Delft University of Technology

MDO for Aerospace Applications

AE4205

Assignment Part 1

Authors:

S. Nolet (4535677)

E.C. Peeters (4547322)

November 17, 2019

Nomenclature Table

Symbol	Meaning	Unit
A	Area	m^2
b	Wing Span	m
C_L	Lift Coefficient	-
c	Chord Length	m
c_r	Root Chord	m
c_t	Tip Chord	m
C_T	Specific Fuel Consumption	N $(N\ s)^{-1}$
CST_n	The n^{th} CST coefficient	-
D	Aerodynamic Drag Force	N
f_{tank}	Correctional Value for Fuel Tank Volume	-
i_r	Root Incidence Angle	deg
i_t	Tip Incidence Angle	deg
L	Aerodynamic Lift Force	N
Λ	Leading Edge Sweep Angle	deg
M	Aerodynamic Moment	N m
n_{max}	Maximum Load Factor	-
R	Range	m
ρ_{cruise}	Cruise Air Density	kg m^{-3}
ρ_{fuel}	Fuel Density	kg m^{-3}
S	Wing Area	m^2
V_{cruise}	Cruise Airspeed	m s−1
V_{tank}	Fuel Tank Volume	m^3
W	Weight	kg
W_{fuel}	Aircraft Fuel Weight	kg
$W_{TO_{max}}$	Maximum Take-Off Weight	kg
W_{A-W}	Aircraft Total Weight Excluding Wing and Fuel Weight	kg
$\boldsymbol{x}$	The Design Vector	-
x	Coordinate in the X-Direction	m
y	Coordinate in the Y-Direction	m
z	Coordinate in the Z-Direction	m

1 Parameterisation

In the following section, the wing of the Tupolev TU-334 is parameterised. The parameterisation was split into the parameterisation of the wing planform and the parameterisation of the airfoil.

1.1 Tupolev Tu-334

The aircraft that was assigned to us was the Tupolev Tu-334, which is portrayed in Figure 1.

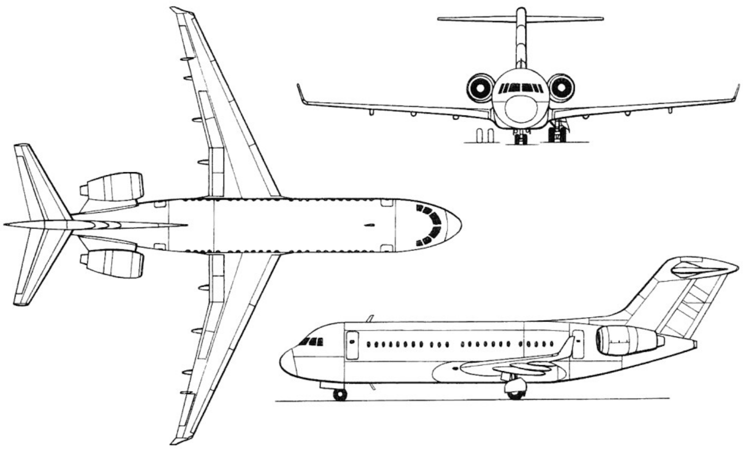


Figure 1: Blueprint of Tupolev Tu-334 aircraft[1].

1.2 Wing Planform Geomtery

The parameterisation of the wing planform can be seen in Figure 2 and Figure 3. It consists of the root chord (C_r) in m, the tip chord (C_t) in m, the half-span ($b/2$) in m, the leading edge sweep (Λ) in degrees, the root chord incidence angle (i_r) in degrees and the tip chord incidence angle (i_t) in degrees.

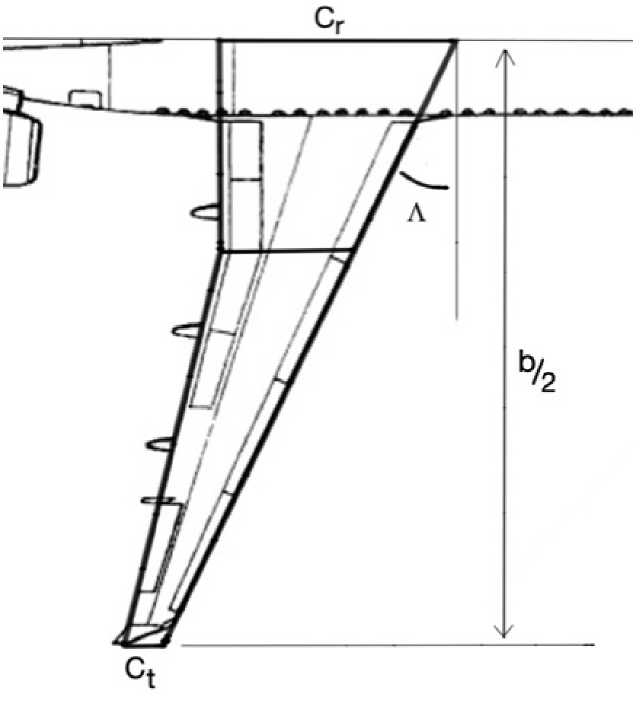


Figure 2: Wing planform parameters

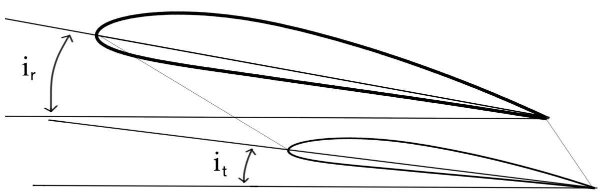


Figure 3: Airfoil incidence angles at root and tip

[1]Retrieved from `https://www.the-blueprints.com/blueprints/modernplanes/tupolev/74222/view/tupolev_tu-334/`

Together with the spanwise position of the kink and the trailing edge sweep of the inboard section of the wing, which are fixed, this parameterisation fully describes the wing planform geometry. The six parameters that determine the wing planform geometry make up the first part of the design vector:

$$\boldsymbol{x_1} = \begin{bmatrix} C_r \\ C_t \\ b/2 \\ \Lambda \\ i_r \\ i_t \end{bmatrix}$$

1.3 Airfoil Geometry

The second part of design vector consists of the parameters that determine the airfoil geometry. The airfoil geometry is parameterised using the Class Shape Transformation (CST) parameterisation method. This method consists of combining a class function with a shape function to approach the geometry of an airfoil. The class function takes two coefficients, N_1 and N_2, which are set constant at 0.5 and 1 respectively. This class function forms the basis of the geometry and is then modified by adding a shape function, a Bernstein polynomial in this case. This Bernstein polynomial needs to be of order 5 at least, so at least 6 Bernstein coefficients are needed. These Bernstein/CST coefficients form the second part of the design vector. In total, 24 CST coefficients are needed to parameterise the airfoils at the root and tip of the wing; 6 for the upper side and 6 for the lower side. The airfoil at the kink is a linear interpolation of the root and tip airfoils. The second part of the design vector is then:

$$\boldsymbol{x_2} = \begin{bmatrix} CST_1 \\ ... \\ CST_{24} \end{bmatrix}$$

1.4 Bounds on Parameterisation

The optimisation that will be used in this assignment does not only need an initial value of the design vector, but also bounds on each individual design parameter. For the initial values, the geometry of the Tupolev 334 is taken. The incidence angles of the root and tip chord are not known, so the initial value is set to 0°. The airfoil of the Tupolev 334 is not known, so the Whitcomb airfoil is chosen and scaled to 14% t/c for the root and 8% t/c for the tip of the wing. These scaled airfoils are then converted to CST coefficients.

Parameter	**Lower bound**	**Initial value**	**Upper bound**
$\boldsymbol{C_r}$	0.5 m	5.6 m	20.0 m
$\boldsymbol{C_t}$	0.1 m	0.9 m	20.0 m
b/2	5.0 m	14.0 m	30.0 m
$\boldsymbol{\Lambda}$	0.5°	26.0°	60.0°
$\boldsymbol{i_r}$	−5.0°	0.0°	5.0°
$\boldsymbol{i_t}$	−5.0°	0.0°	5.0°
$\boldsymbol{CST_n}$	0.5· scaled Whitcomb	scaled Whitcomb	2.0· scaled Whitcomb

The lower and upper bounds are mostly chosen using common sense and engineering instinct. For instance, the minimum root chord cannot be 0, because then the wing cannot connect to the fuselage. One bound is set by the EMWET discipline; the leading edge sweep (Λ) cannot be smaller than 0.5 degrees. If after running the program, one of the optimised parameters takes the value of a lower or upper bound, the bounds have not been set properly. In that case, wider bounds will be implaced if possible and the program will be run again.

2 Optimisation Problem Description

This section describes a description of the optimisation problem. It includes information on the objective function and the constraints on the design. It also presents the different disciplines from which the design will be calculated.

2.1 Objective Function

The maximum take-off weight is determined by Equation 1. As this maximum take-off weight has to be minimised in this assignment, Equation 1 is the objective function for this assignment.

$$W_{TO_{max}}(\boldsymbol{x}) = W_{A-W} + W_{fuel}(\boldsymbol{x}) + W_{str}(\boldsymbol{x}) \tag{1}$$

2.2 Fuel Volume Inequality Constraint

Next to the objective function, two inequality constraints are proposed. The first one has to do with the fuel storage of the aircraft. It is required that the amount of fuel to fly the mission must be able to fit inside wing integrated fuel tanks. This inequality constraint is presented in Equation 2, which states that the volume needed for the fuel can not exceed the available volume in the wings.

$$\frac{W_{fuel}(\boldsymbol{x})}{\rho_{fuel}} \leq V_{tank}(\boldsymbol{x}) \cdot f_{tank} \tag{2}$$

It is assumed that the fuel tank will be placed between the forward and rearward spars. The fuel tank starts at the centerline of the fuselage and stretches to a maximum of 85% of the wing span. The tank inboard of the kink and outboard of the kink can be investigated separately and later added together to get the total fuel tank volume. The spar positions for these three points are shown in Table 1. They can be linearly interpolated to find the spar positions along each point of the fuel tank.

	Root	**Kink**	**0.85b/2**
Front spar	15%	25%	20%
Rear spar	80%	65%	60%

Table 1: Spar positions for the fuel tank in percentage of the local chord.

For the inboard and outboard parts of the tank, the airfoil shape and chord along the span are interpolated linearly. Therefore, the total tank volume can be calculated by Equation 3. From the set of CST coefficients provided in the design vector, coordinate points along the upper and lower surface of the airfoil can be computed at any spanwise position y. The effective areas, usable for the fuel tank, of these positions can then be calculated as proposed in Equation 4, where the subscript i stands for a coordinate index as can be seen in Figure 4. This equation only holds for coordinate index points where $(\frac{x}{c})_{frontspar} \leq (\frac{x}{c})_i \leq (\frac{x}{c})_{rearspar}$.

$$V_{tank}(\boldsymbol{x}) = 2 \cdot \left[y_{kink} \cdot \frac{(A_{eff,root} + A_{eff,kink})}{2} + (y_{0.85b} - y_{kink}) \cdot \frac{(A_{eff,kink} + A_{eff,0.85b/2})}{2} \right] \tag{3}$$

$$A_{eff,y} = \sum_{i=1}^{n} \left[(x_i - x_{i-1}) \cdot \frac{(z_{upper,i} - z_{lower,i}) + (z_{upper,i-1} - z_{lower,i-1})}{2} \right] \tag{4}$$

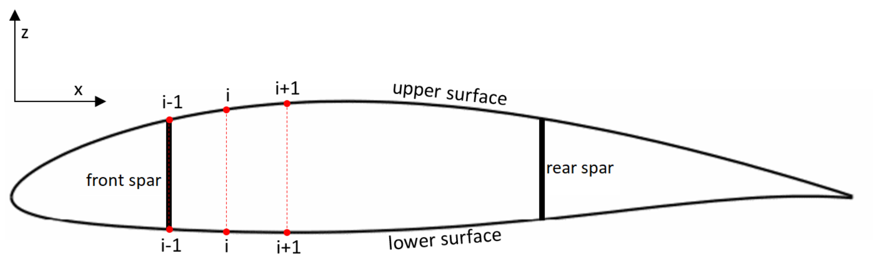


Figure 4: Conventions for computing the useful airfoil area.

2.3 Wing Loading Inequality Constraint

The second inequality constraint comes from the initial design of the aircraft. In the initial design, loading diagrams are made in order to ensure that the design stays within operable ranges in terms of wing and power loading. The wing loading is usually constraint by either a landing or take-off requirement. In order to make sure that the optimised aircraft still meets these requirements, the wing loading of the optimised version cannot exceed the wing loading of the original design. This is also described in Equation 5. The original wing loading is found to be 558.72 kg/m^2.

$$\left(\frac{W}{S}\right)_{optimised} \leq \left(\frac{W}{S}\right)_{original} \tag{5}$$

2.4 Disciplines

The method proposed in this assignment consists of several disciplines in order to optimise the total weight of the aircraft. 4 different disciplines are used, of which two are implemented twice for different applications. Each paragraph gives a short description of each of the 4 disciplines in the XDSM.

Design Point Calculator (critical) (DPC$_{\text{crit}}$)

The Design Point Calculator, or DPC for short, is used in order to find an appropriate value for the lift coefficient of the aircraft. It is used in order to calculate the design point in a mid-cruise condition. This means that the $C_{L_{des}}$ is calculated, aerodynamic forces are then determined by the Q3D solver. The DPC is used two times in the MDO architecture. In this critical case the design point is calculated with an initial guess of the weight components and then scaling the $C_{L_{des}}$ with the maximum load factor. This is done in order for calculating the structural weight under the most critical load case. The equation to calculate the $C_{L_{des}}$ is presented in Equation 7, from which the $C_{L_{crit}}$ is extrapolated as in Equation 8.

$$L_{des}(\boldsymbol{x}) = W_{des}(\boldsymbol{x}) = \sqrt{W_{TO_{max}}(\boldsymbol{x}) \cdot [W_{TO_{max}}(\boldsymbol{x}) - W_{fuel}(\boldsymbol{x})]} \tag{6}$$

$$C_{L_{des}} = \frac{2 \cdot W_{des}(\boldsymbol{x})}{\rho_{cruise} \cdot S \cdot V_{cruise}^2} \tag{7}$$

$$C_{L_{crit}} = C_{L_{des}} \cdot n_{max} \tag{8}$$

Q3DSolver (critical)

The next step in the optimisation is calculating the critical loads that the wing has to be able to endure. This is done by using the pre-defined aerodynamic analysis tool Q3DSolver. As inputs the planform geometry and wing surface definition are used for the root, kink and tip. Also the flight conditions and the critical lift coefficient are used. Lastly, since the structural analysis tool only requires the lift distribution from Q3D, an inviscid approach is selected for a faster computation time. The useful output of this tool is then the spanwise distribution of lift and pitching moment.

EMWET

The EMWET structural analysis tool calculates the structural weight of the wing, this is then also the major output of the tool. As input it again uses the the planform geometry and wing surface definition for the root, the kink and the tip. Several structural parameters have to be inputted, such as spar locations or rib pitch. Next to this, the properties of the used materials are another input. Lastly, the the critical lift distribution and pitching moment along the wing are used, as calculated by the critical Q3DSolver.

Design Point Calculator (DPC$_{\text{cruise}}$)

For the calculation of the fuel weight, an ordinary mission needs to be considered. This means that the design point needs to be evaluated at a mid-cruise condition. Again, Equation 6 and Equation 7 have to be used in order to calculate the desired $C_{L_{des}}$ as input for the aerodynamic tool. The only difference is that this time, instead of a initial guess for the structural weight, the output of the EMWET discipline will be used. This will in term lead to a faster convergence of the MDA loop.

Q3DSolver (cruise)

Similarly as for the critical case, the $C_{L_{des}}$ will be inputted into the Q3DSolver aerodynamic tool. Again, also the wing shape and planform geometry, together with the flight conditions, are used as input. Different from before, the forces will be calculated with a viscous approach as also a proper evaluation of the drag is necessary. The output will be lift and drag forces on the wing.

Breguet Range Equation

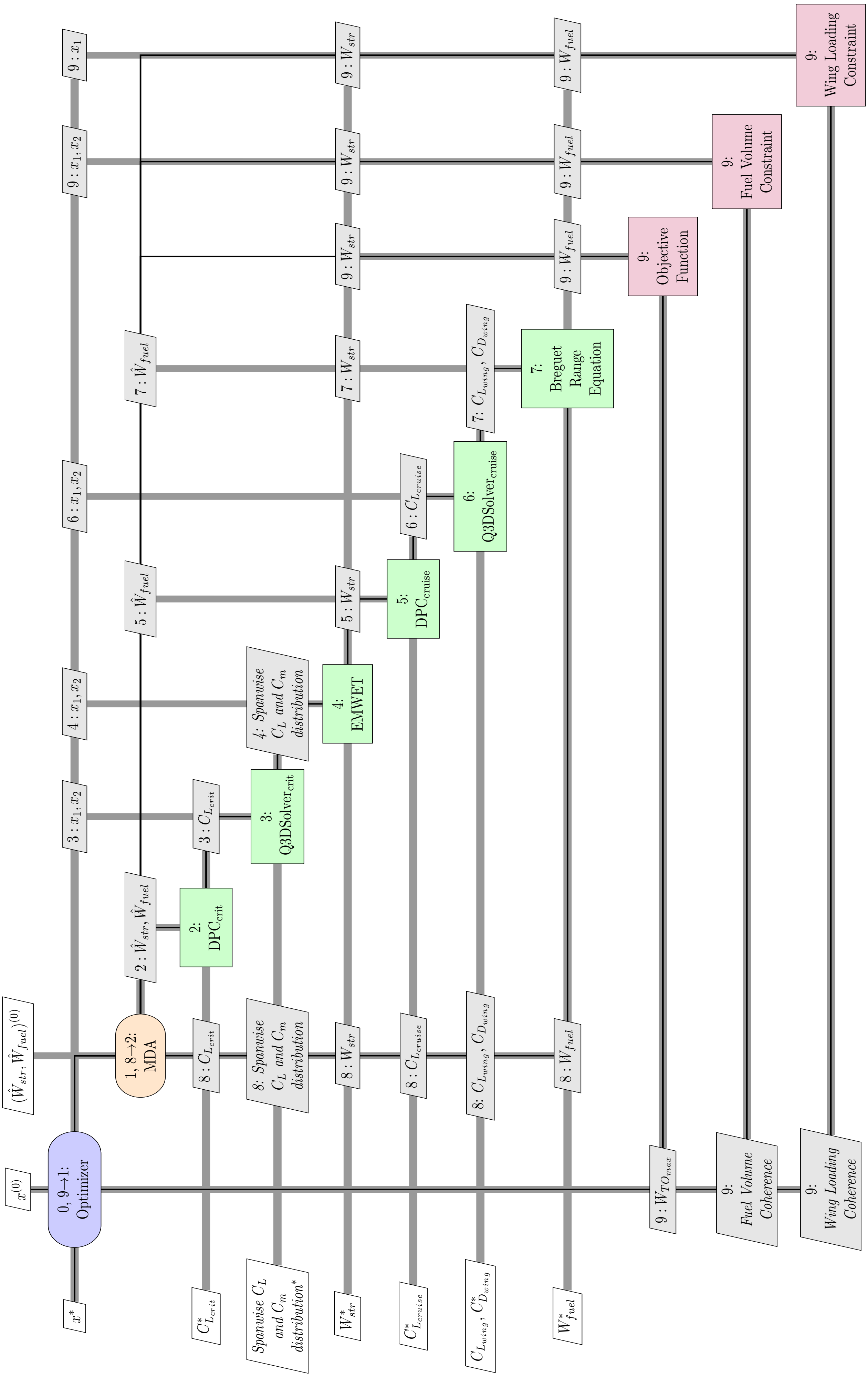
In order to assess the fuel that is needed for a regular mission, the Breguet range equation is used, which is shown in Equation 9. If the design range of the aircraft is used as input, together with the lift and drag forces, the specific fuel consumption and the cruise speed, the fuel fraction during cruise can be calculated. This fuel fraction relates the weight at the start of the cruise phase with the weight at the end of cruise. This fraction can then be used in Equation 10, in order to find the fuel weight for the aircraft. Again, an updated guess on the structural weight can be used from the EMWET discipline in order to fasten up the convergence of the MDA loop. The value of 0.938 in this equation comes from all other fuel fractions outside the cruise, such as taxi, take-off, climb, etc.

$$R = \frac{V}{C_T} \cdot \frac{L}{D} \cdot \ln\left(\frac{W_{\text{start}-cr}}{W_{\text{end}-cr}}\right) \tag{9}$$

$$W_{\text{fuel}} = \left[1 - 0.938 \cdot \frac{W_{\text{end}-cr}}{W_{\text{start}-cr}}\right] \cdot W_{TO_{max}}(\boldsymbol{x}) \tag{10}$$

3 XDSM

The XDSM is shown in the next page. It includes all disciplines, the objective function and the constraints for the optimisation.



$x^{(0)}$
$(\hat{W}_{str}, \hat{W}_{fuel})^{(0)}$
x^*
0, 9→1: Optimizer
1, 8→2: MDA
2: DPC$_{\text{crit}}$
3: Q3DSolver$_{\text{crit}}$
4: EMWET
5: DPC$_{\text{cruise}}$
6: Q3DSolver$_{\text{cruise}}$
7: Breguet Range Equation
9: Objective Function
9: Fuel Volume Constraint
9: Wing Loading Constraint
$2 : \hat{W}_{str}, \hat{W}_{fuel}$
$3 : x_1, x_2$
$4 : x_1, x_2$
$6 : x_1, x_2$
$9 : x_1, x_2$
$9 : x_1$
$5 : \hat{W}_{fuel}$
$7 : \hat{W}_{fuel}$
$3 : C_{L_{crit}}$
$8 : C_{L_{crit}}$
4: Spanwise C_L and C_m distribution
8: Spanwise C_L and C_m distribution
$5 : W_{str}$
$7 : W_{str}$
$8 : W_{str}$
$9 : W_{str}$
$6 : C_{L_{cruise}}$
$8 : C_{L_{cruise}}$
7: $C_{L_{wing}}$, $C_{D_{wing}}$
8: $C_{L_{wing}}$, $C_{D_{wing}}$
$8 : W_{fuel}$
$9 : W_{fuel}$
$9 : W_{TO_{max}}$
9: Fuel Volume Coherence
9: Wing Loading Coherence
$C^*_{L_{crit}}$
Spanwise C_L and C_m distribution*
W^*_{str}
$C^*_{L_{cruise}}$
$C_{L_{wing}}, C^*_{D_{wing}}$
W^*_{fuel}