UIT-RGPV (Autonomous) Bhopal
Department of Petrochemical Engineering

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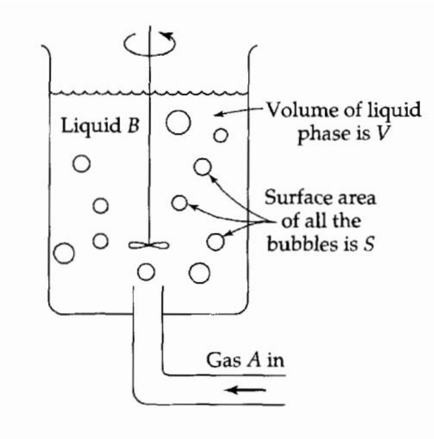
Semester: VII
Subject code – PC 702 **Subject: Transport Phenomena**

Unit: -V
Lecture no.28

Mass transfer

1. Gas Absorption with Chemical Reaction in Agitated Tank

Problem: Consider the diagram shown below. Assume that each gas bubble is surround by a stagnant liquid film of thickness δ,which is small compared to the bubble diameter. Assume a quasi-steady concentration profile is quickly established in the liquid film after the bubble is formed. The gas A is only sparingly soluble in the liquid, so we can neglect the convection term. The liquid outside the stagnant film is at concentration $c_{A\delta}$ and is constant. Even though this is a spherical bubble, it is a thin shell, so you can treat it as a slab. For the full problem statement ·



1. The setup is the same as before, but the boundary conditions are at $z=0$, $\xi=0$,$c_A=c_{A0}$, $\Gamma=1$, and at $z=\delta$, $\xi=1$, $c_A=c_{A\delta}$, $\Gamma=B$ if we state that $B=\dfrac{C_{A\delta}}{C_{A0}}$. Note that the dimensionless length should be redefined accordingly as $\xi=\dfrac{z}{\delta}$ and the Thiele modulus is redefined as $\phi=\sqrt{k_1'''\delta^2/\mathscr{D}_{AB}}$

2. From the previous problem,
$$\Gamma = C_1\cosh(\phi\xi)+C_2\sinh(\phi\xi)$$

3. Applying boundary condition 1 yields
$$C_1=1$$

4. Applying boundary conditions 2 yields
$$C_2=\frac{B-\cosh\phi}{\sinh\phi}$$

5. This means
$$\Gamma=\cosh(\phi\xi)+\frac{B-\cosh\phi}{\sinh\phi}\sinh(\phi\xi)=\frac{\sinh\phi\cosh(\phi\xi)+(B-\cosh\phi)\sinh(\phi\xi)}{\sinh\phi}$$

6. Now equate A entering the liquid at $z = \delta$ to amount consumed in bulk:

$$-S\mathscr{D}_{AB}\frac{dc_A}{dz}\bigg|_{z=\delta} = Vk_1^{'''}c_{A\delta}$$

7. We need the $\frac{ac_A}{dz}\Big|_{z=\delta}$ term. This can be rewritten as

$$\frac{dc_A}{dz} = \frac{dc_A}{d\xi}\frac{d\xi}{dz} = \frac{dc_A}{d\xi}\frac{1}{\delta}$$

8. Therefore,

$$\frac{dc_A}{dz}\bigg|_{z=\delta} = \frac{c_{A0}}{\delta}\left(\frac{\phi\sinh^2\phi - \phi\cosh^2\phi + B\phi\cosh\phi}{\sinh\phi}\right)$$

   using the identity $\cosh x^2 - \sinh x^2 = 1$ yields

$$\frac{dc_A}{dz}\bigg|_{z=\delta} = \frac{c_{A0}}{\delta}\left(\frac{B\phi\cosh\phi - \phi}{\sinh\phi}\right)$$

9. So,

$$-S\mathscr{D}_{AB}\frac{c_{A0}}{\delta}\left(\frac{B\phi\cosh\phi - \phi}{\sinh\phi}\right) = Vk_1^{'''}c_{A\delta}$$

10. This can be solved for B as

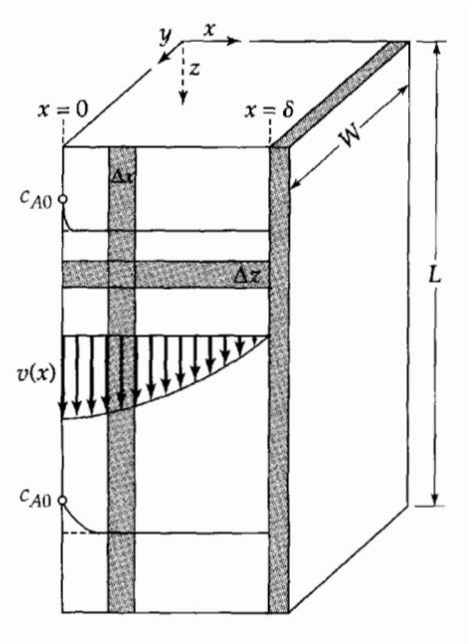
$$B = \frac{1}{\cosh\phi + \frac{V}{S\delta}\phi\sinh\phi}$$

11. The total rate of absorption is

$$\check{N} \equiv \frac{N_{A,z}|_{z=0}\,\delta}{c_{A0}\mathscr{D}_{AB}} = \frac{\phi}{\sinh\phi}\left(\cosh\phi - \frac{1}{\cosh\phi + \frac{V}{S\delta}\phi\sinh\phi}\right)$$

2. Diffusion into a Falling Liquid Film (Gas Absorption)

Problem: Consider the absorption of A into a falling film of liquid B.



1. The velocity profile is found from Transport I as

$$v_z\left(x\right) = v_{max}\left[1-\left(\frac{x}{\delta}\right)^2\right]$$

2. The concentration will change in the x and z direction, so

$$\left.N_{A,z}\right|_z W\Delta x - \left.N_{A,z}\right|_{z+\Delta z} W\Delta x + \left.N_{A,x}\right|_x W\Delta z - \left.N_{A,x}\right|_{x+\Delta x} W\Delta z = 0$$

at steady state

3. This then yields

$$\frac{\partial N_{A,z}}{\partial z} + \frac{\partial N_{A,x}}{\partial x} = 0$$

4. We now want expressions for the molar mass flux:

$$N_{A,z} = -\mathscr{D}_{AB}\frac{\partial c_A}{\partial z} + x_A\left(N_{A,z}+N_{B,z}\right)$$

which reduces to the following because the transport of A in the z direction will be primarily by convection (not diffusion)

$$N_{A,z} = x_A\left(N_{A,z}+N_{B,z}\right) \approx c_A v_z\left(x\right)$$

and in the x direction we have

$$N_{A,x} = -\mathscr{D}_{AB}\frac{\partial c_A}{\partial z}$$

since there is mostly diffusion in the x direction (not convection)

5. Therefore,

$$v_z\frac{\partial c_A}{\partial z} = \mathscr{D}_{AB}\frac{\partial^2 c_A}{\partial x^2}$$

6. Inserting the velocity component yields

$$v_{z,max}\left[1-\left(\frac{x}{\delta}\right)^2\right]\frac{\partial c_A}{\partial z}=\mathscr{D}_{AB}\frac{\partial^2 c_A}{\partial x^2}$$

7. The boundary conditions are: at $z=0$, $c_A=0$ and $x=0$,$c_A=c_{A0}$ and $x=\delta$, $\frac{\partial c_A}{\partial x}=0$ since there is pure B at the top, the liquid-gas interface is determined by the solubility of A in B, and A can't diffuse through the wall

8. We shall use the limiting case of the Penetration Model, which states that there is only penetration in the outer layers of the film such that $v_z \approx v_{z,max}$. This means,

$$v_{z,max}\frac{\partial c_A}{\partial z}=\mathscr{D}_{AB}\frac{\partial^2 c_A}{\partial x^2}$$

and the third boundary condition is changed to at $x=\infty$, $c_A=0$

9. This looks like a semi-infinite solid problem, so

$$\frac{c_A}{c_{A0}}=1-\operatorname{erf}\left(\frac{x}{\sqrt{4\mathscr{D}_{AB}z/v_{z,max}}}\right)$$

10. The local mass flux at the gas-liquid interface may be found by

$$N_{A,x}|_{x=0}=-\mathscr{D}_{AB}\frac{\partial c_A}{\partial x}\bigg|_{x=0}=c_{A0}\sqrt{\frac{\mathscr{D}_{AB}v_{max}}{\pi z}}$$

11. The total molar flow across the surface at $x=0$ is

$$W_A=\int_0^W\int_0^L N_{A,x}|_{x=0}\,dz\,dy=WLc_{A0}\sqrt{\frac{4\mathscr{D}_{AB}v_{max}}{\pi L}}$$