WHAT DO WE KNOW ABOUT EXPOSURE AT DEFAULT ON CONTINGENT CREDIT LINES? - A SURVEY OF THE LITERATURE, EMPIRICAL ANALYSIS AND MODELS

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What do We Know About Exposure at Default on Contingent Credit Lines? – A Survey of the Literature, Empirical Analysis and Models

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Abstract:
Exposure at Default (EAD) quantification for the large exposures to contingent credit lines (CCLs) is a critical for models of credit risk amongst financial institutions. This includes expected loss calculations for loan provisions, economic credit capital as well as regulatory capital under the Basel II advanced Internal Ratings Based (IRB) framework. However, banks struggle in quantifying EAD due to limited empirical evidence and industry benchmarks, unavailable or inconsistent internal data and paucity of practical models. This study contributes to this modelling effort by surveying the existing literature and consolidating the empirical evidence on EAD. We consider recent extensions of prior empirical work that considers alternative determinants and measures of EAD risk in addition to the traditional approaches, including regression models and actuarial based models of EAD. We illustrate these new EAD paradigms through an empirical investigation using a sample of Moody's rated defaulted firms, first the construction of a predictive econometric model in the generalized linear model class, followed by the calibration of an EAD model similar to basic CreditRisk+ type using Fast Fourrier transforms to convolute portfolio segments.

Keywords: Exposure at Default, Recoveries, Default Risk, Bankruptcy, Credit Risk, Basel II

JEL Classification: G33, G34, C25, C15, C52

1. Introduction

Committed revolving credit facilities, or contingent credit lines (CCLs), offer borrowers an option to draw funds up to specified limits according to changing circumstances. CCLs appeal to a clientele of borrowers with particular financing strategies and an attractive return profile from investor's perspective. A great interest in analyzing these stems from the unique characteristics relative to other credit products. Understanding these are of high importance to loan structurers in pricing, credit risk managers in computing limits or credit Value-at-Risk, as well as banking supervisors assessing capital adequacy. This can also play a vital role in liquidity risk management of CCLs. There is potential relief on regulatory capital relative to similar investments. However, these instruments present a great challenge in valuation and risk management, as estimates of revolving credits facility's expected exposure at default (EAD) needs to be formed. We recognize this to be a key parameter in estimating expected loss and credit risk capital for unfunded commitments. Financial institutions have a great

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interest in estimating such quantities either from their internal histories or from available benchmark data-sets, to parameterize credit risk models, as well as to satisfy supervisory requirements such as the Basel II advanced internal ratings based (IRB) framework for regulatory capital (BCBS, 2006).

Two features about committed lines worth mentioning are various fees, which must be paid over the life of the commitment, and the material adverse change (MAC) clause—which essentially means that the issuing authority can unconditionally cancel the commitment. The fee structure generally has three associated types: an up-front fee for establishing the commitment, a usage fee levied on the unused portion of the line and a service fee on the used portion of the line. A study of a sample of 1,347 loans and find that only 46 percent had a commitment fee, 38 percent had an annual fee, and 69 percent had a usage fee (Booth and Chua, 1995).

Currently many financial institutions find it challenging to quantify EAD for various reasons: limitations of empirical evidence (either internally or through industry benchmarks), outright unavailability or inconsistency of internal data, unsuitability of external data and paucity of practical EAD risk models. This study contributes to the effort of modelling EAD by surveying the existing prior literature and consolidating the available empirical evidence on EAD. We consider recent extensions of prior empirical work that considers alternative determinants and measures of EAD risk in addition to the traditional approaches (Asarnow and Marker, 1995; Araten and Jacobs, 2001), including regression5 and actuarial based models6 of EAD. We then illustrate these new EAD paradigms through an empirical investigation using a sample of Moody's rated defaulted firms, first the construction of a predictive econometric model in the generalized linear model class, followed by the calibration of an EAD model in basic CreditRisk+ model and Fast Fourier Transforms.

2. Alternative Measurement Frameworks for EAD

Let t denote the current time, T a fixed horizon (maturity or ex post calendar time of default) and τ a random time of default. $\mathbf{X}_t$ is a vector of obligor or facility characteristics (e.g., risk rating, product type, financial ratios) observed at time t. Dollar exposure (i.e., used or draw amount) and limit (i.e., availability or commitment) at time t are denoted by E_t and L_t, respectively. The time t expected exposure at horizon T, conditional upon default occurring before the horizon and upon the vector of covariates $\mathbf{X}_t$, is denoted by $EAD_{\mathbf{X}_t,t,T}$ and satisfies the relations:

$$EAD_{\mathbf{X}_t,t,T} = \mathrm{E}_t\left(L_{\mathbf{X}_T,T} \mid \tau \le T, \mathbf{X}_t\right) = \mathrm{E}_t\left(E_{\mathbf{X}_T,T} \mid \tau \le T, \mathbf{X}_t\right) \tag{2.1}$$

Note that this assumes that lines are fully drawn at default, or if not that limits are reduced by the bank to the outstanding amount, so that observationally the drawn amount coincides with the line's limit:

$$L_{\mathbf{X}_\tau,\tau} = E_{\mathbf{X}_\tau,\tau} \tag{2.2}$$

Traditionally the dollar EAD at default is estimated through a loan equivalency (LEQ) factor (Araten and Jacobs, 2001), denoted $LEQ^f_{\mathbf{X}_t,t,T}$, that is applied to the current unused amount $L_t - E_t$ (or availability, "headroom"):

$$EAD_{\mathbf{X}_t,t,T} = E_t + LEQ^f_{\mathbf{X}_t,t,T} \times \left(L_t - E_t\right) \tag{2.3}$$

The LEQ factor may be thought of as the expected portion of the unused drawn down upon in the event of default conditional upon default occurring within horizon and the vector of explanatory variables:

$$LEQ^f_{\mathbf{X}_t,t,T} = \mathrm{E}_t\left(\frac{E_\tau - E_t}{L_t - E_t} \mid \tau \le T, \mathbf{X}_t\right) \tag{2.4}$$

Note that by the properties of conditional expectation, substituting (3.4) in (3.3) yields (3.1). Depending upon how we model the dependency of the LEQ factor upon the risk drivers, there are various ways in which we can estimate (2.4). A common and rather simplistic practice is to let X index a segment homogenous with respect to EAD risk, within which LEQ is constant, and can be estimated by observations of changes in utilization over unused to default:

$$LEQ_{\mathbf{X}}^{f} = \frac{1}{N_{\mathbf{X}}}\sum_{i=1}^{N_{\mathbf{X}}}\left(\frac{L_{\mathbf{X}_T,T_i^D} - E_{\mathbf{X}_t,t_i}}{L_{\mathbf{X}_t,t_i} - E_{\mathbf{X}_t,t_i}}\right) \tag{2.5}$$

Where i indexes an observation of a credit line at time t_i, defaulting at time T_i^D, conditional on an vector $\mathbf{X}$ that indexes a segment. Alternatively, we may also think of (3.5) as a regression of observed LEQ factors in a reference data-set upon a vector of covariates $\mathbf{X}$. Taken to the opposite extreme of generality, consider solving for LEQ as a general function of the risk drivers, such that an objective is maximized:

$$LEQ_{\mathbf{X}}^{f} = \underset{LEQ(\cdot)}{\arg\min}\left\{\mathrm{E}^{F^*}\left\|\frac{L_{\mathbf{X}_{T_i^D},T_i^D} - E_{\mathbf{X}_{t_i},t_i}}{E_{\mathbf{X}_{t_i},t_i} - E_{\mathbf{X}_{t_i},t_i}} - LEQ\left(\mathbf{X}_{t_i}\right)\right\|\right\} \tag{2.6}$$

Where $\|\ \|$ denotes some distance norm, E^{F^*} is expectation with respect to the empirical distribution and $LEQ(\cdot)$ denotes a function; in the case where the norm is quadratic and the function is linear, we have ordinary linear regression of observed LEQ factors upon risk drivers. An alternative approach estimates an expected credit conversion factor (CCF), denoted $CCF^{f}_{\mathbf{X}_t,t,T}$, that is applied to the current used amount5:

$$EAD_{\mathbf{X}_t,t,T} = CCF_{\mathbf{X}_t,t,T}^{f} \times E_t \tag{2.7}$$

The CCF is simply the expected gross percent change in the utilized amount between the observation and default date:

$$CCF_{\mathbf{X}_t,t,T}^{f} = \mathrm{E}_t\left(\frac{L_\tau}{E_t} \mid \tau \le T, \mathbf{X}_t\right) = \mathrm{E}_t\left(\frac{E_\tau}{E_t} \mid \tau \le T, \mathbf{X}_t\right) \tag{2.8}$$

As in (2.5), assuming CCF to be constant in the appropriate segment, it can be estimated by averaging the observed percent changes in commitment within an EAD segment indexed by $\mathbf{X}$:

$$CCF_{\mathbf{X}}^{f} = \frac{1}{N_{\mathbf{X}}}\sum_{i=1}^{N_{\mathbf{X}}}\frac{E_{\mathbf{X}_{T_i^D},T_i^D}}{E_{\mathbf{X}_{t_i},t_i}} \tag{2.9}$$

As with the LEQ factor, this can be thought of in a regression framework, or even more generally as in the framework of (2.6):

$$CCF_{\mathbf{X}}^{f} = \underset{CCF(\cdot)}{\arg\min}\left\{\mathrm{E}^{F^*}\left\|\frac{E_{\mathbf{X}_{T^D},T^D}}{E_{\mathbf{X}_t,t}} - CCF\left(\mathbf{X}_t\right)\right\|\right\} \tag{2.10}$$

Yet another alternative is the expected exposure-at-default (EAD) factor, denoted by $EAD^{f}_{t,T}$, models expected dollar EAD as the expected availability at default (Jacobs, 2010):

$$EAD_{\mathbf{X}_t,t,T} = \mathrm{E}_t\left(L_{\mathbf{X}_T,T} \mid \tau \le T, \mathbf{X}_t\right) = L_{\mathbf{X}_t,t} \times \mathrm{E}_t\left(\frac{L_{\mathbf{X}_T,T}}{L_{\mathbf{X}_t,t}} \mid \tau \le T, \mathbf{X}_t\right) \tag{2.11}$$

Therefore, the dollar EAD factor may be factored into the product of the current utilization and an EAD factor:

$$EAD_{\mathbf{X}_t,t,T} = L_{\mathbf{X}_t,t} \times EAD_{\mathbf{X}_t,T\text{-}t}^{f} \tag{2.12}$$

The EAD factor is the expected gross percent change in availability:

$$EAD_{\mathbf{X}_t,t,T}^{f} = E_t\left(\frac{L_\tau}{L_t} \mid \tau \le T, \mathbf{X}_t\right) \tag{2.13}$$

As in (2.5) and (2.9), most simply the EAD factor may be estimated as the average product of the changes in the limit from the point of observation to that of default:

Table 1. Average Revolver Utilization Rates and Loan Equivalency Factors*

Debt Rating	Average Revolver Utilization	Loan Equivalency Factors
AAA	0.1%	69.0%
AAA	1.6%	73.4%
AAA	4.6%	72.3%
BBB	20.0%	72.0%
BB	46.8%	74.5%
B	63.7%	81.1%
CCC	75.0%	86.0%

* Reproduced from Asarnow and Marker (1995), based upon 50 defaulted large corporate loans extended by Citibank in the period 1988-1993.

$$EAD_{\mathbf{X}} = \frac{1}{N_{\mathbf{X}}} \sum_{i=1}^{N_{\mathbf{X}}} \frac{L_{\mathbf{X}_{T_i^D}, T_i^D}}{L_{\mathbf{X}_{t_i}, t_i}} \tag{2.14}$$

Furthermore, as with the LEQ and CCF factors, we may look at the most general case of this:

$$EADF_{\mathbf{X}}^{f} = \underset{EADF(\cdot)}{\arg\min} \left\{ \mathrm{E}^{F^*} \left\| \frac{L_{\mathbf{X}_{T^D}, T^D}}{L_{\mathbf{X}_t, t}} - EADF\left(\mathbf{X}_t\right) \right\| \right\} \tag{2.15}$$

Finally, we may consider directly predicting dollar EAD, which subsumes all of the factors considered herein (Moral, 2008):

$$EAD_{\mathbf{X}}^{f} = \underset{EAD(\cdot)}{\arg\min} \left\{ \mathrm{E}^{F^*} \left\| L_{\mathbf{X}_{T^D}, T^D} - EAD\left(\mathbf{X}_t\right) \right\| \right\} \tag{2.16}$$

3. Review of the Literature and Comparison of Empirical Results

In this section we review the existing empirical literature on EAD. While for the most part this has been for the most limited to internal bank and trade journal studies that have focused on the level of partial draw-down, there is a recent and growing literature that has began to transcend this.

The earliest known study is an analysis conducted by Chase Manhattan Bank, with the assistance of Oliver Wyman Mercer (Chase Manhattan Bank, 1994). Drawdowns are studied on 104 revolving credit facilities downgraded in the period 12/91-12/93. The analysis is divided into three parts: 6 month commitments ("short-term"), 1-year commitments ("long-term") and a blend of the two (more specifically, an average for the even years.) LEQs are directly estimated for facilities for which defaults are observed in the sample, for speculative ratings (BB and below.) However, for investment grade commitments, the migration method is used, which extrapolates factors for better ratings from worse with less time to default using estimated cutback and drawdown rates. Estimated LEQ factors were found to increase with worse risk rating and longer tenor.

Attempts to directly estimate partial draw-down has also been undertaken in previous empirical literature. The earliest published set of results on EAD (Asarnow and Marker, 1995) analyzes utilization patterns on a monthly basis for revolving commitments in the period 1/87-12/93, for credit lines issued by Citibank to companies having an S&P rating history. They find a downward sloping pattern of usage level from high rated obligors to low rated obligors (i.e., lower rated firms would have already consumed their credit lines earlier than when it approaches default.) However, results by subgrade are not statistically meaningful due to thin data. Utilization rates by rating are computed for 84 months and averaged by rating category. An unpublished version of this study (Marker , 2000) analyzes empirical LEQs based upon subset of 50 facilities downgraded to BB/B or worse in the period 1991-1993. LEQs are extrapolated to the better risk grades due to lack of investment grade downgrades, and not averaged across facilities, but across quarterly total used and unused for each rating category. Unlike the first Chase study (Araten and Jacobs, 2001), estimated LEQs are found to decrease with increasing credit quality. In Table 1, we reproduce these results, which show both estimated LEQs and utilization rates to increase with decreasing credit quality, contrary to most of the subsequent empirical evidence. But there are several caveats to consider with respect to data and methodology that call these results into question. First, these are not directly estimated averages of LEQ, but rather exposures amounts at default expressed as a percentage of the normally unused commitments. Second, the sample is rather limited, having only 50 facilities which were generally rated BB/B or worse, and furthermore results are extrapolated to better rated facilities.

Table 2. Summary Statistics of LEQ for Revolving Credits

Panel 2.1: Sample Distribution			**Panel 2.2: Distributional Statistics**	
LEQ Group	Count	Percent of Total	Statistic	Value
[0%, 10%)	323	38.8%	Average	43.4%
[10%, 20%)	34	4.1%	Standard Deviation	41.4%
[20%, 30%)	40	4.8%	Median	35.2%
[30%, 40%)	42	5.0%	Percent Truncated from Above	13.8%
[40%, 50%)	44	5.3%	Percent Truncated from Below	27.7%
[50%, 60%)	34	4.1%	Percent non-Truncated	58.5%
[60%, 70%)	31	3.7%	Average of Non-Truncated	50.6%
[70%, 80%)	30	3.5%	Standard Deviation of Non-Truncated	35.1%
[80%, 90%)	40	4.8%	Number of Observations	834
[90%, 100%)	48	6.0%	Number of Obligors	309
100%	168	19.9%	Number of Faciities	317
Total	834	100.0%		

*Reproduced from Araten et al (2001).

Table 3. Average LEQ by Facility Risk Grade and Time-to-Default fdor Revolving Credits*
(number of observations in parentheses)

		Time-to-Default				Total
		1	2	3	4-6	
Facility Risk Grade	1-3 (AAA/BBB)	82.5% (4)	56.7% (14)	48.8% (6)	100% (2)	62.2% (26)
	4 (BBB+/BBB-)	54.8% (18)	52.1% (20)	41.5% (9)	62.5% (5)	52.2% (52)
	5 (BB)	32.0% (81)	44.9% (84)	62.1% (45)	74.6% (21)	46.4% (231)
	6 (BB-/B+)	39.6% (129)	49.8% (100)	74.2% (37)	67.8% (29)	50.1% (295)
	7 (B/B-)	26.5% (86)	39.7% (22)	37.3% (5)	97.8% (2)	30.7% (115)
	8 (CCC)	24.5% (100)	26.7% (14)	9.4% (1)	N/A (0)	24.6% (115)
Total		32.9% (418)	46.6% (254)	62.1% (103)	71.8% (59)	43.4% (834)

*Reproduced from Araten et al (2001).

Another study in this period (Kaplan and Zingales, 1997) find that the un-drawn portion of credit lines decreases when firms are more liquidity constrained. On the other hand, a subsequent study (Gatev and Strahan. 2006) fluctuations, which subsequently inspired Jones and Wu (2009) to use the same for modeling framework for partial draw-downs (See Section 5).

A widely cited bank study previously referred to (Araten and Jacobs, 2001) documents a similar trend of partial draw-downs decreasing as a firm approaches default, which we reproduce in Tables 2 and 3. They study direct estimates of LEQ factors for 834 observations of eventually defaulted CCLs (408 facilities of 399 borrowers) at a quarterly frequency in the period 1Q95-4Q00 for J.P. Morgan Chase borrowers. Given the set of all revolving commitments and advised lines eventually having facility grades rated (accruing) substandard or worse, they track the rating history and usage. The sampling methodology involves stepping back in time from the point of default, calculating the LEQ as change in usage to default relative to unused at a given point in time, at either 1 year intervals or at the time of a grade change. The main result of this analysis is estimated LEQs increasing with time-to-default and with diminished credit risk (i.e., better risk grades.) A pronounced increase in estimated LEQs with tenor was found, with one and five year revolving credits having averages of 32% and 72%, respectively. The decrease in LEQ by grade (worsening credit quality) was found to be milder: 62% for BBB and better, 48% for BBB- to B+, and 27% for B and worse. The overall average is 43.4%, with relatively high 41.4% standard deviation, and a "Barbell" shaped distribution with significant point masses at 0% & 100%. The latter distributional feature is largely an artifact of the truncation of the LEQ estimates, as calculated LEQs greater (less) than 100% (0%) were capped (floored) at 100% (0%). There is a lack of statistical differentiation by other demographics: lending organization, commitment type or size, geography or industry. LEQs are found to decrease with percent usage, but this is highly correlated with risk rating. However, lower LEQs are found for Advised Lines – 17% for one year, but having a similar pattern by grade and time-default. The array of statistical and conceptual issues encountered in this study include outliers, high volatility (on the order of the mean), lack of statistical significance by risk drivers, paucity of data at the investment grade, default definition, sensitivity of estimates to small unused, non-normality, judgmental recoding and data management.

A subsequent study of HELC utilization in the U.S. market (Agarwal and Ambrose, 2006) and document similar patterns to the earlier bank studies just described (Asarnow and Marker, 1995; Araten and Jacobs, 2001), confirming that borrowers with deteriorating credit quality increase their utilization. It is argued in several of aforementioned studies that the level of usage is mainly affected by two distinct forces, that the lender might detect deteriorating credit quality of the borrower and cut back the limits (thereby increasing the utilization ratio), or that the borrower may actually use up the line before the lender realizes deteriorating credit quality. As indicated by in a recent study of credit card usage in the U.S.(Qi, M. 2009), borrowers are more active than lenders in this game of "race to default." This adds another dimension to the problem of EAD estimation, namely that lender behaviour impacts the utilization rates and subsequent EAD. Hence EAD benchmarks which might have been derived from large bank which has suitable early warning signals in place may be very different from ratios actually being realized by a mid-sized bank in the same region. This property of EAD makes using suitable benchmarks almost impossible in practice.

An academic paper that is rather similar to this line of research (Sufi, 2008) empirically examines use of bank lines of credit to corporations, using annual 10K filing data. The author finds that the flexibility afforded firms by use of unfunded commitments creates a moral hazard problem, which is mitigated by banks imposing strict covenants, and lending to borrowers with historically high profitability. Table 4 reproduces coefficient estimates from the study, an empirical specification that examines the ratio of bank liquidity to total liquidity ratio as the dependent variable. We see in column (1) the effect of firm profitability on the bank liquidity to total liquidity ratio is positive and statistically significant at the 1 percent level, showing that more profitable firms have a higher proportion of their total liquidity in the form of bank lines of credit. This supports the supports the claim that higher firm profitability increases the willingness of banks to supply lines of credit.

Another recent manuscript circulating in the regulatory channels (Moral, 2006), reviews different methodologies and proposes an optimal framework (from the regulatory viewpoint) for estimating EAD factors. He shows that it is possible to directly impose constraints in the EAD estimation that reflect the preferences of supervisors for not underestimating regulatory capital. It is further shown that a special case of this general problem reduces to a quantile regression of dollar EAD on covariates, which is tractable has several desirable properties.

Table 4. Bank Liquidity to Total Liquidity Ratio and Borrower Characteristics

	Sample:	(1) Full	(1) High D/A	(1) Low D/A	(1) High PD	(1) Low PD
Earnings Measures	Average of 3-year lagged EBITDA/assets	0.208	0.783	0.120	0.255	0.030
		(0.076)	(0.119)	(0.090)	(0.085)	(0.229)
	EBITDA variance	-0.233	0.207	-0.216	-0.11	-0.560
		(0.123)	(0.301)	(0.118)	(0.115\)	(0.243)
Measures of Informational Asymetry	Equity traded over the counter	0.050	-0.059	0.084	0.024	0.126
		(0.044)	(0.065)	(0.052)	(0.048)	(0.088)
	NOT in S&P 500, mid 400, or small 600	0.067	-0.016	0.114	0.031	0.097
		(0.018)	(0.037)	(0.052)	(0.055)	(0.049)
	Ln[1+Firm age (years since	-0.026	-0.026	-0.002	-0.018	-0.061
		(0.017)	(0.019)	(0.025)	(0.025)	(0.0274)
Other Firm Characteristics	Market to book ratio (lagged)	-0.021	-0.025	-0.013	-0.011	-0.034
		(0.006)	(0.013)	(0.005)	(0.005)	(0.011)
	Tangible assets to total assets ratio (lagged)	0.239	0.011	0.250	0.145	0.475
		(0.079)	(0.082)	(0.131)	(0.096)	(0.132)
	Ln[Total sales ($M)] (lagged)	0.069	0.047	0.052	0.057	0.074
		(0.009)	(0.010)	(0.015)	(0.011)	(0.015)
Number of Observations		1916	958	958	930	931
R-squared		0.37	0.32	0.30	0.40	0.32

* Reproduced from Sufi (2005). A random sample of 300 firms from 1996 through 2003, for a total unbalanced panel of 1,916 firm-year observations. Data on lines of credit and the portfolio of debt come from direct examination of annual 10-K SEC filings. This table presents coefficient estimates for pooled regressions relating the unused line of credit to unused line of credit plus balance sheet cash ratio (the bank liquidity to total liquidity ratio) to characteristics of the firm. Column (1) reports coefficient estimates from a regression using the entire sample; columns (2) and (3) split the sample based on the debt to total assets ratio (D/A). Columns (4) and (5) split the sample based on Altman's Z-Score (1968) as a measure of the probability of default. All regressions include year and industry indicator variables. Standard errors are heteroskedasticity-robust, clustered at the firm.

Another recent study documents (Jiminez, Lopez and Saurina. 2008) credit line usage, LEQ factors for revolving commitments, of different firms granted by banks in Spain between 1984 and 2005, using the Spanish Credit Register , providing a census of all corporate lending within the country over the last twenty years, which is unique in that is includes both defaulting and non-defaulted firms. The length and breadth of this dataset allows the authors to provide one of the most comprehensive overviews of corporate credit line use and EAD to date. The final data-set consists of 696,445 credit lines granted to 334,442 firms by 404 banks and their final estimation results are presented in Table 5. They report a variety of factors such as commitment size, collateralization and maturity of CCLs that affect the usage level. They also report a statistically significant higher usage rate for firms that eventually default at least 3 years prior to default and that the usage monotonically increases as these firms approach default. It is important to point out that Jiménez et. al. (2008) use utilization rates as their response variable, as opposed to an LEQ factor. One distinction in their modeling technique is that the authors use both defaulted and non-defaulted firms in their analysis and include an indicator variable to account for the differences.

We conclude our review of empirical EAD studies with one of the most extensive examinations of the U.S. large corporate market to date (Jacobs, Jr., 2010). The author builds an EAD database from probably the most extensive loss severity database of defaults (bankruptcies and out-of-court settlements) and

Table 5. Regression Models for Utilization Rates*

Model:	Censored OLS	Tobit with Random Effects	Groups (Fixed Effects)
Intercept	57.909***	57.077***	50.615***
Default Indicator	18.061***	23.693***	---------
Years from Default	6.705***	10.365***	6.166***
Years from Default^2	0.661***	1.071***	0.288**
Age of Loan	-4.631***	-6.052***	-4.678***
Age of Loan*Default Indicator	0.961*	3.425*	---------
Long Term Indicator	2.585***	2.906***	---------
Collateral Indicator	-0.529***	-0.243**	---------
Log (Commitment Size)	-0.041**	0.369***	1.238***
Firm Risk Indicator	-3.708***	-4.599***	-5.551***
Log(1+Years with Lender)	-3.691***	-3.974***	-1.125***
Log(# of Bank Relationships)	3.089***	2.355***	-0.694***
Main Bank Indicator	3.363***	2.522***	2.063***
Bank Market Share	-3.945***	-4.233***	-0.654***
Bank NPL Ratio	0.018	0.027	0.025
Savings Bank Indicator	-4.047***	-4.450***	---------
Credit Cooperative Indicator	-1.824***	-1.751***	---------
Spanish GDP Growth	-0.937***	-0.992***	-0.569***
Credit Line/Firm/Bank Fixed	No	No	No
F-Statistic	0.00	0.01	0.01

* Reproduced from Jiménez et al (2008). Note that for the within-groups (fixed effects) model, time invariant explanatory variables are not included in the model. ***, ** and * indicate statistical significance at the 1%, 5% and 10% levels, respectively.

recoveries, Moody's Ultimate Recovery Database™ (February 2008 release; "MURD"). Most of the issuers in MURD have rated instruments (S&P or Moody's) at some point prior to default, and traded equity, largely representative of the U.S. large corporate loss experience. The author merges MURD with various public sources of information (www.bankruptcydata.com, Edgar SEC filings, LEXIS/NEXIS, Bloomberg, Compustat and CRSP). The basis of the research database contains data on 3,886 defaulted instruments from 1985-2007 for 683 borrowers, for which there is information on all classes of debt in the capital structure at the time of default. All instruments are detailed by facility type, seniority ranking, collateral type, position in the capital structure, original and defaulted amount, resolution outcome, instrument price or value of securities at the resolution of default (emergence from bankruptcy, Chapter 7 liquidation, acquisition or out-of-court settlement). The latter includes either the prices of pre-petition instruments at the time of emergence from bankruptcy or new instruments received in settlement of bankruptcy or other distressed restructuring. In a sub-set of observations, we can obtain the price of traded debt, the equity prices or financial statement data at around the time of default. A smaller sub-set of observations considered in this study consist of revolving loans, for which we can trace the outstanding amounts, limits and ratings in SEC filings (10K and 10Q reports). This subset of MURD includes 496 obligors, 504 defaults and 544 facilities.

First, we discuss the various exposure and EAD risk measures, shown in Table 6 as reproduced from Table 1 of the study[5]. The raw LEQ, having a mean of 63.7%, exhibits extreme variation, ranging in (-21,000%, 106,250%), having a standard deviation (2,759.7%), on the order of100 times the mean. The Winsorized version

Table 6 - Summary Statistics on EAD Risk Measures (S&P and Moodys Rated Defaulted Borrowers Revolving Lines of Credits 1985-2007)*

	Cnt	Average	Standard Deviation	Minimum	5th Prcntl	25th Prcntl	Median	75th Prcntl	95th Prcntl	Maximum	Skew	Kurtosis	Corr LEQ_{COLL}	Corr CCF_{WIND}	Corr $EADF_{WIND}$
Exposure at Default (EAD)[0]	530	133,140	295,035	158	1,656	20,725	50,000	116,234	508,232	4,250,000	7.5099	82.1857	N/A	N/A	N/A
Dollar Change in Drawn to EAD (DCDE)[1]	2118	48,972	279,972	(3,177,300)	(3,177,300)	(2,056)	7,514	36,617	275,400	4,250,000	6.8444	116.0538	30.07%	35.02%	24.38%
LEQ (Raw)[2]	1582	63.72%	2759.66%	-21000.00%	-21000.00%	-12.75%	33.28%	87.64%	231.76%	106250.00%	35.7617	1391.0651	9.47%	15.34%	11.49%
LEQ (Collared)[3]	1582	42.21%	40.92%	0.00%	0.00%	0.00%	33.28%	87.64%	100.00%	100.00%	0.3054	-1.5700	100.00%	55.06%	77.09%
LEQ (Winsorized)[4]	1582	16.80%	210.38%	-1165.74%	-1165.74%	-12.75%	33.28%	87.64%	231.76%	804.43%	-1.9084	13.5038	58.59%	35.70%	53.93%
CCF[5]	1330	1061.8%	20032.7%	0.47%	0.47%	85.30%	111.11%	198.86%	860.29%	704054.38%	32.9416	1145.3158	1.89%	15.37%	-2.10%
CCF (Winsorized)	1330	190.4%	203.4%	26.29%	26.29%	85.30%	111.11%	198.86%	855.66%	860.29%	2.27	4.45	55.06%	100.00%	38.44%
EAD Factor[6]	1587	143.40%	2666.07%	0.37%	0.37%	42.46%	70.67%	95.96%	152.86%	106250.00%	39.80	1584.89	4.95%	41.74%	7.48%
EAD Factor (Winsorized)	1587	70.76%	36.94%	11.24%	11.24%	42.46%	70.67%	95.96%	152.39%	152.86%	0.29	-0.39	77.09%	38.44%	100.00%
Utilization[7]	1621	45.85%	32.85%	0.00%	0.00%	14.00%	48.04%	74.27%	95.00%	100.00%	-0.06	-1.35	-33.50%	-61.58%	1.03%
Commitment[8]	1621	184,027	383,442	217	217	40,000	80,000	176,400	570,000	4,250,000	6.24	48.28	2.51%	-4.41%	-6.88%
Drawndown Rate[9]	879	0.39%	7.00%	-0.10%	-0.10%	-0.02%	0.01%	0.05%	0.41%	181.97%	23.17	561.82	-4.38%	-2.80%	-2.76%
Cutback Rate[10]	1126	88.50%	2791.11%	-96.07%	-96.07%	0.00%	0.00%	0.00%	66.67%	93650.00%	33.54	1125.34	4.51%	1.52%	3.60%
Drawn[11]	1621	71,576	163,029	0	0	5,557	26,463	76,900	260,000	3,090,000	8.41	107.87	-14.69%	-18.58%	-5.85%
Undrawn[12]	773	112,450	329,695	0	0	13,082	34,099	82,300	396,500	4,250,000	7.79	73.49	9.54%	12.53%	-5.08%

* Reproduced from Table 1 of Jacobs (2010). 496 (504) defaulted borrowers (instances of default), having 544 revolving credit exposures and sampled prior to default at one year anniversarries, changes in risk rating prior or other signifcant events prior to default

0 - Dollar EAD (or outstanding amount at default)

1 - Change from outstanding amount at an observation date to dollar EAD

2 - Empirically measured Loan Equivalent Exposure where $LEQ_{t,T}$ = ($Drawn_T$ - $Drawn_t$)/Undrawnt, T (t) = default (observation) date

3 - LEQ floored (capped) at 0% (100%) = max(min(LEQ,1),0)

4 - LEQ floored (capped) at the 1st (99th) percentiles = max(min(LEQ,516.95%),-874.57%)

5 - Credit Conversion Factor: $CCF_{t,T}$ = $Drawn_T$/$Drawn_t$, T (t) = default (observation) date.

6 - Exposure at Default Factor: $EAD_{t,T}$ = $Exposure_T$/$Exposure_t$, T (t) = default (observation) date.

7- $Utilization_t$ = $Drawn_t$/$Commitment_t$ where Commitmentt = Drawnt + Undrawnt

8 - Commitmentt = Total legal commitment or limit on credit line at observation date t ($000s)

9 - Percent change in drawn from date prior until observation date

10 - Percent change in commitment from date prior until observation date

11 - Drawnt = Total amount outstanding on line at time t

12 - Undrawnt = Total undrawn commitment (legal commitment minus drawn) on line at time t

of the LEQ, which floors (caps) variable at the 5th (95th) quantile of its ordered value (a standard technique for making statistical inference in the presence of extreme outliers and possibly contaminated data) is little better in terms of stability: a mean of 16.8%, but still varying in a huge domain of -1,165.7% to 804.4%, and standard deviation of 210.4% (about 10 times the mean.) The collared LEQ, restricted to the unit interval, averages 42.2%, with standard deviation on the order of the mean of 41.0%. Interestingly, all 3 measures share a median of 33.3%, suggesting that LEQ may be a candidate for the application of robust statistics (e.g., MAD regression). All versions of the LEQ are highly correlated with CCF and EADF, the highest being collared LEQ, with respective rank order correlations of 55.1% and 77.1%. There is an outlier problem with the CCF as well – it averages 1,061.8% and has a maximum of 704,054.4% (note that there is a "natural flooring" at 0 in the case of the CCF factor, as well as with the EADF). The Winsorized version is more reasonable, an average of 190.4%, showing that by this measure that on average drawn amounts are 90% higher at default as compared to earlier points in time. The average EADF of 143.4% suggests that typically line limits *increase* about 43% leading to default; however, average Winsorized EADF of 70.8% suggests that typically lines are reduced by about 30% leading to default. Note that there are slightly different counts for these (1582, 1330 and 1587 for LEQ, CCF and EADF, respectively), as there are different data requirements for each (i.e., non-zero unused and used for LEQ and CCF, respectively.) The severity of the outlier problem associated with the LEQ factor can be more easily seen through visual inspection of the raw, Winsorized and collared distributions in Figures 1.1-1.3. The distributions of CCF and EADF are shown in Figures 2.1-2.2 and Figure 3.1-3.2, respectively.

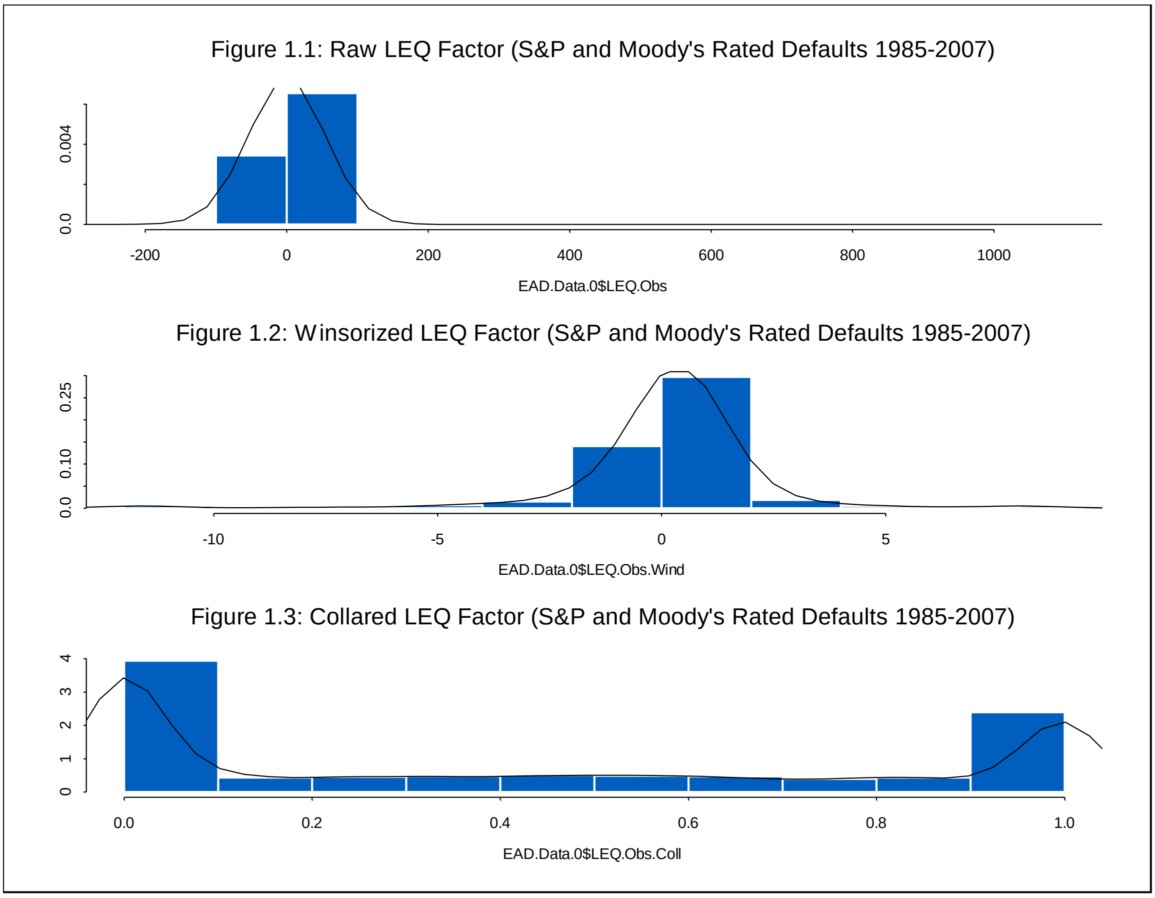


Figure 1.1: Raw LEQ Factor (S&P and Moody's Rated Defaults 1985-2007)
0.0
0.004
-200
0
200
400
600
800
1000
EAD.Data.0$LEQ.Obs
Figure 1.2: Winsorized LEQ Factor (S&P and Moody's Rated Defaults 1985-2007)
0.0
0.10
0.25
-10
-5
0
5
EAD.Data.0$LEQ.Obs.Wind
Figure 1.3: Collared LEQ Factor (S&P and Moody's Rated Defaults 1985-2007)
0
1
2
3
4
0.0
0.2
0.4
0.6
0.8
1.0
EAD.Data.0$LEQ.Obs.Coll

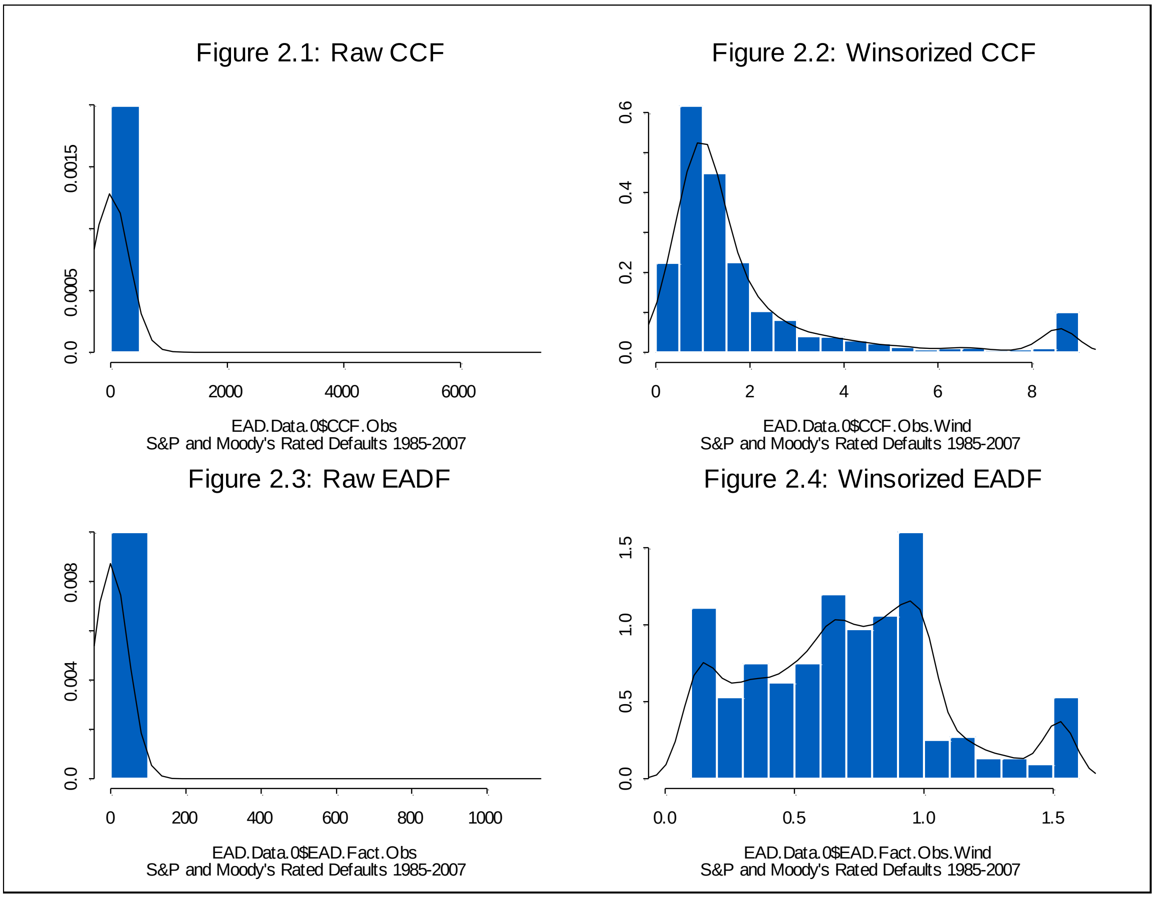


Figure 2.1: Raw CCF
0.0
0.0005
0.0015
0
2000
4000
6000
EAD.Data.0$CCF.Obs
S&P and Moody's Rated Defaults 1985-2007
Figure 2.2: Winsorized CCF
0.0
0.2
0.4
0.6
0
2
4
6
8
EAD.Data.0$CCF.Obs.Wind
S&P and Moody's Rated Defaults 1985-2007
Figure 2.3: Raw EADF
0.0
0.004
0.008
0
200
400
600
800
1000
EAD.Data.0$EAD.Fact.Obs
S&P and Moody's Rated Defaults 1985-2007
Figure 2.4: Winsorized EADF
0.0
0.5
1.0
1.5
0.0
0.5
1.0
1.5
EAD.Data.0$EAD.Fact.Obs.Wind
S&P and Moody's Rated Defaults 1985-2007

Both the raw and Winsorized LEQ distributions are extremely heavy tailed and skewed, more like something more out of the Stable family (possibly with undefined 1st and 2nd moments) than a normal or even spherical distribution. On the other hand, the collaring of LEQ in Figure 1.3 yields a bimodal distribution, having point masses at 0 and 1, and approximately uniform between these boundaries. This appears to be potentially well approximated by a beta distribution, and resembles empirical distributions of ultimate Loss-Given-Default (LGD) observed in the literature (Jacobs, Jr. and Karagozoglu, 2010; Araten, Jacobs, Jr., and Varshney. 2004).

We first turn our attention to features of the data that characterize the pattern of usages and limits. Average percent utilization ("UTIL") in the data-set is 45.9%, with a median of 48.0%, and a standard deviation of 32.9%. UTIL is inversely correlated with two of the EAD risk measures, Spearman correlation coefficients of -33.5% and -61.6% for LEQ and CCF, respectively; however, it has negligible correlation with EADF of 1.0%. The average undrawn amount ("UNDRN") is $112.5 Million, and has positive Spearman rank correlations the same two EAD risk measures as well, coefficients of 9.5% and 12.5% for LEQ and CCF, respectively; however, it has negative correlation with EADF of -5.1%. The drawn amount ("DRAWN"), averaging $71.6 Million, is most strongly inversely related to LEQ and CCF, respective correlation of -14.7% and -18.6%, and a little less so for EADF (-5.9%.) The average cutback rate ("CR"), or percentage change in commitment across subsequent observation dates (a measure of the speed with which banks clamp down on commitments as borrowers approach default), exhibits massive upward skew with an average of 88.5%, median of 0.0% and maximum of 93,650.0%. The CR exhibits a mild positive relationship with the LEQ and EADF, respective correlations of 4.5% and 3.6% with those variables, and less so but still positive with respect to CCF, a coefficient of 1.5%. On the other hand, the drawdown rate ("DR"), a measure of the aggressiveness with which obligors tap lines on their way to default, exhibits a more stable looking yet positively skewed distribution, averaging 0.4% and ranging in -0.1% to 182.0%. Perhaps counter-intuitively, correlations with all EAD risk parameters are negative but of mild magnitude (-4.4%, -2.8% and -2.8% with LEQ, CCF and EADF, respectively.) `

Table 7, reproduced from Table 8 of the study (Jacobs, Jr., 2010), presents the estimation results for the beta link generalized linear model (BLGLM.) Various sets of independent variables were analyzed in a series of univariate and multivariate regressions. The final set chosen for each EAD risk measure was determined based upon a partially quantitative, and partially judgmental, process that weighed the following (sometimes competing) considerations. First, measures of in-sample model performance, predictive accuracy (or goodness-of-fit) such as McFadden Pseudo-Rsquared (MPR2) and log-likelihood (LL), versus a Spearman rank correlation (SRC) measure of rank ordering (or discriminatory) accuracy. Second, the author considers the signs and significance levels of independent variables. Finally, there is an attempt to find a parsimonious representation that has a large number of variables in common across models.

The strongest results to emerges, generally ranking highest in statistical significance (in terms of p-values, or PVs) as well as partial effect (PEs) magnitudes, is the inverse relationship between UTIL (percent utilization) and two of the EAD risk measures: PEs (PVs) of -0.35 and -0.39 (2.5E-06 and 6.5E-6) for LEQ and CCF, respectively. However, UTIL does not enter the EADF model; rather the CR (cutback rate) enters this model and not the others, having a marginal PE of -0.02 and PV of 0.07. While all parameters are directly related with the UNDRN, the partial effects are all rather small, and significance is only marginal in the case of EADF: PEs (PVs) of 3.3E-5, 2.2E-5 and 7.5E-5 (7.4E-3, 2.8E-6 and 0.04) for LEQ, CCF and EADF, respectively. On the other hand, DRWN only enters the model for CCF, and while highly significant (PV of 9.17E-7), the economic significance is questionable (PE of only -0.02); this too is in line with the negative univariate Spearman correlation. However, LEQ had a reasonably sized negative correlation with DRAWN in that analysis, but this did not enter the regression model; while the correlation with EADF was the opposite sign, it was rather small.

Let us know consider the key variables of the bank study (Araten, Jacobs, Jr., 2001), the TTD (time-to-default measured in years) and ORR (the obligor risk rating measured as dummy variables.) We see consistently across models that TTD is directly associated with EAD risk, but that the relationship is stronger for LEQ and CCF, having much higher PEs (0.05 and 0.35, respectively) and much lower PVs (1.7E-5 and 1.6E-6, respectively), as opposed to EADF, having a PE (PV) of only 0.02 (0.10) and in fact being nearly insignificant. ORR has negative coefficient estimates across all EAD risk measures. However, in some instances they are only marginally or not statistically significant, such as rating CCC-CC in the CCF model (PV = 0.23), or ratings AAA-BBB and BB in the EADF model (PVs = 0.13 and 0.36, respectively.) Generally, the pattern in the magnitudes of the partial effects is decreasing as ratings worsen, albeit non-monotonically.

Table 7 - Beta Link Generalized Linear Model Multiple Regression Models for EAD Risk Measures (S&P and Moodys Rated Defaulted Borrowers Revolving Lines of Credits 1985-2007)*

	LEQ[2]		CCF[3]		EADF[4]	
	Partial Effect	P-Value	Partial Effect	P-Value	Partial Effect	P-Value
Utilization[5]	-0.3508	2.53E-06	-0.3881	6.52E-06		
Commitment[6]	3.64E-05	0.0723				
Cutback Rate[7]					-1.74E-03	0.0658
Drawn[8]			-0.0191	5.53E-07		
Undrawn[9]	3.27E-05	7.42E-03	2.20E-05	2.81E-06	7.45E-05	0.0441
Time-to-Default[10]	0.0516	1.72E-05	0.3462	1.58E-06	0.0225	2.08E-03
Rating 1[11]	-0.1442	0.0426	-0.2440	0.1015	-0.0503	0.1267
Rating 2[11]	-0.0681	6.20E-03	-0.1511	0.0835	-0.0093	0.3581
Rating 3[11]	-0.0735	1.03E-05	-0.1895	3.70E-03	-0.0079	0.0634
Rating 4[11]	-0.0502	2.08E-04	-0.1591	0.0977	-0.0135	0.0910
Rating 5[11]	-0.0110	0.1003	-0.0277	0.2278	-0.0068	0.1195
Leverage 1[12]	-0.0515	0.0714	-0.1332	0.0276		
Leverage 2[13]					-0.0922	0.0065
Size[14]	0.1154	2.63E-03	0.1855	0.0655	0.0463	0.1081
Intangibility[15]	0.0600	0.0214			0.0483	0.0878
Liquidity[16]	-0.0366	0.0251	-0.1110	0.0845	-0.0264	0.0960
Profitabilty[17]	-6.59E-04	0.0230	-5.79E-04	0.0265	-7.46E-05	0.0996
Colllateral Rank[18]	0.0306	3.07E-03	0.0816	0.0277	0.0111	0.1027
Debt Cushion[19]	-0.2801	5.18E-06	-0.5193	0.0122	-0.3073	7.34E-06
Speculative Default Rate[20]	-0.9336	0.0635	-0.0928	0.0960	-0.1766	5.03E-04
Percent Bank Debt[21]	0.2854	5.61E-06	0.3859	0.0928	0.3868	8.09E-03
Percent Secured Debt[22]	0.1115	2.65E-03			0.1830	2.71E-03
Degrees of Freedom	455		457		456	
Likelihood Ratio P-Value	7.48E-12		1.66E-19		7.62E-09	
Pseudo R-Squared	0.2040		0.2336		0.1611	
Spearman Rank Correlation	0.4670		0.5618		0.4115	
MSE of Forecasted EAD	2.74E+15		7.53E+15		2.23E+17	

*Reproduced from Table 10 of Jacobs (2010)

1 - Defaulted borrowers, having a revolving credit exposure prior to default, sampled at one year anniversaries or changes in risk rating prior to default. Reproduced from Table 8 of Jacobs (2010).

2 - Empirically measured Loan Equivalent Exposure where LEQt,T = (DrawnT - Drawnt)/Undrawnt, T (t) = default (observation) date floored (capped) at 0% (100%) = max(min(LEQ,1),0)

3 - Credit Conversion Factor: CCFt,T = DrawnT/Drawnt, T (t) = default (observation) date floored (capped) at the 1st (99th) percentiles = max(min(LEQ,516.95%),-874.57%)

4 -Exposure at Default Factor: EADt,T = ExposureT/Exposuret, T (t) = default (observation) date

5 - Utilization = Drawnt/Commitmentt where Commitmentt = Drawnt + Undrawnt

6 - Commitmentt = Total legal commitment or limit on credit line at observation date t ($000s)

7 - Percent change in drawn rom date prior until observation date

8 - Drawnt = Total amount outstanding on line at time t

9 - Undrawnt = Total undrawn commitment (legal commitment minus drawn) on line at time t

10 - Time from observation date to default of revolving credit (years)

11 - Numeric codes for major S&P rating: 1 = AAA-BBB, 2 = BB, 3 = B, 4 = CCC-CC, 5 = C

12 - Leverage measured by the ratio of long term debt to the market value of equity observed at 1 and 2 years prior to default

13 - Leverage measured by the ratio of total debt to the book value of assets observed at 1 and 2 years prior to default
14 - Company size measured by the logarithm of book value observed at 1 and 2 years prior to default
15 - Intangibility of assets measured by the ratio of intangible to total assets observed at 1 and 2 years prior to default
16 - Liquidity measured by the current ratio (current assets to current liabilities) observed at 1 and 2 years prior to default
17 - Profitability measured by the profit margin (ratio of net income to net sales) observed at 1 and 2 years prior to default
18 - Ranking of collateral quality
19 - Proportion of debt in the capital structure subordinated to the instrument.
20- Moody's trailing 1-year default rate calculated for quarterly cohorts of speculative grade rated issuers
21 -Proportion of bank debt in the capital structure
22 -Proportion of secured debt in the capital structure

Now considering the financial ratio variables, we see that five of six dimensions of the ratios from the univariate analysis survive in the multivariate regressions (measures of leverage, size, liquidity, intangibility and profitability), and have signs consistent with such across all 3 models; whereas a measure of cash flow does not enter any of the models. LTD/MVE, or leverage as measured by the ratio of long-term debt to the market value of equity, is negatively related to EAD risk and at least marginally significant in the LEQ and CCF models: PEs of -0.05 and -0.13 (PVs of 0.07 and 0.03), respectively. However, in the EADF model LTD/BVE, the accounting measure of leverage (long-term debt ratio to book value of total assets) enters, having a PE of -0.09 and a PV of 0.01. This is in line with the univariate results, and consistent with our hypothesis that more highly levered firms may be under closer scrutiny, and hence less able to draw down on unused lines.

The BVTA measure of company size (the logarithm of the book value of assets) has positive coefficients across all models (PEs of 0.12, 0.19 and 0.05 for LEQ, CCF and EADF, respectively); however, it is only highly statistically in one of the models (PE = 2.6E+4 for LEQ), marginally significant in another model (PE = 0.07 for CCF) and just short of significant in the third model (PE = 0.11 for EADF.) This result is what we saw in the correlation analysis, and may be explained by a tendency of banks to monitor larger companies less intensively, as they may be perceived as less likely to require use of their lines.

The CR liquidity measure, as in the univariate analysis, is consistently negative across models (PEs of -0.04, -0.11 and -0.03 for LEQ, CCF and EADF, respectively), in line with the univariate correlation analysis; but it is marginally significant in 2 of the models (PVs of 0.085 and 0.096 in the CCF and EADF models), and only significant at the 5% level in the LEQ model (PV = 0.03.) As alluded to before, this may be considered a reasonable result, as less liquidity constrained firms may draw less aggressively on their lines as they approach distress. Similarly, the PM (profit margin) profitability measure is consistently negative across models (albeit with small PEs of -6.6E-4, -5.8E-4 and -7.3E-5 for LEQ, CCF and EADF, respectively), in line with the univariate correlation analysis, and the expectation that less unprofitable firms on their way to default may pose lower EAD risk; but it is significant at the 5% level in only 2 of the models (PVs of 0.02 and 0.03 in the LEQ and CCF models), being just marginally significant in the EADF model (PV = 0.10.) Finally for the financials, the INTA measure of intangibility (ratio of intangible to total assets) enters only the LEQ and EADF models, having positive PEs (0.06 and 0.05, respectively), as well as moderate significance levels (PVs of 0.02 and 0.05, respectively.)

Now let us discuss results regarding measures of instrument-level characteristics. The COLL measure of collateral quality is present in all, and the CRED measure of seniority in none, of the regressions, consistent with the larger univariate correlations observed in the former as opposed to the latter. While the signs of the coefficient estimates for COLL are positive across models (PEs of 0.03, 0.08 and 0.01 for LEQ, CCF and EADF, respectively), only for LEQ do we observe high significance (PV = 3.1E+03), while for CCF significance is just at the 5% level (PV = 0.03), and for EADF we are just shy of significance at the 10% level (PV = 0.103.) Second, the CUSH measure of tranche safety attributable to the revolving credit is inversely related to EAD risk across regression models, as in the univariate analysis. In this case, PEs are relatively strong, as compared to some other variables: -0.28, -0.52 and -0.31 for LEQ, CCF and EADF, respectively. In this case, significance levels are also notably high, at much better than the 1% level in 2 cases (PVs of 5.2E-6 and 7.3E-6 for LEQ and EADF, respectively), and at the 5% level in another (PV = 0.03 for CCF). These results suggest that while superior collateral does seem to mitigate EAD risk, above and beyond this there is a beneficial effect to be had from greater debt cushion.

Among capital structure variables, only the PERCBNK (percent bank debt) and the PERCSEC (percent secured debt) enters the leading regression models. In the case of PERCBNK, which enters all models, coefficient estimates are economically significant and of positive sign (PEs of 0.29, 0.39 and 0.39 for LEQ, CCF and EADF, respectively), in line with the observed correlations. However, while highly statistically significant for LEQ and EADF (with PVs of 5.6E-6 and 8.1E-3, respectively), this variable barely achieves such status in the CCF model (PV = 0.09.) Nonetheless, this result has a rationale in a story that when more banks are present in the creditor group, there may be coordination problems (e.g., this may be associated with a larger syndicate.) Alternatively and has been argued in prior literature (Carey and Gordy, 2007), through the economic incentives of banks at the top of the capital structure, the optimal foreclosure boundary may be set higher than otherwise, and to the extent that lower LGD rates may be associated with this, an inverse correlation with EAD (if we believe that a tradeoff exists) may be consistent with the empirical result that we are finding. On the other hand, PERCSEC appears in only two models, LEQ and EADF, significantly (PVs of 0.03 in both) and having positive signs (PEs of 0.11 and 0.18, respectively.)

The final variable that we consider is a measure of the economic cycle that made it into the final regression models, the MSG12MTDR (speculative default rate.) This is expected, as the univariate Spearman correlations for MSG12MTDR were generally higher than for MAC12MTDR (All-corporate default rate.) and SPR (S&P 500 index return) across all EAD risk measures. We have evidence of counter-cyclicality, as all partial effects are negative (-0.93, -0.09 and -0.18 for LEQ, CCF and EADF, respectively.) However, in only the EADF model do we have a high degree of statistical significance (PV = 50E-4); whereas in the LEQ and CCF models, significance is marginal (PVs of 0.064 and 0.96. respectively.) Therefore, we can regard this as only limited evidence against "downturn EAD" or for a counter-cyclicality in EAD risk. If we are willing to put some stock in these results, what economic rationale could be put forward? We could ascribe this to an "LGD-EAD" tradeoff: as the cycle turns downward and banks anticipate both higher default and higher recovery risk, they clamp down on revolving credit exposures, thereby reducing EAD even as loss severities may be rising.

Finally, we discuss in-sample measures of model quality in Table 2, measures of predictive accuracy (Log-Likelihood Ratio – LLR and McFadden Pseudo R-Squared – MPR2), discriminatory power (Spearman Rank correlation between actual and predicted values – SRC) and in-sample forecasting ability for dollar EAD (Mean Squared Error of forecasted dollar exposure at default, or MSE-EAD.) The LLR and MPR2 are standard diagnostics assessing in-sample fit in non-linear models, having potentially non-normal errors. The SRC here is meant to mimic the Area Under the Receiving Operating Curve (AUROC) or Accuracy Ratio (AR) statistics calculated in binary dependent models, such of probability of default (PD) prediction, and is in fact a generalization of the concept. The MSE-EAD measure is a bit non-standard, in that instead of focusing on how the models can predict or rank order the EAD risk measures, we focus on how the predicted parameters can forecast dollar EAD. For example, estimated EAD prior to default would equal the outstanding at some horizon to default (i.e., a year), plus estimated LEQ (based upon the known values of explanatory variables at that time and the regression relationship) times the unused amount, and similarly for CCF and EADF.

Indeed, in the final specifications we observe that most variables are statistically significant, although it is about evenly split between very high levels of significance, and in some cases only marginally significance. However, there is much variation amongst in-sample performance measures, and we see in Table 7 that by these the CCF model performs best, and the EADF model ranks worst: the former model has the highest MPR2 of 0.30 (vs. 0.20 and 0.16 for LEQ and EADF, respectively) and highest SRC of 0.56 (vs. 0.47 and 0.41 for LEQ and EADF, respectively), as well as the smallest p-value on the likelihood ratio test of 1.7E-19 (vs. 7.5E-12 and 7.6E-9 for LEQ and EADF, respectively.) However, in the exercise of forecasting the dollar EAD based upon these measures, in terms of MSE-EAD the LEQ measure performs best (2.7E+15), followed closely by CCF (7.5E+15), while EADF performs far worse than the other two (2.2E+17), the latter underperforming by about a factor of 100.

4. Theoretical Models of EAD and an Actuarial Framework

There have also been some notable academic studies with a greater focused on pricing of CCLs. One early study employs an option-theoretic approach to pricing contingent loan commitments (Thakor and Udell, 1987). In this setting, the bank buys a put option from the obligor and the obligor is sells its debt to the bank through offering credit lines on pre-specified terms and conditions. The authors proceed to measure the sensitivity of the loan values of such to changes in interest rates and partial draw-downs on a CCL are explained through interest elasticity of demand for borrowed funds and bank customer relationship dynamics. If a firm has infinite opportunities for investment and no restriction on capital structure or leverage, then the interest elasticity will be perfectly elastic, and vice-versa if there are no such opportunities, in which case it will be perfectly inelastic.

Alternatively, if looking into the bank customer relationship framework, the later will try to minimize its expected cost of renewal of the line in next year and opportunity for loss by not utilizing the full facility this year when availing the line. A recent related study in the option pricing literature (Loukoianova , Neftci and Sharma,. 2007) develops a theoretical pricing model for CCLs. These are widely used in bank lending and instrumental in the functioning of short-term capital markets, which are closely related to the instruments considered herein. The authors use a financial engineering approach to analyzing the structure of simple CCLs, applying various derivative pricing methods, and discuss issues in the hedging of CCL portfolios. At this point, we should highlight a few key differences between traded options and CCL, as the latter may not be necessarily used in full (i.e., partial drawdowns on CCL). There is also the material adverse change clause (MAC) in place, such that the CCL can be denied at any time, whereas an option will remain valid till its expiry. Finally, in general CCL will have different pricing (fee) structure as compared the one upfront payment of premiums in the case of options.

In another study (Jones and Wu, 2009), motivated by empirical studies indicate presence of a pricing incentive for partial draw-downs induced by interest rate fluctuations (Kaplan and Zingales, 1997; Gatev and Strahan. 2006), the authors build a model with credit quality as a jump-diffusion process, giving rise to partial draw-downs and CCL pricing as a function of the dynamic credit state. The proportion of the credit line drawn is modelled as function of the difference between an alternative opportunity rate and the marginal cost of line borrowings. This opportunity rate is defined as the rate of interest charged to the borrower outside the purview of the defined credit line. In order to incorporate linking of the loan spread to the credit default swap spread of the borrower, the marginal cost of borrowing is defined as function of the reference rate, contractual spread over the reference rate and proportion of the excess that is added to current period loan. Apart from the interest rate differential, the sensitivity of drawdown to the interest rate differential is included to model the amount of partial draw-down, a sensitivity is similar to the interest elasticity proposed previously (Thakor and Udell, 1987).

Both of these approaches (Thakor and Udell, 1987; Jones and Wu, 2009) appear intuitive and convincing, but implementing such in banks where most CCLs are extended to unrated obligors whose market spread may not be easily available might pose problems of parameterization. Some parameters, such as interest elasticity, may be affected by firm-specific behavior as well as present macroeconomic factors. Another approach that has been attempted to estimate usage of limits is in a continuous-time model, where the credit provider and the credit taker interact within a game-theoretic framework (Leippold, Ebnoether and Vanini. 2003.)

Other authors have tried to analyze the pricing of CCL from the demand side of firms and show that credit line usage depends on the business growth potential of the firm as well as the uncertainty involved in those investment opportunities (Martin., Spencer and Santomero, 1997). External macroeconomic variables - such as size of credit line, collateralization, etc. - are also found to be associated with the level of partial draw-downs of obligors on CCLs.

Another study uses numerical procedures to obtain simulated values of early exercise of capped variable-rate loan commitments (Château, 1990). An extension of this introduces dynamics for the indebtedness value (Chateau and Wu, 2004), the market value of commitment as defined previously in the literature (Thakor and Udell, 1987), providing an alternative approach to compute banks' exposure to commitment risk as mandated by the Bank for International Settlement (BIS) in Basel II and provided an analytic solution to price the commitment put as an American option. They use prior empirical evidence[4] for determining the partial draw-down proportions with which to parameterize their model.

Finally, a recent study attempts (Bag and Jacobs, Jr., 2010) to build an easy to implement, pragmatic and parsimonious yet accurate model to determine the EAD distribution of a CCL portfolio, modeling each revolver as a portfolio of a large number of option instruments which can be exercised by the borrower, determining the level of usage. Using an algorithm similar to the basic CreditRisk+ and Fourier Transforms, they arrive at a portfolio level probability distribution of usage, and perform a simulation experiment in which they illustrate the convolution of two portfolio segments to derive an EAD distribution. The authors illustrate such calculations using two sample segments of 13 obligors each, chosen randomly from Moody' Default Risk Service (DRS) database of CCLs rated as of 12/31/2008, details of which we reproduce in Table 8 from Table 1 of the sudy[29]. The segments are investment grade (rated Moody's Baa3 and higher) and junk grade (rated Moody's Ba1 and lower), respectively, with limits of each obligor varying from \$25 MM to \$235 MM. The values of the LEQ factors are sourced from Jacobs (2010) based upon estimated additional drawdowns on unused limits for Moody's rated CCLs 1987-2009 defaulting within a 1-year horizon in Moody's Ultimate Recovery Database (MURD.) These are 65% and 40%, for investment grade and junk grade rated loans, respectively.

Table 8 - Portfolios of Moody's Rated Contingent Credit Lines (4/31/2010 Release of the Moody's Default Risk Service Database - obligors Rated as of 12/31/08)*

Segment	Issuer Number	Issuer Name	Unused Limit ($ '000)	Number of Puts	Put Size	Moody's Senior Unsecured Credit Rating	Moody's Broad Industry Category	Debt Type Description
Investment Grade	153000	Central Maine Power Company	50,000	1000	50	Baa1	PUBLIC UTILITY	Revolving Credit Facility
	191670	Commercial Metals Company	235,000	1000	235	Baa2	INDUSTRIAL	Revolving Credit Facility
	232000	Detroit Edison Company (The)	68,750	1000	69	Baa1	PUBLIC UTILITY	Revolving Credit Facility
	252000	Duquesne Light Company	100,000	1000	100	Baa2	PUBLIC UTILITY	Revolving Credit Facility
	404000	Indianapolis Power & Light Company	120,600	1000	121	Baa2	PUBLIC UTILITY	Revolving Credit Facility
	490000	Michigan Consolidated Gas Company	81,250	1000	81	A3	PUBLIC UTILITY	Revolving Credit Facility
	576000	Orange and Rockland Utilities, Inc.	100,000	1000	100	Baa1	PUBLIC UTILITY	Revolving Credit Facility
	687000	South Carolina Electric & Gas Company	75,000	1000	75	Baa1	PUBLIC UTILITY	Revolving Credit Facility
	769000	Tucson Electric Power Company	120,000	1000	120	Baa3	PUBLIC UTILITY	Sr. Sec. Revolving Credit Facility
	600045390	IDACORP, Inc.	250,000	1000	250	Baa2	PUBLIC UTILITY	Revolving Credit Facility
	600050191	Rayonier Forest Resources, L.P.	50,000	1000	50	Baa3	REAL ESTATE FIN	Gtd. Revolving Credit Facility
	600064222	Michigan Electric Transmission Company, LLC	25,000	1000	25	Baa1	INDUSTRIAL	Sr. Sec. Revolving Credit Facility
	808653810	NASDAQ OMX Group, Inc. (The)	150,000	1000	150	Baa3	SECURITIES	Sr Sec 1st Lien Rev Credit Facility
Junk Grade	600040059	Accuride Corporation	212,000	1000	212	C	INDUSTRIAL	Sr. Sec. Revolving Credit Facility/Gtd 1st Lien Sr Sec Revolver
	809883143	Peach Holdings, Inc.	35,000	1000	35	C	FINANCE	Gtd. Sr. Sec. Revolving Credit Facility
	820360433	Bravo Health, Inc.	25,000	1000	25	B2	INSURANCE	Sr. Sec. Revolving Credit Facility
	199515	Conseco, Inc.	80,000	1000	80	Caa3	INSURANCE	Sr. Sec. Revolving Credit Facility
	600058771	GSCP (NJ), L.P.	60,000	1000	60	C	OTHER NON-BANI	Gtd. Sr. Sec. Revolving Credit Facility
	566800	Covanta Energy Corporation	100,000	1000	100	Ba3	PUBLIC UTILITY	Gtd 1st Lien Sr Sec Revolver
	807760066	Interstate Operating Company, L.P.	140,000	1000	140	Caa3	REAL ESTATE FIN	Gtd. Sr. Sec. Revolving Credit Facility
	820399560	TPG-Austin Portfolio Holdings LLC	100,000	1000	100	Ca	REAL ESTATE FIN	Gtd. Sr. Sec. Revolving Credit Facility
	431200	Kansas City Southern Railway Company (The)	100,000	1000	100	B2	TRANSPORTATIO	Gtd. Sr. Sec. Revolving Credit Facility
	809492974	Standard Steel, LLC	20,000	1000	20	Caa1	TRANSPORTATIO	Gtd 1st Lien Sr Sec Revolver
	600038850	AEP Industries, Inc.	100,000	1000	100	B2	INDUSTRIAL	Gtd. Revolving Credit Facility
	600042238	Alliance Laundry Systems LLC	230,000	1000	230	B3	INDUSTRIAL	Gtd. Sr. Sec. Revolving Credit Facility
	44000	American Greetings Corporation	75,000	1000	75	B1	INDUSTRIAL	Gtd. Sr. Sec. Revolving Credit Facility

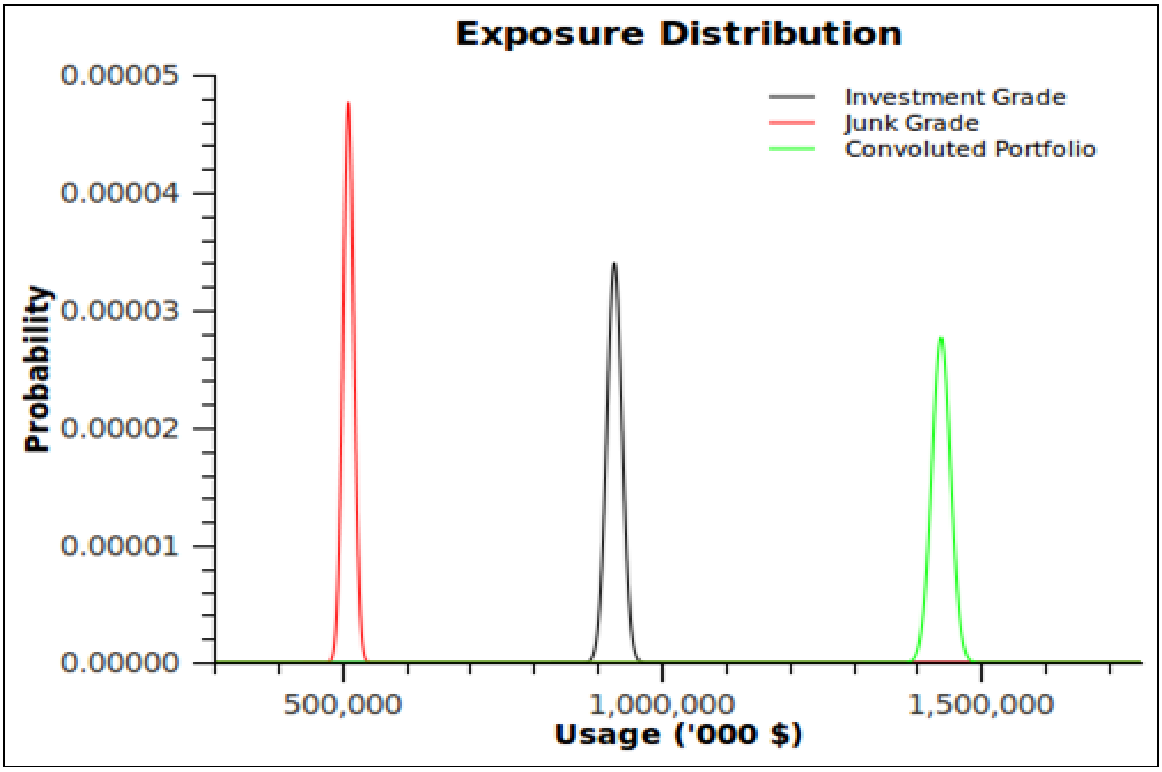
* Reproduced from Table 1 of Bag and Jacobs (2010)

Table 9 - Descriptive Statistics of Sample Portfolio (4/31/2010 Release of the Moody's Default Risk Service Database - obligors Rated as of 12/31/08)*

	Investment Grade Portfolio	Junk Grade Portfolio	Convoluted Portfolio
LEQ Factor	65%	40%	
Mean ('000 $)	926,640	510,800	1,437,400
Standard Deviation ('000 $)	11,735	8,374	14,417
Skewness	0.015698	0.020199	0.012426
Kurtosis	3.0003	3.0005	3.0000

* Reproduced from Table 3 of Bag and Jacobs (2010)

Figure 3 - Exposure Distribution of Sample and Convoluted Portfolios (4/31/2010 Release of the Moody's Default Risk Service Database - obligors Rated as of 12/31/08)



* Reproduced from Figure 1 of Bag and Jacobs (2010)

Each of the obligor's limit is divided into 1,000 puts each put size of $1000. Table 9, reproduced from Table 3 of the sudy[29], shows summary statistics of the results. This is also shown graphically in in Figure 3, which shows the usage distribution of each segment and the final convoluted portfolio. To explore the effect of choice of n in the exposure distribution they use a hypothetical five obligor portfolio A with the random unused limits. The portfolio and results of the experiment are summarized in Table 3 in appendix C of the sudy. As noted, the standard deviation of the usage distribution decreases as they increase the number of puts used. This may be explained by the fact that we are implicitly assuming a zero standard deviation and as the number of puts increases, i.e. closer to real life it shows up in the calculation.

5. Conclusion

We argued that all banking risk managers would agree that EAD quantification for CCLs is a critical for the construction of credit risk models of varied kinds (e.g., loan loss provisions and credit capital, including Basel II advanced IRB.) We have recognized the great challenge that this has been for banks due to paucity of historical data and benchmark models, and have contributed to the efforts of the industry, by reviewing the existing literature on both models and empirical evidence on EAD. We considered recent extensions of prior empirical work, which study alternative risk drivers and measures of EAD risk, including multiple regression empirically-based models as well as actuarial-based theoretical models of EAD. On the empirical side, we focused on a study of a sample of Moody's rated CCLs, which involved building a predictive econometric GLM model in the generalized linear model class. In the case of theoretical models, we studied the calibration of a framework for understanding distributions of EAD in a CreditRisk+ framework involving fast Fourrier transforms technology for the convolution of portfolio segments.

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