Synthesis Imaging in Radio Astronomy Reading Note

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Contents

1 Coherence in Radio Astronomy **4**
1.1 Introduction 4
1.2 Form of the Observed Electric Field 5
1.3 Spatial Coherence Function of the Field 6
1.4 The Basic Fourier Inversions of Synthesis Imaging 6
1.4.1 Measurements confined to a plane 7
1.4.2 All sources in a small region of sky 7
1.4.3 Effect of discrete sampling 8
1.4.4 Effect of the element reception pattern 8
1.5 Extensions to the Basic Theory 9
1.5.1 Spectroscopy 9
1.5.2 Polarimetry 10

2 Fundamentals of Radio Interferometry **12**
2.1 Introduction 12
2.1.1 Response of an Interferometer 13
2.1.2 Effect of Bandwidth in a Two-Element Interferometer 14

Preface

The Summer School on Synthesis Imaging held in Socorro, New Mexico from June 17 to June 23, 1998 was the sixth in a series held approximately every three years. This Volume contains the edited texts of lectures from the series, and succeeds the previous collection from the Third Synthesis Imaging Summer School published in 1989. It is intended for **serious students of synthesis imaging and image processing.**

Purpose of the course

The NRAO operates two of the world's most powerful synthesis radio telescopes, the Very Large Array (VLA) and the Very Long Baseline Array (VLBA), a synthesis tele- scope.
(Now in 2019, there are three telescopes, including ALMA, https://public.nrao.edu/)
The major goal of this course, like that of its predecessors beginning in 1982 was to **inform potential users of these two synthesis telescopes about the principles of these instruments' operation, about subtleties of data acquisition, calibration and processing with such instruments, and about techniques for obtaining the best results from them.**

As such, the course is aimed first at radio astronomers who need synthesis techniques and instruments for their research. Get practical experience of synthesis imaging, Fourier methods and coherence. Many of the exciting developments in data processing that have made synthesis telescopes so powerful. We therefore have set out to **discuss the subject as fully as necessary for those who wish to use the NRAO's radio synthesis instruments for their own research**. Our goal is to **discuss the subject in enough detail that the student can appreciate both the strengths and limitations of the synthesis technique**, and so begin to evaluate how much, or how little, credence to give individual synthesis images. **To exploit an instrument fully for frontier research, the user must understand it thoroughly enough to distinguish unexpected instrumental errors from unexpected discoveries about the cosmos. A firm understanding of synthesis instruments involves understanding their operating principles (and the assumptions that underlie them), their hardware and the algorithms and software used in the data reduction.** All these topics are covered here.

Although synthesis imaging is a specialized skill even within radio astronomy, it is grounded in mathematical and physical principles that have applications in other fields. Interpret images made by synthesis telescopes. It may also interest researchers who **use Fourier methods or deconvolution techniques for imaging in physics, medicine, remote sensing, seismology or radar.**

Subject matter

The first segment of the course contains sixteen lectures that describe the fundamentals of synthesis imaging and which could be read as a stand-alone course by the beginning student. The second segment consists of seventeen lectures on more advanced and specialized topics.

NRAO lore

AIPS
https://public.nrao.edu/
The on-line bibliographic Abstract Service of the Astrophysics Data System at Harvard University.

This book is dedicated to the memory of Daniel S. Briggs. Dan was killed in a skydiving accident near Chicago just a few weeks after the summer school. Dan was an enthusiastic teacher and researcher who will be sorely missed. His 'Robust' weighting scheme has become the standard for synthesis imaging. At the summer school he gave an excellent lecture on imaging and deconvolution (see Lectures 7 and 8 of this volume contributed by Dan and collaborators). He entered thoroughly into the spirit of the summer school — attending all the lectures and interacting with many of the participants. Dan also influenced the style of this book by making many helpful suggestions. We are in his debt.

Chapter 1

Coherence in Radio Astronomy

In this lecture the **main principles of synthesis imaging are derived.**

1.1 Introduction

A survey of the derivation of the main principles of synthesis imaging, and the assumptions that go into them. This is because a substantial number of the lectures to follow will discuss the problems which arise when these assumptions are violated under the conditions of the observation the astronomer wants to make. At the same time, I will cast this introduction into the terminology of modern optics, in an attempt to stay abreast of current fashions in physics.

Some reference books, e.g. The alternate viewpoint on radio interferometers, from the perspective of the electrical engineers who originally developed them, is explicated in Swenson & Mathur (1968).
The interferometer in radio astronomy, IEEE, 1968

other:
SparseRI: A Compressed Sensing Framework for Aperture Synthesis Imaging in Radio Astronomy
The Sensitivity of Synthetic Aperture Radiometers for Remote Sensing Applications From Space
Basics of Interferometer, PPT
Introduction to Radio Interferometry-NRAO
Introduction to Radio Interferometers - ASTRON

1.2 Form of the Observed Electric Field

Assumptions:

1. scalar
2. far away
3. space empty
4. radiation not spatially coherent

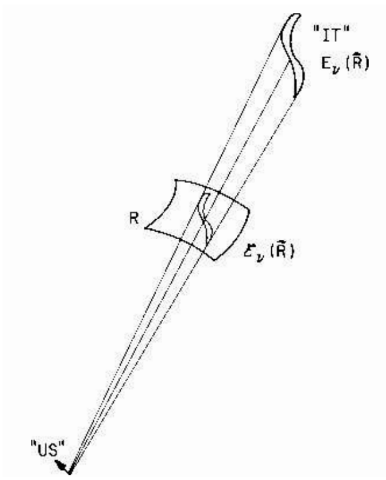

Figure 1.1: The passage of radiation from a distant source through an imaginary sphere at radius $|\mathbf{R}|$ defines the electric field distribution $\epsilon_\nu(\mathbf{R})$ on this surface. For the purposes of synthesis imaging, all astronomical sources may be considered to lie on this sphere, provided only that their real distances greatly exceed B^2/λ, where $B = |\mathbf{r_1} - \mathbf{r_2}|$ is the baseline length.

$$E_\nu(r) = \int \epsilon_\nu(\mathbf{R}) \frac{e^{2\pi i\nu|\mathbf{R}-\mathbf{r}|/c}}{|\mathbf{R}-\mathbf{r}|} dS \tag{1.1}$$

Here dS is the element of surface area on the celestial sphere.
Equation 1.1 is the general form of the **quasi-monochromatic component** of the electric field at frequency ν due to all sources of cosmic electromagnetic radiation. **This is all we have, we can measure only the properties of this field $E_\nu(\mathbf{r})$ to tell us about the nature of things at large in the universe.**

1.3 Spatial Coherence Function of the Field

Among the properties of $E_\nu(\mathbf{r})$ is the correlation of the field at two different locations.
The **correlation** of the field at points $\mathbf{r_1}$ and $\mathbf{r_2}$ is defined as the expectation of a product, namely $V_\nu(\mathbf{r_1}, \mathbf{r_2}) = \langle E_\nu(\mathbf{r_1}) E_\nu^*(\mathbf{r_2}) \rangle$, the raised asterisk indicates the complex conjugate. Use equation 1.1 to substitute for $E_\nu(\mathbf{r})$ and use assumption 4 to derive/rearrange the formula and get:

$$V_\nu(\mathbf{r_1}, \mathbf{r_2}) = \int I_\nu(\mathbf{s}) e^{-2\pi i \nu \mathbf{s} \cdot (\mathbf{r_1} - \mathbf{r_2})/c} d\Omega \tag{1.2}$$

$\mathbf{s}$ is the unit vector $\mathbf{R}/|\mathbf{R}|$, $I_\nu(\mathbf{s})$ is the observed ***intensity*** $|\mathbf{R}|^2 \langle |\epsilon_\nu(\mathbf{s})|^2 \rangle$

Observe that Equation 1.2 depends only on the separation vector $\mathbf{r_1} - \mathbf{r_2}$ of the two points, not on their absolute locations $\mathbf{r_1}$ and $\mathbf{r_2}$(两个观测台站). **Therefore, we can find out all we can learn about the correlation properties of the radiation field by holding one observation point fixed and moving the second around;** we do not have to measure at all possible pairs of points. This function V, of a single (vector) separation $\mathbf{r_1} - \mathbf{r_2}$ is called the **spatial coherence function**, or the **spatial autocorrelation function**, of the field $E_\nu(\mathbf{r})$. It is all we have to measure.

An interferometer is a device for measuring this spatial coherence function.

1.4 The Basic Fourier Inversions of Synthesis Imaging

A second interesting point about Equation 1.2 is that the equation is, within reasonable, well-defined limits, invertible. The intensity distribution of the ra- diation as a function of direction $\mathbf{s}$ can therefore be deduced in certain cases by measuring the spatial coherence function V as a function of $\mathbf{r_1} - \mathbf{r_2}$ and performing the inversion.

There are two special cases of a great deal of interest. In fact, it is usually argued that any actual case is so close to one of these two special cases that the invertibility properties (although not necessarily the effort required to perform the inversion) must be essentially similar. Since there are two forms of interest, there are two alternate forms of our fifth (and final) simplifying assumption:

1.4.1 Measurements confined to a plane

First, we could choose to make our measurements only in a plane; that is, in some favored coordinate system, the vector spacing of the separation variable in the coherence function, conveniently measured in terms of the wavelength $\lambda = c/\nu$, is $\mathbf{r_1} - \mathbf{r_2} = \lambda(u, v, 0)$.
In this same coordinate system, the components of the unit vector $\mathbf{s}$ are $(l, m, \sqrt{1-l^2-m^2})$. Inserting these, and explicitly showing the form, in this coordinate system, of the element of solid angle, we have

$$V_\nu(u, v, \omega \equiv 0) = \iint I_\nu(l, m) \frac{e^{-2\pi i(ul+vm)}}{\sqrt{1-l^2-m^2}} dl dm \tag{1.3}$$

Equation 1.3 is, clearly, a Fourier transform relation between the spatial coherence function $V_\nu(u, v, \omega \equiv 0)$ (with separations expressed in wavelengths), and the modified intensity I, $I_\nu(l, m)/\sqrt{1-l^2-m^2}$ (with angles expressed as direction cosines). Now we are home free. **Mathematicians have devoted decades of work to telling us when we can invert a Fourier transform, and how much information it requires.**

1.4.2 All sources in a small region of sky

The alternate form of the fifth simplifying assumption is to consider the case where all of the radiation comes from only a small portion of the celestial sphere. That is, to take $s = s_0 + \sigma$, and neglect all terms in the squares of the components of σ. In particular, the statement that both s and s_0 are unit vectors implies that:

$$1 = |s| = s \cdot s \approx 1 + 2s_0 \cdot \sigma$$

i.e., s_0 and σ are perpendicular. If we again introduce a special coordinate system such that $s_0 = (0, 0, 1)$, then we have a slightly different offspring of equation 1.2:

$$V'_\nu(u, v, \omega) = e^{-2\pi i\omega} \iint I_\nu(l, m) e^{-2\pi i(ul+vm)} dl dm \tag{1.4}$$

Here, the components of the vector $r_1 - r_2$ have been denoted by (u, v, ω). It is customary to absorb the factor in front of the integral in Equation 1.4 into the left hand side, by considering the quantity $V_\nu(u, v, \omega) = e^{2\pi i\omega} V'_\nu(u, v, \omega)$, which we see is independent of ω:

$$V_\nu(u, v) = \iint I_\nu(l, m) e^{-2\pi i(ul+vm)} dl dm \tag{1.5}$$

$V_\nu(u, v)$ is the coherence function relative to the direction s_0, which is called the ***phase tracking center***.
Since equation 1.5 is a Fourier transform, we have the direct inversion:

$$I_\nu(l, m) = \iint V_\nu(u, v) e^{2\pi i(ul+vm)} dudv \tag{1.6}$$

1.4.3 Effect of discrete sampling

In practice the spatial coherence function V is not known everywhere but is sampled at particular places on the u-v plane. The sampling can be described by a sampling function $S(u, v)$, which is zero where no data have been taken. One can then calculate a function

$$I_\nu^D(l, m) = \iint V_\nu(u, v) S(u, v) e^{2\pi i(ul+vm)} dudv \tag{1.7}$$

Radio astronomers often refer to $I_\nu^D(l, m)$ as the **dirty image**; its relation to the desired intensity distribution $I_\nu(l, m)$ is (using the convolution theorem for Fourier transforms):

$$I_\nu^D = I_\nu * B \tag{1.8}$$

where the in-line asterisk denotes convolution
上面是时域的卷积，下面是频域的乘积：

$$V_\nu^D(u, v) = V_\nu(u, v) S(u, v)$$

and:

$$B(l, m) = \iint S(u, v) e^{2\pi i(ul+vm)} dudv \tag{1.9}$$

is the **synthesized beam** or **point spread function** corresponding to the **sampling function** $S(u, v)$. Equation 1.8 says that I^D is the **true intensity distribution** I convolved with the synthesized beam B. Lecture 8 discusses methods for undoing this convolution.

1.4.4 Effect of the element reception pattern

An additional minor alteration must be made to the above for convenience in practical calculation. In practice, the interferometer elements are not point probes which sense the voltage at that point, but are elements of finite size, which have some sensitivity to the direction of arrival of the radio radiation. That is, there is an additional factor within the integral of Equation 1.1 (,and hence of other equations) of $A_\nu(s)$ (the **primary beam** or **normalized reception pattern** of the interferometer elements) describing this sensitivity as a function of direction. For explicitness, Equation 1.5 is rewritten below, with this factor included:

$$V_\nu(u,v) = \iint A_\nu(l,m) I_\nu(l,m) e^{-2\pi i(ul+vm)} dl dm \tag{1.10}$$

The $V_\nu(u,v)$ so defined is normally termed the **complex visibility** relative to the chosen phase tracking center.

Although the factor A_ν, looks like merely a nuisance, it is actually the reason that the second form of the final assumption (used in Section 4.2) is so acceptable in many practical cases—$A_\nu(s)$ falls rapidly to zero except in the vicinity of some s_0, the pointing center for the array elements.

1.5 Extensions to the Basic Theory

1.5.1 Spectroscopy

With current technology, it is attractive to implement the latter portions of the interferometer in digital hardware. In this technology, it is quite inexpensive to add additional multipliers to calculate the correlation as a function of lag. Admitting a range of quasi-monochromatic waves to the interferometer, we can write an expression for the correlation as a function of lag, noting that for each quasi-monochromatic wave, a lag is equivalent to a phase shift, i.e., a multiplication by a complex exponential

$$V(u,v,\tau) = \int V(u,v,\nu) e^{2\pi i \tau \nu} d\nu$$

The above is clearly a Fourier transform, with complementary variables ν and τ, and can be inverted to extract the desired $V(u,v,\nu)$ (Spectroscopy). Since, in this digital technology, one is dealing with sampled data, I give the sampled form of the inversion below:

$$V(u,v,j\Delta\nu) = \sum_k V(u,v,k\Delta\tau) e^{-2\pi i jk\Delta\nu\Delta\tau}$$

The fact that we are dealing with sampled data is of some interest, and we should stop and inquire about how the Fourier sampling theorem is to be applied. Examining the above, in its full complex form, we see that the replication interval is

$1/\Delta\tau$ in frequency, so that the band of frequencies must be limited, before sampling, to a total bandwidth of less than this, to avoid loss of information in the sampling process.

This is different from the statement one usually encounters, in which a prefiltering to $1/2\Delta\tau$ is required to preserve the information in the sampling process for a signal (actually it is usually stated, equivalently, as requiring a sampling interval of $1/2B$, where B is the prefilter bandwidth). This factor of two difference is due to the complex nature of the quantities we have been dealing with—the $V(u, v, \nu)$ are complex numbers, calculated by a complex multiplication of the complex field quantities. Complex multipliers and complex samplers require at least twice as many electronic components as devices that produce a real number, and the resulting doubling of the hardware permits us to sample a factor of two less often. (是说采样的数据点是复数的，所以Nyquist采样定理得出的最小采样间隔这时候不需要以前那么小，只需要一般情况下的2倍?)

Finally, if one derives the spectrum in this manner, one can, clearly, convert back to the single continuum channel at zero lag simply by summing the derived frequency-dependent V. This process clearly results in a complex number, even though each measurement was only a real number. The process of transforming a real function into a complex one by Fourier transforming and then transforming back on half the interval is called a **Hilbert transform**, and is an alternate method to implementing complex correlators.

1.5.2 Polarimetry

Actually, the electromagnetic field is a vector phenomenon, and the polarization properties carry interesting physical information. For the case of noise emission, one must be a bit careful about the definition of polarization. A monochromatic wave is always completely polarized, in some particular elliptical polarization, in that a single number describes the variation of the fields everywhere. For electromagnetic noise, polarization is defined by a correlation process. One picks two orthogonal polarizations and analyzes the radiation of the quasi-monochromatic waves into the components in these two polarizations. Then the polarization of the quasi-monochromatic wave is described by the 2×2 matrix of correlations between these two resolutions into orthogonal components. For instance, if we pick right and left circular polarization as the two orthogonal modes, then the matrix

$$\begin{bmatrix} RR^* & RL^* \\ LR^* & LL^* \end{bmatrix}$$

describes the polarization. This can be related to more familiar descriptions of polarization. For instance, the **Stokes parameters** have the intensity I, two linear polarization parameters Q and U, and a circular polarization parameter V related to the above numbers in simple (and more or less obvious) linear combinations:

$$\begin{bmatrix} I+V & Q+iU \\ Q-iU & I-V \end{bmatrix}$$

The complex correlation functions on the celestial sphere are preserved in the spatial coherence functions that interferometers measure. That is, one can derive, for instance, the distribution of $\langle RR^* \rangle$ on the sky by measuring the coherence function of R on the ground, and so forth for the other components of the matrix. Since the intensity is the quantity in which one is always interested, one usually forms the sum of the R and L coherence functions before transforming to the sky plane, which one can always do, since the relations are linear. One can choose to do the same with the other Stokes parameters, or one can calculate the transforms of the mutual coherence between R and L to find the distribution of $\langle RL^* \rangle$ on the sky, and later note that this is, in terms of the Stokes parameters, $Q + iU$.

Chapter 2

Fundamentals of Radio Interferometry

The practical aspects of interferometry are reviewed, starting with a two element interferometer.

2.1 Introduction

In the first lecture, it was shown that images of a distant radio source can be made by measuring the mutual coherence function of the electric fields at pairs of points in a plane normal to the direction to the source. This process can be envisioned by considering a large flat area on the Earth's surface on which antennas are located, and an electronic system including correlators to measure the coherence of the received signals. Now suppose that the Earth does not rotate, and that the source under observation is at the zenith. We can measure the coherence as a function of the spacing between pairs of antennas, independent of the absolute location of the antennas in the antenna plane. Next suppose that the Earth remains fixed relative to the sky, but that the source under investigation is not at the zenith. A wavefront from the source meets the plane in a line that progresses across the plane, and thus does not reach all antennas simultaneously. As a result, we need to include delay elements in the electronic system to ensure that the signals received by different antennas from the same wavefront arrive at the correlators at the same time. **The spacings in (u,v) coordinates are now the components of the baselines projected onto a plane normal to the direction of the source.** Finally, consider the effect of the rotation of the Earth. It is necessary for the antenna pointing and the time delays to be continuously adjusted to follow the source across the sky. Further, although at any instant the baselines lie within a plane, this plane is carried through space by the Earth, and its position relative to the source is continuously changing. The non-coplanar distribution ot the baselines which thus generally occurs during an extended period of observation can complicate the inversion of the coherence data to obtain an image. On the other hand, the motion of the baseline vectors has the useful effect of reducing the number of antenna locations on the Earth required to measure the coherence over the necessary range of (u,v) coordinates.

2.1.1 Response of an Interferometer

When taking observations to make an interferometric image of a radio source, it is usual to specify a position on which the synthesized field of view is to be centered. This position is commonly referred to as the **phase tracking center or phase reference position**.

We now introduce the **visibility**, which is a measure of the coherence discussed in Lecture 1. The term visibility was first used in interferometry by Michelson (1890) to express the relative amplitude of the optical fringes that he observed. As used in radio astronomy, visibility is a complex quantity, the magnitude of which has the dimensions of spectral power flux density ($Wm^{-2}Hz^{-1}$). It can be regarded as an unnormalized measure of the coherence of the electric field, modified to some extent by the characteristics of the interferometer. The complex visibility of the source is defined as

$$V \equiv |V|e^{i\phi V} = \int_S \mathcal{A}(\sigma)I(\sigma)e^{-2\pi i\nu b\cdot\sigma/c}d\Omega \tag{2.1}$$

where $\mathcal{A}(\sigma) \equiv A(\sigma)/A_0$ is the normalized antenna reception pattern, A_0 being the response at the beam center.

$$r = A_0\Delta\nu|V|\cos(\frac{2\pi\nu b\cdot s_0}{c} - \phi_V) \tag{2.2}$$

In the interpretation of interferometer measurements the usual procedure is to measure the amplitude and phase of the fringe pattern as represented by the cosine term in Equation 2.2, and then derive the amplitude and phase of V by appropriate calibration. The brightness distribution of the source is obtained from the visibility data by inversion of the transformation in Equation 2.1. Thus V must be measured over a sufficiently wide range of $\nu b\cdot\sigma/c$, which is the component of the baseline normal to the direction of the source and measured in wavelengths. This component can be envisaged as the baseline viewed from the direction of the source.

2.1.2 Effect of Bandwidth in a Two-Element Interferometer

Since the frequency of the cosine fringe term in Equation 2.2 is proportional to the observing frequency v, observing with a finite bandwidth $\Delta\nu$ results, in effect, in the combination of fringe patterns with a corresponding range of fringe frequencies. For a rectangular frequency passband, the interferometer response is:

$$r = A_0|V|\Delta\nu \frac{\sin \pi\Delta\nu\tau_g}{\pi\Delta\nu\tau_g} \cos(2\pi\nu_0\tau_g - \phi_V) \tag{2.3}$$

where ν_0 is the center frequency of the observing passband. Thus in the system that we are considering the fringes are modulated by a sinc-function envelope, sometimes referred to as the **bandwidth pattern**. The full fringe amplitude is observed only when the source is in a direction normal to the baseline so that $\tau_g = 0$. The range of τ_g for which the fringe amplitude is within, say, 1% of the maximum value can be obtained by writing:

$$\frac{\sin \pi\Delta\nu\tau_g}{\pi\Delta\nu\tau_g} \approx 1 - \frac{(\pi\Delta\nu\tau_g)^2}{6} > 0.99 \tag{2.4}$$

which yields $|\Delta\nu\tau_g| < 0.078$, where the approximation in Equation 2.4 is valid for $\pi\Delta\nu\tau_g \ll 1$. **The angular range of τ_g, within this limit depends upon the length and orientation of the baseline**: for example, with $\Delta\nu = 50$ MHz and $|b| = 1$ km, the response falls by 1% when the angle θ in Fig. 2-1 is 2 arcmin. In order to observe a source over a wide range of hour-angle, it is necessary to include within the system a computer-controlled delay to compensate for τ_g.