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Downturn Credit Portfolio Risk, Regulatory Capital and Prudential Incentives*

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ABSTRACT

This paper analyzes the level and cyclicality of bank capital requirement in relation to (i) the model methodologies through-the-cycle and point-in-time, (ii) four distinct downturn loss rate given default concepts, and (iii) US corporate and mortgage loans. The major finding is that less accurate models may lead to a lower bank capital requirement for real estate loans. In other words, the current capital regulations may not support the development of credit portfolio risk measurement models as these would lead to higher capital requirements and hence lower lending volumes. The finding explains why risk measurement techniques in real estate lending may be less developed than in other credit risk instruments. In addition, various policy recommendations for prudential regulators are made.

I. INTRODUCTION

The current financial crisis provides ample evidence of the close links between financial markets. Credit loan portfolios are generally exposed to counterparty credit risk and asset value risk, which is conditional on the occurrence of credit default events. Credit default and asset value risk are highly dependent. For example, during economic downturns asset values decrease and credit default events increase, which amplifies realized loss rates (e.g., compare Altman et al. 2005; Acharya et al. 2007).

In order to cover increased loss rates in their credit portfolios during economic downturns, financial institutions are required to maintain minimum

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capital levels. These rules are also known as Basel II regulations and are implemented today in most countries. Under the Internal Ratings-Based Approach, the 1 year, 99.9% Credit Value-at-Risk is the benchmark for capital and is calculated by multiplying the 99.9th percentile (i.e., a 'downturn' scenario) of the probability of default (PD) with a loss rate given default (LGD) given an economic downturn and subtracting the current level of provisioning. Laeven and Majnoni (2003) provide an overview on the relationship between bank capital and provisioning.

Given these regulations, financial institutions face the crucial modeling choice, whether econometric properties should be reflected in the risk models that derive estimates for the PD and LGD parameters. One argument in support of such an approach would be the accurate measurement of credit portfolio risks given the current state of the business cycle. One argument against such an approach would be that with the measurement of credit portfolio risks, the regulatory capital requirement will be measured given the current state of the economy. This implies a pro-cyclical capital requirement, which results in the need to raise capital during periods of higher risk such as economic downturns when share prices are low, or alternatively, a decrease in lending volumes. For evidence on pro-cyclicality of capital requirements and potential solutions for dampening cyclicality see Gordy and Howells (2006).

The present paper analyzes various PD and LGD models as well as the resulting Basel Credit Value-at-Risk. The literature in relation to PD estimation has a rich pedigree. Merton (1974), Ohlson (1980), Leland (1994), Jarrow and Turnbull (1995), Longstaff and Schwartz (1995), Madan and Unal (1995), Leland and Toft (1996), Jarrow et al. (1997), Duffie and Singleton (1999), Shumway (2001), McNeil and Wendin (2007) and Duffie et al. (2007) address the default likelihood. Dietsch and Petey (2004) and McNeil and Wendin (2007) model the correlations between default events.

The literature in relation to LGDs is more recent and various approaches have been developed. The first generation of contributions identified the factors driving the values including correlations between PDs and LGDs (compare Carey 1998; Altman et al. 2005; Cantor and Varma 2005; Schuermann 2005; Acharya et al. 2007; Pan and Singleton 2008; Qi and Yang 2009; Grunert and Weber 2009).

The second generation developed and empirically applied frameworks to quantify the correlation between PDs and LGDs (compare Frye 2000; Pykhtin 2003; Düllmann and Trapp 2005; Rösch and Scheule 2005).

The latest generation derives concepts to stress LGDs for economic downturns, which is relatively new as the analysis of economic downturns is traditionally done on a portfolio level and not a parameter level. However, recent proposals by the Basel Committee on Banking Supervision have created the need to stress the loss given default or in other words calculate the 'Downturn LGD'. Barco (2007) extends work by Miu and Ozdemir (2006) empirically by first calculating the economic capital (based on the dependence structure between LGDs and PDs) and then deriving the Downturn LGD (based on the assumption of independence between LGDs and PDs). The findings are that the Downturn LGD depends on the expected LGD, correlation between PDs and LGDs and the confidence level at which the economic capital is sufficient to cover future credit losses. Extending this work, Rösch and Scheule (2009) present an econometric approach to specify the Downturn LGD and show that the Downturn LGD depends on the sensitivity to the macro economy and the correlation between the default and recovery process of a loan.

The present paper analyzes the level and cyclicality of capital requirement in relation to

- Two model methodologies [through-the-cycle (TTC) and point-in-time (PIT)];
- Four LGD concepts. These concepts are based on (i) benchmarks provided by the Basel Committee on Banking Supervision: 10% for real estate loans and 45% for corporate loans, (ii) the expected LGD for downturn years, (iii) the Downturn LGD model by Department of the Treasury, Federal Reserve System and Federal Insurance Corporation (2006) and (iv) the Downturn LGD model by Rösch and Scheule (2009);
- Two main loan segments of US financial institutions (business loans and real estate loans).

The research results are of highest importance to the accurate allocation of bank capital in order to balance bank stability and bank lending. The recent bank failures have given rise to the concern that both the current capital requirements and lending volumes are insufficient. The paper analyzes the level and cyclicality of regulatory bank capital for various risk modeling approaches and aims to answer whether financial institutions provide on average sufficient capital to cover losses. It provides guidance to financial institutions in choosing a concept for modeling regulatory capital and for regulators in limiting the choices given to financial institutions. In addition, the paper analyzes the regulatory LGD benchmarks provided by the Basel Committee on Banking Supervision.

The contributions to the academic literature are fourfold. Firstly, the paper analyzes the impact of model methodology, LGD definition and risk segment on (i) the level, as well as (ii) the cyclicality of regulatory capital. Two Downturn LGD concepts have not been previously explored: (i) the benchmark loss rates given default provided by the Basel Committee on Banking Supervision and (ii) the specification of Downturn LGD on a subset of past downturns. In particular the study of models that are calibrated to past economic downturns is interesting from a post crisis perspective as it may reveal how the output of future risk models will change by the incorporation of the current financial crisis into benchmark data sets. The two other Downturn LGD concepts have been analyzed by Rösch and Scheule (2009) with regard to their capital level but not the degree of cyclicality.

Secondly, the paper compares two model methodologies, namely PIT and TTC for the various downturn LGD definitions and controls for a consistent calculation of provisions. In particular the contribution by Rösch and Scheule (2009) does not control for the level of provisioning.

Thirdly, the paper estimates models for non-marketable corporate and real estate loan portfolios of US firms. Both exposure classes account for the majority of loans on the books of US financial institutions. Almost all previous studies focus on default and loss experience in relation to bond issues and bond market prices after default. The empirical study extracts loss rates given default from accounting values of delinquency and charge-off rates.

Fourthly, the data covers 21 years and includes particularly the years 2007 and 2008. It may therefore provide an explanation of the systemic failure of risk models during the current financial crisis. The analysis shows that real estate credit risk models that rely on TTC models result in a lower regulatory capital requirement. This may have been an important negative incentive for financial institutions to develop forward-looking (predictive) risk models in the area.

The remainder of this paper is organized as follows. Section II provides a framework for the measurement of credit portfolio risks and introduces four concepts for the specification of the LGD. Section III describes the data, estimates risk models and calculates and analyzes the impact on the Basel Credit Value-at-Risk. Section IV discusses the major ramifications of the empirical results and provides suggestions for a new stability framework for financial markets, institutions and instruments.

II. FRAMEWORK

A. Default and loss given default risk modeling

A latent asset value process is used for modeling discrete-time default events based on the pioneering work by Merton (1974). The approach has been used in many credit risk models applied in the financial industry as well as in the Basel II framework and put forward in the financial and credit risk literature. Examples are Gordy (2000), Gordy (2003), Heitfield (2005) and McNeil and Wendin (2007).

The asset return of borrower i ($i = 1, \ldots, N_t$) in time period t ($t = 1, \ldots, T$) is a linear combination of a systematic risk factor F_t which is specific to the time period under consideration, and an idiosyncratic factor U_{it}. Both are standard normally distributed, independent from each other and over time. The idiosyncratic factors are independent between borrowers. Denoting the factor weight of the systematic factor by ω gives the normalized asset return

$$S_{it} = -\omega F_t + \sqrt{1-\omega^2}U_{it} \tag{1}$$

($i = 1, \ldots, N_t; t = 1, \ldots, T$). Note that the factor weights are chosen such that the correlation between asset returns between borrowers i and j ($i \neq j$) at time t is ω^2 which is also known as 'asset correlation'.

Borrower i defaults in time period t when the asset return hits a threshold γ, i.e., $S_{it} < \gamma \Leftrightarrow D_{it} = 1$ where

$$D_{it} = \begin{cases} 1 & \text{borrower } i \text{ defaults in } t \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

is a default indicator variable. The PD is due to the normalization of S_{it}

$$PD_{it} = P(D_{it} = 1) = P(S_{it} < \gamma) = \Phi(\gamma) \tag{3}$$

where $\Phi(.)$ is the standard normal cumulative density function. Conditional on a particular state of the economy f_t, e.g., an economic downturn, the conditional PD (CPD) is

$$CPD_{it} = P(D_{it} = 1|f_t) = P(S_{it} < \gamma|f_t) = \Phi\left(\frac{\gamma + \omega f_t}{\sqrt{1-\omega^2}}\right). \tag{4}$$

The recovery rate R_{it} is modeled by a nonlinear transformation of a normally distributed variable $\widetilde{Y}_{it}$ which is given by

$$\widetilde{Y}_{it} = \widetilde{\beta} + \widetilde{b}X_t + \widetilde{\delta}Y^*_{it} \tag{5}$$

where X_t and Y^*_{it} are standard normally distributed and independent systematic and idiosyncratic risk factors driving the recovery, and $\widetilde{\beta}$, $\widetilde{b}$ and $\widetilde{\delta}$ are parameters. The nonlinear transformation may be a logistic or probit transformation, see Schönbucher (2003) and Düllmann and Trapp (2005). A probit transformation is used

$$R_{it} = \Phi(\widetilde{Y}_{it}). \tag{6}$$

The loss given default is defined as

$$LGD_{it} = 1 - R_{it}. \tag{7}$$

As the data relates to an aggregated level, the average recovery is calculated as $\bar{R}_t = \frac{1}{N_t}\sum_{i=1}^{N_t} R_{it}$. Assuming a large number of borrowers, $\bar{R}_t$ is approximated by

$$R_t(X_t) \equiv \operatorname*{plim}_{n\to\infty} \bar{R}_t = \Phi(\beta + bX_t) \tag{8}$$

which is the conditional recovery (conditional on the systematic factor) where $\beta = \widetilde{\beta}/\sqrt{(1+\widetilde{\delta}^2)}$ and $b = \widetilde{b}/\sqrt{(1+\widetilde{\delta}^2)}$. Equation (8) has expectation

$$R_t = \Phi\left(\beta\sqrt{\frac{1}{1+b^2}}\right) \tag{9}$$

and the expected loss given default (ELGD) is $ELGD_t = 1 - R_t$

Downturn risk relates to a downturn state of the economy, i.e., on the systematic risk factor. Hence, only the common risk factors are considered and treat borrowers within a segment as homogeneous with respect to the parameters. The correlation between the systematic recovery and the PD is introduced by assuming that the risk factors F_t and X_t are jointly normally distributed with correlation ρ.

The approach can be extended to a dynamic econometric model by including observable macroeconomic risk factors. Let z^D_{t-1} and z^R_{t-1} denote vectors of macroeconomic variables driving the default and the recovery process, respectively. The default probability can be modeled as

$$PD_t = \Phi(\gamma_0 + \gamma_1' z_{t-1}^D) \tag{10}$$

where vector γ_1 contains parameters for the exposures to the macroeconomic factors and γ_0 is a constant. Similarly the expected recovery rate can be modeled as

$$R_t = \Phi\left((\beta_0 + \beta_1' z_{t-1}^R) \cdot \sqrt{\frac{1}{1+b^2}} \right) \tag{11}$$

where the vector β_1 contains the respective coefficients and β_0 is a constant. Note that the macroeconomic factors are included with a time lag and are therefore known before the modeling period. The random effects F_t and X_t are contemporaneous and hence unknown. The estimation is done via maximum-likelihood as described in Rösch and Scheule (2005).

B. Downturn PD

The stylized model framework in Basel II makes specific assumptions about the downturn state for the systematic factor F_t in its capital curve: the factor is stressed such that $f_t = \Phi^{-1}(0.999)$ giving the CPD in dependence of the unconditional PD (and omitting the subscript i) as

$$CPD_t(f_t = \Phi^{-1}(0.999)) = \Phi\left(\frac{\Phi^{-1}(PD_t) + \omega\Phi^{-1}(0.999)}{\sqrt{1-\omega^2}} \right). \tag{12}$$

C. Downturn LGD

In relation to the Downturn loss rate given default (Downturn LGD), the Basel Committee on Banking Supervision (2006) specifies:

> A bank must estimate a LGD for each facility that aims to reflect economic downturn conditions where necessary to capture the relevant risks. This LGD cannot be less than the long-run default-weighted average loss rate given default calculated based on the average economic loss of all observed defaults within the data source for that type of facility. In addition, a bank must take into account the potential for the LGD of the facility to be higher than the default-weighted average during a period when credit losses are substantially higher than average [. . .].

In other words, LGD rates should lie between the ‘long-run default-weighted’ LGD and one. Various approaches to specify the Downturn LGD are common in industry. The empirical analysis will compare the implications of four concepts on the regulatory capital:

- Basel LGD (BLGD): Basel Committee on Banking Supervision (2006) specifies within the Internal Ratings Based (IRB) approach a benchmark LGD for corporate senior unsecured loans (45%) and a floor for real estate secured loans (10%):

$$BLGD = \begin{cases} 45\% & \text{for corporate senior unsecured loans} \\ 10\% & \text{for real estate secured loans} \end{cases}. \quad (13)$$

Please note that a replacement of the 10% floor for real estate loans by bank-internal estimates has been discussed and may occur in the future.

- Downturn LGD 1: Financial institutions may apply the expected LGD of selected historic downturn periods. For example, the empirical analysis investigates periods where the loss rate is below the median loss rate:

$$DLGD1_t = 1 - \Phi\left((\beta_{0,\text{downturn}} + \beta'_{1,\text{downturn}} z^R_{t-1}) \cdot \sqrt{\frac{1}{1 + b^2_{\text{downturn}}}} \right). \quad (14)$$

- Downturn LGD 2: the Department of the Treasury, Federal Reserve System and Federal Insurance Corporation (2006) propose a linear relationship between the Downturn LGD and the Expected LGD (ELGD) with a floor of 8% and a cap of 100%. This formula aims to account for the correlation between PDs and LGDs:

$$DLGD2_t = 0.08 + 0.92 \times ELGD_t. \quad (15)$$

- Downturn LGD 3: Rösch and Scheule (2009) introduce an econometric model to estimate the sensitivity of LGDs to the business cycle (b) as well as the correlations between PDs and LGDs (ρ) and apply the same stress scenario which is used by the Basel Committee on Banking Supervision (2006) and presented in equation (12):

$$DLGD3_t = \Phi\left(\frac{\Phi^{-1}(ELGD_t) \cdot \sqrt{1 + b^2} - b\rho\Phi^{-1}(0.999)}{\sqrt{1 + b^2(1 - \rho^2)}} \right). \quad (16)$$

DLGD3 is a new concept where the downturn LGD depends on empirical properties such as the ELGD, the sensitivity b to a random systematic factor and the correlation ρ between the random systematic factors driving the default and recovery process. In the special case where the systematic factors are uncorrelated, the downturn parameter drops to the expected LGD. Alternatively, a joint 99.9% confidence interval for both systematic risk factors may be constructed according to the Bonferroni principle. However, this may result in a downturn PD which is inconsistent with the Basel II downturn definition for conditional PDs.

Figures 1 and 2 show the dependence of DLGD3 on the correlation ρ of the systematic risk factors of the default and recovery process. In each figure, constellations of 0.2, 0.5, and 0.8, respectively, are assumed for the ELGD. Figure 1 assumes a higher dependence of DLGD3 on the economy (b=1) and Figure 2 assumes a lower dependence (b=0.2).

Hence, a lower risk factor correlation and/or a higher economic dependency implies a higher level of the downturn loss given default.

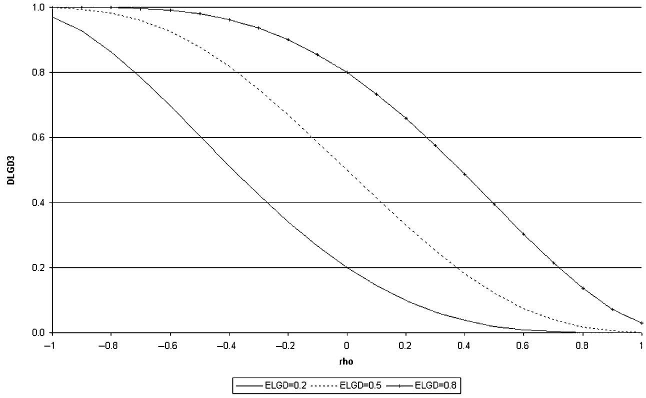

Figure 1 DLGD3 in Dependence of Risk Factor Correlation; High Dependence on the Economy ($b = 1.0$)

Notes: DLGD3 decreases with ρ and the expected LGD (ELGD).

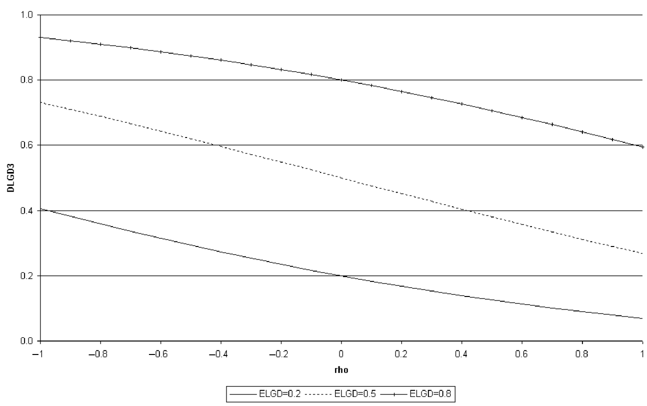

Figure 2 DLGD3 in Dependence of Risk Factor Correlation; Low Dependence on the Economy ($b = 0.2$)

Notes: DLGD3 decreases with ρ and the expected LGD (ELGD). The decrease is less pronounced for a low sensitivity to the business cycle (this figure) than for a high sensitivity to the business cycle (compare Figure 1).

III. EMPIRICAL ANALYSIS

A. Data

The data was provided by the Federal Reserve (http://www.federalreserve.gov) and relates to business loans (Corporate) and loans secured by real estate (Real Estate) of all US commercial banks. The data consists of annual observations during the years 1987–2008. The estimation period ranges from 1988 to 2008 due to the use of time-lagged explanatory variables in PIT models.

The default rate is represented by the delinquency rate, the loss rate is represented by the charge-off rate and the LGD by the ratio of charge-off rate and delinquency rate. The charge-off and delinquency rates are calculated from data available in the Report of Condition and Income, filed each quarter by all US commercial banks with their regulator. Charge-off rates are defined as the flow of a bank's net charge-offs (gross charge-offs less recoveries) during a quarter divided by the average level of its loans outstanding over that quarter. The delinquency rate for any loan category is the ratio of the dollar amount of a bank's delinquent loans in that category to the dollar amount of total loans outstanding in that category.

Please note that delinquency and charge-off rates are proxies for default probabilities and loss rates given default. The charge-off rate may be considered as a slightly conservative value for the expected loss due to the conservative stance of current accounting rules. The delinquency rate may also be a conservative measure for default probability compared with the Basel Committee on Banking Supervision event definition, which includes the criteria delinquency of more than 90-days as well as unlikeliness to pay. In particular, the unlikeliness to pay involves a certain degree of arbitrariness for financial institutions. In terms of overall risk measurement the consequences are limited as lower default probabilities are generally accompanied by higher loss rates given default and vice versa. Furthermore, the resulting average proxies for LGD (Corporate: 0.2675; Real Estate: 0.0920, compare Table 1) are convincingly close to empirical values of bank portfolios and reflect the

Table 1 Descriptive statistics for default rates, loss rates given default and loss rates, US corporate and real estate backed loans, 1988–2008 and downturn years

	Corporate			Real estate		
	DR	LGD	LR	DR	LGD	LR
All years						
Mean	0.0294	0.2675	0.0080	0.0327	0.0920	0.0034
Median	0.0222	0.2399	0.0070	0.0236	0.0885	0.0019
Minimum	0.0123	0.1104	0.0023	0.0138	0.0266	0.0006
Maximum	0.0601	0.5511	0.0175	0.0732	0.2324	0.0113
Std. Dev.	0.0155	0.1206	0.0051	0.0181	0.0520	0.0034
Downturn years						
Mean	0.0406	0.3368	0.0112	0.0474	0.1306	0.0059
Median	0.0441	0.3127	0.0108	0.0485	0.1228	0.0052
Minimum	0.0153	0.2036	0.0049	0.0227	0.0675	0.0009
Maximum	0.0601	0.5511	0.0175	0.0732	0.2324	0.0113
Std. Dev.	0.0149	0.1190	0.0044	0.0158	0.0474	0.0034

Notes: The data is provided by the Federal Reserve (http://www.federalreserve.gov) and relates to business loans and loans secured by real estate of all US commercial banks. The default rate is represented by the delinquency rate, the loss rate is represented by the charge-off rate and the loss rate given default by the ratio of charge-off rate and delinquency rate. The downturn data set is restricted to the ten years with the highest loss rates (i.e., years 1988–1992, 2000–2003 and 2008 for corporate loans and years 1988–1995 and 2007–2008 for real estate loans).

Table 2 Correlation coefficients for default rates, loss rates given default and loss rates, US corporate and real estate backed loans, 1988–2008 and downturn years

	Corporate			Real estate		
	DR	LGD	LR	DR	LGD	LR
All years						
Corporate						
DR	1.0000	0.0087	0.7543	0.8044	0.4923	0.6762
LGD		1.0000	0.2003	−0.0906	0.4355	−0.2713
LR			1.0000	0.4596	0.4852	0.5804
Real Estate						
DR				1.0000	0.7256	0.8425
LGD					1.0000	0.6129
LR						1.0000
Downturn years						
Corporate						
DR	1.0000	−0.8037	0.5513	0.6188	−0.2552	0.2114
LGD		1.0000	−0.3379	−0.5791	0.2069	−0.3592
LR			1.0000	−0.0170	−0.3458	−0.1206
Real Estate						
DR				1.0000	0.4540	0.6607
LGD					1.0000	0.1824
LR						1.0000

Notes: The data is provided by the Federal Reserve (www.federalreserve.gov) and relates to business loans and loans secured by real estate of all US commercial banks. The default rate is represented by the delinquency rate, the loss rate is represented by the charge-off rate and the loss rate given default by the ratio of charge-off rate and delinquency rate. The downturn data set is restricted to the ten years with the highest loss rates (i.e., years 1988–1992, 2000–2003 and 2008 for corporate loans and years 1988–1995 and 2007–2008 for real estate loans).

observation that real estate loans had historically lower severities (collateralized loans) than corporate loans.

Tables 1 and 2 describe the default rate (DR), LGD and 1-year lagged loss rate (LR) for the two risk segments Corporate and Real Estate for the whole data set, as well as downturn years. A downturn year is defined as a year in which the loss rate is below its median (i.e., years 1988–1992, 2000–2003 and 2008 for Corporate; and years 1988–1995 and 2007–2008 for Real Estate).

Figure 3 (Corporate) and Figure 4 (Real Estate) show the default and LGD over time. Credit portfolio risk changes over time and is cyclical. High-risk years were 1991 (First Gulf War), 2002 (after the US terrorist attacks) and most recently 2008 (Global Financial Crisis).

B. Joint PD and LGD model

Two models are estimated for the two risk segments Corporate and Real Estate:

- TTC model: the model does not include any time varying information and estimates (i) the average PD, (ii) the average LGD and (iii) the correlation between the periodic PD and LGD;

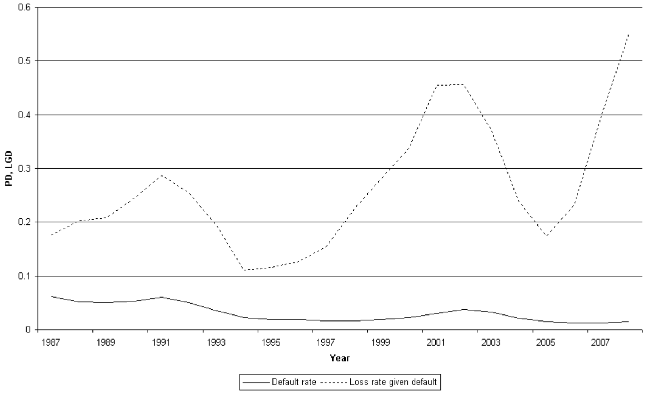

Figure 3 Default Rate and Loss Rate Given Default, US Corporate Loans, 1987–2008
Notes: The default and loss rate given default for US corporate loans increased in 1991, 2002 and most recently 2008.

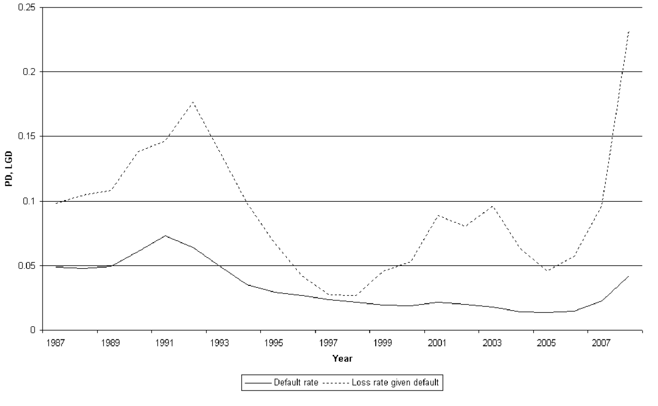

Figure 4 Default Rate and Loss Rate Given Default, US Real Estate Backed Loans, 1987–2008
Notes: The default and loss rate given default for US real estate backed loans increased in 1991/92, 2002 and most recently 2008.

- PIT model: the model includes time varying information and estimates (i) one PD per period, (ii) one LGD per period and (iii) the correlation between the periodic PD and LGD. Consistent with previous research (compare Altman et al. 2005), the loss rate lagged by 1 year was found to be a good explanatory variable for both the PD and the LGD. Other economic variables such as CPI, GDP or the unemployment rate were found to be correlated but less significant.

Table 3 shows the parameter estimates. In the upper panel the results for the static TTC model are given. The asset correlation is $\omega^2 = 0.2025^2 = 0.0410$ and

Table 3 Estimation results, through-the-cycle model and point-in-time, 1988–2008

Parameter	Corporate			Real estate		
	Estimate	SE	Significance	Estimate	SE	Significance
TTC						
γ_0	−1.8904	0.0493	***	−1.8451	0.0477	***
ω	0.2025	0.0345	***	0.2121	0.0343	***
β_0	0.6580	0.0786	***	1.3926	0.0656	***
b	0.3600	0.0555	***	0.3083	0.0470	***
ρ	−0.0480	0.2368		−0.7641	0.1060	***
PIT						
γ_0	−2.1937	0.0616	***	−2.0702	0.0432	***
γ_1	34.3654	6.2382	***	56.6819	8.5191	***
ω	0.1170	0.0255	***	0.1056	0.0240	***
β_0	0.7962	0.1441	***	1.5990	0.0745	***
β_1	−17.1981	15.2129		−60.6162	15.6964	***
b	0.3495	0.0539	***	0.2358	0.0364	***
ρ	0.2918	0.2553		−0.5506	0.2146	**

Notes: The through-the-cycle (TTC) model is specified according to equations (4) and (8). The point-in-time model is specified according to equations (10) and (11). SE indicates the standard error.
***Significant at 1%.
**Significant at 5%.
*Significant at 10%.
The Akaike information criterion (smaller is better) is 195.2 for TTC (Corporate), 178.8 TTC (Real Estate), 163.5 for PIT (Corporate) and 158.5 for PIT (Real Estate).

the standard deviation of the systematic factor for the LGD is $b = 0.3600$ for Corporate and is of a similar size for Real Estate. The correlation between the risk factor for the PD and the recovery model is slightly negative for Corporate and highly negative for Real Estate. This indicates the cyclical nature of the collateral real estate. The systematic factor affects the conditional PD (equation 4) and the recovery (equation 8) with a positive sign. This specification implies that PD and recovery are negatively related. Consequently, PD and LGD are positively related.

In the PIT model, the lagged loss rate is included as regressor for both PD and recovery. The respective coefficient is (as expected) positive for PD and negative for the recovery. Consistent with other studies (compare Rösch and Scheule 2005), the asset correlation (i.e., ω^2) decreases when the cyclical movement of PDs is explained by observable information. Likewise the respective recovery parameter (i.e., b) decreases. The correlation between the PD and recovery factor changes from −0.048 to +0.2918 for Corporate and from −0.7641 to −0.5506 for Real Estate. Note that the residual correlation is rendered less pronounced (and even positive for Corporate) because both PD and recovery are modeled using the same observable variables (i.e., the lagged loss rate) as explanatory variable. As the lagged loss rate is included in both equations it absorbs part of

Table 4 Estimation results, through-the-cycle model and point-in-time, downturns years

Parameter	Corporate			Real estate		
	Estimate	SE	Significance	Estimate	SE	Significance
TTC						
γ_0	−1.7438	0.0562	***	−1.6703	0.0502	***
ω	0.1631	0.0429	***	0.1416	0.0390	***
β_0	0.4387	0.0978	***	1.1472	0.0649	***
b	0.3109	0.0679	***	0.2053	0.0459	***
ρ	0.8818	0.1044	***	−0.6110	0.2346	**
PIT						
γ_0	−2.0356	0.1381	***	−1.8672	0.0808	***
γ_1	25.3407	11.4338	*	31.8164	11.7487	**
ω	0.1299	0.0380	***	0.0986	0.0327	**
β_0	0.1741	0.2652		1.2436	0.1301	***
β_1	23.7905	22.2036		−16.3125	19.2908	
b	0.2948	0.0659	***	0.1983	0.0443	***
ρ	0.9221	0.1054	***	−0.6266	0.2659	**

Notes: The through-the-cycle (TTC) model is specified according to equations (4) and (8). The point-in-time model is specified according to equations (10) and (11). The data set is restricted to the ten yearswith the highest loss rates (i.e., years 1988–1992, 2000–2003 and 2008 for corporate; and years 1988–1995 and 2007–2008 for real estate loans). SE indicates the standard error.
***Significant at 1%.
**Significant at 5%.
*Significant at 10%.
The Akaike Information Criterion (smaller is better) is 87.5 for TTC (Corporate), 85.8 TTC (Real Estate), 86.3 for PIT (Corporate) and 83.9 for PIT (Real Estate).

the correlation between PD and recovery as a higher value of lagged losses implies c.p. both a higher PD and lower recovery. In this respect the PIT model already includes a 'downturn' effect when the lagged loss rate is high.

In addition, the same model is estimated for downturn years. A downturn year is defined as a year in which the loss rate is lower than the median loss rate (i.e., years 1988–1992, 2000–2003 and 2008 for Corporate; and years 1988–1995 and 2007–2008 for Real Estate). Table 4 shows the parameter estimates. The parameters γ_0 and β_0 increase for all risk segments and model methodologies. For example, γ_0 increases from -1.8904 (Table 3) to -1.7438 (Table 4) for Corporate. Interestingly, this may imply that credit risk models will lead to higher risk measures after the current financial crisis, as the high realizations of risk will be reflected in the benchmark data and hence included into statistical models which are based on such data.

Please note that the reason for the requirement of a downturn LGD is the correlation between the default and the recovery process. This correlation ρ is negative in the TTC model for all observation years, which is consistent with previous contributions in literature. However, after the state of the economy is included for the PD and LGD (or recovery) model the residual correlation may be different. The state of the economy may be included by (i) including time-varying covariates (PIT) or (ii) estimating models for economic downturn years only or (iii) both.

C. Bank capital

The regulatory capital can be derived from the model outputs. The regulatory capital is measured by the Credit Value-at-Risk (CVaR), i.e., the difference between the Basel Value-at-Risk and provisions. The Basel Value-at-Risk is calculated by multiplying the Basel CPD (compare equation 12) and LGD. For the LGD, multiple concepts will be analyzed: BLGD based on equation (13), DLGD1 based on equation (14), DLGD2 based on equation (15) and DLGD3 based on equation (16). The level of provisions is approximated by the loss rate (compare equation 17 and 18).

For TTC models, the mean loss rates are taken as a proxy for the level of provisioning in the respective risk segments in the calculation of the required capital:

$$Provisions = \begin{cases} 0.81\% & \text{for corporate senior unsecured loans} \\ 0.37\% & \text{for real estate secured loans} \end{cases}. \tag{17}$$

For PIT models, the periodic loss rates are taken as a proxy for the level of provisioning in the respective risk segments in the calculation of the required capital:

$$Provisions_t = EL_t = PD_t \cdot LGD_t. \tag{18}$$

Please note that this paper does not include or analyze the maturity adjustment of corporate loans as well as asset correlation for small and medium-sized entities (compare Basel Committee on Banking Supervision 2006).

For the TTC model, Table 5 computes the PD, CPD based on the empirical asset correlation, Basel CPD based on the Basel asset correlation, empirical asset correlation and Basel asset correlation. The Basel asset correlation is higher than the empirical asset correlation and the resulting Basel CPD higher than the CPD. Table 6 shows the four LGDs: BLGD, DLGD1, DLGD2 and DLGD3 and resulting capital requirement. The resulting capital requirement is highest for BLGD for Corporate and DLGD3 for Real Estate.

Table 5 PD, CPD and asset correlation estimates, through-the-cycle model

	Corporate	Real estate
PD	0.0294	0.0325
CPD	0.0983	0.1117
Basel CPD	0.2233	0.2410
Asset correlation	0.0410	0.0450
Basel asset correlation	0.1477	0.1500

Notes: PD is calculated using equation (4). CPD is calculated using equation (12). Basel CPD is calculated by replacing the estimated parameter $\hat{\omega}$ by the square root of the asset correlation proposed by Basel Committee on Banking Supervision (2006).

Table 6 LGD and VaR estimates, through-the-cycle model

	Corporate	Real estate
BLGD	0.4500	0.1000
DLGD1	0.3376	0.1305
DLGD2	0.3265	0.1643
DLGD3	0.2847	0.2572
CVaR BLGD	0.0924	0.0204
CVaR DLGD1	0.0673	0.0277
CVaR DLGD2	0.0648	0.0359
CVaR DLGD3	0.0555	0.0583

Notes: DLGD1 is based on equation (14), BLGD is based on equation (13), DLGD2 is based on equation (15) and DLGD3 is based on equation (16). The Credit Value-at-Risk (CVaR) is calculated by multiplying the respective LGD with the Basel CPD and subtracting the loss according to equation (17).

Table 7 PD, CPD and asset correlation estimates, 2006 and 2008, point-in-time model

	Corporate			Real estate		
	2006	2008	Average	2006	2008	Average
PD	0.0177	0.0214	0.0294	0.0209	0.0262	0.0328
CPD	0.0397	0.0469	0.0614	0.0429	0.0523	0.0630
Basel CPD	0.1807	0.1958	0.2205	0.1817	0.2102	0.2350
Asset correlation	0.0137	0.0137	0.0137	0.0111	0.0111	0.0111
Basel asset correlation	0.1684	0.1611	0.1521	0.1500	0.1500	0.1500

Notes: PD is calculated using equation (10). CPD is calculated using equation (12). Basel CPD is calculated by replacing the estimated parameter $\hat{\omega}$ by the square root of the asset correlation proposed by Basel Committee on Banking Supervision (2006). In a point-in-time model, risk measures may vary periodically. The periods 2006 and 2008 were chosen as examples. The average parameter is calculated by the average over the periodic parameter estimates during the observation period.

For the PIT model, Table 7 computes the PD, CPD based on the empirical asset correlation, Basel CPD based on the Basel asset correlation, empirical asset correlation and Basel asset correlation. The Basel asset correlation is higher than the empirical asset correlation and the resulting Basel CPD higher than the CPD. Table 8 shows the four LGDs: BLGD, DLGD1, DLGD2 and DLGD3 and resulting capital requirement for two selected years 2006 and 2008 as well as the average over the observation period. The resulting capital requirement is highest for BLGD for Corporate and DLGD3 (Average) for Real Estate. In the instance of Corporate and DLGD3 the changed sign of the correlation now results in a capital relief as a large part of the downturn effect is already captured by modeling the time-varying PD and ELGD as a function of the lagged loss rate.

Moreover, it is obvious that PIT models lead to a dynamic capital requirement. Figure 5 shows the resulting capital requirements for the different LGD concepts for Corporate and Figure 6 for Real Estate.

Table 8 LGD and VaR estimates, 2006 and 2008, point-in-time model

	Corporate			Real estate		
	2006	2008	Average	2006	2008	Average
BLGD	0.4500	0.4500	0.4500	0.1000	0.1000	0.1000
DLGD1	0.4102	0.3902	0.3639	0.1132	0.1184	0.1223
DLGD2	0.3000	0.3107	0.3265	0.1392	0.1515	0.1647
DLGD3	0.1358	0.1443	0.1576	0.1207	0.1422	0.1626
CVaR BLGD	0.0784	0.0796	0.0913	0.0173	0.0111	0.0198
CVaR DLGD1	0.0712	0.0679	0.0707	0.0197	0.0150	0.0258
CVaR DLGD2	0.0513	0.0524	0.0650	0.0244	0.0219	0.0372
CVaR DLGD3	0.0216	0.0198	0.0276	0.0211	0.0200	0.0380

Notes: DLGD1 is based on equation (14), BLGD is based on equation (13), DLGD2 is based on equation (15) and DLGD3 is based on equation (16). The Credit Value-at-Risk (CVaR) is calculated by multiplying the respective LGD with the Basel CPD and subtracting the loss according to equation (18). In a point-in-time model, risk measures may vary periodically. The periods 2006 and 2008 were chosen as examples. The average parameter is calculated by the average over the periodic parameter estimates during the observation period.

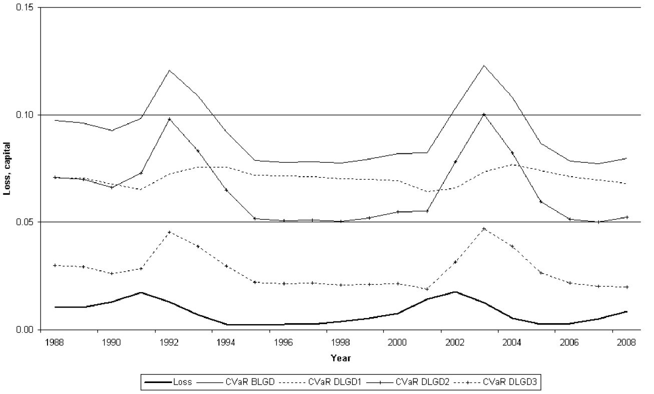

Figure 5 Loss and Capital Requirement Based on Point-in-Time Model, US Corporate Loans, Full Sample Estimation, 1988–2008
Notes: The capital requirement based on the Basel LGD is the highest and Downturn LGD 3 the lowest.

Table 9 summarizes the results and provides the average capital requirement as well as variation coefficient (VC) for the two model methodologies, four LGD definitions and two risk segments. The capital requirement is stationary for the TTC model and thus the VC is equal to zero. The capital requirement for the PIT model is cyclical and thus the VC is greater than zero. In addition, the minimum and maximum capital requirements are reported. The table reveals the following insights:

Firstly, capital requirements differ significantly between the various LGD concepts. For example, the capital levels for the TTC corporate model are BLGD:

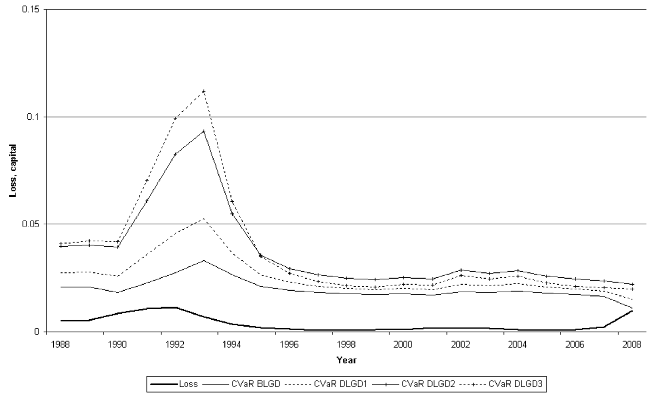

Figure 6 Loss and Capital Requirement Based on Point-in-Time Model, US Real Estate Backed Loans, Full Sample Estimation, 1988–2008

Notes: The capital requirement based on the Downturn LGD 2 and Downturn LGD 3 are the highest and the Basel LGD the lowest.

Table 9 Comparison of level and cyclicality of regulatory capital requirement, through-the-cycle model (TTC) and point-in-time model (PIT)

	Through-the-cycle			Point-in-time		
	Average	VC	Average	VC	Minimum	Maximum
Corporate						
CVaR BLGD	0.0924	0.0000	0.0913	0.1593	0.0773	0.1228
CVaR DLGD1	0.0673	0.0000	0.0707	0.0472	0.0643	0.0768
CVaR DLGD2	0.0648	0.0000	0.0650	0.2429	0.0502	0.1001
CVaR DLGD3	0.0555	0.0000	0.0276	0.3071	0.0189	0.0471
Real Estate						
CVaR BLGD	0.0204	0.0000	0.0198	0.2316	0.0111	0.0330
CVaR DLGD1	0.0277	0.0000	0.0258	0.0258	0.3661	0.0150
CVaR DLGD2	0.0359	0.0000	0.0372	0.5336	0.0219	0.0934
CVaR DLGD3	0.0583	0.0000	0.0380	0.6908	0.0200	0.1118

Notes: DLGD1 is based on equation (14), BLGD is based on equation (13), DLGD2 is based on equation (15) and DLGD3 is based on equation (16). The Credit Value-at-Risk (CVaR) is calculated by multiplying the respective LGD with the Basel CPD and subtracting the loss according to equation (17) for the through-the-cycle model and equation (18) for the point-in-time model. The variation coefficient is the ratio of the empirical standard deviation and mean.

0.0924, DLGD1: 0.0673, DLGD2: 0.0648 and DLGD3: 0.0555. This may give financial institutions the liberty to 'select' the capital requirement. Financial Institutions may choose a combination of DLGD3 and PIT for Corporate and a combination of BLGD and PIT for Real Estate if capital can be raised and released without problems and lending volumes do not change.

Secondly, the difference between a TTC and PIT model is limited on average with the exception of DLGD3, which implies that financial institutions may lower their capital requirement significantly on average when choosing a PIT concept for the Downturn LGD in the instance of corporate loans.

Thirdly, the difference of the regulatory capital requirements between TTC models and PIT models is fundamentally different for given periods as PIT models lead to a time-varying and pro-cyclical capital requirement. In other words, financial institutions are required to hold more capital during economic downturns and less capital during economic booms. The VC is reported and shows that the regulatory capital based on DLGD3 is most cyclical for Corporate (VC=0.3071) and Real Estate (VC=0.6908). Thus, regulators may have to provide guidance on the degree of cyclicality that is desired in a financial system. Cyclicality impacts lending volumes of financial institutions and financial institutions may have to derive target-lending volumes. Both bank solvency and stable lending volumes will have to be balanced by these stakeholders.

Fourthly, the models are more reflective of the embedded risk if bank-internal models are built (e.g., DLGD3) and Basel parameters are not chosen. The capital requirement for corporate loans may be seen as an incentive for financial institutions to measure credit portfolio risks accurately as this may lower their capital requirement. However, the capital requirements for Real Estate loans may achieve the opposite as the most accurate model (DLGD3) results in the highest capital requirement. In particular the BLGD concept, which is often applied by medium-sized banks raises the question on its adequacy as the average for all US financial institutions is very close to the realized loss rate of 2008. Note that the 10% number is a lower boundary for the Downturn LGD.

Last but not least, all model methodologies and LGD concepts provide regulatory capital in excess of the current and past loss rates. A more detailed investigation may be warranted for (i) individual institutions, (ii) multiple year horizons and (iii) future years (i.e., post 2008) in relation to the current financial crisis.

IV. DISCUSSION

This paper analyzed whether capital requirements support the accurate measurement of credit portfolio risks. It found that the Basel II rules do support more accurate modeling of corporate credit portfolio risks, as, by doing so, financial institutions may be able to lower their capital requirement. However, for real estate loans, financial institutions may have an incentive to opt for the Basel benchmark parameters rather than building internal risk models.

Important policy implications may be drawn from the findings of this paper. Firstly, all four LGD concepts (BLGD, DLGD1, DLGD2 and DLGD3), all model methodologies (TTC and point in time) and risk segments (Corporate and Real Estate) provide sufficient capital to cover losses in all years including 2007 and 2008. The findings do not give a reason to reject a concept on the grounds of insufficient capital.

Secondly, PIT models are cyclical. If risk models are forward looking then the capital requirement may be counter-cyclical. If risk models are contemporary then the capital requirement may be pro-cyclical. The results are changes in bank stability (high capital requirement implies high stability) and lending volumes (high capital requirement implies low lending volumes) and the current financial crisis shows that both aspects are important for a financial system. The study has measured the degree of cyclicality of various approaches. Thus, a public discussion is needed on the importance of stationary versus dynamic capital and provisioning systems. Prudential regulators may favor stationary systems while Keynesian economists may favor dynamic systems. Different risk segments exhibit not only different risk characteristics but also different cyclicality patterns. In other words, as Basel II differentiates capital requirements by risk segments, potential cyclicality dampening mechanisms should not be designed as an 'one-fits-all' approach. Bank capital derived from the PIT approach is obviously more cyclical but may require a lower stress for economic downturns downturns.

Thirdly, the Basel benchmarks for the LGD of 45% for corporate loans is sufficient to reflect an economic downturn LGD, while the application of a 10% benchmark for real estate loans may be too low. Please note that the number presents a floor and that empirical values applied by financial institutions may be higher. Interestingly, Australia has implemented a 20% floor for mortgage loans.

Naturally, the study is limited by the data that is analyzed. The data covers various business cycles of the major loan categories of US financial institutions and finds that regulatory capital is provided on average in excess of the current and past loss rates. However, individual financial institutions may deviate significantly from this average. Thus an analysis for the individual institution may be interesting. Other areas of future research may involve the analysis of multiple year risk horizons and the extension of this study to future years as well as other countries, financial markets, institutions and instruments.

Going forward, the paper may provide a stimulus for an increase of transparency in relation to the regulatory requirements of loan portfolios with regard to the level and cyclicality of bank capital and a fertile ground for industry and regulatory discussions and decisions.

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APPENDIX A: ROBUSTNESS OF APPLIED CREDIT PORTFOLIO RISK MODEL

This section discusses the robustness of the credit portfolio risk model with regard to the sample size and the large borrower number approximation of the recovery rate given default in equation (8).

A Monte-Carlo simulation is run as a robustness test to check the small sample properties of the asymptotic estimators, see e.g., Gordy and Heitfield (2000). The Monte-Carlo simulation is carried out in the following steps:

(1) Set-up the ‘real world’ by defining a model with given parameters, i.e., the parameters of the default and recovery process.
(2) Generate random cross-sections and time-series of defaults and recoveries.
(3) Estimate the model parameters under a particular model-specification.

(4) Repeat steps two and three a sufficiently large number of times and analyze the resulting sample of parameter estimates. The level of sufficiency is determined by the stability of simulation results.

A first important question is the impact of sample size used in the model estimation as the empirical data encompasses the years 1987– 2008. The default threshold $\gamma_0 = -2.32$ corresponds to a default probability of 1%. The other parameters are $\omega = 0.2$, $\beta_0 = 0.5$, $b = 0.5$, and $\rho = -0.5$. Table A1 shows the Monte-Carlo results for time-series lengths of $T \in \{20, 100\}$ years. As a result, the parameter estimates are on average close to their true values when the sample size is large (i.e., 100 years). However, the parameter estimates differ from their true counterparts for small sample sizes. This holds particularly for ω, γ and the correlation ρ. However, the large standard deviations imply that the true counterparts are well within a reasonable confidence interval. Thus, the analysis shows that estimation error, reflected by large standard deviations, is an important issue when sample sizes are small. An extension by Rösch and Scheule (2007) incorporates the estimation error into a stress-testing framework.

Next, the empirical analysis averages over individual data due to data availability. Therefore a large number approximation for the LGDs (or recoveries, respectively) is used. Another important question is therefore how the model performs when the number of borrowers is limited and the PDs and recoveries are driven by idiosyncratic variables.

To do a robustness check, a Monte-Carlo simulation is run where a limited number of borrowers with different PDs and recoveries in the 'real world' (the simulation relates to a portfolio of 10,000 loans) is assumed, and the model is estimated under the above simplified assumptions where only DRs and average recoveries are observable. The average default probability within the portfolio is given as 1% and the individual default probabilities differ by using a random 'contamination'. Therefore, a parameter δ is introduced which is the standard deviation of the idiosyncratic component of the recoveries and leads to borrower-specific recoveries.

Table A1 Robustness check I: number of observation periods

	$T=20$		$T=100$	
	Average estimate	Standard deviation	Average estimate	Standard deviation
γ_0	−2.2556	0.1139	−2.2932	0.0687
ω	0.2506	0.0858	0.2087	0.0295
β_0	0.4647	0.1558	0.4657	0.0827
b	0.5284	0.1262	0.5038	0.0416
ρ	−0.5579	0.2127	−0.5162	0.0880

Notes: Results from Monte-Carlo simulations for robustness check. Table shows average parameter estimates and standard deviations. True parameters are: $\omega = 0.2$, $b = 0.5$, $\gamma = -2.32$, $\beta_0 = 0.5$, $\rho = -0.5$.

Table A2 Robustness check II: dispersion of loss rates given default

	$\delta = 0.5$		$\delta = 1$	
	Average estimate	Standard deviation	Average estimate	Standard deviation
γ_0	−2.3337	0.0329	−2.3361	0.0286
ω	0.1894	0.0166	0.1877	0.0149
β_0	0.4523	0.0513	0.3606	0.0392
b	0.4456	0.0320	0.3599	0.0260
ρ	−0.4847	0.0782	−0.4752	0.0820

Notes: Results from Monte-Carlo simulations for robustness check. Table shows average parameter estimates and standard deviations. True parameters are: $\omega = 0.2$, $b = 0.4472$ (for $\delta = 0.5$), $b = 0.3535$ (for $\delta = 1$), $\gamma = -2.32$, $\beta_0 = 0.4472$ (for $\delta = 0.5$), $\beta_0 = 0.3535$ (for $\delta = 1$), $\rho = -0.5$.

In one setting, δ is set to 0.5, which assigns the same weight to the idiosyncratic component and the systematic component. In another setting, δ is set to 1, which weights idiosyncratic risk more heavily. Then we obtain $\beta_0 = \tilde{\beta}_0/\sqrt{1+\delta^2} = \frac{0.5}{\sqrt{1+0.5^2}} = 0.4472$ (for $\delta = 0.5$) and $\beta_0 = \frac{0.5}{\sqrt{1+1^2}} = 0.3535$ (for $\delta = 1$). A similar transformation applies for b. The sample size is set to 100 in order to separate the effect from the one in relation to small sample sizes. Table A2 shows that the deviations from the true values are small.