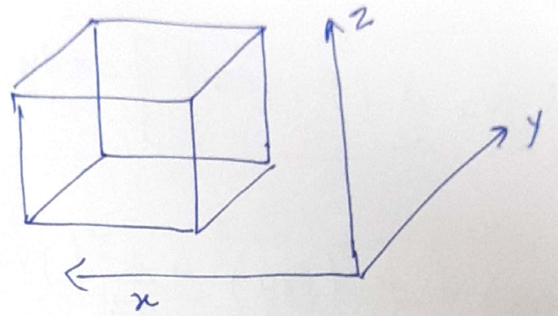
Continuity equation (conservation of mass)

We apply principle of conservation of mass to differential control volume dx, dy, dz through which fluid flows take place.

This volume is imaginary volume. We choose a Cartesian coordinate system (x, y, z) and three velocity component in x, y & z directions are V_x, V_y, V_z

The velocity & density of fluid ρ are function of position and time.



The flow of mass per unit time & per unit area through a surface is product of density and velocity normal to surface.

Thus x component of mass flux entering the surface at x is ρV_x. flux changes from point to point.

Leaving the surface at x+dx. is $\rho V_x + \frac{d}{dx}(\rho V_x)dx$

The Rate of Mass flow into control volume is

$$\rho V_x \, dy dz + \rho V_y \, dx dy + \rho V_z \, dx dy$$

★ Better

The gross rate of mass out flow is

$$\left\{\rho v_x + \frac{d}{dx}(\rho v_x)\,dx\right\}dy\,dz + \left[\rho v_y + \frac{d}{dy}(\rho v_y)\,dy\right]dx\,dz$$

$$+\left\{\rho v_z\ \frac{d}{dz}(\rho v_z)\,dz\right\}dx\,dy$$

The net rate of mass inflow, found by subtracting the outflow from inflow in 3-D –

$$\left[\rho v_x - \left\{\rho v_x \frac{d}{dx}(\rho v_x)\,dx\right\}\right]dy\,dz + \left[\rho v_y - \frac{d}{dy}(\rho v_y)\,dy\right]dx\,dz$$

$$+\left[\rho v_z - \left\{\rho v_z\ \frac{d}{dz}(\rho v_z)\,dz\right\}\right]dx\,dy$$

$$\Rightarrow\ -\left[\frac{d}{dx}(\rho v_x) + \frac{d}{dy}(\rho v_y) + \frac{d}{dz}(\rho v_z)\right]dx\,dy\,dz$$

⟶ Rate of mass increase in control volume.

Equation expression for net rate of mass inflow in 3D with rate of mass increase in control volume & then dividing by control volume, dx, dy, dz we obtained.

$$\frac{\partial \rho}{\partial t} = -\left(\frac{\partial(\rho v_x)}{\partial x} + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z)\right\} \quad \text{—(1)}$$

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This eqn is called continuity equation.

This eqn describes the rate of change density at fixed point resulting from change in mass flux.

This eqn applied to a pure liquid or fluid mixture

$$\boxed{\frac{\partial \rho}{\partial t} = \nabla \cdot \rho \vec{v}}$$

↳ Euler equation significance

(1)

$$\boxed{\rho \frac{D\vec{v}}{Dt} = -\nabla p + \rho \vec{g}}$$

(2) Applies to the motion of a fluid for which forces due to viscous friction are negligible compared with pressure and gravity forces.

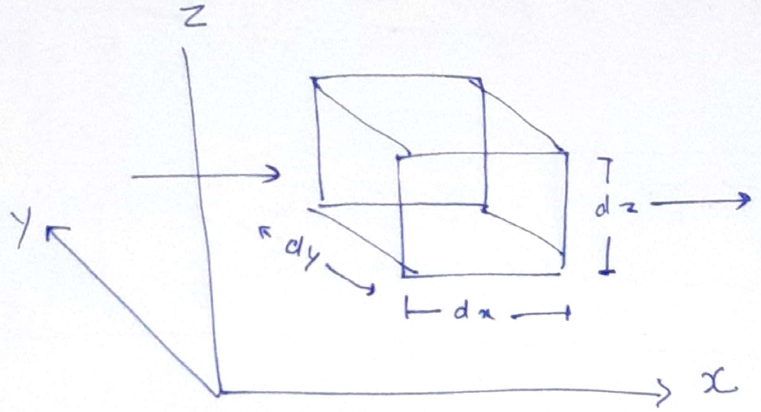
(3) Eulers equations applied to incompressible and to compressible flow.

Assuming the flow velocity is a solenoidal field, or using another appropriate energy equation

(4). The simplest form for Eulers equation being the conservation of the specific entropy.

→ Equation of motion (navier stoke)

consider a differential control volume dx, dy, dz



Momentum enters and leaves the control volume by two mechanisms advection & molecular transfer.

→ The gross rate at which x-component of flow momentum enters into control volume.

$$\rho V_x V_x \, dy\,dz + \rho V_y V_x \, dx\,dz + \rho V_z V_x \, dy\,dz$$

→ The gross rate at which x component of flow momentum leaves the control volume is

$$\left[\rho V_x V_x + \frac{\partial}{\partial x}(\rho V_x V_x)\,dx\right] dy\,dz + \left[\rho V_y V_x + \frac{\partial}{\partial y}(\rho V_y V_x)\,dy\right] dx$$

$$+ \left[\rho V_z V_x + \frac{\partial}{\partial z}(\rho V_z V_x)\,dz\right] dx\,dy$$

The gross rate at which x- component of ~~flow~~ momentum enters the control volume by molecular transport is.

$$\tau_{xx}\, dy\, dz + \tau_{xy}\, dx\, dz + \tau_{xz}\, dx\, dy$$

The gross rate at which x- component of momentum leaves the control volume by molecular transport is.

$$\left[\tau_{xx} + \frac{\partial}{\partial x}(\tau_{xx})dx\right] dy\,dz + \left[\tau_{yx} + \frac{\partial}{\partial y}(\tau_{xy})dy\right] dx\,dz$$

$$+ \left[\tau_{zx} + \frac{\partial}{\partial z}(\tau_{zy})\,dz\right] dx\,dy$$

net Rate at which x- component of momentum enters the control volume by molecular transport.

$$- \left[\frac{\partial}{\partial x}(\tau_{xx}) + \frac{\partial}{\partial y}(\tau_{xy}) + \frac{\partial}{\partial z}(\tau_{zx})\right] dx\,dy\,dz$$

If the pressure and gravitational forces are only forces acting on control volume, then the resultant of these two forces in x- direction is.

$$\left\{ p - \left(p + \frac{\partial p}{\partial x} dx\right)\right\} dy\,dz + \rho g_x\, dx\,dy\,dz$$

$\boxed{g_x \rightarrow x \text{ component of acceleration due to gravity}}$

The rate at which x- component of momentum is accumulated within the control volume

$$\frac{\partial}{\partial t}(\rho v_x)\, dx\,dy\,dz$$

Subsituting these terms in general statement of conservation of Momentum. (dividing both side by control volume we get.

$$\frac{\partial}{\partial t}(\rho v_x) = -\left\{\frac{\partial}{\partial x}(\rho v_x^2) + \frac{\partial}{\partial y}(\rho v_y v_x) + \frac{\partial}{\partial z}(\rho v_z v_x)\right\}$$

$$-\left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right) - \frac{\partial P}{dx} + \rho g_x$$

for y component

$$\frac{\partial}{\partial t}(\rho v_y) = -\left(\frac{\partial}{\partial x}(\rho v_x v_y) + \frac{\partial}{\partial y}(\rho v_y^2) + \frac{\partial}{\partial z}(\rho v_z v_x)\right\}$$

$$-\left(\frac{\delta \tau_{xy}}{\partial x} + \frac{d \tau_{yy}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right) - \frac{\partial p}{\partial y} + \rho g_y$$

for z component

$$\frac{\partial}{\partial t}(\rho v_z) = -\left(\frac{\partial}{\partial} \rho v_x v_z\right) + \frac{\partial}{\partial y}(\rho v_y v_z) + \frac{\partial}{\partial z}(\rho v_z^2)\Big)$$

$$-\left(\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\delta y} + \frac{\partial \tau_{zz}}{\partial z}\right) - \frac{\partial P}{\partial z} + \rho g_z$$

eqn simplified by using continuity eqn.

$$\rho \cdot \frac{Dv_x}{\partial t} = -\frac{\partial p}{\partial x} - \left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} \right) + \rho g_x$$

for y component

$$\rho \cdot \frac{Dv_y}{\partial t} = -\frac{\partial p}{\partial y} - \left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} \right) + \rho g_y$$

for z component

$$\rho \cdot \frac{Dv_z}{\partial t} = \frac{\partial p}{\partial z} - \left(\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z} \right) + \rho g_z$$

↳ for newtonian fluids the relationship b/w stresses and velocity gradients.

$$\underline{\tau_{xx}} = -2\mu \frac{\partial v_x}{\partial x} + \frac{2}{3}\mu (\vec{\nabla} \cdot \vec{u})$$

$$\tau_{yy} = -2\mu \frac{\partial v_y}{\partial y} + \frac{2}{3}\mu (\vec{\nabla} \cdot \vec{u})$$

$$\tau_{zz} = -2\mu \frac{\partial v_z}{\partial z} + \frac{2}{3}\mu (\vec{\nabla} \cdot \vec{u})$$

$$\tau_{xy} = \tau_{xy} = \mu \left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial \cancel{x}} \right)$$

$$\tau_{yz} = \tau_{zy} = -\mu \left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y} \right)$$

$$\tau_{zx} = \tau_{xz} = -\mu \left(\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z} \right)$$

Substituting eqn in.

$$\rho \frac{Dv_x}{Dt} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left[2\mu \frac{\partial v_x}{\partial x} - \frac{2}{3}\mu (v \cdot u)\right] + \frac{\partial}{\partial y}\left[\mu\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right)\right] + \frac{\partial}{\partial z}\left[\mu\left(\frac{\partial v_z}{\partial x} + \frac{\partial v_z}{\partial z}\right)\right] + \rho g_x$$

$$\rho \frac{Dv_y}{Dt} = -\frac{\partial p}{\partial y} + \frac{\partial}{\partial x}\left[\mu\left(\frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y}\right)\right] + \frac{\partial}{\partial y}\left[2\mu \frac{\partial v_y}{\partial y} - \frac{2}{3}\mu (v \cdot u)\right] + \frac{\partial}{\partial z}\left[\mu\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial z}\right) + \rho g_y\right.$$

$$\rho \frac{Dv_z}{Dt} = -\frac{\partial p}{\partial z} + \frac{\partial}{\partial x}\left[\mu\left(\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z}\right)\right] + \frac{\partial}{\partial y}\left[\mu\left(\frac{\partial u_z}{\partial y}\right.\right.$$

Navier eqn reveal how oil is expected to flow through a well or pipeline.

Momentum Balance for steady flow

$$\begin{Bmatrix}\text{Rate of momentum} \\ \text{- in} \\ \text{by convective} \\ \text{Transport}\end{Bmatrix} - \begin{Bmatrix}\text{Rate of momentum} \\ \text{- out} \\ \text{by convective} \\ \text{Transport}\end{Bmatrix} + \begin{Bmatrix}\text{Rate of} \\ \text{Momentum} \\ \text{in} \\ \text{by molecular} \\ \text{Transport}\end{Bmatrix}$$

$$- \begin{Bmatrix}\text{rate of Momentum} \\ \text{- out} \\ \text{by molecular} \\ \text{Transport}\end{Bmatrix} + \text{gravity} = 0.$$

Non- Newtonian fluids

Newtonian fluids are opposite to non newtonian fluids, the viscosity of the fluid changes.

When shear is applied to non- newtonian fluids, the viscosity of the fluid changes.

The behaviour of fluids can be described one of four ways:-

Dilatant - viscosity of fluid increases when shear is applied

EX - Cornflour & water

Pseudoplastic - opposite to dilatant, the more shear applied, the less viscous it becomes.

EX - Ketchup.

This chart shows how viscosity changes in respect to the amount of shear or stress applied to fluid.

Rheopectic – similar to dilatant in that when shear is applied, viscosity increases.

The diff is that viscosity increase is time-dependent

Ex – Gypsum paste.
– cream.

Thixotropic – fluids with thixotropic properties decrease in viscosity when shear is applied.

This is a time dependent property.

Ex – paint
– cosmetics
– Asphalt
– Glue

shear increases | viscosity increases

→ Dilatant → Rheopectic → This is time dependent

→ Pseudoplastic → Thixotropic ↳ shear → viscosity decreases.

↓

Tim

① Partial time derivative

This means variation of variable (c) with time with respect to fixed position (x, y, z)

$$\frac{\partial c}{\partial t}$$

② Total time derivative, $\frac{dc}{dt}$

$$\frac{dc}{dt} = \frac{\partial c}{\partial t} + \frac{\partial c}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial c}{\partial y} \cdot \frac{\partial y}{dt} + \frac{\partial c}{\partial z} \cdot \frac{\partial z}{dt}$$

③ Substantial Time derivative :-

Total time derivative.

$$\frac{Dc}{Dt}$$

Rate of change of property of fluid particle as it moves through the flow.

Ex - velocity
- Acceleration of fluid particle

Unit :- 1

- What is TP?

↳ Momentum, heat and mass transfer are called T.P

fluid dynamics - include transport of momentum

Heat Transfer - Transfer of energy.

M.T = Transport of mass.

Que. Diff b/w Laminar & Turbulent.

Laminar	Turbulent
↳ No lateral mixing	There is mixing (zigzag)
↳ No cross current	cross current.
↳ velocity gradient is high	velocity gradient lower than turbulent flow.
↳ Low Reynold no.	High Reynolds no.
↳ Ex- oil flow through a thin tube, Blood flow through capillaries	Ex - Blood flow in arteries, oil transport in pipeline.

Newtonian / Non-Newtonian.

According to Newton's law of viscosity fluid are classified based on their rheological behaviour :-

(a) Newtonian fluid : obey newton law of viscosity (Ratio of shear stress and strain are constant) means viscosity is independent of shear strain.

↳ Newton's law of viscosity

Newton law of viscosity states that shear stress b/w adjacent fluid layers is proportional to the velocity gradient b/w two layers.

$$\tau = \mu \frac{du}{dy}$$

$\tau = \frac{F}{A} =$ shear stress

$\frac{du}{dy} =$ Rate of shear deformation

$\mu =$ viscosity.

$\boxed{\text{Newton 2nd law of Motion}}$:

Force acting on body of mass m is proportional to time rate of change of its momentum.

If mass constant, the force is proportional to product of mass and acceleration.

$$F = Kd\left(\frac{mv}{dt}\right)$$

$$F = K\,ma$$

$\frac{dv}{dt} = a.$

$K =$ constant.

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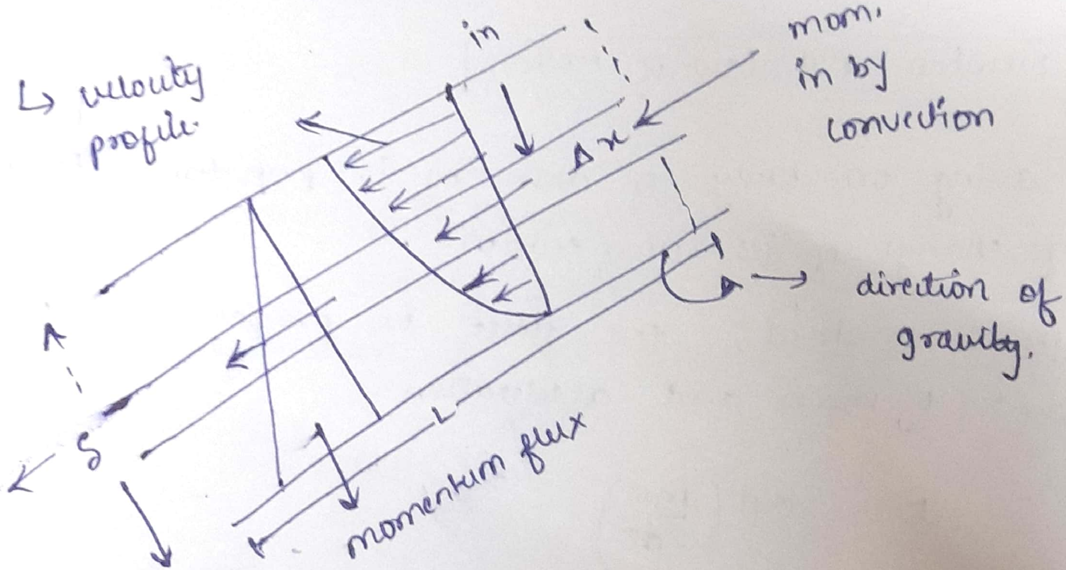
Shell Momentum Balance for falling film

Consider the flow of fluid as a film under laminar condition, along an inclined flat surface

falling film have been used to study various phenomenon in gas absorption, evapouration & coating on surface.

Consider viscosity & density become constant.

consider control volume of thickness Δx, length L
velocity $v_y(x)$ is independent of position y.



Rate of y momentum across surface at x by M.T

$$= (LW)\, \tau_{xy}\big|_x$$

↳ Rate of y momentum at $x+\Delta x$

$$= (LW)\tau_{xy}\big|_{x+\Delta x}$$

↳ Rate of y momentum surface at $y=0$

$$= (W\Delta x\, v_y)(\rho v_y)\big|_{y=0}$$

↳ Rate of y momentum $y=L$

$$= (W\Delta x\, v_y)(\rho v_y)\big|_{y=L}$$

↳ gravity force on fluid

$$= (LW\Delta x)(\rho g \cos a)$$

Substituting in momentum balance –

$$LW\tau_{xy}\big|_x - LW\tau_{xy}\big|_{x+\Delta x} + W\Delta x\,\rho v_y^2\big|_{y=0} - W\Delta x\,\rho v_y^2\big|_{y=L}$$

$$+ LW\Delta x\,\rho g\cos\alpha = 0 \quad — ①$$

The third & fourth term cancel one another v_y at $y=0$ is equal to v_y at $y=L$. for each value of x for constant density fluid.

$$\lim_{\Delta x\to 0}\left(\frac{\tau_{xy}\big|_{x+\Delta x} - \tau_{xy}\big|_x}{\Delta x}\right) = \rho g\cos\alpha \quad — ②$$

$$\frac{d\tau_{xy}}{dx} = \rho g\cos\alpha \quad — ③$$

differential eqn mom. flux.

$$\tau_{xy} = \rho g\cos\alpha + C_1 \quad — ④$$

Boundary condition

$x = 0, \quad \tau_{xy} = 0$

subsituting boundary condition $c_1 = 0$

$$\tau_{xy} = \rho g x \cos\alpha$$

This indicates that momentum flux distribution is linear & max value at solid surface

According to newton law of viscosity

$$\tau_{xy} = -\mu \frac{dv_y}{dx} \quad — \quad (5)$$

combining (4) & (5)

$$\frac{dv_y}{dx} = -\left(\frac{\rho g \cos\alpha}{\mu}\right) x \quad — \quad (7)$$

integrating (7)

$$v_y = -\left(\frac{\rho g \cos\alpha}{2\mu}\right) x^2 + c_2$$

Boundary condition $v_y = 0$ at $x = \delta$

$$c_2 = \frac{\rho g \cos \delta^2}{2\mu}$$

$$v_y = \frac{\rho \delta^2 g \cos\alpha}{2\mu}\left[1 - \left(\frac{x}{\delta}\right)^2\right] \quad — 8$$

Means velocity profile is parabolic

Max velocity at $x = 0$

$$v_{y_{max}} = \frac{\rho g \delta^2 \cos\alpha}{2\mu}$$