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Semester: VII
Subject code – PC 702 **Subject: Transport Phenomena**
Unit: -IV
Lecture no.22

Heat Conduction with a Nuclear Heat Source

6 Heat Conduction with a Nuclear Heat Source

Problem: Consider a double-layer spherical nuclear fuel element. There is a core spherical nuclear material surrounded by a spherical coating. The radius of the nuclear material is given as R_f, and the radius from the center to the outer edge of the coating is given as R_c. Find the heat flux and temperature profile, assuming that the source energy is not constant and is given as

$$S_n = S_{n0}\left[1 + b\left(\frac{r}{R_f}\right)^2\right]$$

where S_{n0} and b are constants. The outer temperature of the system is also at T_0.
Solution:

1. Spherical coordinates are best used here. The temperature and heat flux are only functions of r, and the spherical shell will have a thickness that approaches zero in the radial direction. The only terms are conduction and source for the heat flux from the nuclear fission. However, there is also another expression for the heat flux through the coating part. The two expressions will be identical, but the heat flux through the coating will not have a source.

2. Writing the shell energy balance for the fission heat flux yields with $A = 4\pi r^2$ (the surface area of a sphere) and $V = 4\pi r^2 \Delta r$,

$$4\pi\left[\left(r^2 q_r^f\right)\big|_r - \left(r^2 q_r^f\right)\big|_{r+\Delta r}\right] + 4\pi r^2 \Delta r S_n = 0$$

3. Dividing by $4\pi\Delta r$ and rearranging the terms to make use of the definition of the derivative yields

$$\frac{d\left(r^2 q_r^f\right)}{dr} = r^2 S_n$$

   (a) Since S_n is a function of r, the expression will need to be substituted in for proper integration. This yields

$$\frac{d\left(r^2 q_r^f\right)}{dr} = r^2 S_{n0}\left[1 + b\left(\frac{r}{R_f}\right)^2\right]$$

4. As stated before, the heat flux through the coating will be identical but without a source, so it is

$$\frac{d\left(r^2 q_r^c\right)}{dr} = 0$$

5. Integrating the equations in Step 3(a) and Step 4 independently yields

$$q_r^f = S_{n0}\left(\frac{r}{3} + \frac{br^3}{5R_f^2}\right) + \frac{C_1^f}{r^2}$$

$$q_r^c = \frac{C_1^c}{r^2}$$

6. The value of $r = 0$ is only applicable for the inner fissionable material (since the radial values of the coating are $r = R_f$ to $r = R_c$ instead of $r = 0$ to $r = R_f$). It is clear that $r = 0$, the C_1^f terms becomes unphysical, so $C_1^f = 0$ and thus

$$q_r^f = S_{n0}\left(\frac{r}{3} + \frac{br^3}{5R_f^2}\right)$$

7. At the interfaces, the heat flux must be continuous, so at $r = R_f$, $q_r^c = q_r^f$ such that

$$C_1^c = q_r^f R_f^2 = S_{n0}\left(\frac{R_f^3}{3} + \frac{bR_f^3}{5}\right)$$

8. Substituting the expression for C_1^c yields

$$q_r^c = \frac{R_f^3}{r^2} S_{n0}\left(\frac{1}{3} + \frac{b}{5}\right)$$

9. Now that both components of the heat flux are obtained, the temperatures can be found by using Fourier's Law:

$$-k^f \frac{dT^f}{dr} = S_{n0}\left(\frac{r}{3} + \frac{br^3}{5R_f^2}\right)$$

and

$$-k^c \frac{dT^c}{dr} = \frac{R_f^3}{r^2} S_{n0}\left(\frac{1}{3} + \frac{b}{5}\right)$$

10. Integrating the above equations yields

$$T^f = -\frac{S_{n0}}{k^f}\left(\frac{r^2}{6} + \frac{br^4}{20R_f^2}\right) + C_2^f$$

and

$$T^c = \frac{S_{n0}}{k^c}\left(\frac{1}{3} + \frac{b}{5}\right)\frac{R_f^3}{r} + C_2^c$$

11. At the interfaces, the temperature must be continuous, so at $r = R_f$, $T^f = T^c$, and at $r = R_c$, $T^c = T^0$. Therefore, applying these boundary conditions and solving for the constants yields

$$T^f = \frac{S_{n0}R_f^2}{6k^f}\left[\left[1 - \left(\frac{r}{R_f}\right)^2\right] + \frac{3}{10}b\left[1 - \left(\frac{r}{R_f}\right)^4\right]\right] + \frac{S_{n0}R_f^2}{3k^c}\left(1 + \frac{3b}{5}\right)\left(1 - \frac{R_f}{R_c}\right) + T_0$$

$$T^c = \frac{S_{n0}R_f^2}{3k^c}\left(1 + \frac{3b}{5}\right)\left(\frac{R_f}{r} - \frac{R_f}{R_c}\right) + T_0$$

7 Thermal Resistance

(a) Rectangular R_{th}

- It turns out that many physical phenomena can be described by Flow Rate = Driving Force/Resistance
- Consider heat flowing in the x direction through a rectangular object with length B and area A. If the temperature change is linear,

$$q_x = k\frac{\Delta T}{B} \therefore Q = kA\frac{T_1 - T_2}{B}$$

- Since the change in temperature is the driving force, and Q is the flow rate, the thermal resistance in a rectangular system can be defined as

$$R_{th} = \frac{B}{kA}$$

(b) Defining a General Q Using R_{th}

- Using the above expression for R_{th}, Q can be rewritten as

$$Q = \frac{T_1 - T_2}{R_{th}} = \frac{\Delta T}{R_{th}}$$

  - The above expression holds true for systems in any coordinate system as long as R_{th} is appropriately redefined
  - Here, ΔT is not final minus initial. It is always a positive quantity, and it it is frequently $T_{hot} - T_{cold}$

(c) Cylindrical Shell R_{th}

Problem: Consider a cylindrical tube (with a hollowed out center) with inner radius R_1 and outer radius R_2. The temperature of the innermost surface is T_1 and outermost surface is T_2. Assume heat flows radially and the cylinder has length L. Derive R_{th}.

Solution:

1. Using cylindrical coordinates, the microscopic energy balance can be rewritten as

$$\frac{1}{r}\frac{d}{dr}\left(kr\frac{dT}{dr}\right) = 0$$

2. Integrating once yields

$$\frac{dT}{dr} = \frac{C_1}{r}$$

3. Integrating again yields

$$T = C_1 \ln r + C_2$$

4. The temperatures were defined at the interfaces, so at $r = R_1$, $T = T_1$, and at $r = R_2$, $T = T_2$ such that

$$T_1 = C_1 \ln R_1 + C_2$$

and

$$T_2 = C_1 \ln R_2 + C_2$$

5. Solving for C_1 yields

$$C_1 = -\frac{T_1 - T_2}{\ln\left(R_2/R_1\right)}$$

   (a) Note that C_2 is not needed (but can easily be solved for) since we are looking for heat flux, and that simply requires C_1 in Step 2 to be found

6. Substituting C_1 in Step 2 and using Fourier's Law yields

$$q_r = \frac{k\left(T_1 - T_2\right)}{r\ln\left(R_2/R_1\right)}$$

7. The heat flow can then be found as

$$Q = q_r A_r = \frac{2\pi Lk\left(T_1 - T_2\right)}{\ln\left(R_2/R_1\right)}$$

8. Since the temperature is the driving force, the thermal resistance can be defined as

$$R_{th} = \frac{\ln\left(R_2/R_1\right)}{2\pi kL}$$

(d) **Spherical Shell** R_{th}

- For radial conduction in a spherical system, a similar procedure can be used to find that

$$R_{th} = \frac{R_2 - R_1}{4\pi k R_1 R_2}$$