# AP Physics C Notes

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# Contents

# 1 Review

## 1.1 Derivatives Review

Mathematically velocity is referred to as the derivative of position with respect to time. Acceleration can also be referred to as the second derivative of position with respect to time.

$$a = \frac{d^2x}{dt^2}$$

Evaluating Polynomial Derivates:

$$x = At^n$$
$$v = \frac{dx}{dt} = nAt^{n-1}$$

## 1.2 Kinematic Equation Review

$$v = v_o + at$$
$$x = x_o + v_o t + \frac{1}{2}at^2$$
$$v^2 = v_o^2 + 2a(\Delta x)$$

# 2 Vectors in 2-D Motion

## 2.1 Scalars vs Vectors

- **Scalars** have magnitude only. (distance, speed, time, mass)
- **Vectors** have both magnitude and direction (displacement, velocity, acceleration)
- **Vectors** have both magnitude and direction (displacement, velocity, acceleration)
- The direction of a vector is represented by the direction in which the ray points and is typically given by an angle.
- The magnitude of a vector is the size of whatever the vector represents. It is represented by the length of the vector and is often represented as —A—.
- Equal vectors have the same length and direction and represent the same quantity (such as force or velocity).
- Inverse vectors have the same length but opposite direction.
- Vectors are added graphically together head-to-tail. The sum is called the resultant. The inverse of the sum is called the equilibrant.

## 2.2 Unit Vectors

Unit vectors are quantities that specify direction only. They have a magnitude of exactly one, and typically point in the x, y, or z directions.

î points in the x direction   ĵ points in the y direction   k̂ points in the z direction

# 3 Position, Velocity, and Acceleration Vectors in Multiple Dimensions

## 3.1 Derivates still work

$$v = \frac{dr}{dt}$$

$$\frac{d}{dt}(xi + yj + zk)$$

$$\frac{dx}{dt}i + \frac{dy}{dt}J + \frac{dz}{dt}k$$

$$v_x i + v_u j + v_z k$$

## 3.2 Kinematics Equations

$$v = v_o + at$$

$$r = r_i + v_o t + \frac{1}{2}at^2$$

$$v^2 = v_o^2 + 2a\Delta r$$

1 Dimension:
x = position $\Delta x$: displacement v:velocity a: acceleration

2 or 3 Dimensions:
r: position $\Delta r$ + displacement v: velocity a: acceleration

## 3.3 Unit Vector Notation

$$r = xi + yj + zk$$

$$\Delta r = \Delta xi + \Delta yj + \Delta zk$$

$$v = v_x i + v_y + j + v_z k$$

$$a = a_x i + a_y j + a_z k$$

# 4 2D Motion or Projectile Motion

## 4.1 Projectile Motion

Can be 1, 2 or 3-Dimensional motion. Something is fired, thrown, shot or hurried near the earth's surface. Horizontal velocity is constant. Vertical velocity is accelerated. Air resistance is still ignored.

## 4.2 Horizontal Component of Velocity

is not accelerated
not included by gravity

## 4.3 Vertical Component of Velocity

accelerated by gravity

$$V_y = V_o - gt$$

# 5 Newton's Laws

## 5.1 Force

Force is a push or pull on an object. Unbalanced forces cause an object to accelerate. (ie speed up, slow down, or change directions). Force is also a vector

## 5.2 Types of Forces

1. Contact forces: involves contact between bodies (ie normal/friction)
2. Field forces: act without necessity of contact (ie gravity/electromagnetic)

## 5.3 Forces and Equilibrium

If the net fore on a body is zero, it is in **equilibrium**. An object in equilibrium may be moving relative to us (dynamic equilibrium) or may appear to be at rest (static equilibrium).

## 5.4 Newton's First Law: The Law of Inertia

**A body in motion stays in motion in a straight line unless acted upon by an external force.**

This law is commonly applied to the horizontal component of velocity, which is assumed not to change during the flight of a projectile.

### 5.5 Newton's Second Law: $F = m * a$

$$\sum F = m * a$$

A body accelerates when acted upon by a net external force. The acceleration is proportional to the next force and is in the direction which the net force acts. This law is commonly applied to the vertical component of velocity in projectiles.

### 5.6 Newton's Third Law

For every action there exists an equal and opposite reaction. If A exerts a force F on B, then B exerts a force of -F on A

### 5.7 Commonly Confused Terms

1. **Inertia** or the resistance of an object to being accelerated
2. **Mass:** the same thing as inertia (to a physicist)
3. **Weight:** gravitational attraction.

$$\text{inertia} = \text{mass } \alpha \text{ weight}$$

### 5.8 Normal Force

Normal force is the force that keeps one object from invading another object. Our weight is the force of attraction of our body for the center of the planet.

$$N = mg * \cos\theta$$

### 5.9 Tension

Tension is a pulling force. Generally it exists in a ripe, spring, or cable. It arises at the molecular level, when a rope string, or cable resists being pulled apart.

The horizontal and vertical components of the tension are equal to zero if the system is not accelerating.

### 5.10 Atwood Machine

An Atwood machine is a device used for measuring g. If m1 and m2 are nearly the same, it slows down freefall such that acceleration can be measured. Then g can be measured.

## 6 Forces

### 6.1 Variable Forces

Forces can vary with time, velocity and with position.

### 6.2 Calculus Concepts

Differentiation gives you the tangent of a function:
Position $\rightarrow$ Velocity $\rightarrow$ Acceleration

Integration gives you the area under a curve:
Acceleration $\rightarrow$ Velocity $\rightarrow$ Position

If $a(t) = t^n$ then $\int t^n dt = \frac{t^{n+1}}{n+1} + C$

### 6.3 Drag Forces

Drag forces slow an object down as it passed through a fluid. They act in the opposite direction to velocity, are functions of velocity, and impose terminal velocity.

### 6.4 Drag as a Function of Velocity

$$f_D = bv + cv^2$$

$b$ and $c$ depend upon shape and size of the object and the properties of fluid. $b$ is important at low velocity while $c$ is important at high velocity.

### 6.5 Drag Forces in Free Fall

When $f_D$ equals $mg$, terminal velocity has been reached.

For Slow moving objects: $F_D = BV$
For fast moving objects: $F_D = cv^2$

$$c = \frac{1}{2} D * p * A$$

Where: $D$ = drag coefficient, p = density of fluid, and A = cross-sectional area

## 7 Introduction to Work

### 7.1 Energy and Work

- A body experiences a change in energy when one of more forces do work on it.
- A force does positive work on a body when the force and the displacement are least partially aligned.
- Maximum positive work is done when a force and displacement are exactly the same direction.
- If a force causes no displacement, it does no work (ie normal forces or centripetal forces).
- Forces can do negative work if they are pointed opposite the direction of the displacement.

### 7.2 Calculating Work

Work, which is a scalar resulting from the interaction of two vectors, is the dot product of force and displacement.

$$W = F \cdot r$$

$$W = Fr\cos\theta$$

$$W = F_x r_x + F_y r_y + F_z r_z$$

### 7.3 Units of Work

- Si System: Joule (N m)
- British System: foot-pound
- cgs System: erg (dyne-cm)
- Atomic Level: electron volt

### 7.4 Work in a Pulley System

A pulley system, which has at least one pulley attached to the load, can be used to reduce the force necessary to lift a load. Amount of work done in lifting the load is not changed. The distance the force is applied over is increased, thus the force is reduced since $W = Fd$.

### 7.5 Work and Variable Forces

- For constant forces: $W = F \cdot r$
- For variable forces, you can't move far until the force changes. The force is only constant over an infinitesimal displacement: $dW = F \cdot dr$
- To calculate work for a larger displacement you have to take an integral: $W = \int dW = \int F \cdot dr$

The area under the curve of a graph of force *vs* displacement gives the work done by the force.

$$W = \int_{x_a}^{x_b} F(x)dx$$

## 8 Net Work and Kinetic Energy

### 8.1 Net Work or Total Work

An object can be subject to many forces at the same time, and if the object is moving the work done on each force and be individually determined. At the same time one force does positive work on the object, another force may be doing negative work, and yet another force may be doing no work at all. The net work, or total, work done on the object is the scalar sum of the work done on an object by all forces acting upon the object.

$$W_{net} = \sum W_i$$

$$W_{\text{net}} = \sum W_i$$

### 8.2 The Work Energy Theorem

$$W_{\text{net}=\Delta K}$$

When net work due to all forces acting upon an object is positive, the kinetic energy of the object will increase. When the net work due to all forces acting upon an object is negative, the kinetic energy of the object will increase. When there is no net work acting upon an object, the kinetic energy of the object will be unchanged.

### 8.3 Kinetic Energy

Kinetic energy is one form of mechanical energy, which is energy we can easily see and characterize. Kinetic energy is due to the motion of an object.

$$K = \frac{1}{2}mv^2$$

$$K = \frac{1}{2}mv \cdot v$$

### 8.4 Power

Power is the rate of which work is done, No matter how fast we get up the stairs, our work is the same. If you run then your power is higher than if you walked.

$$P_{\text{ave}} = \frac{W}{t}$$

$$P_{\text{inst}} = \frac{dW}{dt}$$

### 8.5 Units of Power

1. Watt= J/s
2. ft lb/s
3. horsepower (1 HP = 747 Watts)

## 9 Potential Energy

### 9.1 Types of Forces

Conservative foces:

- Work in moving an object is path independent

- Work in moving an object along a closed path is zero.
- Work is directly related to a negative change in potential energy.

Non-Conservative Foces:

- Work is path dependent
- Work along a closed path is not zero
- Work may be relayed to a change is mechanical energy or thermal energy.

### 9.2 Potential Energy

A type of mechanical energy possessed by an object by virtue of its position or configuration. It is represent by the latter $U$. The work done by conservative forces is the negative of the potential energy change.

$$W = -\Delta U$$

### 9.3 Gravitational Potential Energy

The change in gravitational potential energy is the negative of the work done by gravitational force on an object when it is moved. For objects near the earth's surface, the gravitational pull of the earth is roughly constant, so the force necessary to lift an object at constant velocity is equal to the weight, so we can say:

$$\Delta U_g = -W_g = mgh$$

### 9.4 Spring Potential Energy, $U_s$

Springs obey Hooke's Law and unlike gravitational potential energy, we know where the zero potential energy point is for a spring.

$$F_s(x) = -kx$$

$$W_s = \int F_s(x)dx = -k \int xdx = -\frac{1}{2}kx^2$$

$$U_s = \frac{1}{2}kx^2$$

## 10 Law of Conservation of Energy

If a system is isolated and the boundary allows no exchange. with the environment then: $E = U + K + E_{\text{int}} = \text{Constant}$

$$E = U + K = C$$

$$\Delta E = \Delta U + \Delta K = 0$$

For Gravity:

$$\Delta U_g + mgh_f - mgh_i$$

$$\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

For Springs:

$$\Delta U_s = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$$

$$\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

### 10.1 Pendulum Energy

For any two points in the pendulum's swing:

$$\frac{1}{2}mv_1^2 + mgh_1 = \frac{1}{2}mv_2^2 + mgh_2$$

### 10.2 Spring Energy

$$\frac{1}{2}kx_1^2 + \frac{1}{2}mv_1^2 = \frac{1}{2}kx_2^2 + \frac{1}{2}mv_2^2$$

### 10.3 Forces and Potential Energy

In order to discuss the relationship between displacment and forces, we need to know a couple of equations:

$$W = \int F(x)dx = -\int dU = \Delta U$$

$$\int dU = \int F(x)dx$$

$$F(x) = -dU(x)/dx$$

In stable equilibrium, the forces are trying to bring the object back to equilibrium. If the forces are trying to move the object away from equilibrium then it at unstable equilibrium. When the system is displaced from equilibrium and it just stays there, it is at neutral equilibrium.

## 11 Oscillatory Motion

### 11.1 Periodic Motion

**Periodic motion** is motion of an object that regularly returns to its given position after a fixed time interval.

A special kind of periodic motion occurs in mechanical systems when the force acting on the object is proportional to the position of the object relative to some equilibrium position.

If the force is always directed toward the equilibrium position, the motion is called **simple harmonic motion**.

### 11.2 Motion of a Spring Mass System

A block of mass m is attached to a spring and the block is free to move on a frictionless horizontal surface. When the spring is neither stretched nor compressed, the block is at the equilibrium position.

### 11.3 Hooke's Law

$$F_x = -kx$$

- $F_x$ is the restoring force and is always directed toward the equilibrium position and opposite the displacement from equilibrium.
- $k$ is the spring constant
- $x$ is the displacment

### 11.4 Acceleration

$$a_x = \frac{k}{m}x$$

The acceleration is proportional to the displacement of the block and the direction of the acceleration is opposite the direction of the displacement from equilibrium.

The acceleration is not constant. When the block passes though the equilibrium position, a=0. The block continues to $x = -A$ where its acceleration is $+KA/m$

### 11.5 Mathematical Representation of Simple Harmonic Motion

- Acceleration $a = \frac{d^2x}{dt^2} = -\frac{k}{m}x$
- We let $\omega^2 = \frac{k}{m}$
- Then a $= -\omega^2 x$

### 11.6 Graphical Representation of Simple Harmonic Motion

A cosine curve can be used to give physical significance to these constants

$$x(t) = A\cos(\omega t + \phi)$$

- $A$ is the amplitude of the motion

- $\omega$ is called the angular frequency and is in rad/s
- $\phi$ is the phase constant or the initial phase angle

$A$ and $\phi$ are determined uniquely at the position and the velocity of the particle at t=0

- If the particle is at $x = A$ at $t = 0$, then $\phi = 0$
- The phase of the motion is the quantity $(\omega t + \phi)$
- $x(t)$ is the periodic and its value is the same each time $\omega t$ increase by $2\pi$ radians

## 11.7 Period

The period T is the time interval required for the particle to go through one full cycle of its motion

$$T = \frac{2\pi}{\omega}$$

## 11.8 Frequency

The frequency is the inverse of the period and represents the number of oscillations the particle undergoes per time interval.

$$F = \frac{1}{T} = \frac{\omega}{2\pi}$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

The frequency and period only depend on the mass of the particle and the force constant of the spring and do not depend on the parameters of motion.

## 11.9 Motion Equations for Simple Harmonic Motion

$$x(t) = A\cos(\omega t + \phi)$$

$$v = \frac{dx}{dt} = -\omega A\sin(\omega t + \phi)$$

$$a = \frac{d^2x}{dt^2} = -\omega^2 A\cos(\omega t + \phi)$$

## 11.10 Maximum Values of $v$ and $a$

Because the sine and cosine functions oscillate between $\pm 1$, we can easily find the maximum values of velocity and acceleration for an object in SHM.

$$v_{\text{max}} = \omega A = \sqrt{\frac{k}{m}}A$$

$$a_{\text{max}} = \omega^2 A = \frac{k}{m}A$$

### 11.11 Energy of the SHM Oscillator

$$K = \frac{1}{2}mv52 + \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \phi)$$
$$U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2 \cos^2(\omega t + \phi)$$
$$E = K + U = \frac{1}{2}kA^2$$

Energy can also be used to find velocity:

$$v = \pm\sqrt{\frac{k}{m}(A^2 - x^2)} = \pm\omega^2\sqrt{A^2 - x^2}$$

### 11.12 Simple Pendulum

A simple pendulum also exhibits periodic motion. The motion occurs in the vertical plane and is drive by gravitational force and is very close to that of the SHM oscillator if the angle is less than 10 degrees

The forces acting on the object are the tension and the weight. $T$ is the force exerted on the bob by the string and $mg$ is the gravitational force. The tangential component of the gravitational force is a restoring force.

In the tangential direction:

$$F_t = -mg\sin\theta = m\frac{d^2s}{dt^2}$$

The length, $L$, of the pendulum is constant and for small values of $\theta$:

$$\frac{d^2\theta}{dt^2} = \frac{-g}{L}\sin\theta = \theta$$

$$\theta = \theta_{max}\cos(\omega t + \phi)$$

The angular frquency is:

$$w = \sqrt{\frac{g}{L}}$$

The period is:

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$$

### 11.13 Physical Pendulum

If a hanging object oscillates about a fixed axis that does not pass through the center of mass and the object cannot be approximated as a particle, the system is called a **physical pendulum**.

$$-mgf\sin\theta = I\frac{d^2\theta}{dt^2}$$

Assuming $\theta$ is small, this becomes:

$$\frac{d^2\theta}{dt^2} = -\theta\frac{mgd}{I} = \omega^2\theta$$

The angular frequency is: $\omega = \sqrt{\frac{mgd}{I}}$

The period is $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{mgd}}$

### 11.14 Torsional Pendulum

Assume a rigid object is suspended from a wire attached at its top to a fixed support. The twisted wire exerts a restoring torque on the object that is proportional to its angular position. The restoring torque is: $\tau = \kappa\theta$

Newton's Second Law gives:

$$\tau = \kappa\theta = I\frac{d^2\theta}{dt^2}$$

$$\frac{d^2\theta}{dt^2} = -\frac{\kappa}{I}\theta$$

## 12 Momentum and Impulse

### 12.1 Linear Momentum

The linear momentum of a particle is the product of its mass and velocity.

$$p = mv$$

A system has a linear momentum that ie equal to the vector sum of the momenta of the particles

$$P = \sum p_i$$

The linear momentum of a system is conserved unless the system experiences an external force.

### 12.2 Impulse (J)

Impulse is the internal of force over a period of time

$$J = \int F dt$$

The change in momentum of the particle is equal to the impulse acting on it

$$\Delta p = J$$

$$dp = F dt$$

### 12.3 Collisions

Momentum is conserved so $P_{\text{before}} = P_{\text{after}}$. In elastic collisions, kinetic energy is also conserved. In inelastic collisions, kinetic energy is lost. In perfectly inelastic collisions, kinetic energy is still lost but the objects now stick together. In explosions, which is the reverse of perfectly inelastic collisions, kinetic energy is gained.

## 13 Rotational Motion

### 13.1 Rotational Kinematics

In translational motion, the position is represented by a point such as x. In rotational motion, the position is represented by an angle such as $\theta$, and a radius $r$.

Linear displacement is represented by the vector $\Delta x$. Angular displacement is represented by $\Delta\theta$, which is not a vector, but behalves like one for small values. The right-hand rule determines direction.

A particle that rotates through an angle $\Delta\theta$ also translates through a distance $s$, which is the length of the arc defining its path. This distance $s$ is related to the angular displacement $\Delta\theta$ by the equation $s = r\Delta\theta$.

The instantaneous velocity has magnitude $v_T = \frac{ds}{dt}$ and is tangent to the circle. The same particle rotates with an angular velocity with $\omega = \frac{d\theta}{dt}$. The direction of the angular velocity is given by the right-hand rule. Tangential and angular speeds are related by the equation $v = r\omega$.

### 13.2 Acceleration

Tangential acceleration is given by $a_T = \frac{dv_t}{dt}$. This acceleration is parallel or antiparallel to the velocity.

Angular acceleration of this particle is given by $\alpha = \frac{d\omega}{dt}$. Angular acceleration is parallel or anti-parallel to the angular velocity.

Tangental and angular acceleration are related by the equation $a = r\alpha$.

Centripetal acceleration always points to the center of the circle and is $a_c = \frac{v^2}{r}$ or $a_c = r\omega^2$.

### 13.3 Kinematic Equations

$$\omega = \alpha t$$
$$\theta = \theta_o + \omega_o t + \frac{\alpha}{t^2}$$
$$\omega^2 = \omega_o^2 + 2\alpha(\omega - \omega_o)$$

### 13.4 Rotational Energetics

### 13.5 Inertia and Rotational Inertia

In linear motion, inertia is equivalent to mass. Rotating systems have "rotational inertia."

$$I = \sum mr^2$$

- I: rotational inertia ($kgm^2$)
- m: mass ($kg$)
- r: radius of rotation ($m$)

### 13.6 Kinetic Energy

$$K_{\text{trans}} = \frac{1}{2}mv^2$$
$$K_{\text{rot}} = \frac{1}{2}I\omega^2$$
$$K_{\text{rot}} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

### 13.7 Rotational Inertia Calculations

$$\text{System of Particles: } I = \sum mr^2$$
$$\text{Solid object: } I = \int r^2 dm$$
$$\text{Parallel Axis Theorem: } I = I_{\text{cm}} + mh^2$$

# 14 Torque

## 14.1 Torque as a vector

Torque is the rotational Analog of force that causes rotation to begin. The magnitude of the torque is proportional to that of the force and moment arm.

$$\tau = Fr \sin\theta$$

$$\tau = rxF$$

## 14.2 Equilibrium

Equilibrium occurs when there is no net force and no net torque on a system. Static equilibrium occurs when nothing in the system is moving or rotating your reference frame. Dynamic equilibrium occurs when the system is translating at constant velocity and/or rotation at constant rotational velocity.

$$\sum F = 0$$

$$\sum \tau = 0$$

## 14.3 Torque and Newton's $2^{\text{nd}}$ Law

$$\sum \tau = I\alpha$$

## 14.4 Work in Rotating Systems

$$W = F \cdot \Delta r$$
$$W_{\text{rot}} = \tau \cdot \Delta\theta$$
$$W_{\text{net}} = \Delta K$$
$$W_c = -\Delta U$$
$$W = \Delta E$$

## 14.5 Power in Rotating Systems

$$P = \frac{dW}{dt}$$
$$P = F \cdot v$$
$$P_{\text{rot}} = \tau \cdot \omega$$

### 14.6 Conservation of Energy

$$E_{\text{rot}} = U + K = \text{Constant}$$

### 14.7 Rolling without Slipping

Total kinetic energy of a body is the sum of the translational and rotational kinetic energies.

$$K = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

When a body is rolling without slipping, another equation holds true:

$$V_{\text{cm}} = \omega\tau$$

Therefore, this equation can be combined with the first one to create the two following equations:

$$K = \frac{1}{2}M^2 + \frac{1}{2}I_{\text{cm}}\frac{v^2}{R^2}$$

$$K = \frac{1}{2}m\omega^2R^2 + \frac{1}{2}I_{\text{cm}}\omega^2$$

## 15 Angular Momentum

### 15.1 Angular Momentum for Particle

Angular momentum is a quantity that tells us how hard it is to change the rotational motion of a particular spinning body. Objects with lots of angular momentum are hard to stop spinning or to turn and have great orientational stability.

For a single particle:

$$L = r\text{x}p$$

- L: angular momentum for a single Particle
- r: distance from particle to point of rotation
- p: linear momentum

### 15.2 Angular Momentum for solid Object

For a solid object, angular momentum is analogous to linear momentum of a solid object

$$P = mv$$

- Replace momentum with angular momentum
- Replace mass with rotational inertia
- Replace velocity with angular velocity

$$L = I\omega$$

### 15.3 Law of Conservation of Angular Momentum

The momentum of a system will not change unless an external force is applied. Angular momentum of a system will also not change unless an external torque is applied.

### 15.4 Angular Momentum and Torque

$$F = \frac{dP}{dt}$$

$$\tau = \frac{dL}{dt}$$

If there is an unbalanced torque, then torque changes L with respect to time. Torque increases angular momentum when the two vectors are parallel and decreases when they are anti-parallel.

Torque changes the direction of the angular momentum vector in all other situations. This results in what is called the **percussion** of spinning tops.

## 16 Charge and Polarization

### 16.1 The Atom

The atom has positive charge in the nucleus, which is located in the protons. The positive charge cannot move from the atom unless there is a nuclear reaction.

The atom has negative charge in the election cloud on the outside of the atom. Electrons can move from the atom to another atom without all that much difficulty.

### 16.2 Charge

Charge comes in two forms, which Ben Franklin designated as positive (+) and negative (-). Charge is quantized and the smallest possible stable charge, which we designate as $e$, is the magnitude of the charge on 1 electron or proton. $e$ is referred to as the electron

### 16.3 Coulomb's Law

Coulomb's Law states that the force between any two objects depends on their charges, the constant $k$, and the distance between the objects.

$$F = \frac{k|q_1||q_2|}{r^2}$$

$$k = 9\text{x}10^9\text{Nm}^2/\text{c}^2$$

$$k = \frac{1}{4\pi\epsilon_o}$$

### 16.4 Superposition

When one or more charge contributes to the electric field, the resultant electric field is the vector sum of the electric fields.

### 16.5 The Electric Field

The presence of + or - charge modifiers empty space. This enables the electrical force to at on charged particles without actually touching them. The electric field is created in the space around a charged particle or a configuration of charges.

If a charged particle is placed in an electric field created by other charges, it will experience a force as a result of the field.

Why use fields?

- Forces exist only when two or more particles are present
- Fields exist even if no force is present
- The field of one particle only can be calculated

The lines of force indicate the direction of the force on a positive charge. These lines of force point away from positive charges and into negative charges.

### 16.6 Field Vectors from Field Lines

The electric field at a given point is not the field line itself, but can be determined by taking the tangent to the field at any point.

### 16.7 Force from Electric Field

$$F = Eq$$

- F: Force (N)
- E: Electric Field (N/C)
- q: Charge (C)

### 16.8 For Spherical Electric Fields

The electric field surrounding a point charge or a spherical charge can be calculated by:

$$E = k\frac{q}{r^2}$$

- E: Electric Field (N/C)
- k: 8.99 x $10^9$
- q: Charge (C)
- r: Distance from center of charge q (m)

# 17 Charge Distribution

## 17.1 Limitations of Coulomb's Law

Coulomb's Law equations for Force and field can only be used directly for point changes or spherically symmetric charges. For more complicated "Continuous charge distributions" we need to need to integrate.

## 17.2 Linear Charge Distribution

When charge resides on a long thin object such as a wire or a ring, we can that a linear charge distribution.

$$\lambda = \frac{Q}{L} = \frac{dQ}{dL}$$

## 17.3 Surface Charge Distribution

When change resides on larger surface, we can it a surface charge distribution. It is sometimes convenient for us to define a surface charge density, $\sigma$, which is charge per unit area.

$$\sigma = \frac{Q}{A} = \frac{dQ}{dA}$$

## 17.4 Volume Charge Distribution

$$= \frac{Q}{V} = \frac{dQ}{dv}$$

## 17.5 General Procedure

$$E = \int dE = \int \frac{kdq}{r^2}$$

You need to integrate of a spatial variable and find a common variable that $r$ and $dq$ both depend on and find the appropriate limits to the integral.

## 17.6 Motion of Charged Particles in Electric Fields

$$F = qE = ma$$

The motion is not unlike projective motion and the electric field is constant so kinematic equations can be employed.

### 17.7 Work Done by Electric Field

$$W = \int_A^B F \cdot r = q \int_A^B$$

$$\Delta U = -q \int E \cdot dr$$

$$\Delta V = -\int E \cdot dr$$

$$V = \frac{kq}{r}$$

### 17.8 Point Charge Potential Derivation

## 18 Electric Flux

### 18.1 Flux

Flux means "flow". You can increase flux in three ways:

- increase the field
- increase the area of the loop
- make sure the loop is appropriately angled

### 18.2 The Area Vector

The area vector is defined as a vector perpendicular to a surface with a magnitude equal to the scalar area of the surface. Flux is proportional to the field vector magnitude, area vector magnitude, and the cosine of the angle between them and the units is $\mathrm{Nm}^2/\mathrm{C}$. For a closed shape each side has its own area vector.

$$\Phi = v \cdot A$$

$$\Phi = E \cdot A$$

### 18.3 Calculation of flux over a closed surface in a vector field

If there is a "source" of the vector field in the closed shape, the flux over the surface is positive. If there is a "sink" of the vector field in the closed shape, the flux over the surface is negative. For a general vector:

$$\Phi = \oint v \cdot dA$$

For an electric field:

$$\Phi = \oint E \cdot dA$$

### 18.4 Gauss's Law of Electricity

$$q = \epsilon_o \Phi_e$$

$q$ is the net charge enclosed inside a given gaussian surface and is the sum of all the + and - charges. $\epsilon_o$ is the electrical permittivity of free space. Other forms include:

$$q = \epsilon_o \Phi_e$$

$$\oint E \cdot dA = \frac{q}{\epsilon_o}$$

$$\nabla \cdot E = \frac{\rho}{\epsilon_o}$$

$$\rho = \frac{q}{v}$$

### 18.5 Choosing Gaussian Surfaces

Conditions when choosing a surface:

- The value of the electric field can be argued by symmetry to be constant over the surface.
- The dot product of $E \cdot dA = E(dA)$ because $E$ and $A$ are in the same direction.
- The dot product of $E \cdot dA = 0$ because $E$ and $a$ are perpendicular.
- The filed can be argued to be zero over the surface.

## 19 Electric Energy and Capacitance

### 19.1 Electric Potential Energy

The electrostatic force is a conservative force and it is possible to define an electrical potential energy function with this force. Woke done by a conservative force ie equal to the negative change in potential energy.

### 19.2 Work and Potential Energy

There is a uniform field between two plates. As a positive charge charge moves from $A$ (high PE) to $B$ (low PE), work is done on it.

$$W = Fd = qE_x(x_f - x_i)$$

$$\Delta PE = -W = -qE_x(x_f - x_i)$$

## 19.3 Potential Difference

The potential difference between points $A$ and $B$ is defined as the change in the potential energy of a charge $q$ that moved from $A$ to $B$ divided by the size of the charge. Potential difference is not the same as potential energy.

$$\Delta V = V_B - V_a = \frac{\Delta PE}{q}$$
$$\Delta PE = q\Delta V$$

Both electric potential energy and potential difference are scalar quantities. The units of potential difference is $V = \frac{J}{C}$.

A special formula can be used when there is a uniform electric field:

$$\Delta V = V_b - V_a = -E\Delta x$$

## 19.4 Energy and Charge Movements

A positive charge gains electrical potential energy when it is moved in a direction opposite the electric field. A negative charge loses electric potential energy when it moves in the direction opposite the electric field.

If a charge is released in the electric field, it experiences a force and accelerates and gains kinetic energy. As it gains kinetic energy, it loses an equal amount of electrical potential energy.

## 19.5 Summary of Positive/Negative Charge Movements and Energy

When a positive charge is placed in an electric field

- it moves in the direction of the field:
- it moves from a point of higher potential to a point of lower potential
- its electric potential energy decreases
- its kinetic energy increases

When a negative charge is placed in an electric field:

- it moves opposite to the direction of the field
- it moves from a point of power potential to a point of higher potential
- its electrical energy increases
- its kinetic energy decreases
- Work had to be done on the charge for it to move from point A to point B

## 19.6 Electric Potential of a Point Charge

The point of zero electric charge is taken to be at an infinite distance from the charge. The potential created by a point charge $q$ at any distance $r$ from the charge is:

$$V = k_e \frac{q}{r}$$

A potential exists at some point in space whether or not there is a test charge at that point.

## 19.7 Electric Field and Electric Potential Depend on Distance

The electric field is proportional to:

$$\frac{1}{r^2}$$

The electric potential is proportional to:

$$\frac{1}{r}$$

## 19.8 Electric Potential of Multiple Point Charges

The superposition principle still applies so the total electric potential at some point $P$ due to serval point charges is the algebraic sum of the electric potentials due to the sum of the individual charges.

## 19.9 Electric Potential Energy of Two Charges

$V_1$ is the electric potential due to $q_1$ at some point $P$. The work required to bring $q_2$ from infinity to $pP$ without acceleration is $q_2 V_1$. This work is equal to the potential energy of the two particle systems is:

$$PE = q_2 V_1 = k_e \frac{q_1 q_2}{r}$$

## 19.10 Notes about Electric Potential Energy of two charges

If the charges have the same sing, PE is positive and work must be done to force the two charges near each other and the like charges would repel.

If the charges have opposite signs, PE is negative and the force would be attractive. Work must be done to hold back the unlike charges from accelerating as they are brought close together.

## 19.11 Potentials and Charged Conductors

Because no work is required to move a charge between to points that are at the same electric $W = 0$ when $V_a = V_b$. All points on the surface of. charged conductor in electrostatic equilibrium are at the same potential. Therefore, the electric potential is a constant everywhere on the surface of a charged conductor in equilibrium.

### 19.12 Conductors in Equilibrium

The conductor has an excess of a positive charge and all of the charge resides at the surface. $E = 0$ inside the conductor. The electric field just outside the conductor is perpendicular to the surface. The potential is constant everywhere on the surface of the conductor. The potential everywhere inside is constant and equal to tits value on the surface.

### 19.13 Electron Volt

The electron volt (eV) is defined as the energy that an electron gains when accelerated through a potential difference of 1 v.

$$1 \text{ eV } = 1.6 * 10^{-19} \text{ J}$$

### 19.14 Equipotential Surface

An equipotential surface is a surface on which all points are at the same potential and no work is required to move a charge at a constant speed on an equipotential surface and the electric field at every point on the surface is perpendicular to the surface.

### 19.15 Equipotentials and Electric Field Lines - Positive Charge

The equipotentials for a point charge are a family of spheres centered on the point charge and the field lines are perpendicular to the electric potential at all points.

### 19.16 Equipotentials and Electric Field lInes - Dipole

Equipotential lines are shown in blue and electric field lines are shown in red. The field lines are perpendicular to the equipotential lines at all points.

### 19.17 Electrical Potential Energy

$$U = qV = \frac{kq_1q_2}{r} = \frac{q_1q_2}{4\pi\epsilon_0 r}$$

This formula works for the potential energy of two point charges. For more than two charges, you must add the potential energy contribution due to each pair.

## 20 Capacitance

### 20.1 Capacitors

A capacitor is a device used in a variety of an electric circuits. The capacitance, $C$, of a capacitor is defined as the ratio of the magnitude of the charge on either conductor (plate) to the magnitude of the potential difference between the conductors (plates). It's units is Farad (F) and $1F = \frac{C}{V}$.

$$C = \frac{Q}{\Delta V}$$

## 20.2 Parallel-Plate Capacitor

The capacitance of a device depends on the geometric arrangement of the conductors. For a parallel-plate capacitor whose plates are separated:

$$C = \frac{k\epsilon_o A}{d}$$

Capacitors consist of two parallel plates with an area $A$ and are separated by a distance $d$ and both plates carry equal and opposite charges. When connected to a battery, charge is pulled off one plate and transferred to the other plate and the transfer stopes when $\Delta V_{\text{cap}} = \Delta V_{\text{battery}}$

## 20.3 Electric Field in a Parallel-Plate Capacitor

The electric field between the plates is uniform near the center and nonuniform near the edges. The filed may be taken as constant thought the region between the plates.

## 20.4 Capacitors in Circuits

A circuit is a collection of objects usually containing a source of electric energy connected to elements that convert it to other forms. A circuit diagram can be used to show the path of the real circuit.

## 20.5 Capacitors in Parallel

When capacitors are first connected in the circuit, electrons are transferred from the left plates through the battery to the right plate, leaving the left plate positively charged and the right plate negatively charged.

The total charge is equal to the sum of the charges on the capacitors:

$$Q_{\text{total}} = Q_1 + Q_2$$

The potential difference across the capacitors is the same and each is equal to the voltage of the battery. The capacitors can also be replaced with one capacitor with a capacitance of $C_{\text{eq}}$. The equivalent capacitor must have exactly the same external effect on the circuit as the original capacitors.

## 20.6 Capacitors in Series

When a battery is connected to the circuit, electrons are transferred from the left plate of $C_1$ to the right plate of $C_2$ through the battery. As this negative charge accumulates on thr right plate of $C_2$m ab equivalent amount of negative charge is removed from the left plate of $C_2$.

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$C = \frac{Q}{\Delta V}$$

$$C_{\text{eq}} = C_1 + C_2$$

Capacitors in parallel have the same voltage differences as does the equivalent capacitance. Capacitors in series all have the same charge as does their equivalent capacitance.

$$\text{Energy Stored} = \frac{1}{2} Q \Delta V$$

$$\text{Energy} = \frac{1}{2} Q \Delta V = \frac{1}{2} C \Delta V^2 = \frac{Q^2}{2C}$$

## 20.7 Capacitors with Dielectrics

A dielectric is an insulating material that, when placed between the plates of a capacitor, increase the capacitance. Dielectrics include rubber, plastic, or waxed paper.

$$C = \kappa C_o = \kappa e_o (A/D)$$

# 21 Important Formulas

## 21.1 Electric Fields

Coulomb's Law: $F = \frac{k|q_1||q_2|}{r^2}$
Force from Electric Fields: $F = Eq$
Spherical Electric Fields: $E = k\frac{q}{r^2}$
Motion of Charged Particles in Electric Fields: $F = qE = ma$
Work of a charge moving between two plates: $W = Fd = qE_x(x_f - x_i)$ and $\Delta PE = -W$

## 21.2 Gauss' Law:

Gauss' Law: $\nabla \cdot E = \frac{\rho}{\epsilon_o}$ and $\rho = \frac{q}{v}$

## 21.3 Electric Potential Energy and Potential Difference

Change in Potential Energy: $\Delta PE = -W = qE_x(x_f - x_i)$
Potential Difference: $\Delta V = V_B - B_a = \frac{\Delta PE}{q}$ and $\Delta PE = q\Delta V$
If you have a uniform electric field $\Delta V = V_b - V_a = -E\Delta x$
Electric Potential of a point charge: $V = k_e\frac{q}{r}$
Electric Potential Energy of two point charges: $PE = q_2V_1 = k_e\frac{q_1q_2}{r}$
Electrical Potential Energy of two point charges $U = qV = k\frac{q_1q_2}{r}$

## 21.4 Capacitance

Capacitance: $C = \frac{Q}{\Delta V}$
Parallel-Plate Capacitance: $C = \frac{k\epsilon_o A}{d}$
Capacitors in Parallel: $Q_{\text{total}} = Q_1 + Q_2 + ...$

## 21.5 Symbols

F is force
E is electric Field
V is electric potential
PE is Potential Energy
q is charge
k is Coulomb's constant
r is the distance between the center of charge
$\epsilon_o$ is the permittivity of free space
C is capicitance

## 21.6 Units

$k = 9\text{x}10^9\text{Nm}^2/\text{c}^2 = \frac{1}{4\pi\epsilon_o}$
1 eV $= 1.6 * 10^{-19}$ J
1 F $= \frac{C}{V}$
$\mu = 10^{-6}$
$n = 10^{-9}$