MA-202 Numerical Techniques (2022)

B. Tech. II year CSE

Name - Rajan Kumar

Roll No - 202051152

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Q1. Write a program docondition() to make use of MATLAB commands cond() and det() to compute the condition number and det(A)det(A−1) where A is the Hilbert matrix defined by:

$$A = [a_{mn}] = \frac{1}{m+n-1}.$$

Increase the dimension of the Hilbert matrix from N = 7 to 12 and see the degree of discrepancy between AA−1 and the identity matrix. Note: The number RCOND following the warning message about near-singularity or ill-condition given by MATLAB is a reciprocal condition number, which can be computed by the rcond() command and is supposed to get close to 1/0 for a well/badly conditioned matrix.

Answer:

```
syms A m n N
N=4;     % Size of the matrix
for i=1:N
    for j=1:N
        A(i,j)= 1/(i+j-1);
    end
end
disp(A)
```

$$\begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} \end{pmatrix}$$

```
Ain=A^-1;
cond(A)
```

```
ans =
```

$$\sqrt{\frac{\sqrt{\sigma_2}}{6\,\sigma_4^{1/6}} + \frac{\sqrt{\frac{628361585696\,\sqrt{6}\,\sqrt{\frac{4162457018}{7815437776125} + \frac{16\,\sqrt{3}\,\sqrt{4621318097}\,\mathrm{i}}{868381975125}}}{148899515625} + \frac{915862834\,\sigma_4^{1/3}\,\sqrt{\sigma_2}}{40516875} -}}{6\,\sigma_4^{1/6}\,\sigma_2^{1/4}}}$$

where

$$\sigma_1 = \sqrt{2858465861238784\,\sigma_3^{1/3} + \sigma_3^{2/3} + 6153440329728000000000000}$$

$$\sigma_2 = \frac{457931417\,\sigma_4^{1/3}}{40516875} + 9\,\sigma_4^{2/3} + \frac{326021}{78764805}$$

$$\sigma_3 = 48269565260952344985600000000000000 + 16698832846848000000000000000\,\sqrt{3}\,\sqrt{462131\ldots}$$

$$\sigma_4 = \frac{2081228509}{211016819955375} + \frac{8\,\sqrt{3}\,\sqrt{4621318097}\,\mathrm{i}}{23446313328375}$$

```
det(A)*det(Ain)
```

ans = 1

Q2. Write a MATLAB program to evaluate the expressions $p(x) = (x-1)^5$ and $q(x) = x^5 - 5x^4 + 10x^3 - 10x^2 + 5x - 1$ for values of $x = \{0, 0.25, 0.5, 0.75, 0.90, 0.95, 0.99\}$. Plot the two expressions and comment upon the results.

Answer:

```
x = [0,0.25,0.5,0.75,0.90,0.95,0.99]
```

```
x = 1×7
         0    0.2500    0.5000    0.7500    0.9000    0.9500    0.9900
```

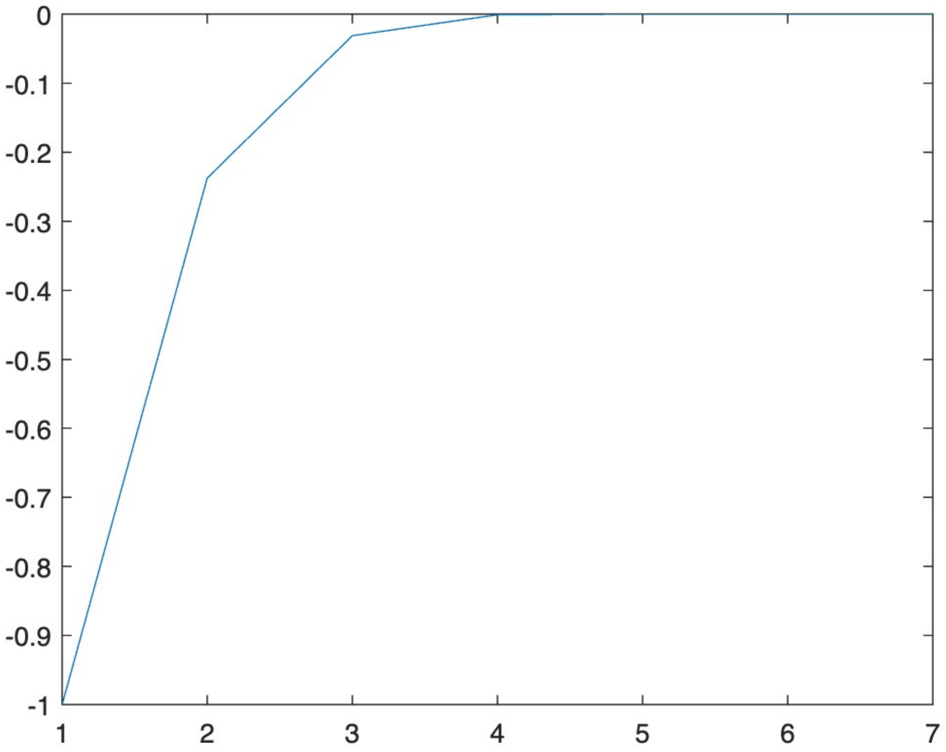
```
p = (x-1).^5
```

```
p = 1×7
   -1.0000   -0.2373   -0.0312   -0.0010   -0.0000   -0.0000   -0.0000
```

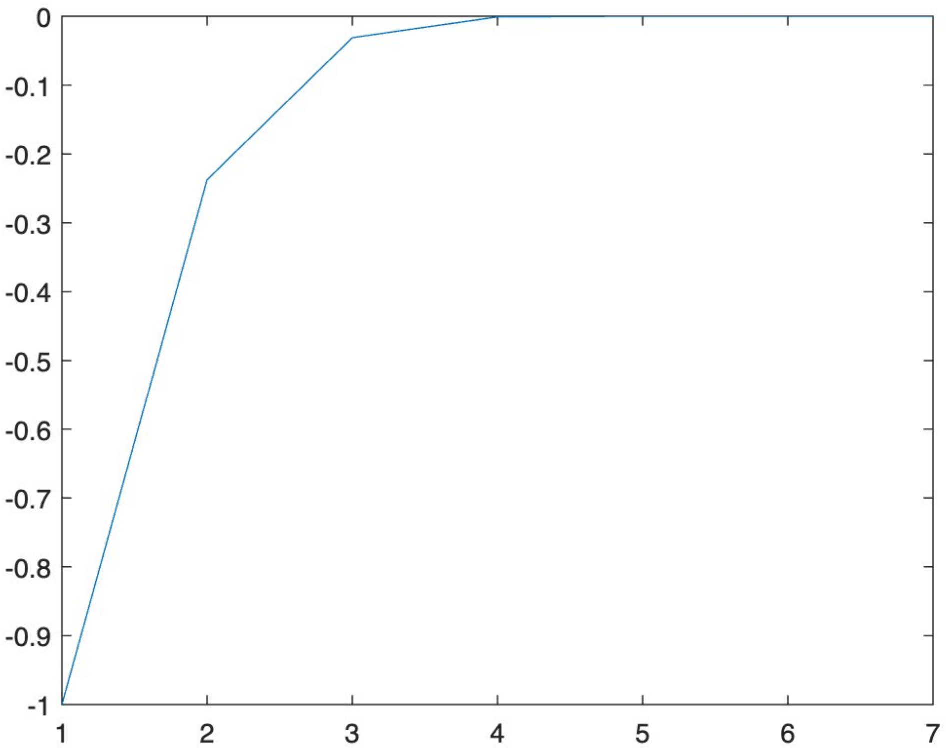
```
q = x.^5-5*x.^4+10*x.^3-10*x.^2+5*x-1
```

```
q = 1×7
   -1.0000   -0.2373   -0.0312   -0.0010   -0.0000   -0.0000   -0.0000
```

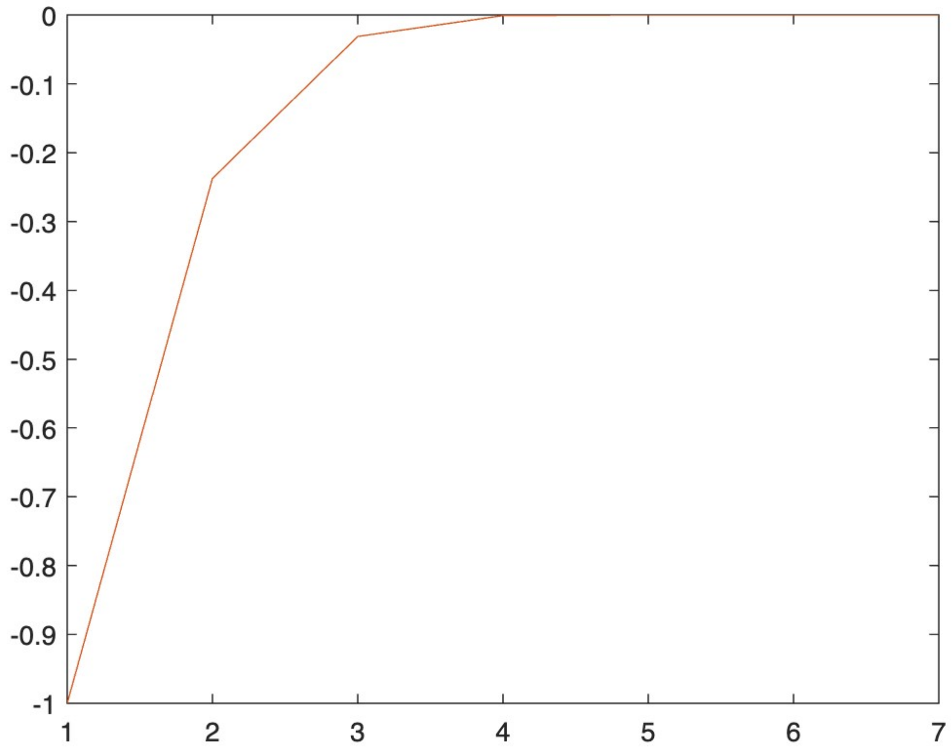
```
plot(p)
```


0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
-1
1
2
3
4
5
6
7

```
plot(q)
```


0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
-1
1
2
3
4
5
6
7

```
plot(1:7,p,1:7,q)
```



Q3. Write a MATLAB program to compute the below two mathematically equivalent expressions for values $x = 1, 10, 100, 10000, 100000$. Also plot them, and based on the plots, find out which is better in terms of resisting the loss of significance.

$A) \sqrt{2x^2+1}+1$

$B) \frac{2x^2}{\sqrt{2x^2+1}+1}$

Answer:

```
% (A.)
x = [1,10,100,10000,100000];
for i = 1:5
    A(i) = sqrt(2*x(i)^2 +1)+1
end
```

```
A =
```

$$\begin{pmatrix} \frac{6152031499462229}{2251799813685248} & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} \end{pmatrix}$$

A =

$$\begin{pmatrix} \frac{6152031499462229}{2251799813685248} & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{8544143013451155}{562949953421312} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} \end{pmatrix}$$

A =

$$\begin{pmatrix} \frac{6152031499462229}{2251799813685248} & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{8544143013451155}{562949953421312} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \\ \frac{5011130385235165}{35184372088832} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} \end{pmatrix}$$

A =

$$\begin{pmatrix} \frac{6152031499462229}{2251799813685248} & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{8544143013451155}{562949953421312} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \\ \frac{5011130385235165}{35184372088832} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} \\ \frac{3887635527594689}{274877906944} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} \end{pmatrix}$$

A =

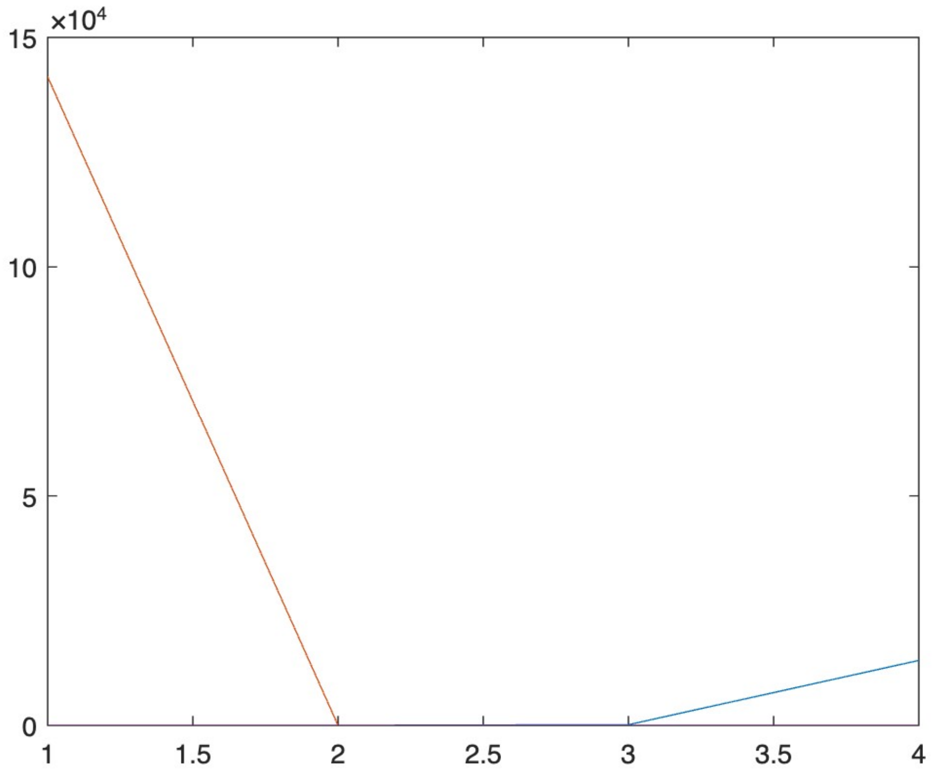
$$\begin{pmatrix} \frac{6152031499462229}{2251799813685248} & \frac{607404394977691}{4294967296} & \frac{1}{3} & \frac{1}{4} \\ \frac{8544143013451155}{562949953421312} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \\ \frac{5011130385235165}{35184372088832} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} \\ \frac{3887635527594689}{274877906944} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} \end{pmatrix}$$

```
%(B.)
for j = 1:5
    B(i) = 2*x(i)^2/(sqrt(2*x(i)^2 +1)+1)
```

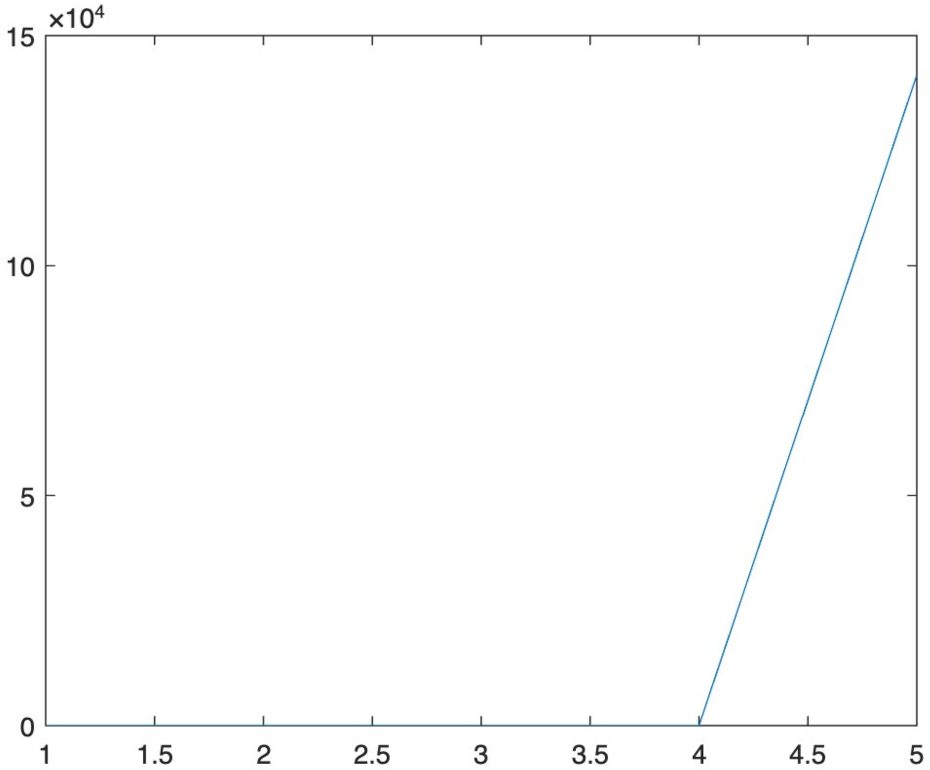
```
end
```

```
B = 1×5
10^5 ×
         0         0         0         0    1.4142
B = 1×5
10^5 ×
         0         0         0         0    1.4142
B = 1×5
10^5 ×
         0         0         0         0    1.4142
B = 1×5
10^5 ×
         0         0         0         0    1.4142
B = 1×5
10^5 ×
         0         0         0         0    1.4142
```

```
plot(A)
```




```
plot(B)
```



Q4. Consider the following two expressions:

$$f(x) = \sqrt{x}\,(\sqrt{x+1} - \sqrt{x}),$$

$$g(x) = \frac{\sqrt{x}}{\sqrt{x+1} + \sqrt{x}}.$$

Write a MATLAB program to compute the values of the two expressions when $x = 1, 10, 100, 10000, 100000$. Observe what happens as increases and what kind of computing error is happening upon execution.

Answer:

```
x = [1,10,100,10000,100000];
f = sqrt(x).*(sqrt(x+1)-sqrt(x))     % Roundoff Error
```

```
f = 1×5
    0.4142    0.4881    0.4988    0.5000    0.5000
```

```
% g = sqrt(x)./(sqrt(x+1)+sqrt(x))   % Roundoff error
```

Q5. Write a MATLAB program to compute $\frac{y^n}{e^{nx}}$ and $\left(\frac{y}{e^x}\right)^n$ for values of $x = 36$ and $y = 1e16$ for values of $n = -20, -19, 19, 20$. Which of these expressions is the right one to avoid overflow/ underflow ?

Answer:

```
n=[-20,-19,19,20];
y=1e16;
x=36;
y.^n ./ exp(n.*x) % Overflow when n = 20 and underflow for n = -20 & -19
```

```
ans = 1×4
10^6 ×
    0.0000    0.0000    8.7614       NaN
```

```
(y./exp(x)).^n    % Underflow for n = -20 & -19
```

```
ans = 1×4
10^7 ×
    0.0000    0.0000    0.8761    2.0322
```

Q6. For $x = 9.8^{201}$ and $y = 10.2^{199}$, evaluate the following two expressions that are mathematically equivalent and tell which is better in terms of the power of resisting the overflow:

$z = \sqrt{x^2 + y^2}$,

$z = y\sqrt{\left(\frac{x^2}{y^2}\right)^2 + 1}$.

. Also evaluate the following two expressions for $x = 9.8^{-201}$ and $y = 10.2^{-199}$, and comment upon the which is better in terms of nature of overflow/underflow.

Answer:

```
x1 = 9.8^201; x2 = 9.8^-201;
y1=10.2^199;  y2=10.2^-199;

z = sqrt(x1^2 + y1^2)
```

```
z = Inf
```

```
z = y1*sqrt(((x1^2)/(y1^2))+1)    % More resistive to overflow but less to underflow
```

```
z = NaN
```

```
z = sqrt(x2^2 + y2^2)
```

```
z = 0
```

```
z = y2*sqrt(((x2^2)/(y2^2))+1)
```

```
z = NaN
```

```
% First term better for underflow and second for overflow
```

System of linear equations:

There are several numerical schemes for solving a system of linear equations of the type: $AX = B$, where A is a $M \times N$ matrix, X matrix is $N \times 1$, and B matrix is $M \times 1$.

We will deal with the three cases:

(i) The case where the number (M) of equations and the number (N) of unknowns are equal (M = N) so that the coefficient matrix A is square.

(ii) The case where the number (M) of equations is smaller than the number(N) of unknowns (M < N) so that we might have to find the minimum-norm solution among the numerous solutions.

(iii) The case where the number of equations is greater than the number of unknowns (M > N) so that there might exist no exact solution and we must find a solution based on global error minimization, like the "least-squares error (LSE) solution.

Q7. Write a MATLAB routine lineqsol() to solve a given set of linear equations, covering all of the above three cases.

Answer:

```
A=[1;2];
b=[2.9;3.9];
X=lineq(A,b);
disp(X);
```

```
    2.1400
```

Q8. Solve the linear equations using command lineqsol() to solve the given set of linear equations, covering all of three cases depicted below:

(a.) A = [1 2; 3 4]; b = [-1;1]

(b.) A = [1 2; 2 4]; b = [-1;1]

(c.) A = [1; 2]; b = 3

(d.) A = [1; 2]; b = [2.1; 3.9]

Answer:

```
A=[1 2; 3 4];
b=[-1;1];
X=lineq(A,b);
disp("a)")
```

```
a)
```

```
disp(X);
```

```
    3.0000
   -2.0000
```

```
A=[1 2;2 4];
b=[-1;1];
```

```
X=lineq(A,b);
disp("b)")
```

```
b)
```

```
disp(X);
```

```
    0.0400
    0.0800
```

```
A=[1 2];
b=3;
X=lineq(A,b);
disp("c)")
```

```
c)
```

```
disp(X);
```

```
    0.6000
    1.2000
```

```
A=[1;2];
b=[2.9;3.9];
X=lineq(A,b);
disp("d)")
```

```
d)
```

```
disp(X);
```

```
    2.1400
```

```
function X=lineq(A,b)
  m=size(A,1);
  n=size(A,2);
  % Critically determined case
  if m==n
    % Checking for singular matrix
    if cond(A) > 1e14
      X=pinv(A)*b;
    else
   %  X = inv(A)*b;
      X = A\b;
    end
  else
    % Underdetermined case
    if m<n
      X=A'*pinv(A*A')*b;
    % Overdetermined case
    else
      X=pinv(A'*A)*A'*b;
    end
  end
end
```