UIT-RGPV (Autonomous) Bhopal
Department of Petrochemical Engineering

Subject In-charge: Prof. M.S. Chouhan
Semester: VII
Subject code – PC 702 **Subject: Transport Phenomena**
Unit: -III

Lecture no.17

Continuity equation Momentum Equation (Euler's Equation)

5.1 EQUATIONS OF STEADY ONE-DIMENSIONAL COMPRESSIBLE FLOW

In many situations of practical importance, compressible flow takes place through variable area duct. So in developing field equations effect of area change is considered for generalization. Basic field equations backed up by thermodynamic relations give complete picture of compressible flow. Development and use of these equations are discussed in the following sections.

5.2 CONTINUITY EQUATION

The continuity equation is derived from the principle of mass-conservation. Fig. 5.2.1 shows the control volumes for mass balance.

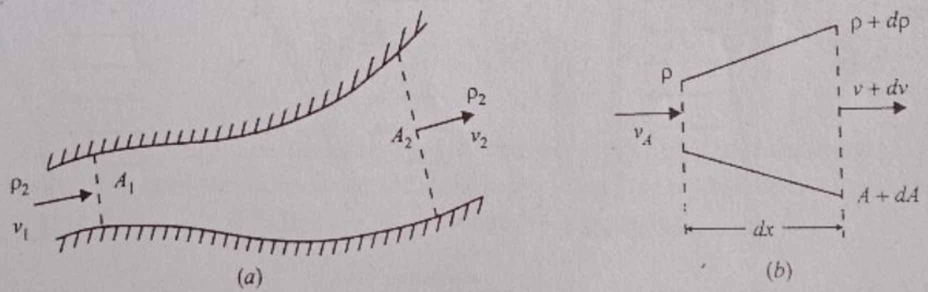

Fig. 5.2.1 Control volume for mass balance. (*a*) Finite control volume (*b*) Differential control volume

At steady state there is no mass accumulation. So mass flow rate at the inlet is equal to mass flow rate at the outlet. So for finite control volume,

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2 \qquad \text{...(5.2.1)}$$

For differential control volume, mass balance may be written as,

$$\rho VA = (\rho + d\rho)(V + dV)(A + dA) \qquad \text{...(5.2.2)}$$

Neglecting differential product terms,

Eq. (5.2.2) is written as,

$$VAd\rho + \rho AdV + \rho VdA = 0 \qquad \text{...(5.2.3)}$$

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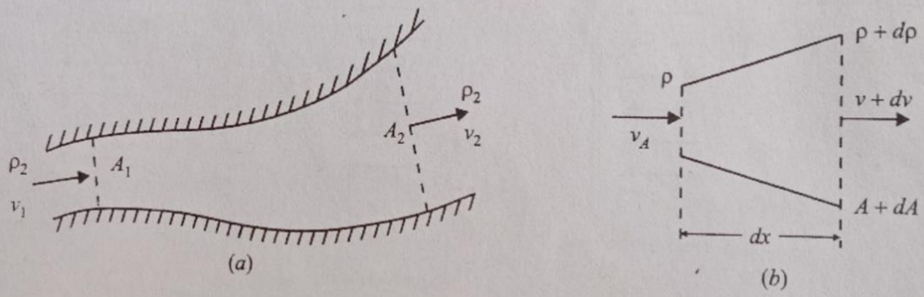

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Dividing Eq. (5.2.3) by ρVA we get,

$$\frac{d\rho}{\rho} + \frac{dV}{V} + \frac{dA}{A} = 0 \quad \text{...(5.2.4)}$$

Eq. (5.2.4) is significant because it implies that for incompressible flow where fractional density change is zero, fractional velocity change and fractional area change bears opposite sign. So if velocity increases, area decreases, and the opposite argument is also true.

But in case of compressible flow fractional density change is significant. In that case relation between velocity change and area change is not simple to predict.

MOMENTUM EQUATION (Euler's Equation)

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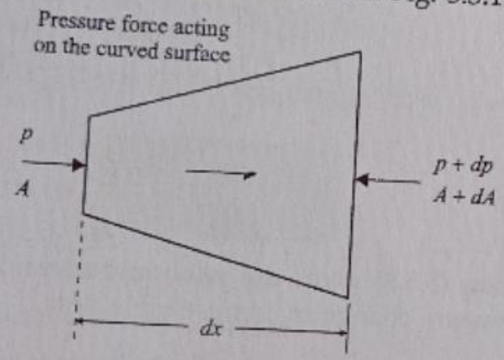
5.3 MOMENTUM EQUATION (Euler's Equation)

One dimensional momentum balance equation, which does not involve friction loss, known as Euler's equation, is obtained by momentum balance over a differential control volume as shown in Fig. 5.3.1.

Fig. 5.3.1 Differential Control volume for momentum balance.



At steady flow condition and neglecting the gravitational force and friction term, pressure force balance gives,

$$\underbrace{pA}_{\text{at inlet surface}} - \underbrace{(p+dp)(A+dA)}_{\text{at outlet surface}} + \underbrace{\frac{1}{2}[p+(p+dp)][(A+dA)-A]}_{\text{at the lateral surface}}$$

It is to be noted that the pressure force on the lateral surface is approximated as the product of average pressure on that surface *i.e.*, $\frac{1}{2}[p+(p+dp)]$ and the incremental area $2\times\frac{1}{2}dA$.

$\therefore$ The pressure force balance term $= -Adp$

In obtaining the pressure force balance term the differential product terms are neglected.

Now rate of momentum flow at any cross-section is given by, the product of mass flow rate and velocity, *i.e.*, $(\rho VA)V$.

So we can write,

Flow momentum in $= (\rho VA)V$

Flow momentum out $= (\rho VA)(V + dV)$

So the difference between momentum outflow and inflow is,

$$(\rho VA)(V + dV) - (\rho VA)V = \rho VA\,dV$$

The above derivation assumes no momentum transfer across the lateral surface.

According to Newton's second law of motion, we can write,

$$-Adp = \rho VAdV \qquad ...(5.3.1)$$

The above equation when rearranged gives one dimensional Euler's equation of motion for steady state.

$$-\frac{dp}{\rho} = VdV \qquad ...(5.3.2)$$

We have denoted V is the velocity in x-direction and always positive in that direction. So any positive change in velocity implies pressure will decrease in the direction of the flow. The opposite is also true.

When Euler's equation is integrated it gives,

$$\frac{V^2}{2} + \int \frac{dp}{\rho} = \text{constant} \qquad ...(5.3.3)$$

The integral only can be evaluated if the density-pressure relationship is known for compressible flow. Otherwise for incompressible flow (constant density flow), Eq. (5.3.3) reduces to Bernoulli's equation

$$\frac{V^2}{2} + \frac{p}{\rho} = \text{constant} \qquad ...(5.3.4)$$

The above argument implies that Bernoulli's equation only can be applied for incompressible flow.

Energy Equation

The steady state energy balance between two stations 1 and 2 in a flowing fluid as shown in Fig. 5.4.1 gives,

$$h_2 + \frac{V_2^2}{2} = h_1 + \frac{V_1^2}{2} + q - w \qquad ...(5.4.1)$$