# Essential Radio Astronomy Lecture Note

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# Contents

# Chapter 1

# Introduction to Radio Astronomy

## 1.1 Related links

- https://www.cv.nrao.edu/course/astr534/PDFnewfiles/Introradastro.pdf
- https://www.astron.nl/eris2017/Documents/ERIS2017_L1_McKean.pdf
  2017 European Radio Interferometry School, a week of lectures and tutorials on how to achieve scientific results from radio interferometry.
  Topics covered:
  1. Calibration and imaging of continuum, spectral line, and polarisation data;
  2. Low-freq. (LOFAR domain), high-freq. (ALMA/IRAM domain), and VLBI-interferometry;
  3. Extracting information from the data and interpreting the results;
  4. Choosing the most suitable array and observing plan for your project.
- http://www.radio-astronomy.org/pdf/sara-beginner-booklet.pdf
- http://egg.astro.cornell.edu/alfalfa/ugradteam/pdf12/radio_lecture_jess_uat12.pdf
- http://www.jb.man.ac.uk/ tob/course/radio2.html
- SKA Shanghai 2018

## 1.2 Software Preparation

ref: 2017 European Radio Interferometry School.
Examples will be drawn from m-, cm- and mm-wave instruments such as LOFAR,

JVLA, EVN, eMERLIN and ALMA.

CDs for advanced users may include tutorials on using the proposal preparation tools (the Proposal Submission Tool for the VLA, GBT, and VLBA, and the Observing Tool for ALMA) and data reduction software (CASA for the VLA and ALMA, GBTIDL for the GBT, and AIPS for VLBA). nrao-cd

### 1.2.1 AIPS

- 1996 The AIPSview Astronomy Visualization Tools, PDF
  AIPSview is a set of two new software tools for visual data analysis being developed by the radio astronomy group at the University of Illinois as part of the AIPS++ project. The tools provide a wide range of functionality for the display and analysis of 2D and 3D astronomical data sets. In this paper we describe how to obtain further information about AIPSview on the WWW including how to obtain executable and source code, and discuss the current functionality of AIPSview and future development plans.

- http://www.aips.nrao.edu/index.shtml
  What is AIPS
  The Astronomical Image Processing System is a software package for calibration, data analysis, image display, plotting, and a variety of ancillary tasks on Astronomical Data. It comes from the National Radio Astronomy Observatory.

  Download install.pl via ftp and the "perl install.pl -n" to install the software.
  ftp host website: ftp://ftp.aoc.nrao.edu/pub/software/aips
  linux下登陆FTP

In terminal, using following command to login:
lftp ftp://ftp.aoc.nrao.edu/pub/software/aips
Download files:
要下载文件首先得设置用于存放下载文件的本地目录。
命令：lcd 本地目录
然后进入待下载文件的目录。根据下载文件类型的不同，命令也不同。
单个文件
命令：get file
get -c file 可以进行断点续传
批量文件
命令： mget *.txt 批量下载目录下的txt文件
mget -c *.txt 断点续传加上批量下载
整个目录
命令：mirror aaa/

```
lftp ftp.aoc.nrao.edu:/pub/software/aips> lcd ./
lcd ok, local cwd=/home/anything/THU/astro/softwares/aips
lftp ftp.aoc.nrao.edu:/pub/software/aips> mirror 31DEC19
`31DEC19.tar.gz' at 1206184 (0%) 47.0K/s eta:58m [Receiving data]
[0] 0:lftp*                                                      "ar
```

```
==================== AIPS 31DEC19 Install Wizard====================

Screen 11: FINAL REVIEW before installing!
------------------------------------------
This is your last, best hope for checking the settings before
committing to the install.  Please check these settings, and
make sure they are what you want:

   AIPS_ROOT (screen 3): /home/anything/THU/astro/softwares/aips/31DEC19
       Group (screen 4): anything
 Group Write (screen 4): YES
Architecture (screen 5): LNX64
   Site name (screen 5): ANY
  AIPS hosts (screen 6): LOCALHOST
  Data areas (screen 7): /home/anything/THU/astro/softwares/aips/31DEC19/DATA/LOCALHOST_1
    Printers (screen 8):   Paper type (screen 8): A4
 Tape drives (screen 9):
  Tape hosts (screen 9): 127.0.0.1
   Advanced (screen 10): (not listed here)
```

```
/home/anything/THU/astro/softwares/aips/31DEC19/31DEC19/LNX64/SYSTEM/ANY/MAKE.MNJ: 69: /home/anything/
THU/astro/softwares/aips/31DEC19/31DEC19/LNX64/SYSTEM/ANY/MAKE.MNJ: /opt/local/compilers/gcc-6/bin/gcc
: not found
MAKE.MNJ -   UPDLSTDAT compiled
MAKE.MNJ - all .OLD files created
MAKE.MNJ - LAST*.DAT files created with begin date of 20190903.000000
MAKE.MNJ - Done.
MAKE.MNJ - The MNJ now uses cvs (http://www.cvshome.org) for updates;
MAKE.MNJ - looking to see if I can find a copy of it....
MAKE.MNJ - I can not seem to find it, either it is not here or it is
MAKE.MNJ - here, but it is installed in a place that is not in your
MAKE.MNJ - search path. If it is not here, you should install it and
MAKE.MNJ - run MAKE.MNJ again. Otherwise, please tell me where you
MAKE.MNJ - put it.
MAKE.MNJ - Full path to cvs: : |
```

install cvs: http://www.cvshome.org, instruction

installing AIPS failed.
2019 install instruction

Installation instructions for Training Week 1

## Installation instructions for Training Week 1

There are two courses in training week 1: radio astronomy and telescope systems, and radio interferometry. Both of these have a hands-on component, and software should be pre-installed on laptops beforehand. The software packages needed are IPython for the IPython notebook component of the radio astronomy course, and AIPS or CASA for radio interferometry. Either MacOS or Linux is needed (i.e. **not Windows**). On Windows laptops or PCs it should be possible to install a VirtualBox `https://www.virtualbox.org/` with Linux as the guest operating system, or alternatively have Linux as a dual boot if you are planning to use it a lot.

### 1. Radio astronomy

This course uses IPython notebooks, so IPython is needed. On Linux systems with `pip`, you may be able to get and install IPython with `pip install ipython`. (IPython does run on Windows in theory, but we have not tested the notebooks with Windows, and MacOS/Linux is needed in any case for part 2).

Probably the easiest (and on some systems the only) way to install IPython is via the installation of Jupyter, which is available on

```
jupyter.readthedocs.org/en/latest/install.html
```

This installs without problems provided that at least Python 2.7 is present. If you do not have Python 2.7, then the best course is to install `anaconda` - a link to installation instructions for this is given on the same webpage. Once you have anaconda, any other packages can be installed with the `conda install` command. In some cases (e.g. jupyter) this can be done directly: `conda install jupyter`. In other cases, all you should need to do is to google `conda [package-name]` for any package that appears to be missing.

You can test your installation by downloading the file

```
http://www.jb.man.ac.uk/~njj/test.ipynb
```

and running it with `jupyter notebook test.ipynb`. You should get a cell of commands with a blue line to the left. Clicking inside the cell and then selecting `cell` and then `Run cell` should produce a plot with blue dots.

### 2. Radio interferometry

*2.1 Major packages*

There are currently two major software packages for radio interferometry, AIPS and CASA. AIPS is an older, Fortran-based system; CASA is a newer, C++/Python-based system which will eventually supersede AIPS. CASA has a fuller suite of imaging features, including much better wide-field mapping and imaging algorithms, but AIPS has some features useful for long-baseline interferometry.

AIPS is the primary software package for VLBI, e-MERLIN and LOFAR (long baselines). CASA, or CASA-based, systems are the primary system for LOFAR (other than long baselines), the JVLA, and ALMA. The syntax of the two packages is not hugely different, and there is a translation available on `https://casaguides.nrao.edu/index.php?title=aips-to-casa_Cheat_Sheet`.

The course will be bilingual (in AIPS and CASA!) so you can choose which one to install. Both are interoperable with Python: CASA natively, and AIPS by an interface (ParselTongue, which is not covered in the first week's course).

Both are distributed as binaries and should be relatively straightforward to install on any version of Linux or any version of MacOS. Please install and test one (or both) as below if you are intending to follow the course. **Note: neither AIPS nor CASA runs on Windows.**

Both packages are available as binary installations for MacOS and Linux. AIPS can be downloaded from

`http://www.aips.nrao.edu/install.shtml`

and CASA from

`http://casa.nrao.edu/casa_obtaining.shtml`

In both cases, the installation should be relatively painless. For CASA, the installation comes as a unix tar file, which can be unzipped in any desired directory. If you start the executable `casapy`, which should be in the top directory, this should start CASA and produce a log window.

For AIPS, the installation is slightly more complicated, but a Perl install wizard guides you through the steps. If you start the executable `START_AIPS`, which should be in the top directory, AIPS should start up and invite you to type in an AIPS number - any number larger than 10 should be fine. If you start it with the argument `tv=local`, then a black TV window should pop up as well.

*2.2 Difmap*

There is also a third useful package which does only some tasks in radio interferometry, but does them very well and very interactively. It is particularly useful for long-baseline interferometry. It's available from `ftp://ftp.astro.caltech.edu/pub/difmap/difmap.html` . It requires the plotting package PGPLOT, which is available from `http://www.astro.caltech.edu/~tjp/pgplot/`.

If there are any problems, I'm happy to try and help: `neal.jackson@manchester.ac.uk` although I may need to consult the local IT support here in complicated cases. Please forward screenshots and/or as much detail of error messages as possible in these cases.

- 天文博客
- How to build AIPS on Ubuntu
- The AIPS++ Project

Finally successfully installed:

```
AipsWiz:  =====> We're DONE!  Let's have a nice Banana Split! <======

AipsWiz: ***************** READ CAREFULLY **************************

AipsWiz: ***************** READ CAREFULLY **************************

AipsWiz: Services should be defined either in /etc/services
 oryour YP/NIS services map (all tcp services)
This may require sudo or root privileges

AipsWiz: ***************** READ CAREFULLY **************************

AipsWiz: Here are the final setup instructions for running AIPS

        1.  Reference the LOGIN.SH file in your .profile file
              or perhaps your .bashrc file
              (dot it now too, via ". ./LOGIN.SH")

        2.  Check that it works:

                aips notv tpok

              (this will not start a TV or tape servers).
            Try 'print 2 + 2' for a very basic test.

        3.  Make a cron entry for the do_daily.LOCALHOST file
              that the MAKE.MNJ created, so you can run the
              AIPS 'midnight job'.  This is optional but
              strongly recommended.

AipsWiz: That's it.  You should now have the latest AIPS!  Enjoy.

# anything @ anything-ThinkPad-E470c in ~/.aips [13:40:30]
$ |
```

```
# anything @ LOCALHOST in ~/.aips [13:48:17]
$ aips notv tpok
START_AIPS: Your initial AIPS printer is the
START_AIPS:  - system name , AIPS type

START_AIPS: User data area assignments:
DADEVS.PL: This program is untested under Perl version 5.026
  (Using global default file /home/anything/.aips/DA00/DADEVS.LIST for DADEVS.PL)
   Disk 1 (1) is /home/anything/.aips/DATA/LOCALHOST_1

Tape assignments:
   Tape 1 is REMOTE
   Tape 2 is REMOTE

START_AIPS: Assuming TPMON daemons are running or not used (you said TPOK)
Starting up 31DEC18 AIPS with normal priority
Begin the one true AIPS number 1 (release of 31DEC18) at priority =   0
AIPS 1: You are NOT assigned a TV device or server
AIPS 1: You are NOT assigned a graphics device or server
AIPS 1: Enter user ID number
?2
AIPS 1:                          31DEC18 AIPS:
AIPS 1:      Copyright (C) 1995-2019 Associated Universities, Inc.
AIPS 1:            AIPS comes with ABSOLUTELY NO WARRANTY;
AIPS 1:                 for details, type HELP GNUGPL
AIPS 1: This is free software, and you are welcome to redistribute it
AIPS 1: under certain conditions; type EXPLAIN GNUGPL for details.
AIPS 1: Previous session command-line history recovered.
AIPS 1: TAB-key completions enabled, type HELP READLINE for details.
AIPS 1: Recovered POPS environment from last exit
>print 2+2
AIPS 1:      4
>|
```

### TWO VLBI TUTORIALS

From http://www.aips.nrao.edu/index.shtml
Two extensive tutorials on VLBI data reduction in AIPS have been prepared, complete with data sets, introductory material about AIPS, and detailed instructions. They are:

- simple VLBA project including self-calibration
- spectral-line VLBA project plus astrometry

Users new to, or rusty in, VLBI data reduction are encouraged to try these tutorials. Appendix C and Chapter 9 of the AIPS CookBook are also recommended.

### 1.2.2 CASA

CASA-obtaining

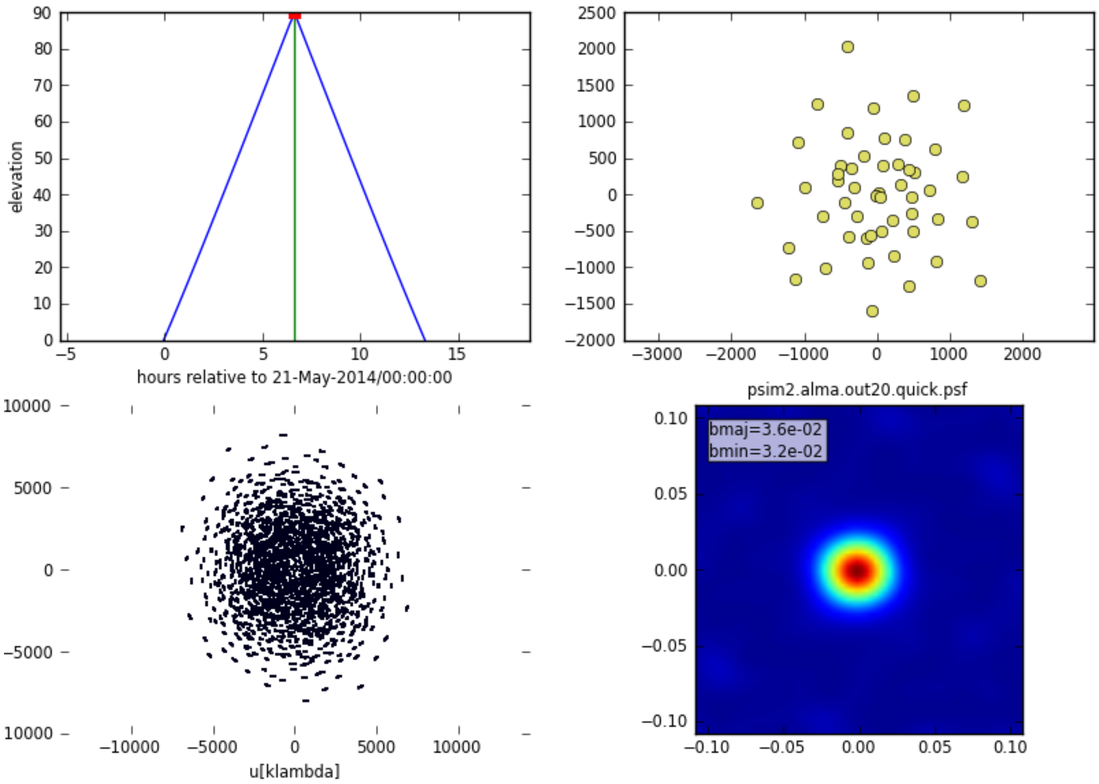
CASA Documentation Homepage

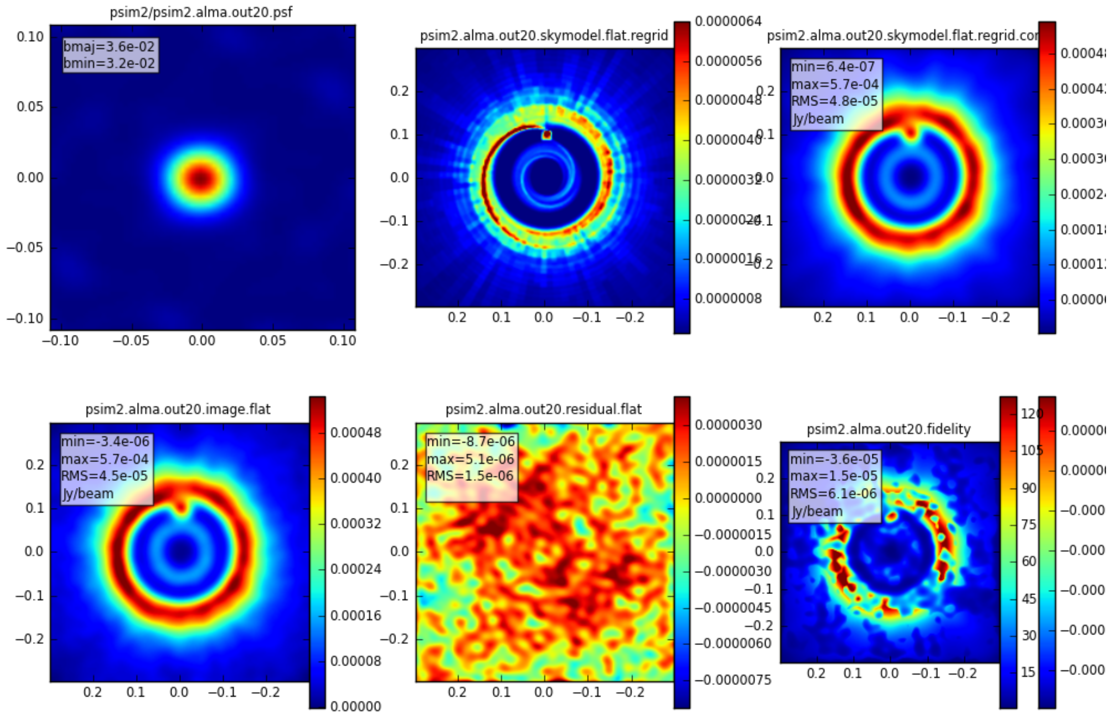
CASA homepage

CASA Guides

**Protoplanetary Disk Simulation Using CASA**

Reference






## 1.3 Introduction to Radio Astronomy - John McKean

The following of this note file is mainly center on 2017 European Radio Interferometry School(which aims to give a general introduction to radio astronomy, focusing on the issues that you must consider for single element telescopes that make up an interferometer) materials.

### 1.3.1 The Radio Window

$\nu \sim 10^7 Hz - 10^{12} Hz, \;\; \lambda \sim 10m - 0.1mm$

The observing window is constrained by atmospheric absorption / emission and refraction.

1. Charged particles in the ionosphere reflects radio waves back into space at $< 10$ MHz.

2. Vibrational transitions of molecules have similar energy to infra-red photons and absorb the radiation at $> 1$ GHz (completely by $\sim 300$ GHz)

### 1.3.2 The low-frequency cut-off

The ionosphere consists of a plasma of charged particles (conducting layers), that has an effective refractive index of,

$$n^2 = 1 - \frac{\omega_p^2}{\omega^2} = 1 - \frac{\lambda^2}{\lambda_p^2}$$

where, the plasma frequency is defined as,

$$\nu_p[Hz] = \frac{\omega_p}{2\pi} = (\frac{N_e e^2}{4\pi^2 \epsilon_0 m})^{1/2} = 8.97 \times 10^3 \sqrt{\frac{N_e}{[cm^{-3}]}}$$

when $\omega < \omega_p \rightarrow n^2 < 0$, there is no propagation, i.e. total reflection. (i.e. low-frequency cutoff)

| |
|---|
| Worked example: What is the cut-off frequency for LOFAR observations carried out when the electron density is $N_e = 2.5 \times 10^5 cm^{-3}$ (night time) and $N_e = 1.5 \times 10^6 cm^{-3}$ (day time)?<br>$\nu_p$[Hz] = 4.5 MHz (night time)<br>$\nu_p$[Hz] = 11 MHz (day time)<br>(at night, the observable frequency by LOFAR can be lower) |

At frequencies,

1. $\omega < \omega_p$: $n^2 < 0$, reflection ($\nu < 10MHz$),
2. $\omega > \omega_p$: $n^2 > 0$, refraction ($10MHz < \nu < 10GHz$),
3. $\omega \gg \omega_p$: $n^2 \rightarrow 1$ ($\nu > 10GHz$).

The observing conditions are dependent on the electron density, i.e. the solar conditions (space weather), since the ionisation is due to the ultra-violet radiation field from the Sun,

$$O_2 + h\nu \rightarrow O_2^{+*} + e^-$$

$$O_2 + h\nu \rightarrow O^+ + O + e^-$$

### 1.3.3 The high-frequency cut-off (absorption)

1. Molecules in the atmosphere can absorb the incoming radiation, but also emit radiation (via thermal emission).

2. Mass absorption co-efficient (k): From atomic and molecular physics, define for various species, i,

$$k_i = \frac{\sigma n_i}{r_i \rho_0}$$

   where: $k_i$ is Mass attenuation coefficient ($cm^2 g^{-1}$), $\sigma$ Cross-section ($cm^2$), $n_i$ is Number density of particles ($cm^{-3}$), $\rho_0$ is Mass density of air ($gcm^{-3}$), $r_i$ is Mixing ratio($= \rho_i/\rho_0$)

3. Optical depth ($\tau$): A measure of the absorption/ scattering (attenuation) of electromagnetic radiation in a medium (probability of an interaction),

$$\tau_i(\lambda, z_0) = \int_{z_0}^{\infty} n_i(z)\sigma dz = \int_{z_0}^{\infty} r_i(z)\rho_0(z)k_i(\lambda)dz$$

   or, in terms of the linear absorption co-efficient ($\kappa$)

$$\tau_i(\lambda, z_0) = \int_{z_0}^{\infty} \kappa(\lambda, z)dz$$

   where $\kappa(\lambda, z) = k_i(\lambda)\rho_i(z)$,
   $\kappa(\lambda, z)$: linear absorption coefficient ($cm^{-1}$)
   $k_i(\lambda)$: Mass attenuation coefficient ($cm^2 g^{-1}$)
   $\rho_i(z)$: Mass density of species i ($gcm^{-3}$)
   $\rho_i(z) = r_i(z)\rho_0(z)$

   The attenuation of an incident ray of intensity $I_0$, received at altitude $z_0$, summed over all absorbing species is,

$$I(z_0) = I_0 exp[-\sum_i \tau(\lambda, z_0)] = I_0 exp[-\tau(z)]$$

   Where, for convenience, we consider all species together and define the optical depth as a function of zenith angle, $\tau(z)$

| |
|---|
| Worked example: What is the optical depth for sky transparencies of 0.5, 0.1 and 0.01? |
| Rearrange, in terms of $\tau$, and evaluate, $\tau = -ln(\frac{I(z_0)}{I_0})$ |
| $tau_{0.5} = -ln(0.5) = 0.69$ |
| $tau_{0.1} = -ln(0.1) = 2.3$ |
| $tau_{0.01} = -ln(0.01) = 4.6$ |
| The smaller the transparancy, the larger the optical depth $\tau$ |

Note that the opacity changes with the path length, and so depends on the airmass X(z), which assuming a plane parallel atmosphere,

$$\tau(z) = \tau_0 X(z), X(z) = sec(z)$$

where $\tau_0$: Optical depth at Zenith, $X(z)$: Airmass, $z$: Zenith angle

The atmosphere is not completely transparent at radio wavelengths, but $\tau(z)$ varies with frequency $\nu$.

Zenith opacity is the sum of several component opacities at cm $\lambda$.

- Broadband (continuum) opacity: dry air. $\tau_z \approx 0.01$ and almost independent of $\nu$
- Molecular absorption: $O_2$ has rotational transitions that absorb radio waves and are opaque ($\tau_z \gg 1$) at 52 to 60 GHz.
- Hydrosols: Water droplets ($radius \leq 0.1mm$) suspended in clouds absorb radiation (proportional to $\lambda^{-2}$).
- Water vapor: Emission line at $\nu \approx 22.235$ GHz is pressure broadened to $\Delta\nu \sim 4GHz$ width + "continuum" absorption from the "line-wings" of very strong $H_2O$ emission at infrared wavelengths (proportional to $\lambda^{-2}$).

  The zenith optical depth is dependent on the path length through the material.

  - Higher altitude: Move above the water vapour layer (> 4 km).
  - Drier locations: Move to regions with low water vapour.

### 1.3.4 The high-frequency cut-off (emission)

A partially absorbing atmosphere also emits radio noise that can de-grade ground based observations. We can define the total system noise power as an equivalent noise temperature

$$P = \frac{E}{\Delta t} = kT\Delta\nu$$

in terms of spectral power,

Spectral power (W Hz$^{-1}$)

System temperature (Receivers; Sky, Ground; etc)

$$P_\nu = k\,T_{\rm sys}$$

Boltzmann constant = 1.38 x 10$^{-23}$ m$^2$ kg s$^{-2}$ K$^{-1}$

where,

Noise from Radio background (Galaxy, CMB, etc)

Noise from ground emission

Noise from injected noise

Noise from the receiver (Dominates)

$$T_{\rm sys} = T_{\rm bg} + T_{\rm sky} + T_{\rm spill} + T_{\rm loss} + T_{\rm cal} + T_{\rm rx}$$

Noise from atmospheric emission

Noise from losses at receiver

The contribution from the sky opacity to the sky temperature is,

Atmospheric kinetic temperature (≡ 300 K)

Emitted sky temperature (K)

$$T_{\rm sky} = T_{\rm atm}\left[1 - \exp(-\tau_\nu)\right]$$

Optical depth

Don't want $T_{sky}$ to dominate our noise budget, need to minimise $T_{atm}$ and $\tau_\nu$ by observing in cold and dry locations (winter; high alt), especially at high frequencies

---

Worked example: Using the total opacity data for the Green Bank Telescope (West Virginia; USA; 2800 m) and $T_{atm} = 288$ K, what is $T_{sky}$ at $\nu = 5$ GHz, 22 GHz and 115 GHz?
How does this compare with the typical receiver temperature, $T_{rx} \sim 30$ K?

- At $\nu = 5$ GHz, $\tau_z \sim 0.007$, $T_{sky} = 288[1 - exp(-0.007)] \sim 2K$ (Good)
- At $\nu = 22$ GHz, $\tau_z \sim 0.15$, $T_{sky} = 288[1 - exp(-0.15)] \sim 40K$ (Bad)
- At $\nu = 115$ GHz, $\tau_z \sim 0.8$, $T_{sky} = 288[1 - exp(-0.8)] \sim 160K$ (Bad)

---

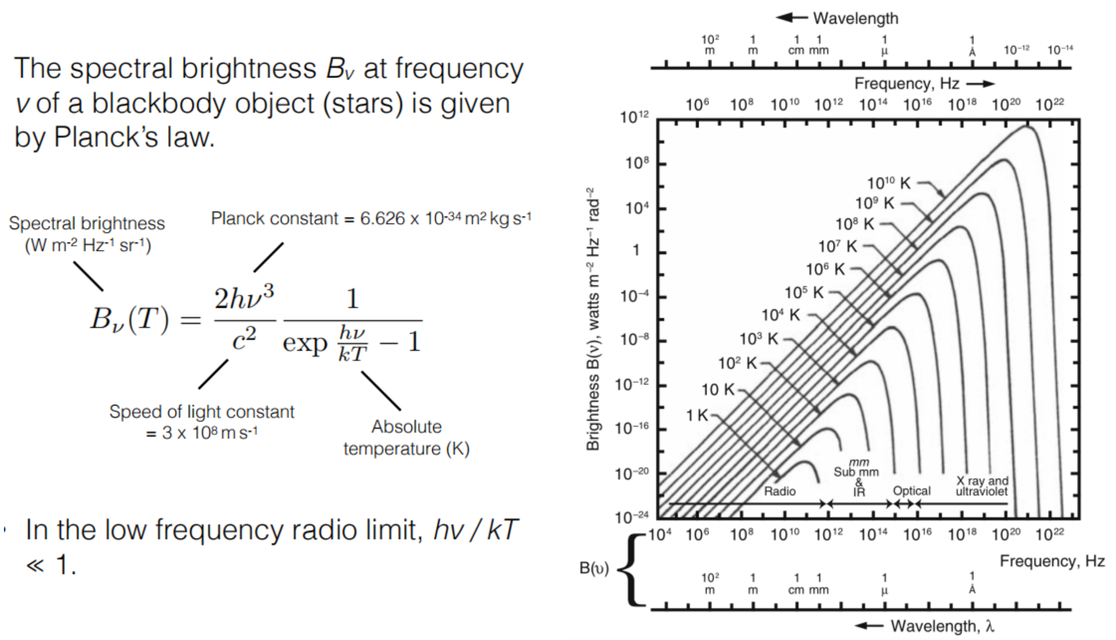
## Key concept: The partially transparent atmosphere

**allows radio waves to be detected from ground-based telescopes, but also attenuates the signal due to absorption/scattering, and also adds noise to the measured signal**

### 1.3.5 Early Radio Astronomy

The first detection of radiation at radio wavelengths was not made until 1932 due to,

1. limitations of technology (our eyes), but then the communication era started,
2. the expectation that celestial objects would be too faint




---

Worked example: Does the low-frequency limit work for the photosphere of the Sun, which has $T = 5800$ K? At $\nu = 1$ GHz,

$$\frac{h\nu}{kT} = \frac{6.626 \times 10^{-34} \times 1 \times 10^9}{1.38 \times 10^{-23} \times 5800} = 8 \times 10^{-6}$$

---

Using this property($h\nu/kT \ll 1$ in the low frequency limit), we can replace the exponential term using the Taylor expansion,

$$exp(h\nu/kT) - 1 \approx 1 + \frac{h\nu}{kT} + ... - 1$$

to give the Rayleigh-Jeans approximation to the Planck function at low-frequencies,

$$B_\nu(T) \approx \frac{2h\nu^3}{c^2}\frac{kT}{h\nu} = \frac{2kT}{\lambda^2}$$

Flux-density ($S_\nu$): The power received per unit detector area in a unit bandwidth ($\Delta\nu = 1$ Hz) at frequency $\nu$. The units are $Wm^{-2}Hz^{-1}$.
The flux-density received from a celestial source of brightness $B_\nu(T)$ and subtending a very small angle $\Omega \ll 1$sr, is approximately,

$$S_\nu = B_\nu\Omega$$

---

Worked example: What is the flux-density at $\nu = 1$ GHz of a black body with temperature T = 5800 K and size $R \approx 7 \times 10^{10}$ cm (the Sun) at about 1 parsec ($d \approx 3 \times 10^{18}$ cm)

$$B_\nu(T) = \frac{2kT\nu^2}{c^2} = 1.78 \times 10^{-18} Wm^{-2}Hz^{-1}sr^{-1}$$

The spectral brightness is an intrinsic property of the source (independent of distance). The solid angle subtended by the source is dependent on the distance

$$\Omega = \frac{\pi R^2}{d^2} \approx 1.71 \times 10^{-15} sr$$

The flux-density is therefore,

$$S_\nu = B_\nu\Omega \approx 3 \times 10^{-33} Wm^{-2}Hz^{-1}$$

This flux density is too small for even todays telescopes to detect (easily), so the thermal emission from stars was thought to be impossible to detect at radio wavelengths, but ...

---

Long distance communication developed by Marconi & Ferdinand Braun - Nobel Prize 1909
Evolution of frequency over the years

- pre-1920: <100 kHz.
- ca. 1920: shift to 1.5 MHz.

- post-1920: 10s of MHz (more voice channels, less effected by the ionosphere and thunderstorms).
- Research labs sprung up in early-1900s

**Karl Jansky (1933, published) discovered a radio signal at 20.5 MHz that varied steady every 23 hours and 56 minutes (Sidereal day).**
**"The data give for the co-ordinates of the region from which the disturbance comes, a right ascension of 18 hours and declination -10 degrees."**
**He had detected the Galactic Centre.**

Grote Reber (1937-39), using his own 10 m telescope, made no detection at 3300 and 910 MHz, **ruling out a Planck spectrum ($B_\nu \propto \nu^2$). Detection made at 150 MHz, confirming Jansky's result and finding the spectrum must be non-thermal**

**Key concept: Radio emission from celestial objects can be measured and it can be both thermal and non-thermal in origin.**

### 1.3.6 Radio Telescopes and interferometers

- Radio telescopes are designed in a different way to optical telescopes, and the radio range is so broad (5 decades in frequency) that different telescope technologies can be used.
- The surface accuracy of a reflector is proportional to $\lambda/16$
  - cm (1 GHz) $\rightarrow$ surface accuracy of $\sim$ 2 cm
  - mm (100 GHz) $\rightarrow$ surface accuracy of $\sim$ 200 $\mu m$.
  - Optical (0.55 $\mu$ m) $\rightarrow$ surface accuracy of $\sim$ 0.034 $\mu m$.

$$\frac{P(\delta)}{P_0} = \eta_s = exp[-(\frac{4\pi\sigma}{\lambda})^2] \tag{1.1}$$

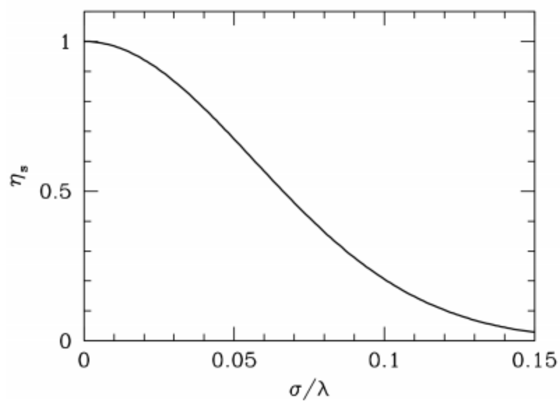



Figure 1.1: Not understand yet

**Not understand equation 1.1 and figure 1.1**

- Large single-element radio telescopes can be constructed cheaply, but have limited spatial resolution, $\theta \approx \lambda/D$

---

Worked example: What is the spatial resolution (in arcseconds) of the $D =$ 300 m Arecibo telescope, operating at $\nu = 5$ GHz?
$\theta \sim 41$ arcsec

---

- Interferometric techniques have been developed to combine several single element telescopes into a multi-element array. Now, the resolution is limited by the distance between the elements

---

What is the spatial resolution (in arcseconds) of the Very Long Baseline Array operating at $\nu = 5$ GHz? The longest distance between telescopes is $D_{max} = 8611$ km.
$\sim 0.00144$ arcsec

---

**Key Concept: Radio interferometry can provide the highest angular resolution imaging possible in astronomy.**

## 1.4 Astrophysical applications

- The large radio window has allowed a wide variety of astronomical sources, thermal and non-thermal radiation mechanisms, and the propagation phenomena to be studied.

1. Discrete cosmic radio sources, at first, supernova remnants and radio galaxies (1948);
2. The 21cm line of atomic hydrogen (1951);
3. Quasi Stellar Objects "Quasars" (1963);
4. The Cosmic Microwave Background (1965);
5. Inter stellar molecules and pronto-planetary discs (1968);
6. Pulsars (1968);
7. Gravitational lenses (1979);
8. The Sunyaev-Zeldovich effect (1983);
9. Distance determinations using source proper motions determined from Very Long Baseline Interferometry (1993); and
10. Molecules in high-redshift galaxies (2005)

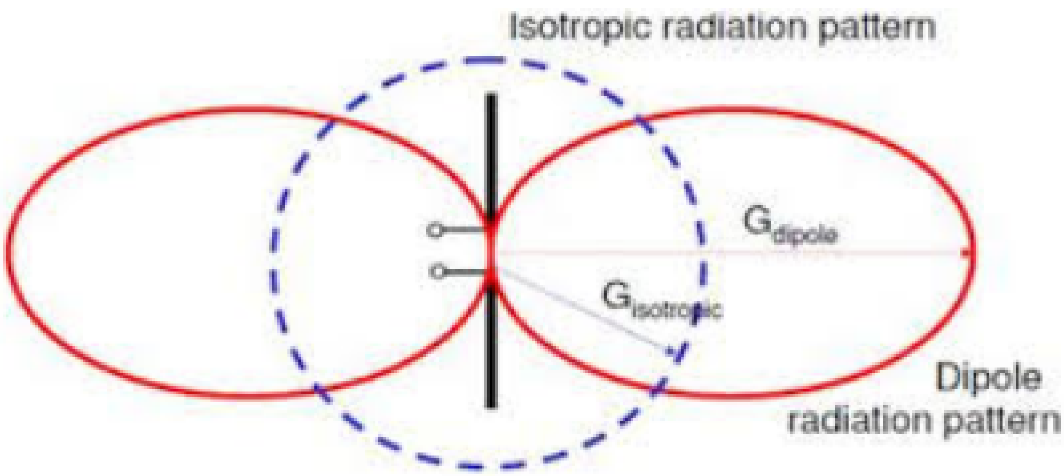
## 1.5 LOFAR

### 1.5.1 Response of the LOFAR antenna

In different frequencies, LOFAR has different Gains as a function of Zenith angle.

### 1.5.2 Power gain

$G(\theta, \phi)$ is the power transmitted per unit solid angle in direction $(\theta, \phi)$ divided by the power transmitted per unit solid angle from an isotropic antenna with the same total power( figure 1.5.2).




The power or gain are often expressed in logarithmic units of decibels (dB):

$$G(dB) = 10 \times log_{10}(G)$$

---

Worked example: What is the maximum and half power of a normalised power pattern in decibels? Maximum power of a normalised power pattern is $P_n = 1$

$$P_n(1) = 10 \times log_{10}(1) = 0dB$$

Half power of a normalised power pattern is $P_n = 0.5$

$$P_n(0.5) = 10 \times log_{10}(0.5) = -3dB$$

---

For a lossless isotropic antenna, conservation of energy requires the directive gain averaged over all directions be,

$$< G >= \frac{\int_{sphere} G d\Omega}{\int_{sphere} d\Omega} = 1$$

Therefore, for an isotropic lossless antena,

$$\int_{sphere} G d\Omega = \int_{sphere} d\Omega = 4\pi$$

and

$$G = 1$$

Lossless antennas may radiate with different directional patterns, but they cannot alter the total amount of power radiated $\rightarrow$ the gain of a lossless antenna depends only on the angular distribution of radiation from that antenna.

**Key Concept: Higher the gain, the narrower the radiation pattern (directivity)**

$$\Delta\Omega \approx \frac{4\pi}{G_{max}}$$

### 1.5.3 Beam solid angle

Beam solid angle: The beam area $\Omega_A$ is the solid angle through which all of the power radiated by the antenna would stream if $P(\theta, \phi)$ maintained its maximum value over $\Omega_A$ and zero everywhere else

Normalised power pattern

Beam solid angle (sr)

$$\boxed{\Omega_{\mathrm{A}} \equiv \int_{4\pi} P_{\mathrm{n}}(\theta, \phi) d\Omega}$$

sky area ($r^2 \sin\theta \, d\theta d\phi$)

The power (and temperature) received is also a function of the power pattern of the antenna. Therefore, the true antenna temperature is,

$$T_{\mathrm{A}} = \frac{A_{\mathrm{e}}}{2k} \int\int I_\nu(\theta, \phi) P_{\mathrm{n}}(\theta, \phi) d\Omega$$

where $P_n(\theta, \phi)$ is the power pattern normalised to unity maximum,

$$P_n = \frac{G(\theta, \Phi)}{G_{max}}$$

### 1.5.4 Response of a reflector antenna

**Paraboloidal reflectors** are useful because,

1. The effective collecting area Ae can approach the geometric area ($= \pi D^2/4$).
2. Simpler than an array of dipoles.
3. Can change the feed antenna to work over a wide frequency range (e.g. for the JVLA 8 receivers on each telescope allow observations from 1–50 GHz)

For a receiving antenna, where the electric field pattern is $f(l)$ and the electric field illuminating the aperture is $g(u)$

$$f(l) = \int_{aperture} g(u)e^{-i2\pi lu}du$$

**Key concept: In the far-field, the electric field pattern is the Fourier transform of the electric field illuminating the aperture**

The radiated power as a function of position

$$P_n(l) = sinc^2(\frac{\theta D}{\lambda})$$

For a one-dimensional uniformly illuminated aperture

$$\theta_{HPBW} \approx 0.89\frac{\lambda}{D}$$

The central peak of the power pattern between the first minima is called the main beam (typically defined by the half-power angular size)

The smaller secondary peaks are called sidelobes.

Main beam solid angle: The area containing the principle response out to the first zero.

Side-lobes: Areas outside the principle response that are non-zero.

$$\Omega_{MB} = \int_{MB} P_n(\theta, \Phi)d\Omega$$

where $\Omega_{MB}$ is the Main beam solid angle (sr)

Main beam efficiency: The fraction of the total beam solid angle inside the main beam

$$\eta_B = \frac{\Omega_{MB}}{\Omega_A}$$

## 1.6 Sensitivity

Our ability to measure a signal is dependent on the noise properties of our complete system ($T_{sys}$), although this is typically dominated by the Johnson noise within the receiver. Our ability to measure a signal is dependent on the noise properties of our complete system (Tsys), although this is typically dominated by the Johnson noise within the receiver.
The variations (uncertainty on some measurement) is estimated by,

1. In time interval $\tau$ there are a minimum $N = 2\Delta\nu\tau$ independent samples of the total noise power $T_{sys}$.

2. The uncertainty in the noise power (from a random gaussian distribution) is $\approx 2^{1/2} T_{sys}$.

3. The rms error in the average of $N \gg 1$ independent samples is reduced by the factor $N^{1/2}$

$$\sigma_T = \frac{2^{1/2} T_{sys}}{N^{1/2}}$$

which gives the (ideal) radiometer equation:

$$\sigma_T \approx \frac{T_{sys}}{\sqrt{\Delta\nu_{RF}\tau}}$$

where $\sigma_T$ is rms uncertaintyK, $T_{sys}$ is System temperature(K), $\Delta\nu_{RF}$ is total bandwidth(Hz), and $\tau$ is total time.

Typically $T_{sys} \gg T_{source}$. Need rms uncertainty in the system temperature to be as low as possible. Increase the observed bandwidth or observing for longer, or decrease the receiver temperature

The signal-to-noise ratio of our target source is:

$$\frac{S}{N} = \frac{T_{source}}{\sigma_T} = \frac{T_{source}}{T_{sys}} \sqrt{\Delta_{RF}\tau}$$

It is convenient to express the rms uncertainty in terms of the system equivalent flux density (SEFD; units of Jy). Recall

$$P_\nu = kT_A = A_e \frac{S_\nu}{2}$$

$$T_A = (\frac{A_e}{2k}) S_\nu$$

where $(\frac{A_e}{2k})$ is called the "forward gain" (K/Jy)

Define the SEFD as,

$$SEFD = \frac{2kt_{sys}}{A_e}$$

---

Worked example: What is the SEFD of a 25-m VLA antenna assuming a system temperature of 55 K and an effective area of 365 $m^2$?

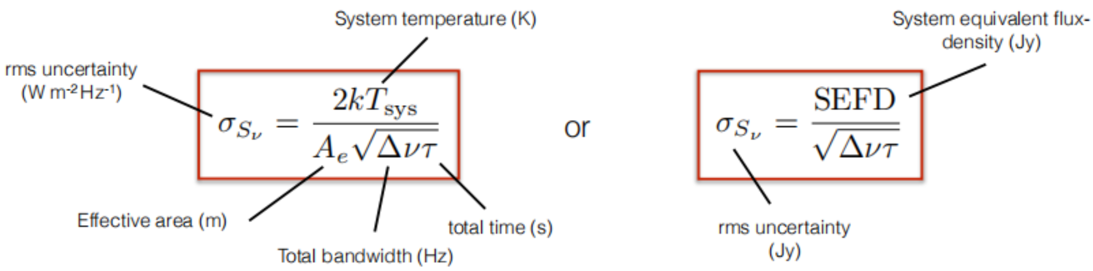
$$SEFD = \frac{2kt_{sys}}{A_e} = \frac{2 \times 1 \times 10^{-23} \times 55}{365} = 300Jy$$

This will scale inversely with the effective area, i.e. a low SEFD suggests a more sensitive telescope

---

The SEFD is a good way to compare the sensitivity of telescopes because it takes the receiver system ($T_{sys}$) and the effective area ($A_e$) into account.

We can define our ideal radiometer equation to determine the sensitivity in terms of fluxdensity,




## 1.7 Summary

1. Radio astronomy covers 5 decades in frequency from ∼10 MHz up to about 1 THz (ground based).

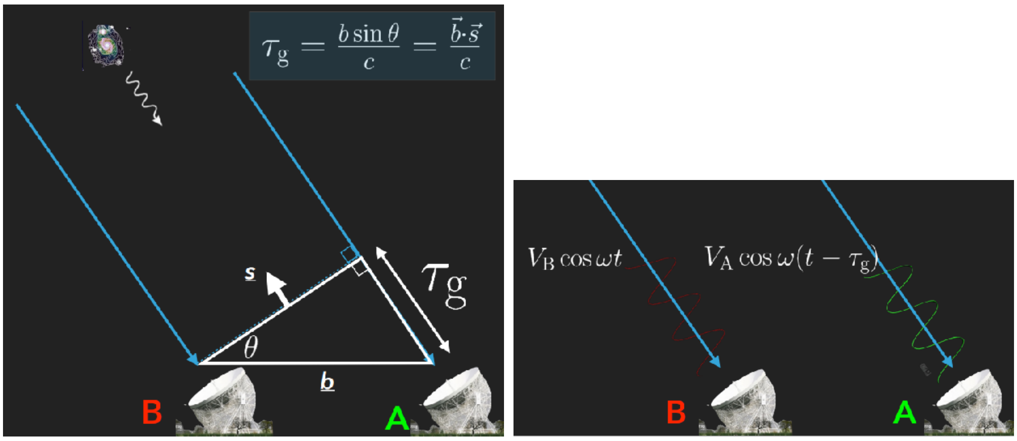
2. It is a well established area of astronomical research that allows for a large number of unique science cases to be investigated (sensitivity and resolution).

3. Keep in mind the properties of the single elements of your interferometer

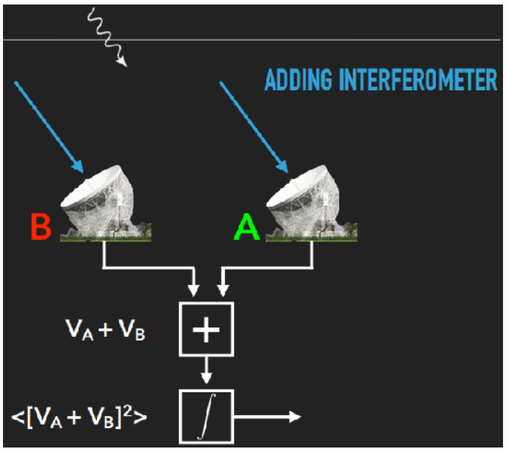
# Chapter 2

# (Gentle) Introduction to Interferometry - ANNA SCAIFE

Radio telescopes measure a voltage due to the incident EM radiation.



## 2.1 Adding Interferometer




$$R \propto < [V_A \cos\omega(t-\tau_g) + V_B \cos(\omega t) + V_{rec,A} + V_{rec,B}]^2 >$$

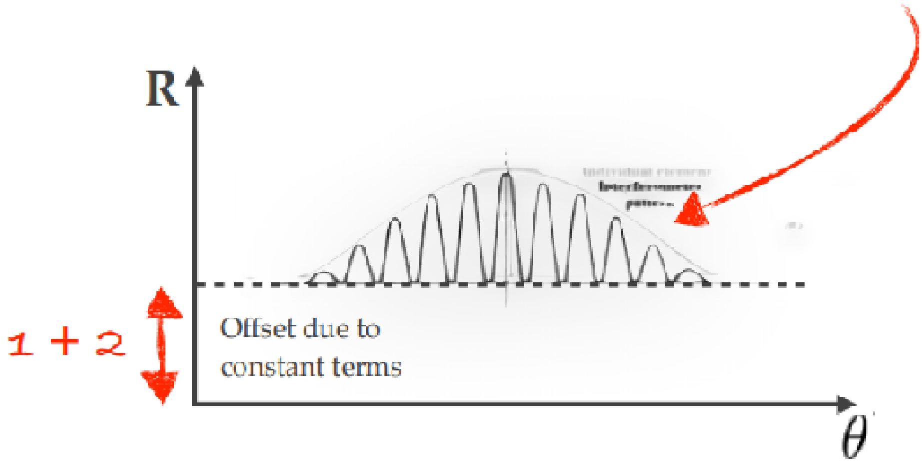
$$R \propto \frac{1}{2}\langle \underbrace{\left[V_{\rm A}^2 + V_{\rm B}^2\right]}_{1} + \underbrace{\left[V_{\rm rec,A}^2 + V_{\rm rec,B}^2\right]}_{2} + \underbrace{[V_{\rm A}V_{\rm B}\cos(\omega\tau_{\rm g})]}_{3}\rangle$$

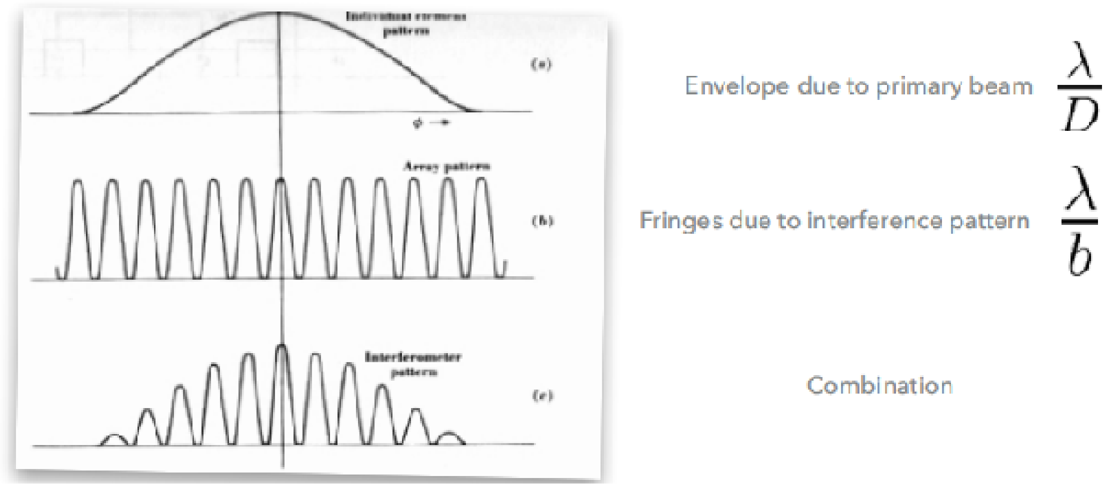
1 Antenna noise powers - usually small

2 Receiver noise powers - usually dominate

3 Interference fringes vary with $\tau_g$

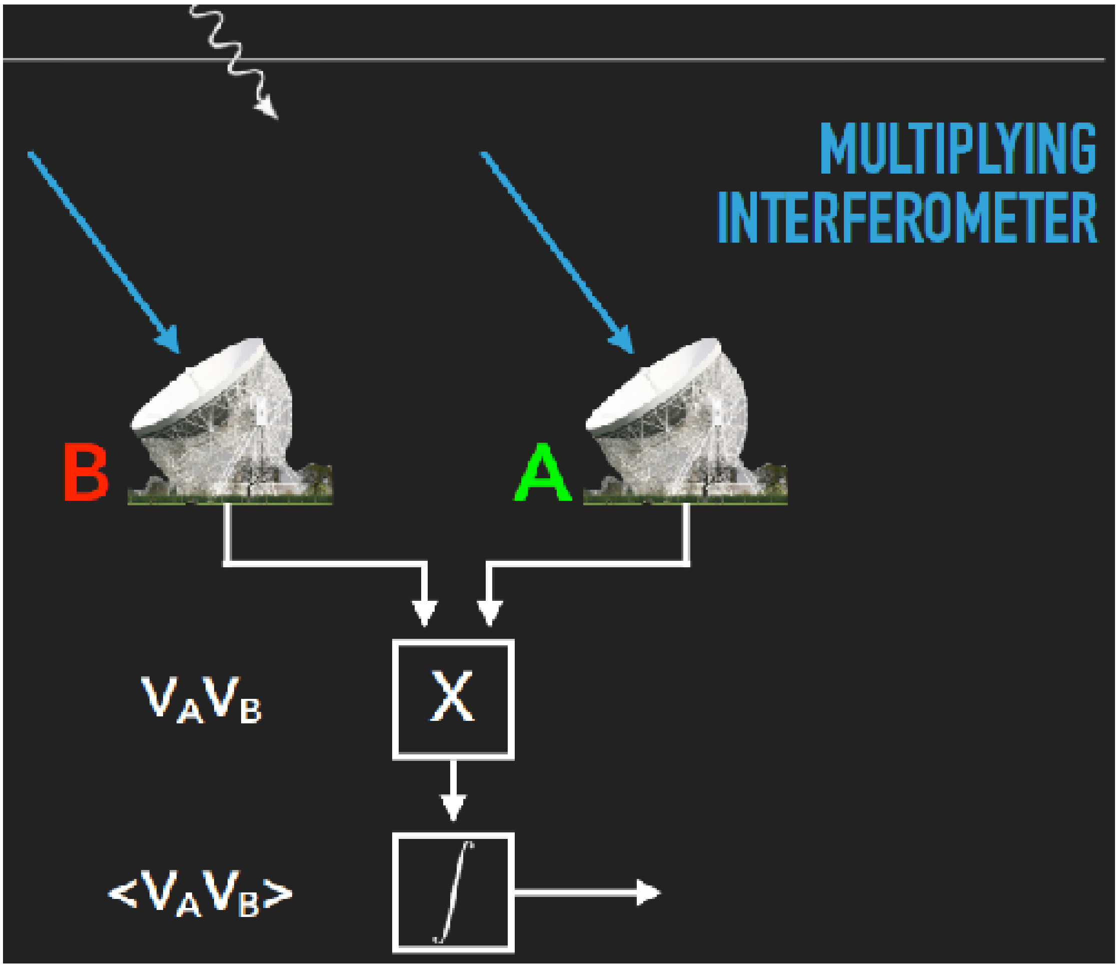
$$R \propto \frac{1}{2}\langle \underbrace{[V_{\rm A}^2 + V_{\rm B}^2]}_{1} + \underbrace{[V_{\rm rec,A}^2 + V_{\rm rec,B}^2]}_{2} + \underbrace{[V_{\rm A} V_{\rm B} \cos(\omega \tau_{\rm g})]}_{3} \rangle$$



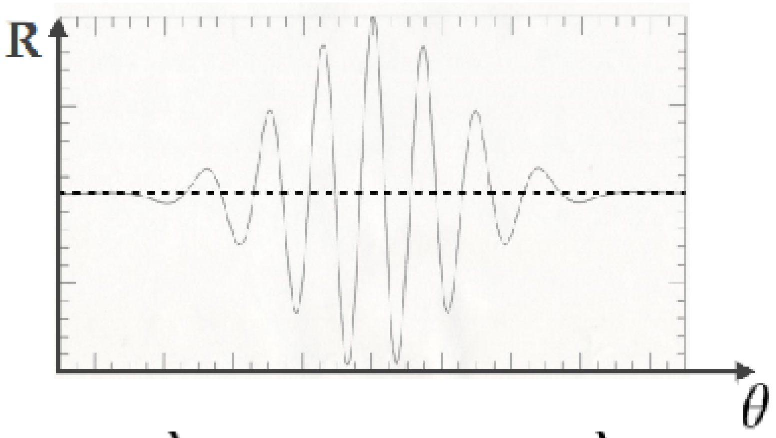




b is the length of the baseline between two telescopes.

## 2.2 Multiplying Interferometer

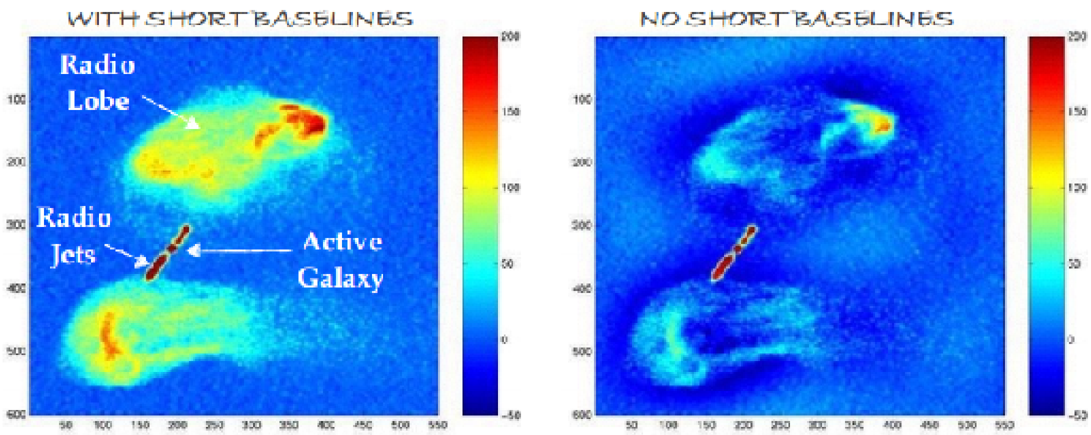


$$R \propto \langle V_A \cos\omega(t-\tau_g)\cdot V_B \cos\omega t\rangle = \frac{1}{2}V_A V_B \cos\tau_g$$

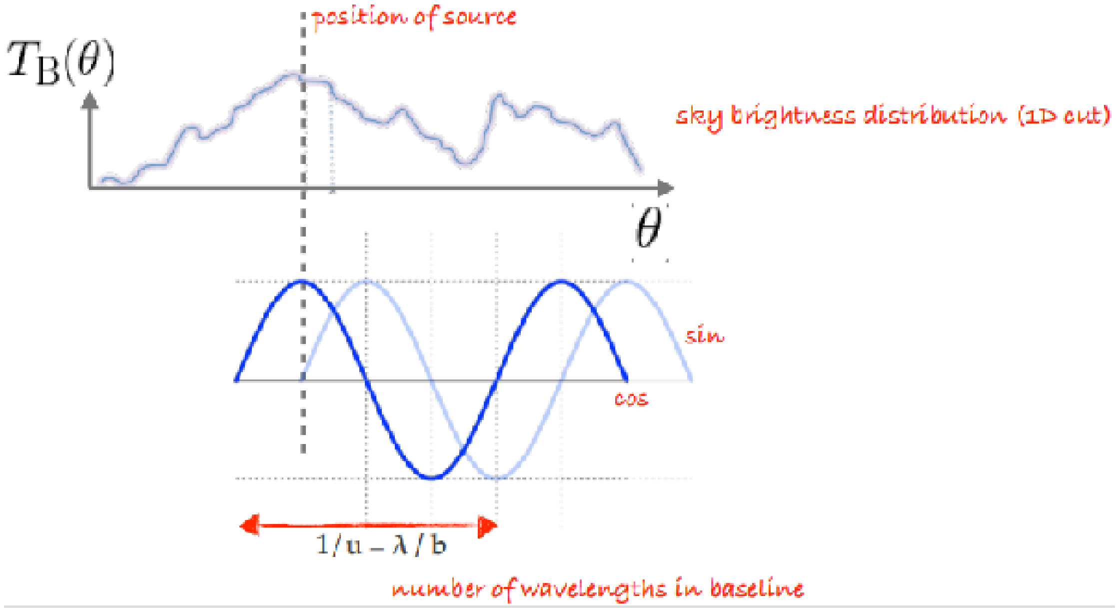



primary beam envelope: $\dfrac{\lambda}{D}$ fringe spacing: $\dfrac{\lambda}{b}$

## 2.3 The Effect of Baseline Length




Missing short baselines results in poor sensitivity to low brightness extended structure. Notice the negative "holes" (darker blue) underlying the bright structure.




Do not understand the above figure

## 2.4 Complex Visibilities

In reality the response will be 2D, but in 1D for simplicity:

$$R_{\cos}(u) = \int_{src} B(\theta)\cos(2\pi u\theta)$$

$$R_{\sin}(u) = \int_{src} B(\theta)\sin(2\pi u\theta)$$

power out as a function of baseline
The sky brightness distribution is **not an even function**. If we want to reconstruct it from its Fourier components then we need **both the cos and sin terms.**

## 2.5 Van Cittert Zernike Function

The 2-D **lateral coherence function** of the radiation field in space is the **Fourier Transform** of the 2-D brightness (or intensity) distribution of the source.

$$\langle V(x_1,t)V(x_2,t)\rangle = \int\int B(\theta,\phi)\mathrm{e}^{-2\pi i(u\theta+v\phi)}\mathrm{d}\theta\mathrm{d}\phi$$

$$u = \frac{(x_1-x_2)}{\lambda} \qquad v = \frac{(y_1-y_2)}{\lambda}$$

The **Visibility Function** is therefore another name for the **spatial correlation function**.

## 2.6 Visibility

For these components we define the **complex fringe visibility** for a particular baseline:

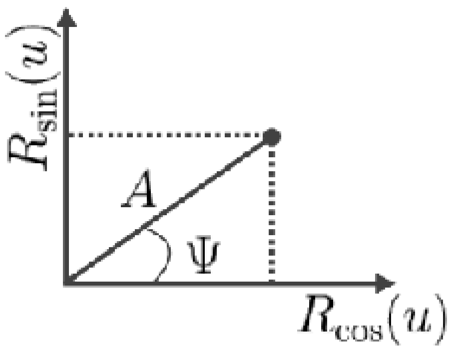
$$A\mathrm{e}^{i\Psi}$$

phase

amplitude

$$\Psi = \tan^{-1}\left[\frac{R_{\sin}}{R_{\cos}}\right]$$

$$A = \left[R_{\cos}^2 + R_{\sin}^2\right]^{1/2}$$




$\Psi$ is the **shift** (in radians) of the Fourier component with respect to the reference position.

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## 2.7 Visibilities and Images

$$I_{means}(l,m) = \frac{1}{M}\sum_{i=1}^{M} V(u_i,v_i)e^{2\pi i(u_i l+v_i m)}$$

This is a **complex** quality, but the sky intensity is **real**.

$$V(-u,-v) = V^*(u,v)$$

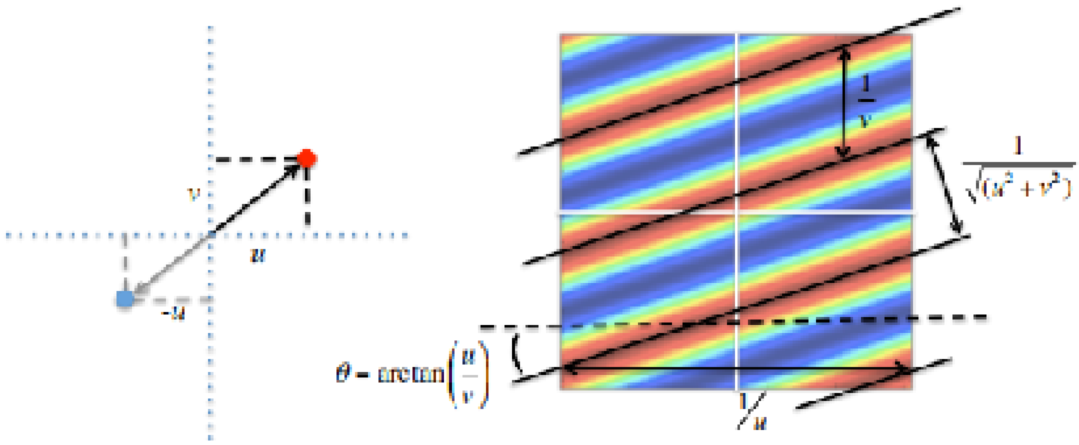
If we change out notation slightly, so that $V = Ae^{i\Phi}$, we can write:

$$I_{means}(l,m) = \frac{1}{M}\sum_{i=1}^{M} A(u_i,v_i)\cos[2\pi(u_i l + v_i m) + \Phi_i]$$

## 2.8 Fourier Components

$$I_{means}(l,m) = \frac{1}{M}\sum_{i=1}^{M} A(u_i,v_i)\cos[2\pi(u_i l + v_i m) + \Phi_i]$$

Writing the equation in this way allows us to visualize how our image is composed.



## 2.9 Conclusions

- The key to interferometer is the geometric delay
- **A 2-element adding interferometer is a direct analogue of Young]s slits**
- The sky is not symmetric - we need both cosine and sine waves to make a picture of it
- **Interferometers measure complex visibilities, which are the Fourier components of the sky brightness.**

# Chapter 3

# Fundamentals of Interferometry - Robert Laing

## 3.1 Objectives

- A more formal approach to radio interferometry using coherence functions 1. A complementary way of looking at the technique 2. Be clear about simplifying assumptions
- Relaxing the assumptions
- How does a radio interferometer work?
  1. Follow the signal path 2. echnologies for different frequency ranges

### 3.1.1 Young's Slit Experiment

- Angular spacing of fringes = $\lambda/d$
- Familiar from optics
- Essentially the way that astronomical interferometers work at optical and infrared wavelength (e.g. VLTI)
- Direct detection

### 3.1.2 Build up an image from many slits

### 3.1.3 How radio interferometers work in practice

But this is not how radio interferometers work in practice

- The two techniques are closely related, and it often helps to think of images as built up of sinusoidal "fringes"
- But radio interferometers collect radiation ("antenna"), turn it into a digital signal ("receiver") and generate the interference pattern in a special-purpose computer ("correlator")
- How does this work?
- I find it easiest to start with the concept of the mutual coherence (or correlation) of the radio signal received from the same object at two different places
- No proofs, but I will try to state the simplifying assumptions clearly and return to them later

## 3.2 The ideal interferometer

- Astrophysical source, location **R**, generates a time-varying electric field **E**(**R**,t). EM wave propagates to us at point **r**.
- In frequency components: $\mathbf{E}(\mathbf{R},t) = \int \mathbf{E}_\nu(\mathbf{R})\exp(2\pi i\nu t)d\nu$

  The coefficients $\mathbf{E}_\nu(\mathbf{R})$ are complex vectors (amplitude and phase; two polarizations)
- Simplification 1: radiation is monochromatic

  $\mathbf{E}_\nu(\mathbf{r}) = \iiint P_\nu(\mathbf{R},\mathbf{r})\mathbf{E}_\nu(\mathbf{R})\, dx\, dy\, dz$

  where $P_\nu(\mathbf{R},\mathbf{r})$ is the propagator
- Simplification 2: scalar field (ignore polarization for now)
- Simplification 3: sources are all very far away
- This is equivalent to having all sources at a fixed distance – there is no depth information

- Simplification 4: space between us and the source is empty
- In this case, the propagator is quite simple (Huygens' Principle), so:
  $E_\nu(\mathbf{r}) = \int E_\nu(\mathbf{R})\{\exp[2\pi i|\mathbf{R}-\mathbf{r}|/c]/|\mathbf{R}-\mathbf{r}|\}\, dA$
  ($dA$ is the element of area at distance $|\mathbf{R}|$)
- We can measure is the correlation of the field at two different observing locations. This is
  $C_\nu(\mathbf{r}_1,\mathbf{r}_2) = \langle E_\nu(\mathbf{r}_1)E^*_\nu(\mathbf{r}_2)\rangle$
  where <> denotes an expectation value and * means complex conjugation.
- Simplification 5: radiation from astronomical objects is not spatially coherent ('random noise').
  $\langle E_\nu(\mathbf{R}_1)E^*_\nu(\mathbf{R}_2)\rangle = 0$ unless $\mathbf{R}_1 = \mathbf{R}_2$

- Now write $\mathbf{s} = \mathbf{R}/|\mathbf{R}|$ and $I_\nu(\mathbf{s}) = |\mathbf{R}|^2\langle|E_\nu(\mathbf{s})|^2\rangle$ (the observed intensity). Using the approximation of large distance to the source again:
  $C_\nu(\mathbf{r}_1,\mathbf{r}_2) = \int I_\nu(\mathbf{s}) \exp[-2\pi i\nu\mathbf{s}.(\mathbf{r}_1-\mathbf{r}_2)/c]\, d\Omega$
  ($d\Omega$ is an element of solid angle)
- $C_\nu(\mathbf{r}_1,\mathbf{r}_2)$, the spatial coherence function, depends only on separation $\mathbf{r}_1-\mathbf{r}_2$, so we can keep one point fixed and move the other around.
- It is a complex function, with real and imaginary parts, or an amplitude and phase.

An interferometer is a device for measuring the spatial coherence function

### 3.2.1 Spatial and Temporal Coherence

Most radio sources, like light bulbs are broad-band, incoherent emitters

### 3.2.2 u,v,w coordinates

We use a coordinate system (u,v,w), where w is along a reference direction to the **phase centre** and (u,v) are in the orthogonal plane, with u East-West and v North-South (the **u-v** plane)

In this coordinate system:

- Baseline vector between antennas $b = (\lambda u, \lambda v, \lambda w)$. Measured in wavelengths.
- Unit vector to the phase centre $s_0 = (0, 0, 1)$
- Unit vector to some point in the field $s = (l, m, n)$, with $l^2 + m^2 + n^2 = 1$

### 3.2.3 The Fourier Relation - The van Cittert - Zernike Theorem

- Simplification 6: receiving elements have no direction dependence
- Simplification 7: all sources are in a small patch of sky
- Simplification 8: we can measure at all values of $\mathbf{r}_1$-$\mathbf{r}_2$ and at all times
- Choose coordinate system so that the phase tracking centre has $s_0$ = (0,0,1) as in previous slide.
- $C(\mathbf{r}_1,\mathbf{r}_2) = \exp(-2\pi i w) V'_\nu(u,v)$
- $V'_\nu(u,v) = \iint I_\nu(l,m) \exp[-2\pi i(ul+vm)]\, dl\, dm$
- This is a Fourier transform relation between the modified complex visibility $V'_\nu$ (the spatial coherence function with separations expressed in wavelengths) and the intensity $I_\nu(l,m)$
- "The Fourier Transform of the spatial coherence function of an incoherent source is equal to its complex visibility": the van Cittert – Zernike Theorem.

### 3.2.4 Fourier Inversion

- This relation can be inverted to get the intensity distribution, which is what we want:

$$I_\nu(l, m) = \iint V'_\nu(u, v) exp[2\pi i(ul + vm)]dudv$$

- **This is the fundamental equation of synthesis imaging**.
- Interferometrists love to talk about the (u,v) plane. Remember that u, v (and w) are measured in wavelengths.
- The vector $b = (u, v, w) = (r_1 - r_2)/\lambda$ is the baseline

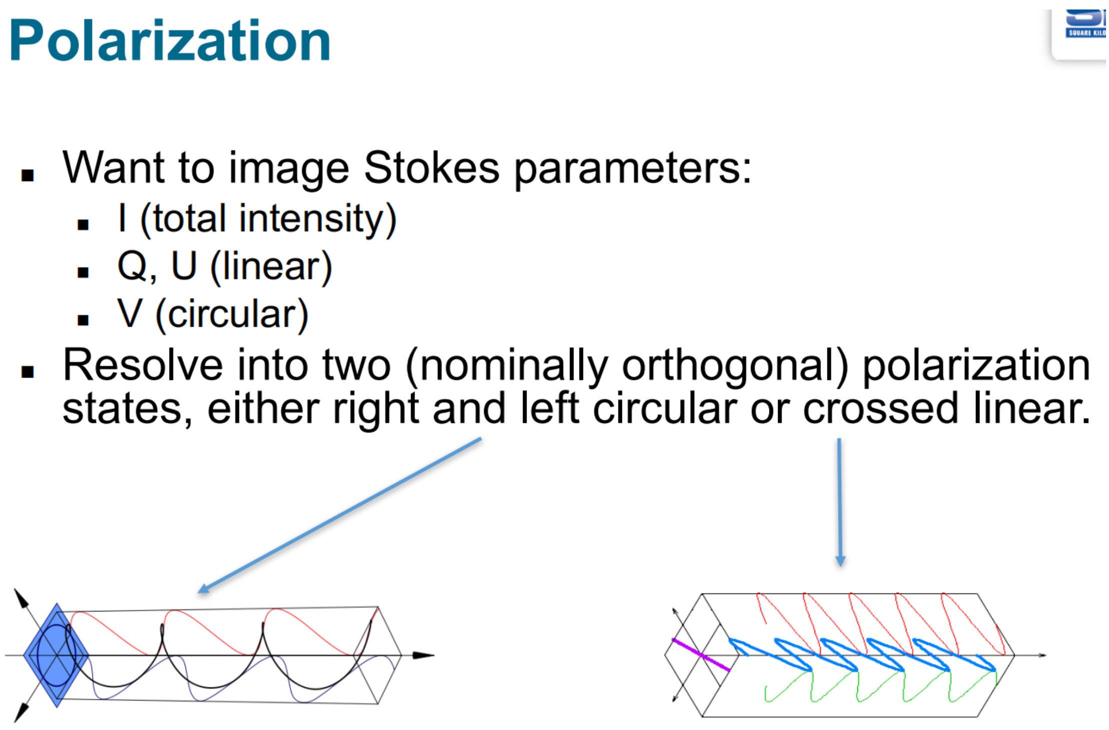
### 3.2.5 Simplifications - Summary

1. Radiation is monochromatic
2. Electromagnetic radiation is a scalar field
3. Sources are all very far away
4. Space between us and the sources is empty
5. Radiation is not spatially coherent
6. Receiving elements have no direction dependence
7. All sources are in a small patch of sky
8. We can measure all baselines at all times

Above, red is Flase,
yellow is sometimes true
green is always true

1. Radiation is monochromatic
   - We observe wide bands both for spectroscopy (HI, molecular lines) and for sensitive continuum imaging, so we need to get round this restriction.
   - In fact, we can easily divide the band into multiple spectral channels (details later)
   - There are imaging restrictions if the individual channels are too wide for the field size – see imaging lectures.
     1.1. Usable field of view $< (\Delta\nu/\nu_0)(l^2 + m^2)^{1/2}$.
     1.2. Not usually an issue for modern instruments, which have large numbers of channels
2. Radiation field is a scalar quantity

- The field is actually a vector and we are interested in both components (i.e. its polarization).
- This makes no difference to the analysis as long as we measure two states of polarization (e.g. right and left circular, or crossed linear) and account for the coupling between states.
- Use the **measurement equation** formalism for this (calibration and polarization lectures)




3. Sources are all a long way away

   - Strictly speaking, in the far field of the interferometer, so that the distance is $> D^2/\lambda$, where D is the interferometer baseline
   - True except in the extreme case of very long baseline observations of solar-system objects

4. Radiation is not spatially coherent

   - Generally true, even if the radiation mechanism is itself coherent (masers, pulsars)
   - May become detectable in observations with very high spectral and spatial resolution

- Coherence can be produced by scattering, since signals from the same location in a sources are spatially coherent, but travel by different paths through interstellar or interplanetary medium

5. Space between us and the source is empty

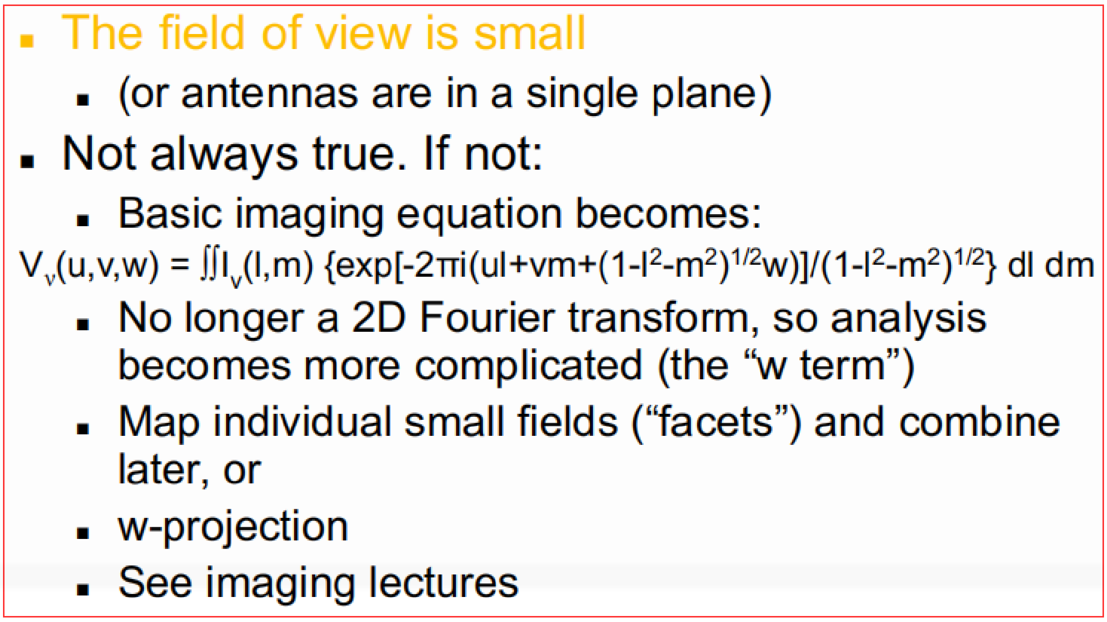
6. The receiving elements have no direction-dependence

- Above two are closely related and not true in general. Examples:
  - Interstellar or interplanetary scattering
  - Tropospheric and (especially) ionospheric fluctuations which lead to path/phase and amplitude errors, sometimes seriously direction-dependent
  - Ionospheric Faraday rotation, which changes the plane of polarization
  - High-frequency antennas are highly directional by design
- Standard calibration deals with the case that there is no direction-dependence (i.e each antenna has a single, time- variable complex gain)
- Direction dependence is becoming more important, especially for low frequencies and wide fields

## Special case: primary beam correction

- If the response of the antenna is direction-dependent, then we are measuring $I_\nu(l,m)\, D_{1\nu}(l,m) D^*_{2\nu}(l,m)$ instead of $I_\nu(l,m)$ (ignore polarization for now)
- An easier case is when the antennas all have the same response
  $A_\nu(l,m) = |D_\nu(l,m)|^2$
- $V'_\nu(u,v) = \iint A_\nu(l,m) I_\nu(l,m) \exp[-2\pi i(ul+vm)]\, dl\, dm$
- We just make the standard Fourier inversion and then divide by the **primary beam** $A_\nu(l,m)$
- $I_\nu(l,m) = A_\nu(l,m)^{-1} \iint V'_\nu(u,v) \exp[2\pi i(ul+vm)]\, du\, dv$

7. The field of view is small (or antennas are in a single plane)




8. We can measure the coherence function for any spacing and time
   Very wrong!

   - We have a number of antennas at fixed locations on the Earth (or in orbit around it)
   - The Earth rotates
   - We make many (usually) short integrations over extended periods, sometimes in separate observations
   - So effectively we sample at discrete u, v (and w) positions.
   - Implicitly assume that the source does not vary: Often a problem when combining observations take over a long time period; some sources vary much faster (e.g. the Sun)
   - Also assume that each integration (time average to get the coherence function) is of infinitesimal duration

- In 2D, this measurement process can be described by a sampling function S(u,v) which is a delta function where we have taken data and zero elsewhere.
- $I^D_\nu(l,m) = \iint V_\nu(u,v)\, S(u,v) \exp[2\pi i(ul+vm)]\, du\, dv$ is the **dirty image**, which is the Fourier transform of the **sampled** visibility data.
- $I^D_\nu(l,m) = I_\nu(l,m) \otimes B(l,m)$, where the $\otimes$ denotes convolution and $B(l,m) = \iint S(u,v) \exp[2\pi i(ul+vm)]\, du\, dv$ is the **dirty beam**
- The process of getting from $I^D_\nu(l,m)$ to $I_\nu(l,m)$ is **deconvolution** (examples in other lectures).
- However, perhaps better to pose the problem in a different way: what model brightness distribution $I_\nu(l,m)$ gives the best fit to the measured visibilities and how well is this model constrained?

**Dirty image, sampled visibility data, dirty beam**

## 3.3 How to build an array

Along the signal path:

1. Antennas
2. Receivers
3. Down-conversion
4. Measuring the correlations
5. Spectral channels
6. Calibration
7. Imaging

What are the key design parameters?

### 3.3.1 Resolution and field

d = baseline, D = antenna diameter

- Science $\rightarrow$ Wavelength, $\lambda$
- Then:
  - Resolution /rad: $\approx \lambda/d_{max}$
  - Maximum observable scale /rad: $\approx \lambda/d_{min}$
  - Primary beam/rad: $\approx \lambda/D$
- Good coverage of the u-v plane (many antennas, Earth rotation) allows high-quality imaging.
- Sources with all brightness on scales $> \lambda/d_{min}$ are resolved out.
- Sources with all brightness on scales $< \lambda/d_{max}$ look like points

### 3.3.2 Noise

$$S_{rms} = \frac{2kT_{sys}}{A_{eff}\sqrt{N_A(N_A-1)t_{int}\Delta\nu}}$$

- RMS noise level $S_{rms}$
  $T_{sys}$ is the system temperature, $A_{eff}$ is the effective area of the antennas, $N_A$ is the number of antennas, $\Delta\nu$ is the bandwidth, $t_{int}$ is the integration time and $k$ is Boltzmann's constant
- For good sensitivity, you need low $T_{sys}$ (receivers), large $A_{eff}$ (big, accurate antennas), large $N_A$ (many antennas), long integrations ($t_{int}$) and, for continuum, large bandwidth $\Delta\nu$.
- Best $T_{rec}$ typically a few to $\sim$30K from $1-100$ GHz, $\sim$100 K at 700 GHz. Atmosphere dominates $T_{sys}$ at high frequencies; foregrounds at low frequencies

### 3.3.3 Antennas collect radiation

Specification, design and cost are frequency-dependent

- High-frequency: steerable dishes ($5-100$ m diameter)
- Low-frequency: fixed dipoles, yagis, ...
- Ruze formula efficiency $= exp[-(4\pi\sigma/\lambda)^2]$

- Surface rms error $\sigma < \lambda/20$
- sub-mm antennas are challenging (surface rms $< 25\ \mu m$ for 12m ALMA antennas); offset pointing $< 0.6$ arcsec rms

### 3.3.4 Receivers

- Detect radiation
- Cryogenically cooled for low noise (at high frequencies)
- Normally detect two polarization states: Separate optically
- Optionally, in various combinations:
  - Amplify RF signal
  - Then either:
    * digitize directly (possible up to ~10's GHz) or
    * **mix** with phase-stable local oscillator signal to make intermediate frequency (IF) → two sidebands (one or both used) → digitize
    * a mixer is a device with a non-linear voltage response that outputs a signal at the difference frequency
- Digitization typically $3 - 8$ bit
- Send to correlator

### 3.3.5 Sidebands

Either separate and keep both sidebands or filter out one

### 3.3.6 Time and Frequency Distribution

- These days, often done over fibre using optical analogue signal
- Master frequency standard (e.g. H maser)
- Must be phase-stable – round trip measurement
- Slave local oscillators at antennas
  . Multiply input frequency
  . Change frequency within tuning range

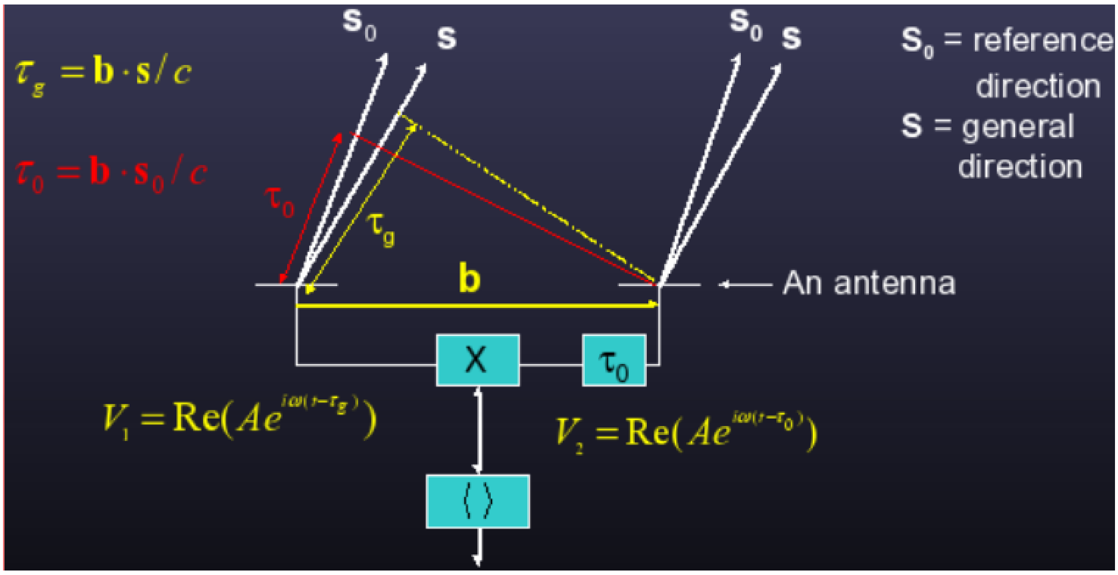
### 3.3.7 Spatial multiplexing

- Focal-plane arrays (multiple receivers in the focal plane of a reflector antenna).
- Phased arrays with multiple beams from fixed antenna elements ("aperture arrays") e.g. LOFAR, MWA
  *a phased array is an array of antennas from which the signals are combined with appropriate amplitudes and phases to reinforce the response in a given direction and suppress it elsewhere*
- Hybrid approach: phased array feeds (= phased arrays in the focal plane of a dish antenna, e.g. APERTIF)

### 3.3.8 Delay

- n An important quantity in interferometry is the time delay in arrival of a wavefront (or signal) at two different locations, or simply the delay, $\tau$.
- This directly affects our ability to calculate the coherence function
- Examples:
  - Constant ("cable") delay in waveguide or electronics
  - geometrical delay (next slide)
  - propagation delay through the atmosphere
- Aim to calibrate and remove all of these accurately
- Phase varies linearly with frequency for a constant delay
  - $\Delta\varphi = 2\pi\tau\Delta\nu$
  - Characteristic signature

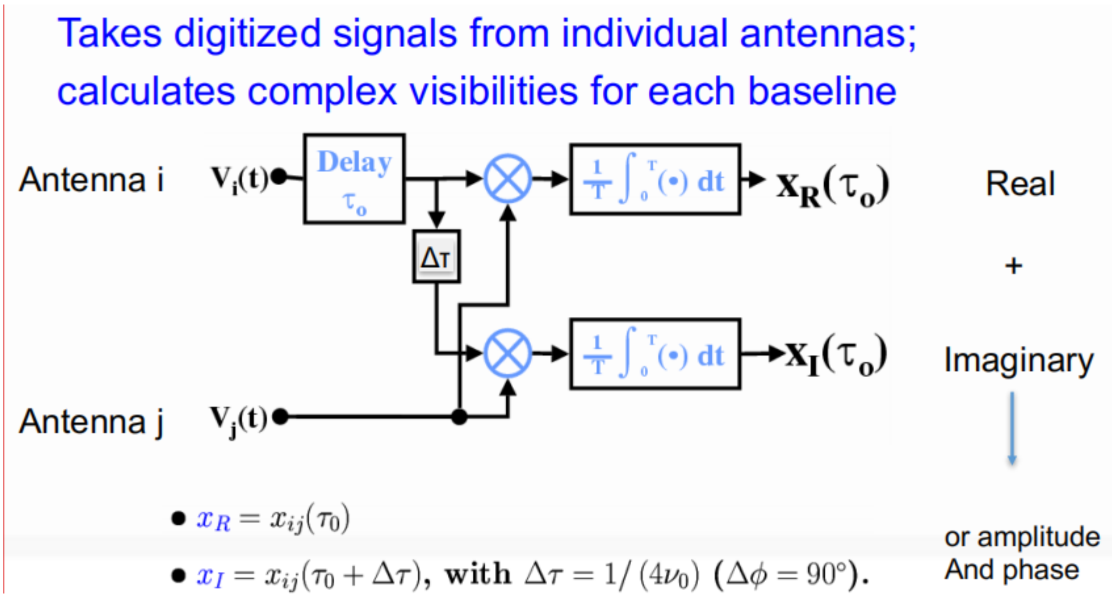
### 3.3.9 Delay Tracking



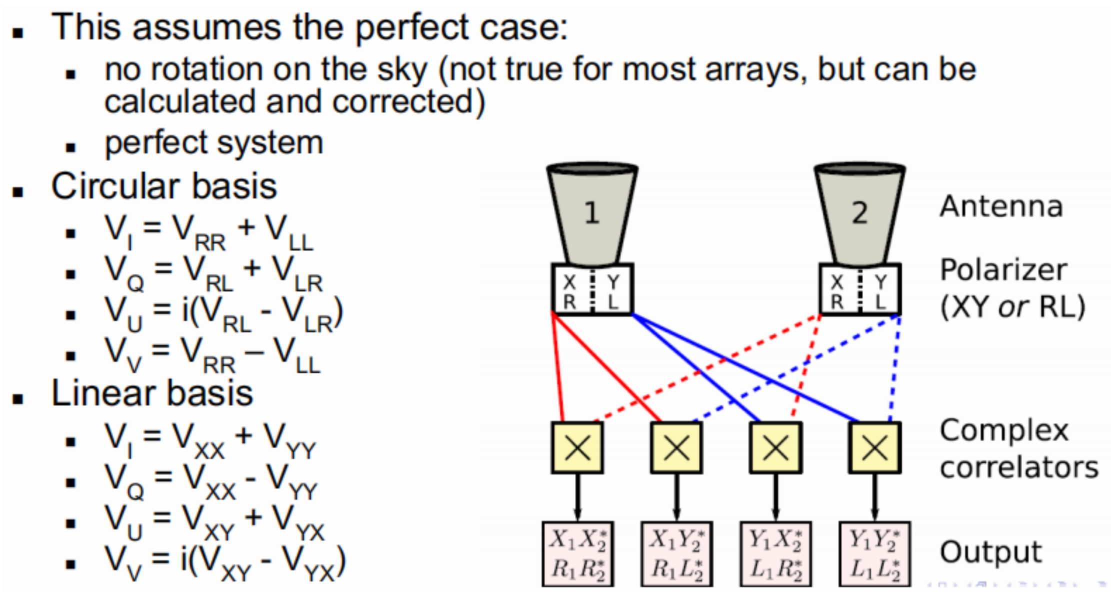

The geometrical delay $\tau_0$ for the delay tracking centre can be calculated accurately from antenna position + Earth rotation model.

Works exactly only for the delay tracking centre. Maximum averaging time is a function of angle from this direction

### 3.3.10 What does a correlator do?




### 3.3.11 Stokes Parameters and Visibilities



### 3.3.12 Spectroscopy

- We make multiple channels by correlating with different values of **lag**, $\tau_0$. This is a delay introduced into the signal from one antenna with respect to another as in the previous slide. For each quasi-monochromatic frequency channel, a lag $\tau$ is equivalent to a phase shift $2\pi\tau\nu$, i.e.
  $V(u,v,\tau) = \int V(u,v,\nu)\exp(2\pi i\tau\nu)\, d\nu$
- This is another Fourier transform relation with complementary variables $\nu$ and $\tau$, and can be inverted to extract the desired visibility as a function of frequency.
- In practice, we do this digitally, in finite frequency channels:
  $V(u,v,j\Delta\nu) = \Sigma_k V(u,v,k\Delta\tau)\exp(-2\pi ijk\Delta\nu\Delta\tau)$
- Each spectral channel can then be imaged (and deconvolved) individually. The final product is a **data cube**, regularly gridded in two spatial and one spectral coordinate.

I have described an "XF" correlator. The Fourier Transform step can be done first ("FX").

### 3.3.13 Calibration Overview

- What we now have is the output from a correlator, which contains signatures from at least:
  - the Earth's atmosphere
  - antennas and optical components
  - receivers, filters, amplifiers
  - digital electronics
  - leakage between polarization states
- What we want is a set of perfect visibilities $V_\nu(u, v, w)$, on an absolute amplitude scale, measured for exactly known baseline vectors (u,v,w), for a set of frequencies, $\nu$, in full polarization.
- Basic idea:
  - Apply a priori calibrations

  - Measure gains for known calibration sources as functions of time and frequency; interpolate to target
  - Refine the calibration on the target

### 3.3.14 A priori and on-line calibrations

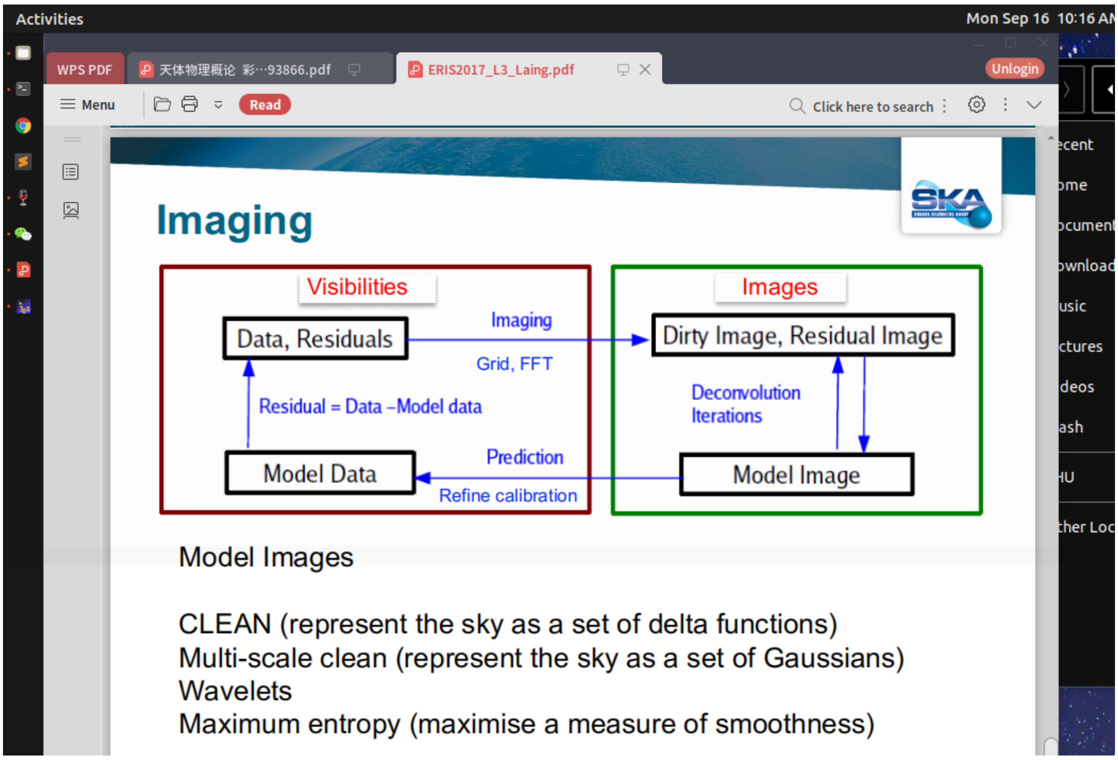
- Array calibrations
  - Antenna pointing and focus model (absolute or offset)
  - Beam forming (aperture arrays and phased-array feeds)
  - Correlator model (antenna locations, Earth orientation and rate, clock, Instrumental delays)
  - Known atmospheric effects (refraction and delay; ionospheric Faraday rotation. . . )
- On-line calibrations (refinement to array calibrations)
  - Reference pointing and focus
  - Attenuator settings
- On-line flags
  - Antennas not on source
  - Receiver broken
  - Radio frequency interference
- Measured/derived from simulations/.. and applied post hoc
  - Antenna gain curve/voltage pattern as function of elevation, temperature, frequency
  - Receiver non-linearity, quantization
  - Noise diode or load calibration to measure gain variations
  - Delay/attenuation due to varying atmospheric water vapour

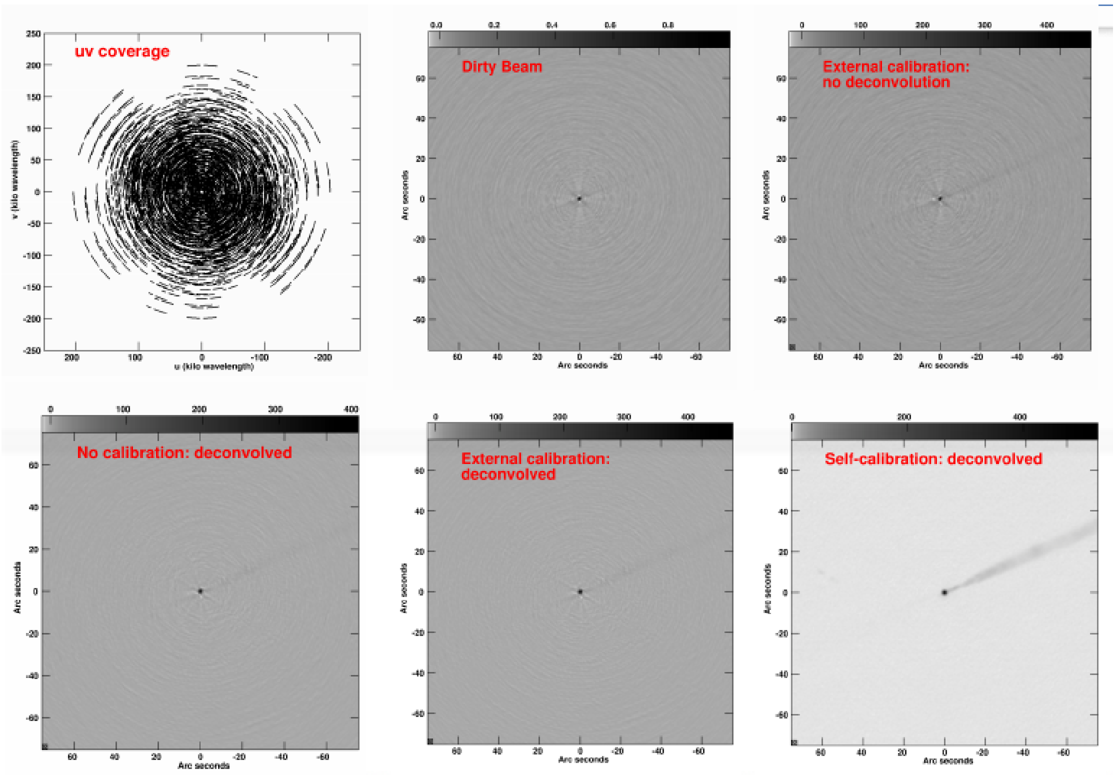
### 3.3.15 Off-line Flagging and Calibration

- Residual delay
- Bandpass: variation of complex gain with frequency
- Variation of complex gain with time (instrument, ionosphere, troposphere)

- Flux density scale (standard calibrator)
- Leakage between polarizations

### 3.3.16 Imaging




uv coverage
Dirty Beam
External calibration: no deconvolution
No calibration: deconvolved
External calibration: deconvolved
Self-calibration: deconvolved

# Chapter 4

# Modern Interferometers - Joe Callingham

## 4.1 What makes an interferometer "modern"?

- Advancements in information, dish, and antenna technology now allow:
  - Aperture arrays (or phased-array feeds)
  - Highly accurate dish shapes for sub-mm observing
  - Complex and high-computing power backends
- Advancements in signal processing now allow much wider bandpasses (e.g. ATCA went from a bandwidth of 128 MHz to 2 GHz, JVLA now at 4 GHz).

## 4.2 Single Pixel Feed