UIT-RGPV (Autonomous) Bhopal
Department of Petrochemical Engineering

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Semester: VII
Subject code – PC 702 **Subject: Transport Phenomena**
Unit: -IV

Lecture no.20

The Overall H. T. Coefficient and H. E. Analysis: Use of the Effectiveness-NTU method

1. The Overall Heat Transfer Coefficient

The Overall Heat Transfer Coefficient

- The heat transfer between the two fluids across the solid wall involves convection of fluid films adjacent to the wall and conduction across the wall.
- The rate of heat transfer can be expressed by a single equation like Newton's law of cooling, with the overall heat transfer coefficient U incorporating convection and conduction terms:

$$q = UA\Delta T_m \tag{1}$$

where,
ΔT_m = mean temperature difference between the two fluids along the exchanger length

- For the *unfinned* tubular heat exchanger, U can be calculated as follows:

$$\frac{1}{UA} = \frac{1}{U_i A_i} = \frac{1}{U_o A_o} \tag{2}$$

$$\frac{1}{UA} = \frac{1}{h_i A_i} + \frac{R_{fi}}{A_i} + \frac{\ln(D_o / D_i)}{2\pi k L} + \frac{R_{fo}}{A_o} + \frac{1}{h_o A_o} \tag{3}$$

where A_i, A_o = inside and outside heat transfer areas
U_i, U_o = overall heat transfer coefficients based on inside and outside surface areas
h_i, h_o = inside (tube-side) convection coefficient and outside (shell-side) convection coefficient
R_{fi}, R_{fo} = fouling resistances at inside and outside surfaces
D_i, D_o = inside and outside diameters of the tube
k = thermal conductivity of the tube wall
L = tube length of heat exchanger

2. Heat Exchanger Analysis: Use of the Log Mean Temperature Difference (LMTD)

To design or to predict the performance of a heat exchanger, it is essential to relate the total heat transfer rate to quantities such as *the inlet and outlet fluid temperatures*, *the overall heat transfer coefficient*, and *the total surface area* for heat transfer. A few steps to design or predict the performance of a heat exchanger:

- **Step 1**
  - Write down the overall energy balances between heat gain of cold fluid, heat loss of hot fluid, and heat transfer across the wall separating the two fluids.

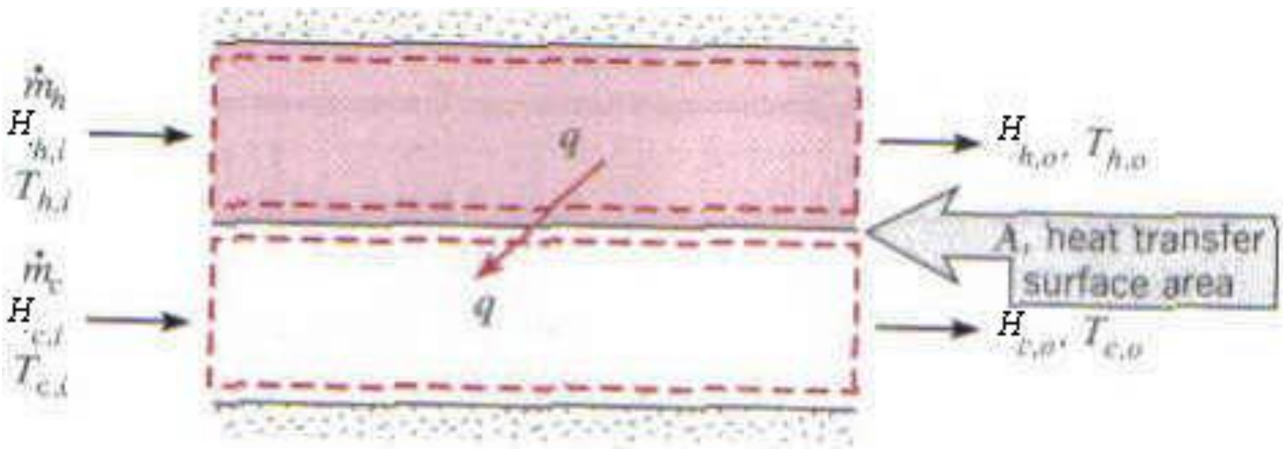
$$q = \dot{m}_c \Delta \hat{H}_c = \dot{m}_h \Delta \hat{H}_h = UA\Delta T_{lm} \tag{4}$$

  - If the fluids are not undergoing a phase change and constant specific heats are assumed, the equation becomes:

$$q = \dot{m}_h C_{p,h}(T_{h,i} - T_{h,o}) = \dot{m}_c C_{p,c}(T_{c,o} - T_{c,i}) = UA\Delta T_{lm} \tag{5}$$

  - Determine any unknown that can be directly calculated from the above relations.

$$q = \text{rate of heat transfer}$$
$$\dot{m}_c, \dot{m}_h = \text{mass flow rate of cold fluid and hot fluid}$$
$$T_{ci}, T_{co}, T_c = \text{inlet, outlet, and mean temperature of cold fluid}$$
$$T_{hi}, T_{ho}, T_h = \text{inlet, outlet, and mean temperature of hot fluid}$$
$$\hat{H}_c, \hat{H}_h = \text{specific enthalpies of cold fluid and hot fluid}$$
$$U = \text{overall heat ransfer coefficient}$$
$$A = \text{heat transfer surface area}$$
$$\Delta T_{lm} = \text{log mean temperature difference (LMTD)}$$
$$= \frac{\Delta T_1 - \Delta T_2}{\ln\left(\Delta T_1 / \Delta T_2\right)}, \text{ where } \Delta T_1 \text{ and } \Delta T_2 \text{ are}$$
temperature differences at the two ends of HE.



$$q = \dot{m}_c \Delta \widehat{H}_c = \dot{m}_h \Delta \widehat{H}_h = UA\Delta T_{lm}$$

Enthalpy change of cold fluid

Enthalpy change of hot fluid

Figure 7.10: Overall Energy Balances of the Hot and Cold Fluids of a Two-fluid Hea Exchanger

- **Step 2**
  - Enthalpies and LMTD depends on fluid temperature behavior:

  (a) Parallel flow

  For parallel flow with no phase change,

$$\Delta H_h = C_{ph}(T_{hi}\text{-}T_{ho}),\ \Delta H_c = C_{pc}(T_{co}\text{-}T_{ci})$$

$$\Delta T_{lm} = \frac{(T_{hi} - T_{ci}) - (T_{ho} - T_{co})}{\ln[(T_{hi} - T_{ci})/(T_{ho} - T_{co})]}$$

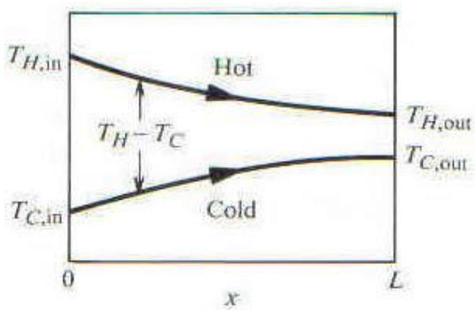


Figure: Temperature scheme for parallel flow

(b) Counterflow

For counterflow with no phase change,

$$\Delta H_h = C_{ph}(T_{hi}\text{-}T_{ho}),\ \Delta H_c = C_{pc}(T_{co}\text{-}T_{ci})$$

$$\Delta T_{lm} = \frac{(T_{ho}-T_{ci})-(T_{hi}-T_{co})}{\ln[(T_{ho}-T_{ci})/(T_{hi}-T_{co})]}$$

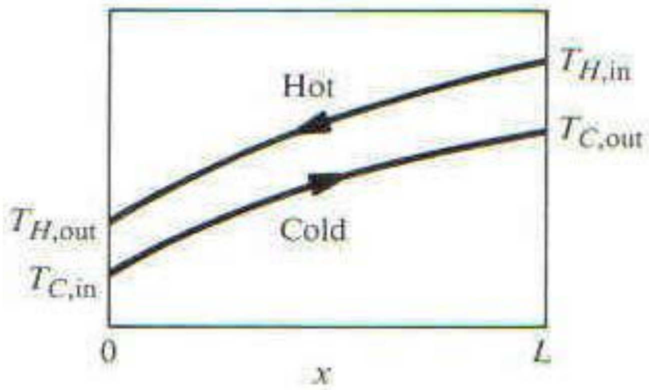


Figure : Temperature scheme for counterflow

(c) Condensers

$$\Delta H_h = \lambda_h,\ \Delta H_c = C_{pc}(T_{co}\text{-}T_{ci})$$

$$\Delta T_{lm} = \frac{T_{co}-T_{ci}}{\ln[(T_h-T_{ci})/(T_h-T_{co})]}$$

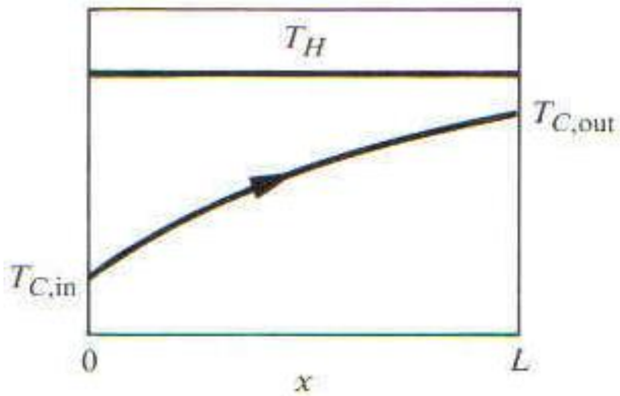


Figure : Temperature scheme for condenser

(d) Evaporators

$$\Delta H_c = \lambda_c,\ \Delta H_h = C_{ph}(T_{hi}\text{-}T_{ho})$$

$$\Delta T_{lm} = \frac{T_{hi} - T_{ho}}{\ln[(T_{hi} - T_c)/(T_{ho} - T_c)]}$$

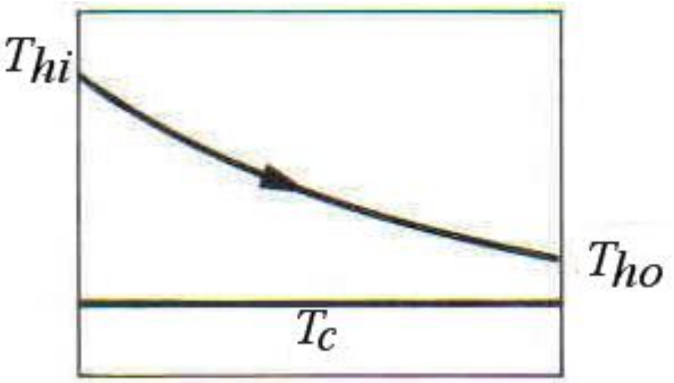


Figure : Temperature scheme for evaporator

where λ= latent heat of vaporization

- Miscellaneous

- Overall heat transfer coefficient (U), if not known, can be determined Equation ·
- Heat transfer area (A) is related to tube length as $A = 2\pi rL$.

- **Step 3**
  - Determine any other unknowns from the overall energy balances
  - Determine the HE effectiveness and number of transfer units.

- If the heat exchanger other than the double pipe is used, the heat transfer is calculated by using a correction factor applied to the LMTD for a counter flow double-pipe arrangement with the same hot and cold fluids temperatures.
- The heat-transfer equation becomes :

$$q = UAF\,\Delta T_m$$

- When phase changed is involved, as in condensation or boiling (evaporation), the fluid normally remains at essentially constant temperature.
- For this condition, P and R becomes zero and F = 1.0 (for boiling and condensation)

Example

Water at the rate 68 kg/min is heated from 35 to 75^{0}C by an oil having specific heat of 1.9 kJ/kg.^{0}C. The fluids are used in a *counterflow double pipe heat exchanger* and the oil enter the exchanger at 110^{0}C and leaves at 75^{0}C. The overall heat-transfer coefficient is 320 W/m^2.^{0}C. Calculate the heat exchanger area.

Solution:
The total heat transfer is determined from the energy adsorbed by the water:

$$\begin{aligned} q &= \dot{m}_w C_w \Delta T_w \\ &= (68)(4180)(75-35) \\ &= 11.37 MJ/\min \\ &= 189.5 kW \end{aligned}$$

Since all the fluid temperature are known, the LMTD can be calculated by using the temperature scheme in Figure 10.12,

$$\Delta Tm = \frac{(110-75)-(75-35)}{\ln(110-75)/(75-35)} = 37.44^0 C$$

Then, since $q = UA\Delta Tm$

$$A = \frac{1.895x10^5}{(320)(37.44)} = 15.82 m^2$$

Example
A cross flow heat exchanger is used to heat an oil in the tubes, c = 1.9 kJ/kg.^{0}C from 15^{0}C to 85^{0}C. Blowing across the outside of the tubes is steam which enters at 130^{0}C and leaves at 110^{0}C with mass flow of 5.2kg/sec. The overall heat transfer coefficient is 275 W/m^2.^{0}C and c for steam is 1.86 kJ/kg.^{0}C. Calculate the surface area of the heat exchanger.

Solution:
The total heat transfer may be obtained from an energy balance on the steam:

$$q = \dot{m}_s c_s \Delta T_s = (5.2)(1.86)(130-110) = 193 kW$$

We can solve for the area from Eq.(8.6). The value of ΔTm calculated is as if the exchanger were counterflow double pipe.
Thus,

$$\Delta T_m = \frac{(130-85)-(110-15)}{\ln\left(\dfrac{130-85}{110-15}\right)} = 66.9^0 C$$

t_1 and t_2 will represent the unmixed fluid (oil) and T_1 and T_2 will represent the mixed fluid (the steam) so that

$T_1 = 130^0$C $T_2 = 110^0$C $t_1 = 15\ ^0$C $t_2 = 85^0$C and we calculate

$$R = \frac{130-110}{85-15} = 0.286$$

$$P = \frac{85-15}{130-15} = 0.609$$

so the area is calculated from

$$A = \frac{q}{UF\Delta T_m}$$

3. Heat Exchanger Analysis: Use of the Effectiveness-NTU method

(a) Introduction

The driving temperature across the heat transfer surface varies with position, but an appropriate mean temperature can be defined. In most simple systems this is the log mean temperature difference (LMTD). Sometimes direct knowledge of the LMTD is not available and the NTU method is used.

- The LMTD approach to heat-exchanger analysis is useful when inlet and outlet temperature are known or are easily determined.

- However, when the inlet or exit temperatures are to be evaluated for a given heat exchanger, the analysis involves an interactive procedure because of the logarithmic function in the LMTD.

- In these cases the analysis is performed more easily by utilizing a method based on the effectiveness of the heat exchanger in transferring a given amount of heat.

- The heat exchanger effectiveness can be define as:

$$Effectiveness, \varepsilon = \frac{\text{Actual rate of heat transfer}}{\text{Maximum possible rate of heat transfer}}$$

- The *actual rate heat transfer, q* may be computed by calculating either the energy lost by the hot fluid or the energy gained by the cold fluid.

- For the parallel-flow exchanger:

$$q = \dot{m}_h c_h (T_{h1} - T_{h2}) = \dot{m}_c c_c (T_{c2} - T_{c1})$$

- For the counter-flow exchanger :

$$q = \dot{m}_h c_h (T_{h1} - T_{h2}) = \dot{m}_c c_c (T_{c1} - T_{c2})$$

- The maximum possible heat transfer, q_{max} is the rate of heat transfer that a heat exchanger of infinite area would transfer with given inlet temperatures, flow rates, and specific heat.

- q_{max} occurs when the fluid with minimum product of flow rate and specific heat changes temperature to the entering temperature of the other fluid.

- Maximum possible heat transfer is expressed as

$$q_{\max} = (\dot{m}\,c)_{\min}(T_{hi} - T_{ci})$$

- The minimun fluid may be either hot or cold fluid depending on *mass-flowrates, m* and *specific heats, c*.

- For the parallel exchanger :

$$\varepsilon = \frac{\dot{m}_h\, c_h (T_{h1} - T_{h2})}{\dot{m}_h\, c_h (T_{h1} - T_{c1})} = \frac{T_{h1} - T_{h2}}{T_{h1} - T_{c1}}$$

$$\varepsilon = \frac{\dot{m}_c\, c_c (T_{c2} - T_{c1})}{\dot{m}_c\, c_c (T_{h1} - T_{c1})} = \frac{T_{c2} - T_{c1}}{T_{h1} - T_{c1}}$$

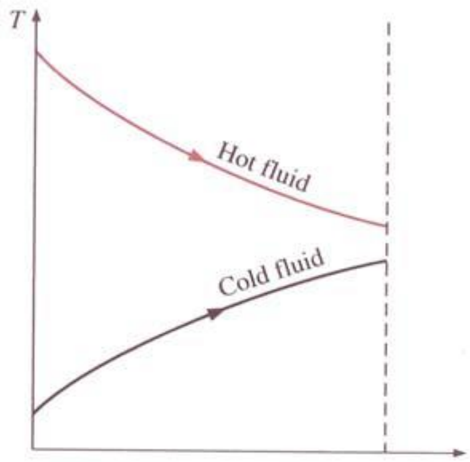


Figure: Temperature profile for parallel exchanger

- In the general way the effectiveness is expressed as :

$$\varepsilon = \frac{\Delta T \text{ (minimum fluid)}}{\text{Maximum temperature difference in heat exchanger}}$$

- The effectiveness is usually written for parallel flow double pipe HE :

$$\varepsilon = \frac{1-\exp[(-UA/\dot{m}_c c_c)(1+\dot{m}_c c_c/\dot{m}_h c_h)}{1+\dot{m}_c c_c/\dot{m}_h c_h}$$

$$\varepsilon = \frac{1-\exp[(-UA/C_{\min})(1+C_{\min}/C_{\max})}{1+C_{\min}/C_{\max}}$$

where $C = \dot{m}c$ = capacity rate

- The number of transfer units (NTU) is indicative of the size of the heat exchanger. $NTU = UA/C_{\min}$

- Figure 7.21 to 7-26 presented effectiveness ratios for various heat exchanger arrangements.
- Table 7.1 and Table 7.2 summarizes the effectiveness relations

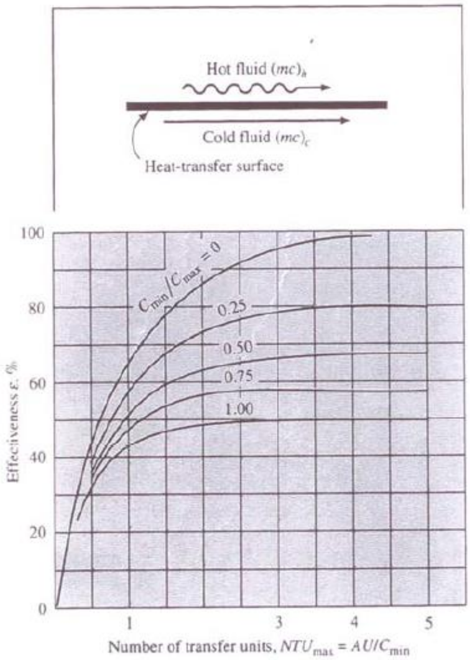


Figure 7.21: Effectiveness for parallelflow exchanger performance

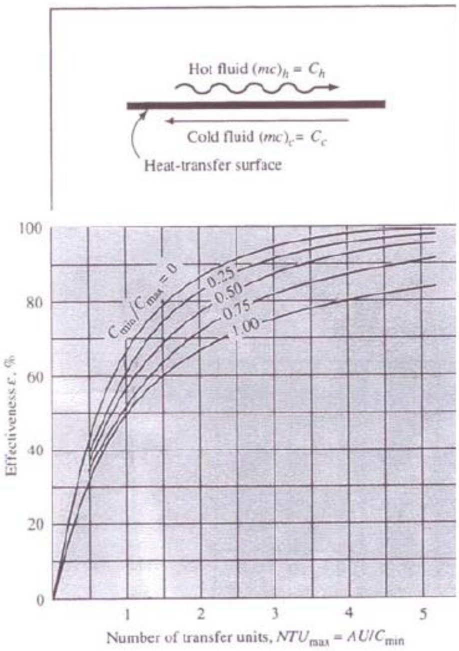


Figure 7.22: Effectiveness for counterflow exchanger performance

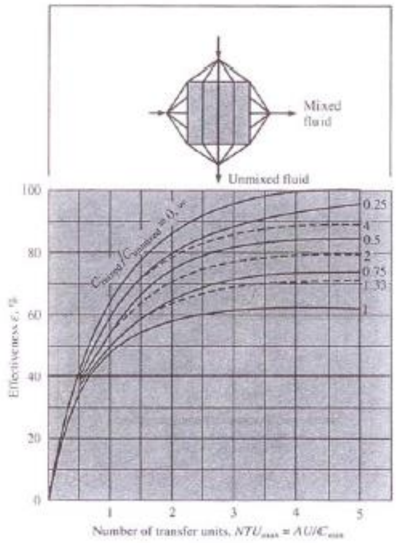


Figure 7.23 : Effectiveness for crossflow exchanger with one fluid mixed

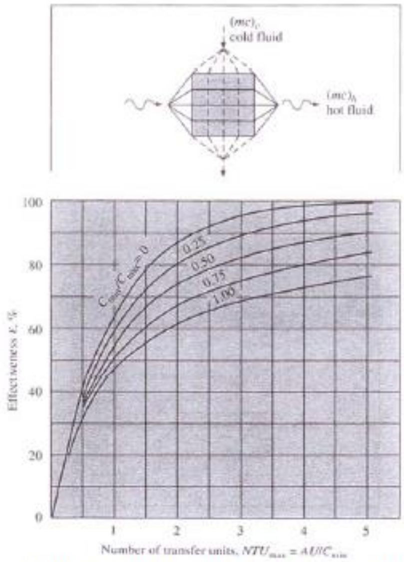


Figure 7.24: Effectiveness for crossflow exchanger with one fluid unmixed

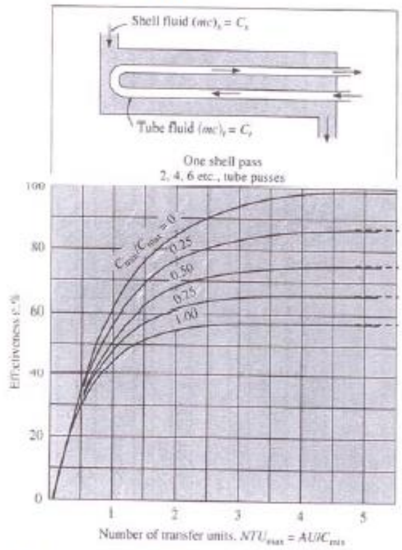


Figure 7.25 : Effectiveness for 1-2 parallel counterflow exchanger performance

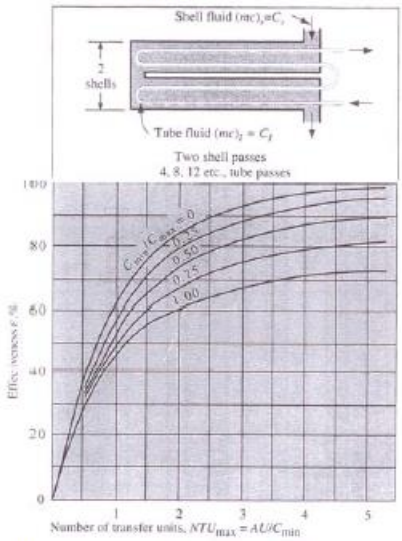


Figure 7.26 : Effectiveness for 2-4 multipass counterflow exchanger performance

Table 7.1: Heat exchanger effectiveness relations

$N = \text{NTU} = \dfrac{UA}{C_{min}}$ $\quad C = \dfrac{C_{min}}{C_{max}}$

Flow geometry	**Relation**
Double pipe:	
Parallel flow	$\epsilon = \dfrac{1 - \exp[-N(1+C)]}{1+C}$
Counterflow	$\epsilon = \dfrac{1 - \exp[-N(1-C)]}{1 - C\exp[-N(1-C)]}$
Counterflow, $C = 1$	$\epsilon = \dfrac{N}{N+1}$
Cross flow:	
Both fluids unmixed	$\epsilon = 1 - \exp\left[\dfrac{\exp(-NCn) - 1}{Cn}\right]$ where $n = N^{-0.22}$
Both fluids mixed	$\epsilon = \left[\dfrac{1}{1-\exp(-N)} + \dfrac{C}{1-\exp(-NC)} - \dfrac{1}{N}\right]^{-1}$
C_{max} mixed, C_{min} unmixed	$\epsilon = (1/C)\{1 - \exp[-C(1 - e^{-N})]\}$
C_{max} unmixed, C_{min} mixed	$\epsilon = 1 - \exp\{-(1/C)[1 - \exp(-NC)]\}$
Shell and tube:	
One shell pass, 2, 4, 6, tube passes	$\epsilon = 2\left\{1 + C + (1+C^2)^{1/2} \times \dfrac{1 + \exp[-N(1+C^2)^{1/2}]}{1 - \exp[-N(1+C^2)^{1/2}]}\right\}^{-1}$
Multiple shell passes, $2n$, $4n$, $6n$ tube passes (ϵ_p = effectiveness of each shell pass, n = number of shell passes)	$\epsilon = \dfrac{[(1-\epsilon_p C)/(1-\epsilon_p)]^n - 1}{[(1-\epsilon_p C)/(1-\epsilon_p)]^n - C}$
Special case for $C = 1$	$\epsilon = \dfrac{n\epsilon_p}{1 + (n-1)\epsilon_p}$
All exchangers with $C = 0$	$\epsilon = 1 - e^{-N}$

Table 7.2: NTU relations for heat exchangers

$C = C_{min}/C_{max}$ ϵ = effectiveness	N = NTU = UA/C_{min}
Flow geometry	**Relation**
Double pipe:	
Parallel flow	$N = \dfrac{-\ln[1-(1+C)\epsilon]}{1+C}$
Counterflow	$N = \dfrac{1}{C-1}\ln\left(\dfrac{\epsilon-1}{C\epsilon-1}\right)$
Counterflow, $C = 1$	$N = \dfrac{\epsilon}{1-\epsilon}$
Cross flow:	
C_{max} mixed, C_{min} unmixed	$N = -\ln\left[1+\dfrac{1}{C}\ln(1-C\epsilon)\right]$
C_{max} unmixed, C_{min} mixed	$N = \dfrac{-1}{C}\ln[1+C\ln(1-\epsilon)]$
Shell and tube:	
One shell pass, 2, 4, 6, tube passes	$N = -(1+C^2)^{-1/2} \times \ln\left[\dfrac{2/\epsilon-1-C-(1+C^2)^{1/2}}{2/\epsilon-1-C+(1+C^2)^{1/2}}\right]$
All exchangers, $C = 0$	$N = -\ln(1-\epsilon)$