theta})}{\partial \theta_j} \right) = \left(\delta_{ij} - \frac{\partial^2 \Psi(\vec{\theta})}{\partial \theta_i \partial \theta_j} \right)$
 $\mu(\theta) = \frac{1}{\det A(\theta)}$
- ▶ Image at $\vec{\theta}$ is magnified by a factor of $|\mu(\vec{\theta})|$
- ▶ Notice that $|\mu(\vec{\theta})|$ diverge at $\det A(\theta) = 0 \rightarrow$ these points in the image plane form closed curves, which is so called **critical lines**.
- ▶ Corresponding curves in the source plane obtained via the lens equation are called **caustics**.

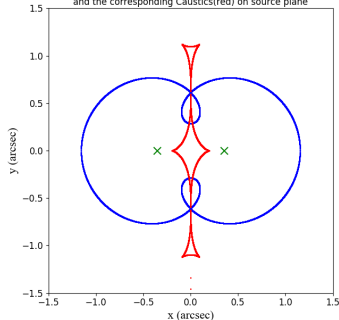
Caustics and Critical lines - Point sources

Separation $0.6''$, $0.7''$:

The Critical Curves(blue) of two point mass(green) lens with $\mu_1 = \mu_2$ and $X = 0.30$ and the corresponding Caustics(red) on source plane



The Critical Curves(blue) of two point mass(green) lens with $\mu_1 = \mu_2$ and $X = 0.35$ and the corresponding Caustics(red) on source plane

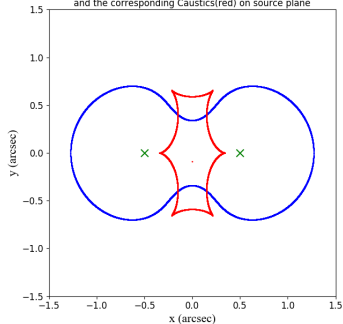


https://github.com/rkkuang/aeroastro/blob/master/gravlen/critical_and_caustics/
The two-point-mass lens - Detailed investigation of a special asymmetric gravitational lens
<http://adsabs.harvard.edu/abs/1986A%26A...164..237S>

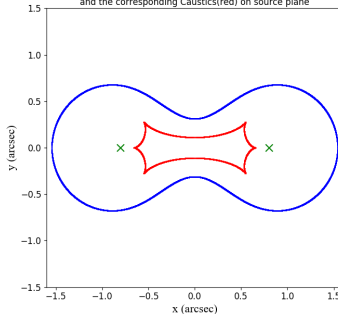
Caustics and Critical lines - Point sources

Separation $1.0''$, $1.6''$:

The Critical Curves(blue) of two point mass(green) lens with $\mu_1 = \mu_2$ and $X = 0.50$ and the corresponding Caustics(red) on source plane

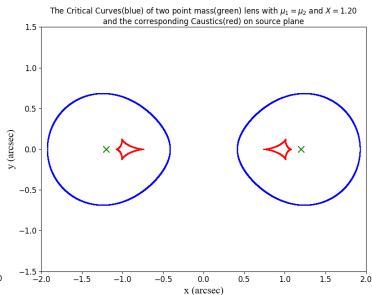
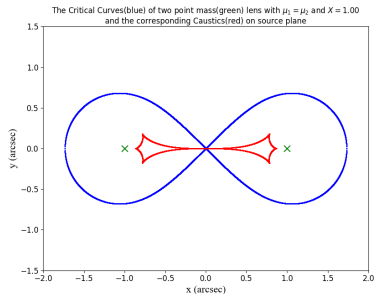


The Critical Curves(blue) of two point mass(green) lens with $\mu_1 = \mu_2$ and $X = 0.80$ and the corresponding Caustics(red) on source plane



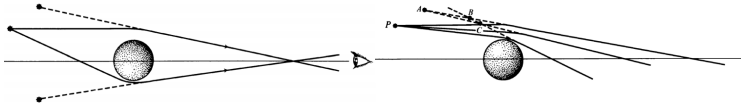
Caustics and Critical lines - Point sources

Separation $2.0''$, $2.4''$:



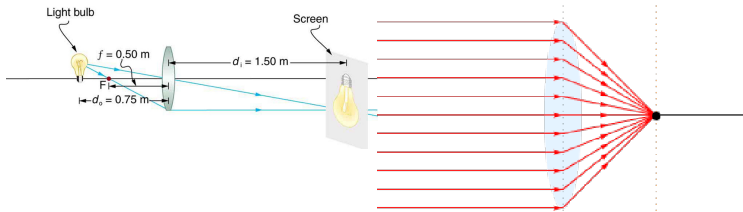
Gravitational Lens vs Genuine Lens

- Grav lens: the observer sees the source at two distinct locations, $\alpha \propto b^{-1}$



Grav lens has no well-defined focal length and cannot produce genuine images, the “images” are corresponds merely to a direction of incidence of light on the observer, not a genuine image in the sky

- Genuine lens, $\alpha \propto b$



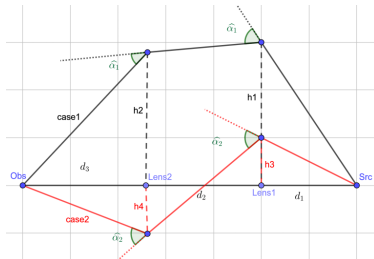
Question 1

A galaxy at redshift 0.5 can be modelled as a singular isothermal sphere; its dispersion is 200km/s . A background source at redshift 2 is lensed by the foreground galaxy into two images with a brightness ratio of 3:1, what are the angular separation and time delay between the two images? You can assume the usual cosmology.

First assume $H_0 = 69.6\text{km s}^{-1}$, $\Omega_m = 0.286$, $\Omega_{vac} = 0.714$ then the distance of $z = 0.5$ and $z = 2$ can be computed, using theories of SIS model and the time delay function we can estimate the angular separation ~ 1.2 arcsec and time delay ~ 45 days

Question 2

Two ($N=2$) galaxies are aligned perfectly with the Earth and a distant quasar. Each galaxy can be modelled as a singular isothermal sphere. How many Einstein rings are formed as a result? How will your results generalize when you have $N > 2$ galaxies?



For N galaxies, the quasar will render 2^{N-1} Einstein rings, and if we consider galaxy lensed by galaxy, will generate (at most) $\sum_{i=1}^{N-1} 2^{i-1}$ more, so the total number of Einstein rings rendered by N galaxies and a quasar will be: $\sum_{i=1}^N 2^{i-1} = 2^N - 1$, at most.

Question 3

A background source is multiply-imaged, can the brightest image arrive last?

Using the relation between effective lensing potential and magnification & time delay

Effective lensing potential:

$$\Psi(\vec{\theta}) = \frac{D_{ds}}{D_d D_s} \frac{2}{c^2} \int \Phi(D_d \vec{\theta}, z) dz, \quad \vec{\nabla}_{\vec{\theta}} \Psi = \vec{\alpha}$$

$$\text{Jacobian matrix: } A = \frac{\partial \vec{\beta}}{\partial \vec{\theta}} = \delta_{ij} - \frac{\partial^2 \Psi(\vec{\theta})}{\partial \theta_i \partial \theta_j} \equiv \delta_{ij} - \Psi_{ij}$$

$$\text{Magnification: } \mu = |\det A|^{-1}$$

$$\text{time delay: } t(\vec{\theta}) = \frac{1+z_d}{c} \frac{D_d D_s}{D_{ds}} \left[\frac{1}{2} (\vec{\theta} - \vec{\beta})^2 - \Psi(\vec{\theta}) \right]$$

$$\mu \leftarrow \Psi \rightarrow t$$

Simulation, under a potential, given a $\vec{\beta}$, solve the lens equation for $\vec{\theta}_i$ see if $\vec{\theta}_k$ which has maximum magnification also have largest time delay

Question 4

Study the caustics and critical curves of two singular isothermal spheres lensing a background quasar. Study two cases 1) Both galaxies are at the same redshift and, 2) these two galaxies are at different redshifts.

Mass density \rightarrow Surface mass density \rightarrow Effective lensing potential \rightarrow Jacobian matrix \rightarrow Magnification draw those points which make magnification largest \rightarrow critical lens \rightarrow caustics

Question 4

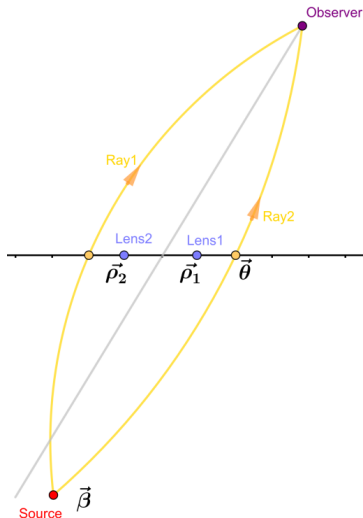
case 1) Both galaxies are at the same redshift

$$\vec{\alpha}(\vec{\theta}) = \theta_{E1} \frac{\vec{\theta} - \vec{\rho}_1}{|\vec{\theta} - \vec{\rho}_1|} + \theta_{E2} \frac{\vec{\theta} - \vec{\rho}_2}{|\vec{\theta} - \vec{\rho}_2|}$$

where $\vec{\rho}_1, \vec{\rho}_2$ are positions of Lens1, Lens2 on the lens plane. $\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta})$

$$A = \frac{\partial \vec{\beta}}{\partial \vec{\theta}} = \left(\delta_{ij} - \frac{\partial \alpha_i(\vec{\theta})}{\partial \theta_j} \right)$$

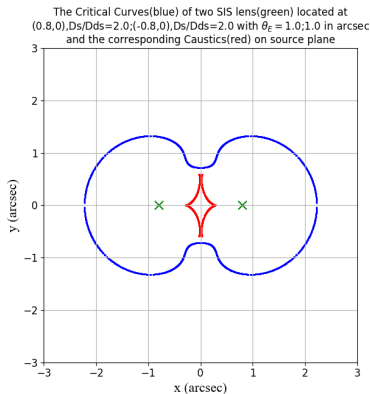
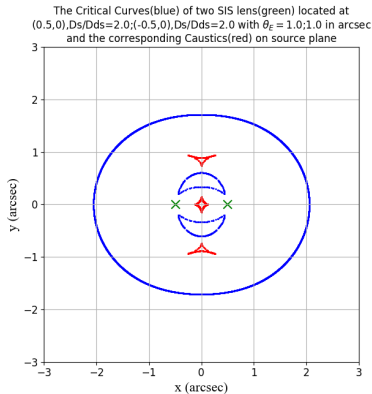
$$\mu(\vec{\theta}) = \frac{1}{\det A(\vec{\theta})}$$



Question 4

case 1) Both galaxies are at the same redshift

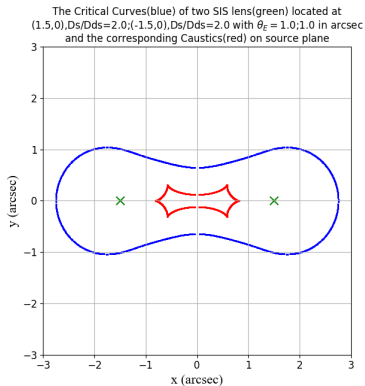
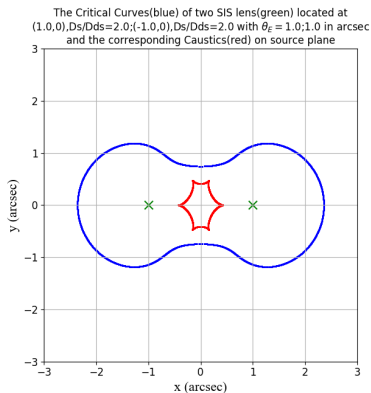
Two SIS lenses ($\theta_E = 1.0''$), separation $1.0'', 1.6''$



Question 4

case 1) Both galaxies are at the same redshift

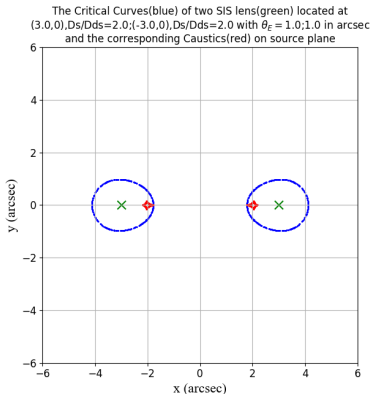
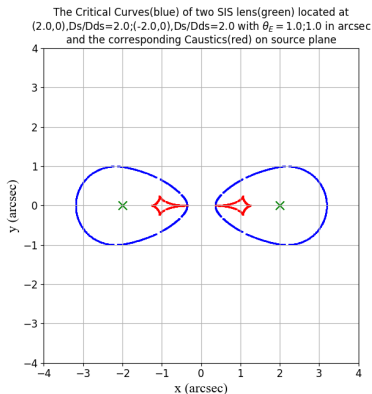
Two SIS lenses ($\theta_E = 1.0''$), separation $2.0'', 3.0''$



Question 4

case 1) Both galaxies are at the same redshift

Two SIS lenses ($\theta_E = 1.0''$), separation $4.0''$, $6.0''$



Question 4

case 2) these two galaxies are at different redshifts

Lens equations of two lens planes system:

$$\vec{\eta} = \frac{D_s}{D_1} \vec{\xi}^{(1)} - D_{1s} \vec{\alpha}^{(1)} \left(\vec{\xi}^{(1)} \right) - D_{2s} \vec{\alpha}^{(2)} \left(\vec{\xi}^{(2)} \right)$$

$$\vec{\xi}^{(2)} = \frac{D_2}{D_1} \vec{\xi}^{(1)} - D_{12} \vec{\alpha}^{(1)} \left(\vec{\xi}^{(1)} \right)$$

For two point mass lenses:

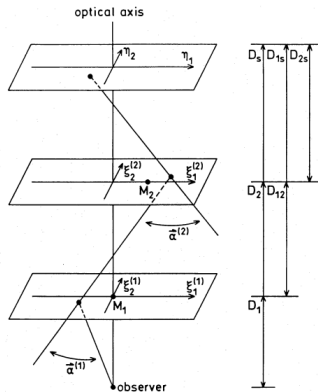
$$\vec{\alpha}^{(1)} \left(\vec{\xi}^{(1)} \right) = \frac{4GM_1}{c^2} \frac{\vec{\xi}^{(1)}}{|\vec{\xi}^{(1)}|^2},$$

$$\vec{\alpha}^{(2)} \left(\vec{\xi}^{(2)} \right) = \frac{4GM_2}{c^2} \frac{\vec{\xi}^{(2)} - \vec{\xi}_{M_2}^{(2)}}{|\vec{\xi}^{(2)} - \vec{\xi}_{M_2}^{(2)}|^2}$$

For two SIS lenses:

$$\vec{\alpha}^{(1)} \left(\vec{\xi}^{(1)} \right) = \frac{4\pi\sigma_{v1}^2}{c^2} \frac{\vec{\xi}^{(1)}}{|\vec{\xi}^{(1)}|},$$

$$\vec{\alpha}^{(2)} \left(\vec{\xi}^{(2)} \right) = \frac{4\pi\sigma_{v2}^2}{c^2} \frac{\vec{\xi}^{(2)} - \vec{\xi}_{M_2}^{(2)}}{|\vec{\xi}^{(2)} - \vec{\xi}_{M_2}^{(2)}|}$$



Question 4

case 2) these two galaxies are at different redshifts

For two SIS lenses:

$$\vec{\alpha}^{(1)} \left(\vec{\xi}^{(1)} \right) = \frac{4\pi\sigma_{v1}^2}{c^2} \frac{\vec{\xi}^{(1)}}{|\vec{\xi}^{(1)}|},$$

$$\vec{\alpha}^{(2)} \left(\vec{\xi}^{(2)} \right) = \frac{4\pi\sigma_{v2}^2}{c^2} \frac{\vec{\xi}^{(2)} - \vec{\xi}_{M_2}^{(2)}}{|\vec{\xi}^{(2)} - \vec{\xi}_{M_2}^{(2)}|}$$

Rearrange these equations:

$$\vec{y} = \vec{x} - m_1 \frac{\vec{x}}{|\vec{x}|} - m_2 \frac{\vec{t} - \vec{\rho}}{|\vec{t} - \vec{\rho}|}$$

$$\vec{t} = \vec{x} - m_1 \beta \frac{\vec{x}}{|\vec{x}|}$$

where $m_1 = D_{1s}\theta_1$, $m_2 = D_{2s}\theta_2$, $\beta = D_{12}/(D_2D_{1s})$,

$$\vec{\xi}^{(2)} = D_2\vec{t}, \quad \vec{\xi}_{M_2}^{(2)} = D_2\vec{\rho}, \quad \vec{\xi}^{(1)} = D_1\vec{x}, \quad \vec{\eta} = D_s\vec{y},$$

$$\theta_i = \frac{4\pi\sigma_{vi}^2}{c^2} = 1'' \cdot 6 \left(\frac{\sigma_{vi}}{200 \text{ km/s}} \right)^2$$

Thus the Jacobian matrix of this two lenses system:

$$A = \frac{\partial \vec{y}}{\partial \vec{x}}$$

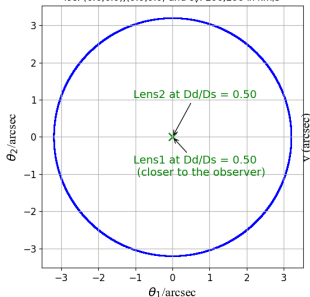
$$\mu(\vec{x}) = \frac{1}{\det A(\vec{x})}$$

Question 4

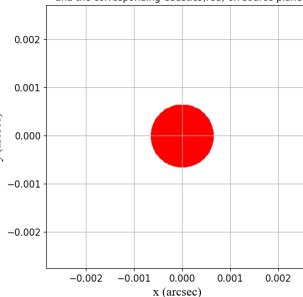
case 2) these two galaxies are at different redshifts

- Obesever, Lens1, Lens2 are on a line, e.g. $\vec{\rho} = (0, 0)$,
and $D_1 = 1, D_2 = 1, D_s = 2$

The Critical Curves(blue) of two SIS lens(green) located at different redshift
loc: (0.0,0.0),(0.0,0.0) and σ_v : 200,200 in km/s



and the corresponding Caustics(red) on source plane

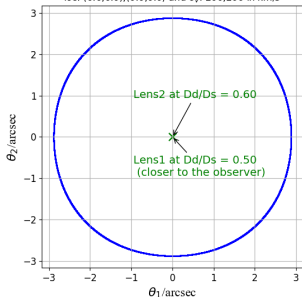


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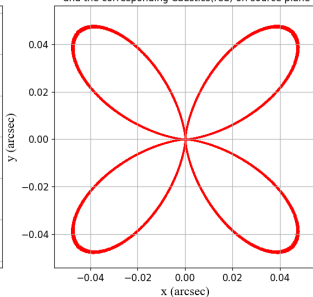
case 2) these two galaxies are at different redshifts

- Obesever, Lens1, Lens2 are on a line, e.g. $\vec{\rho} = (0, 0)$, and $D_1 = 1, D_2 = 1.2, D_s = 2$

The Critical Curves(blue) of two SIS lens(green) located at different redshift
loc: (0,0,0,0),(0,0,0,0) and σ_v : 200,200 in km/s



and the corresponding Caustics(red) on source plane

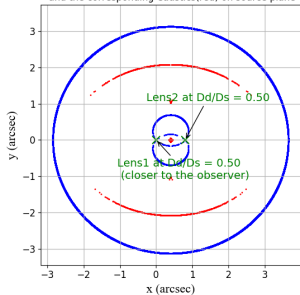


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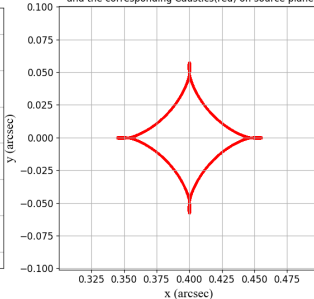
case 2) these two galaxies are at different redshifts

► $\vec{\rho} = (0.8, 0)$, and $D_1 = 1, D_2 = 1, D_s = 2$

The Critical Curves(blue) of two SIS lens(green) located at different redshift
loc: $(0.0, 0.0), (0.8, 0.0)$ and $\sigma_v: 200, 200$ in km/s
and the corresponding Caustics(red) on source plane



and the corresponding Caustics(red) on source plane

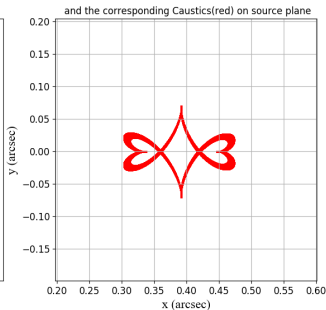
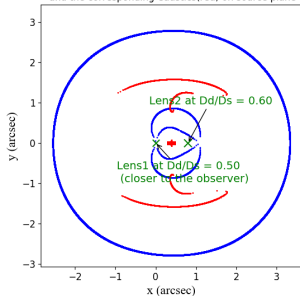


Question 4

case 2) these two galaxies are at different redshifts

- $\vec{\rho} = (0.8, 0)$, and $D_1 = 1, D_2 = 1.2, D_s = 2$

The Critical Curves(blue) of two SIS lens(green) located at different redshift
loc: $(0.0, 0.0), (0.8, 0.0)$ and $\sigma_v: 200, 200$ in km/s
and the corresponding Caustics(red) on source plane

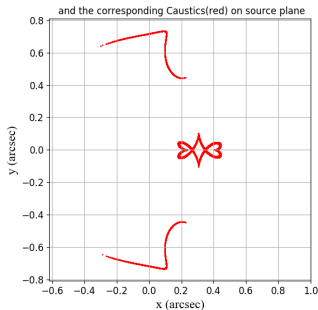
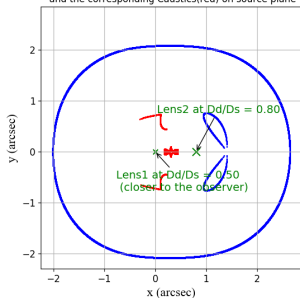


Question 4

case 2) these two galaxies are at different redshifts

- $\vec{\rho} = (0.8, 0)$, and $D_1 = 1, D_2 = 1.6, D_s = 2$

The Critical Curves(blue) of two SIS lens(green) located at different redshift
loc: $(0.0, 0.0), (0.8, 0.0)$ and $\sigma_v: 200, 200$ in km/s
and the corresponding Caustics(red) on source plane

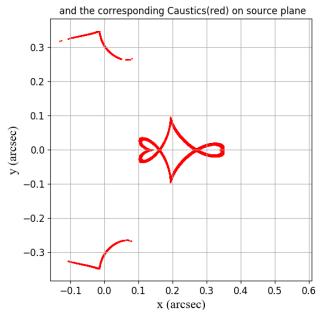
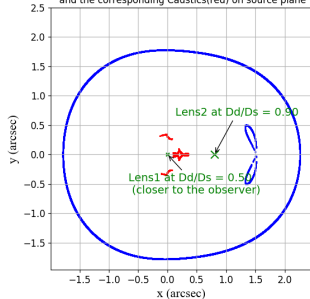


Question 4

case 2) these two galaxies are at different redshifts

- $\vec{\rho} = (0.8, 0)$, and $D_1 = 1, D_2 = 1.8, D_s = 2$

The Critical Curves(blue) of two SIS lens(green) located at different redshift
loc: $(0.0, 0.0), (0.8, 0.0)$ and $\sigma_v: 200, 200$ in km/s
and the corresponding Caustics(red) on source plane

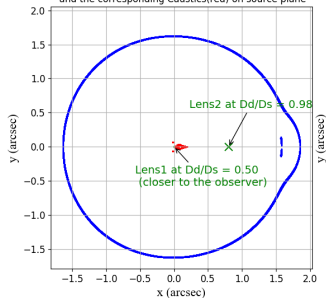


Question 4

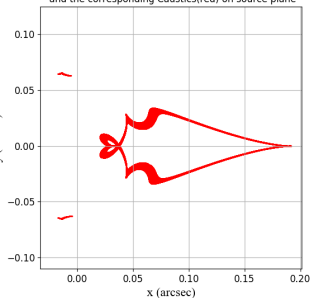
case 2) these two galaxies are at different redshifts

- $\vec{\rho} = (0.8, 0)$, and $D_1 = 1, D_2 = 1.96, D_s = 2$

The Critical Curves(blue) of two SIS lens(green) located at different redshift
loc: $(0.0, 0.0), (0.8, 0.0)$ and $\sigma_v: 200, 200$ in km/s
and the corresponding Caustics(red) on source plane



and the corresponding Caustics(red) on source plane

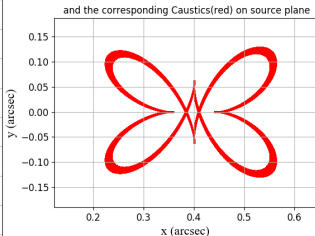
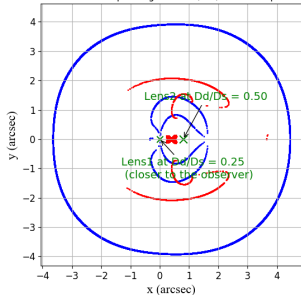


Question 4

case 2) these two galaxies are at different redshifts

- $\vec{\rho} = (0.8, 0)$, and $D_1 = 0.5, D_2 = 1, D_s = 2$

The Critical Curves(blue) of two SIS lens(green) located at different redshift
loc: $(0.0, 0.0), (0.8, 0.0)$ and $\sigma_v: 200, 200$ in km/s
and the corresponding Caustics(red) on source plane

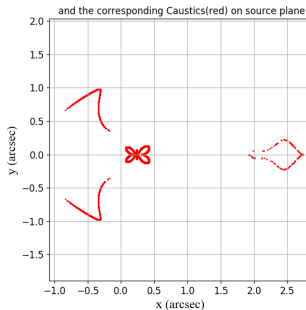
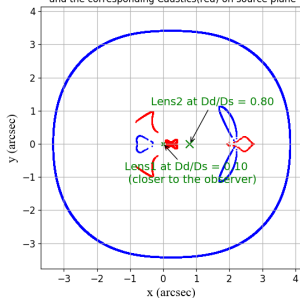


Question 4

case 2) these two galaxies are at different redshifts

- $\vec{\rho} = (0.8, 0)$, and $D_1 = 0.2, D_2 = 1.6, D_s = 2$

The Critical Curves(blue) of two SIS lens(green) located at different redshift
loc: $(0.0, 0.0), (0.8, 0.0)$ and $\sigma_v: 200, 200$ in km/s
and the corresponding Caustics(red) on source plane

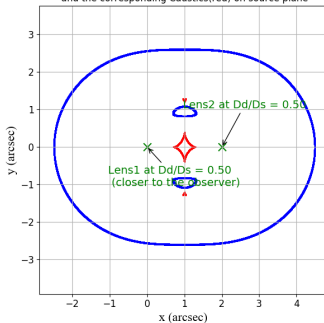


Question 4

case 2) these two galaxies are at different redshifts

- $\vec{\rho} = (2, 0)$, and $D_1 = 1, D_2 = 1, D_s = 2$

The Critical Curves(blue) of two SIS lens(green) located at different redshift
loc: (0.0,0.0),(2.0,0.0) and σ_v : 200,200 in km/s
and the corresponding Caustics(red) on source plane

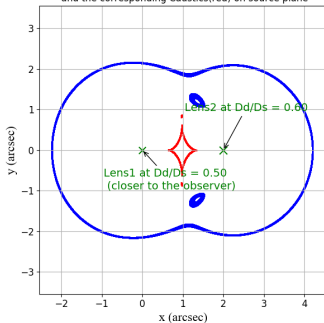


Question 4

case 2) these two galaxies are at different redshifts

- $\vec{\rho} = (2, 0)$, and $D_1 = 1, D_2 = 1.2, D_s = 2$

The Critical Curves(blue) of two SIS lens(green) located at different redshift
loc: (0.0,0.0),(2.0,0.0) and σ_v : 200,200 in km/s
and the corresponding Caustics(red) on source plane

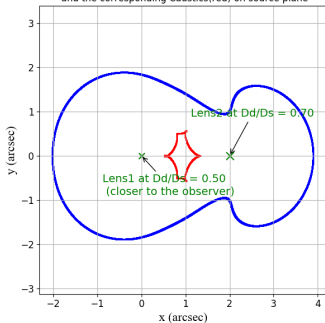


Question 4

case 2) these two galaxies are at different redshifts

- $\vec{\rho} = (2, 0)$, and $D_1 = 1, D_2 = 1.4, D_s = 2$

The Critical Curves(blue) of two SIS lens(green) located at different redshift
loc: (0.0,0.0),(2.0,0.0) and σ_v : 200,200 in km/s
and the corresponding Caustics(red) on source plane

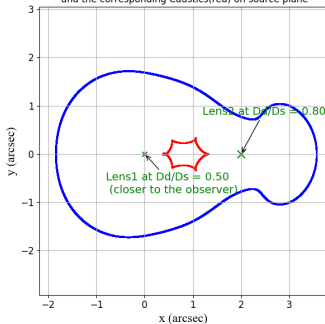


Question 4

case 2) these two galaxies are at different redshifts

- $\vec{\rho} = (2, 0)$, and $D_1 = 1, D_2 = 1.6, D_s = 2$

The Critical Curves(blue) of two SIS lens(green) located at different redshift
loc: (0.0,0.0),(2.0,0.0) and σ_v : 200,200 in km/s
and the corresponding Caustics(red) on source plane

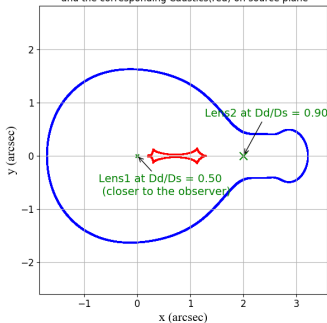


Question 4

case 2) these two galaxies are at different redshifts

- $\vec{\rho} = (2, 0)$, and $D_1 = 1, D_2 = 1.8, D_s = 2$

The Critical Curves(blue) of two SIS lens(green) located at different redshift
loc: (0.0,0.0),(2.0,0.0) and σ_v : 200,200 in km/s
and the corresponding Caustics(red) on source plane

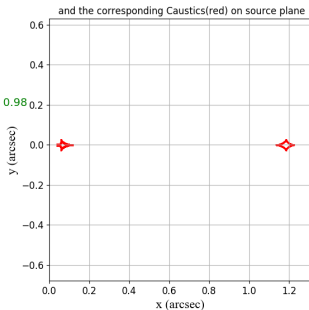
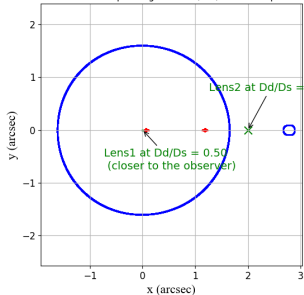


Question 4

case 2) these two galaxies are at different redshifts

► $\vec{\rho} = (2, 0)$, and $D_1 = 1, D_2 = 1.96, D_s = 2$

The Critical Curves(blue) of two SIS lens(green) located at different redshift
loc: $(0.0, 0.0), (2.0, 0.0)$ and $\sigma_v: 200, 200$ in km/s
and the corresponding Caustics(red) on source plane

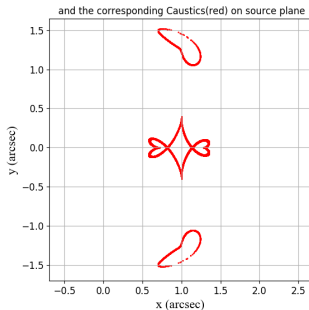
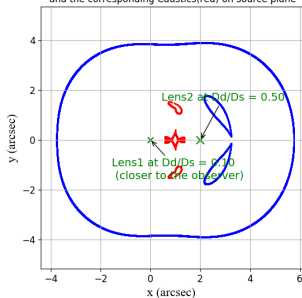


Question 4

case 2) these two galaxies are at different redshifts

- $\vec{\rho} = (2, 0)$, and $D_1 = 0.2, D_2 = 1, D_s = 2$

The Critical Curves(blue) of two SIS lens(green) located at different redshift
loc: $(0.0, 0.0), (2.0, 0.0)$ and $\sigma_v: 200, 200$ in km/s
and the corresponding Caustics(red) on source plane

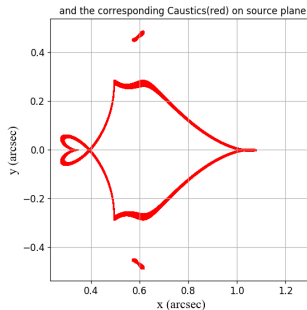
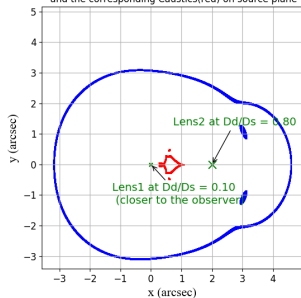


Question 4

case 2) these two galaxies are at different redshifts

- $\vec{\rho} = (2, 0)$, and $D_1 = 0.2, D_2 = 1.6, D_s = 2$

The Critical Curves(blue) of two SIS lens(green) located at different redshift
loc: (0.0,0.0),(2.0,0.0) and σ_v : 200,200 in km/s
and the corresponding Caustics(red) on source plane

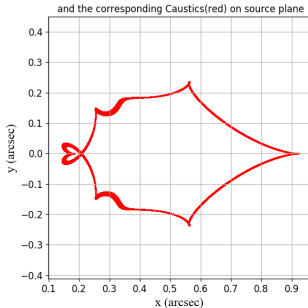
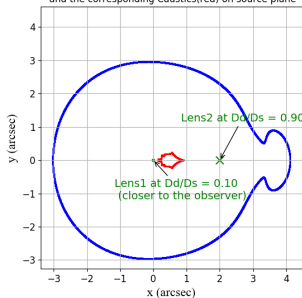


Question 4

case 2) these two galaxies are at different redshifts

► $\vec{\rho} = (2, 0)$, and $D_1 = 0.2, D_2 = 1.8, D_s = 2$

The Critical Curves(blue) of two SIS lens(green) located at different redshift
loc: (0.0,0.0), (2.0,0.0) and σ_v : 200,200 in km/s
and the corresponding Caustics(red) on source plane

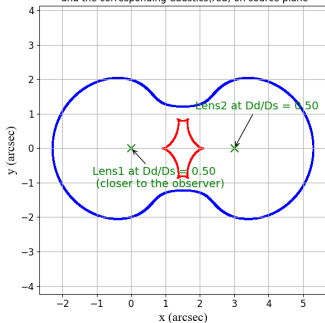


Question 4

case 2) these two galaxies are at different redshifts

- $\vec{\rho} = (3, 0)$, and $D_1 = 1, D_2 = 1, D_s = 2$

The Critical Curves(blue) of two SIS lens(green) located at different redshift
loc: (0.0,0.0),(3.0,0.0) and σ_v : 200,200 in km/s
and the corresponding Caustics(red) on source plane

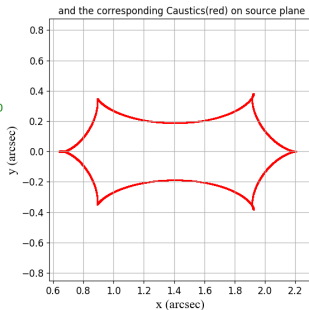
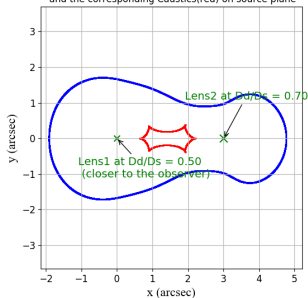


Question 4

case 2) these two galaxies are at different redshifts

- $\vec{\rho} = (3, 0)$, and $D_1 = 1, D_2 = 1.4, D_s = 2$

The Critical Curves(blue) of two SIS lens(green) located at different redshift
loc: $(0.0, 0.0), (3.0, 0.0)$ and σ_v : 200, 200 in km/s
and the corresponding Caustics(red) on source plane

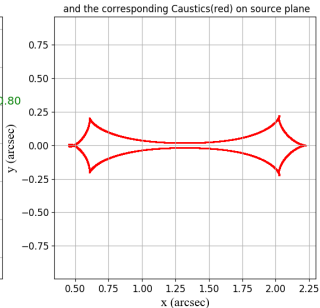
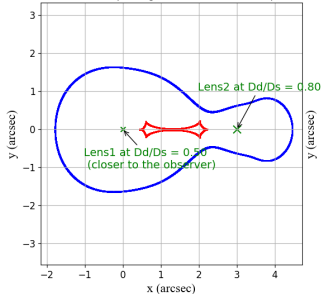


Question 4

case 2) these two galaxies are at different redshifts

► $\vec{\rho} = (3, 0)$, and $D_1 = 1, D_2 = 1.6, D_s = 2$

The Critical Curves(blue) of two SIS lens(green) located at different redshift
loc: $(0.0, 0.0), (3.0, 0.0)$ and $\sigma_v: 200, 200$ in km/s
and the corresponding Caustics(red) on source plane

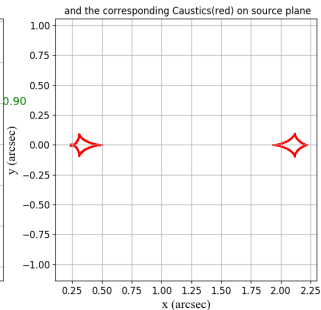
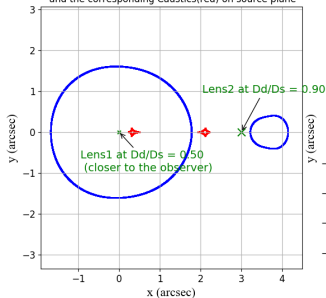


Question 4

case 2) these two galaxies are at different redshifts

- $\vec{\rho} = (3, 0)$, and $D_1 = 1, D_2 = 1.8, D_s = 2$

The Critical Curves(blue) of two SIS lens(green) located at different redshift
loc: $(0.0, 0.0), (3.0, 0.0)$ and $\sigma_v: 200, 200$ in km/s
and the corresponding Caustics(red) on source plane

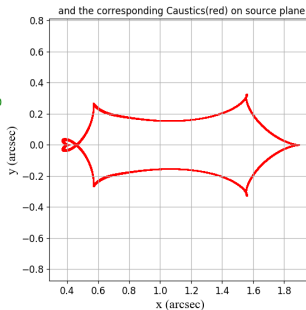
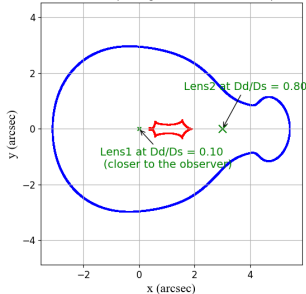


Question 4

case 2) these two galaxies are at different redshifts

- $\vec{\rho} = (3, 0)$, and $D_1 = 0.2, D_2 = 1.6, D_s = 2$

The Critical Curves(blue) of two SIS lens(green) located at different redshift
loc: $(0.0, 0.0), (3.0, 0.0)$ and $\sigma_v: 200, 200$ in km/s
and the corresponding Caustics(red) on source plane

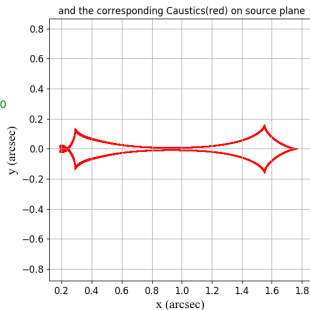
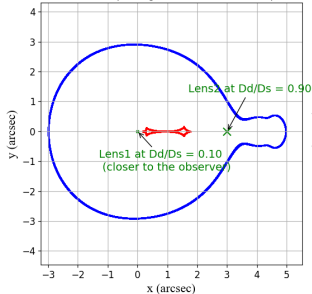


Question 4

case 2) these two galaxies are at different redshifts

- $\vec{\rho} = (3, 0)$, and $D_1 = 0.2, D_2 = 1.8, D_s = 2$

The Critical Curves(blue) of two SIS lens(green) located at different redshift
loc: $(0, 0, 0, 0), (3, 0, 0, 0)$ and $\sigma_v: 200, 200$ in km/s
and the corresponding Caustics(red) on source plane

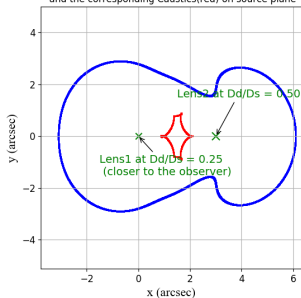


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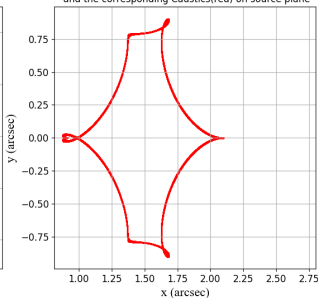
case 2) these two galaxies are at different redshifts

► $\vec{\rho} = (3, 0)$, and $D_1 = 0.5, D_2 = 1, D_s = 2$

The Critical Curves(blue) of two SIS lens(green) located at different redshift
loc: $(0, 0, 0.0), (3.0, 0.0, 0)$ and $\sigma_v: 200, 200$ in km/s
and the corresponding Caustics(red) on source plane



and the corresponding Caustics(red) on source plane

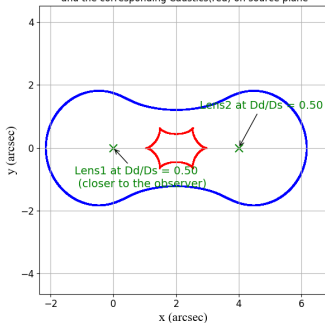


Question 4

case 2) these two galaxies are at different redshifts

- $\vec{\rho} = (4, 0)$, and $D_1 = 1, D_2 = 1, D_s = 2$

The Critical Curves(blue) of two SIS lens(green) located at different redshift
loc: $(0.0, 0.0), (4.0, 0.0)$ and σ_v : 200, 200 in km/s
and the corresponding Caustics(red) on source plane

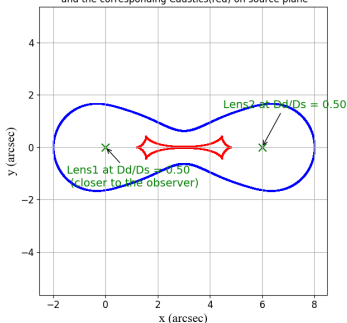


Question 4

case 2) these two galaxies are at different redshifts

- $\vec{\rho} = (6, 0)$, and $D_1 = 1, D_2 = 1, D_s = 2$

The Critical Curves(blue) of two SIS lens(green) located at different redshift
loc: (0.0,0.0),(6.0,0.0) and σ_v : 200,200 in km/s
and the corresponding Caustics(red) on source plane

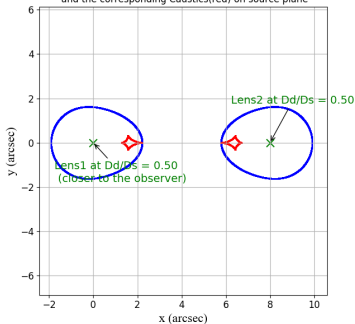


Question 4

case 2) these two galaxies are at different redshifts

- $\vec{\rho} = (8, 0)$, and $D_1 = 1, D_2 = 1, D_s = 2$

The Critical Curves(blue) of two SIS lens(green) located at different redshift
loc: $(0.0, 0.0), (8.0, 0.0)$ and σ_v : 200, 200 in km/s
and the corresponding Caustics(red) on source plane



Gravitational Lensing Applications

GravLens

Renkun Kuang

History

Gravitational
Lensing Theories

Related Questions

Applications

- ▶ Cosmic telescopes: distant, faint objects observation
- ▶ 2-d mass distribution of lenses, dark matter
- ▶ Hubble constant, cosmological constant, density parameter
- ▶

The End, Thanks!

References

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- ▶ *Lectures On Gravitational Lensing*, Narayan & Bartelmann
- ▶ <https://www.cfa.harvard.edu/~dfabricant/huchra/ay202/lectures/lecture12.pdf>
- ▶ <https://web.stanford.edu/~oas/SI/SRGR/notes/SchwarzschildSolution.pdf>
- ▶

Appendix-1, Newtonian prediction

$$\begin{aligned}\phi(b, z) &= -\frac{GM}{(b^2 + z^2)^{1/2}} \\ \alpha &= \frac{v_b}{c} = \frac{1}{c} \int \frac{d\Phi}{db} dt = \frac{1}{c^2} \int \frac{d\Phi}{db} dl \\ &\approx \frac{1}{c^2} \int \frac{d\Phi}{db} dz \\ &= \frac{GMb}{c^2} \int \frac{dz}{(b^2 + z^2)^{3/2}} \\ &= \frac{GMb}{c^2} \left[\frac{z}{b^2 \sqrt{b^2 + z^2}} \right]_{-\infty}^{+\infty} \\ &= \frac{2GM}{c^2 b}\end{aligned}$$

b : impact factor

Gravitational potential simplify the calculation by integrating not along the deflected ray but along z axis

