Measurement of the neutrino magnetic moment at the NOvA experiment

Technical note

Robert Kralik[1]

[1]University of Sussex, Brighton, UK

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Abstract

This is the abstract

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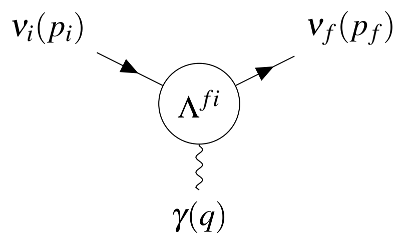

Figure 1: Effective coupling of neutrinos with one photon electromagnetic field.

1 Introduction

(TO DO: *Describe the main motivations for the analysis. Briefly mention that there was a previous study by Biao, what were the results there and what limitations (or maybe talk about this in the Experimental overview?)*)

2 Theoretical overview

In the Standard Model (SM), neutrinos are massless and electrically neutral particles. However, even in the SM neutrinos can have electromagnetic interaction through loop diagrams involving the charged leptons and the W boson. These interactions are described by the neutrino charge radius, described in section 2.2 [1].

To include neutrino masses required by neutrino oscillations, we must go Beyond the Standard Model (BSM), where neutrinos can acquire other electromagnetic properties [2]. In the most general case, considering interactions with a single photon as shown on Fig.1, neutrino electromagnetic interactions can be described by an *effective* interaction Hamiltonian [2]

$$\mathscr{H}_{em}^{(\nu)}(x) = \sum_{k,j=1}^{N} \overline{\nu}_k(x)\,\Lambda_\mu^{kj}\,\nu_j(x)\,A^\mu(x)\,. \tag{1}$$

Here $\nu_k(x), k \in \{1,...,N\}$ are neutrino fields in the mass basis with N neutrino mass states. Λ_μ^{kj} is a general vertex function and $A^\mu(x)$ is the electromagnetic field.

The vertex function $\Lambda_\mu^{fi}(q)$ is generally a matrix and in the most general case can be written in terms of linearly independent products of Dirac matrices (γ) and only depends on the square of the four momentum of the photon $\left(q = p_f - p_i\right)$:

$$\begin{aligned}\Lambda_\mu^{fi}(q) =& \mathbb{F}_1^{fi}\left(q^2\right) q_\mu + \mathbb{F}_2^{fi}\left(q^2\right) q_\mu \gamma_5 + \mathbb{F}_3^{fi}\left(q^2\right) \gamma_\mu + \mathbb{F}_4^{fi}\left(q^2\right) \gamma_\mu \gamma_5 + \\ & \mathbb{F}_5^{fi}\left(q^2\right) \sigma_{\mu\nu} q^\nu + \mathbb{F}_6^{fi}\left(q^2\right) \varepsilon_{\mu\nu\rho\gamma} q^\nu \sigma^{\rho\gamma},\end{aligned} \tag{2}$$

where $\mathbb{F}_i^{fi}\left(q^2\right)$ are six Lorentz invariant form factors [2].

Applying conditions of hermiticity $\left(\mathscr{H}_{em}^{(\nu)\dagger} = \mathscr{H}_{em}^{(\nu)}\right)$ and of the gauge invariance of the electromagnetic field, we can rewrite the vertex function as

$$\Lambda_{\mu}^{fi}(q) = \left(\gamma_{\mu} - q_{\mu}\not{q}/q^{2}\right)\left[\mathbb{F}_{Q}^{fi}\left(q^{2}\right) + \mathbb{F}_{A}^{fi}\left(q^{2}\right) q^{2}\gamma_{5}\right] - i\sigma_{\mu\nu}q^{\nu}\left[\mathbb{F}_{M}^{fi}\left(q^{2}\right) + i\mathbb{F}_{E}^{fi}\left(q^{2}\right)\gamma_{5}\right], \quad (3)$$

where $\mathbb{F}_{Q}^{fi}, \mathbb{F}_{M}^{fi}, \mathbb{F}_{E}^{fi}$ and $\mathbb{F}_{A}^{fi}$ are hermitian matrices representing the charge, dipole magnetic, dipole electric and anapole neutrino form factors. In coupling with a real photon $\left(q^{2} = 0\right)$ these become the neutrino charge and magnetic, electric and anapole moments. The neutrino charge radius corresponds to the second term in the expansion of the charge form factor [2].

We can simplify the above expression as [1]

$$\Lambda_{\mu}^{fi}(q) = \gamma_{\mu}\left(Q_{\nu_{fi}} + \frac{q^{2}}{6}\langle r^{2}\rangle_{\nu_{fi}}\right) - i\sigma_{\mu\nu}q^{\nu}\mu_{\nu_{fi}}, \quad (4)$$

where $Q_{\nu_{fi}}$, $\langle r^{2}\rangle_{\nu_{fi}}$, and $\mu_{\nu_{fi}}$ are the neutrino charge, effective charge radius (also containing anapole moment), and an effective magnetic moment (also containing electric moment) respectively. This is possible thanks to the proportional effect of the neutrino charge radius and the anapole moment, or the neutrino magnetic and electric moment respectively [2]. These quantities (charge, charge radius and magnetic moment) are the three neutrino electromagnetic properties measured in experiments.

2.1 Neutrino electric and magnetic dipole moments

The size and effect of the neutrino electromagnetic properties depends on the specific theory beyond standard model.

Evaluating the one loop diagrams in the minimal extension of the standard model (TO DO: *should I mention here what actually do I mean by the minimal extension of SM? probably this: "introduction of three right-handed neutrinos"*) with right handed (Dirac) neutrinos gives us the first approximation of the electric and magnetic moments $\left(q^{2} = 0\right)$:

$$\left.\begin{matrix}\mu_{kj}^{D} \\ i\varepsilon_{kj}^{D}\end{matrix}\right\} \simeq \frac{3eG_{F}}{16\sqrt{2}\pi^{2}}\left(m_{k} \pm m_{j}\right)\left(\delta_{kj} - \frac{1}{2}\sum_{l=e,\mu,\tau} U_{lk}^{\star}U_{lj}\frac{m_{l}^{2}}{m_{W}^{2}}\right), \quad (5)$$

where m_{k}, m_{j} are the neutrino masses and m_{l} are the masses of charged leptons which appear in the loop diagrams. Higher order electromagnetic corrections were neglected, but those can also have a significant contribution [2]. (TO DO: *ok, so what does that mean? Does that mean that this equation is not actually correct or that it's the higher/lower limit? or something else?*)

(It can be seen that) There are no diagonal electric moments $\left(\varepsilon_{kk}^{D} = 0\right)$ and the diagonal magnetic moments are approximately

$$\mu_{kk}^{D} \simeq \frac{3eG_{F}m_{k}}{8\sqrt{2}\pi^{2}} \simeq 3.2 \times 10^{-19}\left(\frac{m_{k}}{\mathrm{eV}}\right)\mu_{B}, \quad (6)$$

where μ_{B} is the Bohr magneton [2].

The transition magnetic moments are suppressed with respect to the largest of the diagonal magnetic moments by at least a factor of 10^{-4} due to the m_W^2 in denominator and the transition electric moments are even smaller than that due to the mass difference [2]. Therefore an experimental observation of a magnetic moment larger than in eq.6 would indicate physics beyond the minimally extended standard model [3].

Majorana neutrinos can be obtained by either adding a $\mathrm{SU}(2)_L$ Higgs triplet, or right handed neutrinos together with a $\mathrm{SU}(2)_L$ Higgs singlet (TO DO: *is this also considered a minimally extended SM? or is this the only way to get Majorana neutrinos? Most likely the former - from nuElmagInt2015.pdf: the absence of Higgs triplets, without which it is not possible to have Majorana mass terms.*). If we neglect the Feynman diagrams which depend on the model of the scalar sector (TO DO: *what does that mean for the result?*), the magnetic and electric dipole moments are

$$\mu_{kj}^M \simeq -\frac{3ieG_F}{16\sqrt{2}\pi^2}\left(m_k+m_j\right)\sum_{l=e,\mu,\tau}\mathrm{Im}\left[U_{lk}^{\star}U_{lj}\right]\frac{m_l^2}{m_W^2}, \tag{7}$$

$$\varepsilon_{kj}^M \simeq \frac{3ieG_F}{16\sqrt{2}\pi^2}\left(m_k-m_j\right)\sum_{l=e,\mu,\tau}\mathrm{Re}\left[U_{lk}^{\star}U_{lj}\right]\frac{m_l^2}{m_W^2}. \tag{8}$$

These are difficult to compare to the Dirac case, due to possible presence of Majorana phases in the PMNS matrices, but it is clear that they have the same order of magnitude as Dirac transition dipole moments. However, the neglected model dependent contributions can enhance the transition dipole moments [2]. (TO DO: *but how much?*)

It is possible [3] to obtain "natural" upper limits on the size of neutrino magnetic moment by calculating its contribution to the neutrino mass by standard model radiative corrections. For Dirac neutrinos the radiative correction induced by neutrino magnetic moment, generated at an energy scale Λ, to the neutrino mass is generically

$$m_\nu^D \sim \frac{\mu_\nu^D}{3\times 10^{-15}\mu_B}\left[\Lambda\left(\mathrm{TeV}\right)\right]^2 \mathrm{eV}. \tag{9}$$

So for $\Lambda \simeq 1\mathrm{TeV}$ and $m_\nu \lesssim 0.3\mathrm{eV}$ the limit becomes $\mu_\nu^D \lesssim 10^{-15}\mu_B$. This applies only if the new physics is well above the electroweak scale ($\Lambda_{EW} \sim 100\mathrm{GeV}$). It is possible to get Dirac neutrino magnetic moment higher than this limit, for example in frameworks of minimal super-symmetric standard model, by adding more Higgs doublets, or by considering large extra dimensions [2].

The limit for Majorana neutrino magnetic moment is less stringent, due to the antisymmetry condition from eq.**??** and considering $m_\nu \lesssim 0.3\mathrm{eV}$ can be expressed as

$$\mu_{\tau\mu}, \mu_{\tau e} \lesssim 10^{-9}\left[\Lambda\left(\mathrm{TeV}\right)\right]^{-2} \tag{10}$$

$$\mu_{\mu e} \lesssim 3\times 10^{-7}\left[\Lambda\left(\mathrm{TeV}\right)\right]^{-2} \tag{11}$$

which is shown in the flavour basis , which relates to the framework used previously as

$$\mu_{ij} = \sum_{\alpha\beta}\mu_{\alpha\beta}U_{\alpha i}^{\star}U_{\beta j}, \quad \alpha,\beta \in \{e,\mu,\tau\}. \tag{12}$$

This limits imply, that if a magnetic moment $\mu \gtrsim 10^{-15}\mu_B$ would be measured, it is plausible neutrinos are Majorana fermions and the scale of lepton violation would be well below the conventional see-saw scale [3].

2.2 Other neutrino electromagnetic properties

Neutrino electric charge is heavily constraint by the measurements on the neutrality of matter (since generally neutrinos having an electric charge would also mean that neutrons have charge which would affect all heavier nuclei). It is also constrained by the SN1987A, since neutrino having an effective charge would lengthen its path through the extragalactic magnetic fields and would arrive on earth later. It can also be obtained from nu-on-e scatter from the relationship between neutrino millicharge and magnetic moment. [nuElmagInt2015.pdf - sec. VIIA]

The neutrino charge radius is determined by the second term in the expansion of the neutrino charge form factor and can be interpreted using the Fourier transform of a spherically symmetric charge distribution. It can also be negative since the charge density is not a positively defined quantity. In the SM the charge radius has the form of (possible other definitions exist)

$$\langle r^2_{\nu_l}\rangle_{\mathsf{SM}} = \frac{G_{\mathsf{F}}}{4\sqrt{2}\pi^2}\left[3 - 2\log\left(\frac{m_l^2}{m_W^2}\right)\right]. \tag{13}$$

This corresponds to $\langle r^2_{\nu_\mu}\langle_{\mathsf{SM}} = 2.4\times 10^{-33}$ cm^2 and similar scale for other neutrino flavours. [nuElmagInt2015.pdf - sec. VIIB]

[nuElmagInt2015.pdf - sec. VIIB] The effect of the neutrino charge radius on the neutrino-on-electron scattering cross section is through the following shift of the vector coupling constant (Grau and Grifols, 1986; Degrassi, Sirlin, and VMarciano, 1989; Vogel and Engel, 1989; Hagiwara et al., 1994):

$$g_V^{\nu_l} \to g_V^{\nu_l} + \frac{2}{3}m_W^2\langle r^2_{\nu_l}\rangle \sin^2\theta_W \tag{14}$$

[nuElmagInt2015.pdf - sec. VIIB] The current experimental limits for muon neutrinos are from (TO DO: *check the current exp. limits*) Hirsch, Nardi, and Restrepo (2003) who obtained the following 90% C.L. bounds on $\langle r^2_{\nu_\mu}\rangle$ from a reanalysis of CHARM-II (Vilain et al., 1995) and CCFR (McFarland et al.,1998) data:

$$-0.52\times 10^{-32} < \langle r^2_{\nu_\mu}\rangle < 0.68\times 10^{-32}\ \text{cm}^2 \tag{15}$$

In the Standard Model, the neutrino anapole moment is somehow coupled with the neutrino charge radii and is functionally identical. the phenomenology of neutrino anapole moments is similar to that of neutrino charge radii. Hence, the limits on the neutrino charge radii discussed in Sec. VII.B also apply to the neutrino anapole moments multiplied by 6. in the standard model the neutrino charge radius and the anapole moment are not defined separately and one can interpret arbitrarily the charge form factor as a charge radius or as an anapole moment. Therefore, the standard model values for the neutrino charge radii in Eqs. (7.35)–(7.38) can be interpreted also as values of the corresponding neutrino anapole moments. [nuElmagInt2015.pdf - sec. VIIC]

It is possible to consider the toroidal dipole moment as a characteristic of the neutrino which is more convenient and transparent than the anapole moment for the description of T-invariant interactions with nonconservation of the P and C symmetries. the toroidal and anapole moments coincide in the static limit when the masses of the initial and final neutrino states are equal to each other. The toroidal (anapole) interactions of a Majorana as well as a Dirac neutrino are expected to contribute to the total cross section of neutrino elastic scattering off electrons, quarks, and nuclei. Because of the fact that the toroidal (anapole) interactions contribute to the helicity preserving part of the scattering of neutrinos on electrons, quarks, and nuclei, its contributions to cross sections are similar to those of the neutrino charge radius. In principle, these contributions can be probed and information about toroidal moments can be extracted in low-energy scattering experiments in the future. Different effects of the neutrino toroidal moment are discussed by Ginzburg and Tsytovich (1985), Bukina, Dubovik, and Kuznetsov (1998a, 1998b), and Dubovik and Kuznetsov (1998). In particular, it has been shown that the neutrino toroidal electromagnetic interactions can produce Cherenkov radiation of neutrinos propagating in a medium. [nuElmagInt2015.pdf - sec. VIIC]

2.3 Measuring neutrino magnetic moment

(TO DO: *Need to add some general description of the measurements*)

2.3.1 Effective neutrino magnetic moment

(TO DO: *Describe why experiments measure an effective magnetic moment*) What we measure in experiments is an effective "flavour" magnetic moment, which is influenced by mixing of "mass" magnetic moments (and electric moments) and oscillations. In the ultrarelativistic limit this is

$$\mu^2_{\nu_l}(L,E_\nu) = \sum_j \left| \sum_k U^\star_{lk} e^{-i\Delta m^2_{kj} L/2E_\nu} \left(\mu_{jk} - i\varepsilon_{jk}\right) \right|^2 . \tag{16}$$

What is called the effective magnetic moment (often just magnetic moment) therefore contains contributions from both the neutrino magnetic and electric moment [2].

For antineutrinos, the effective magnetic moment is (TO DO: *maybe I should just combine these two equations to avoid overcrowding*)

$$\mu^2_{\bar{\nu}_l}(L,E_\nu) = \sum_j \left| \sum_k U^\star_{lk} e^{+i\Delta m^2_{kj} L/2E_\nu} \left(\mu_{jk} - i\varepsilon_{jk}\right) \right|^2 . \tag{17}$$

So the only difference is in the phase induced by neutrino oscillations.

For experiments with baselines short enough for neutrino oscillations to not develop ($\frac{\Delta m^2 L}{2E_\nu} \ll 1$), such as the NOvA ND, the effective magnetic moment can be expressed as

$$\mu^2_{\nu_l} \simeq \mu^2_{\bar{\nu}_l} \simeq \sum_j \left| \sum_k U^\star_{lk} \left(\mu_{jk} - i\varepsilon_{jk}\right) \right|^2 = \left[U\left(\mu^2 + \varepsilon^2\right) U^\dagger + 2\,\mathrm{Im}\left(U\mu\varepsilon U^\dagger\right) \right]_{ll'} , \tag{18}$$

which is independent of the neutrino energy and of the source to detector distance.

It is important to mention, that since the effective magnetic moment depends on the flavour of the studied neutrino, it is different for different types of neutrino experiment. Also the solar neutrino experiments need to include the effect of the solar matter on the neutrino oscillations. Therefore the reports on the value (or upper limit) of the effective neutrino magnetic moment are not directly comparable between different types of neutrino experiments.

2.3.2 Neutrino-on-electron elastic scattering

The most sensitive method to measure neutrino magnetic moment is the low energy elastic scattering of (anti)neutrinos on electrons [2]. This interaction has two observables, the recoil electron's kinetic energy (T_e) and the recoil angle with respect to the incoming neutrino beam (θ). From simple $2 \to 2$ kinematics we can get

$$(P_\nu - P_{e'})^2 = (P_{\nu'} - P_e)^2\,, \tag{19}$$

$$m_\nu^2 + m_e^2 - 2E_\nu E_{e'} + 2E_\nu p_{e'} \cos\theta = m_\nu^2 + m_e^2 - 2E_{\nu'} m_e. \tag{20}$$

Using the energy conservation

$$E_\nu + m_e = E_{\nu'} + E_{e'} = E_{\nu'} + T_e + m_e \Rightarrow E_{\nu'} = E_\nu - T_e \tag{21}$$

we get

$$E_\nu p_{e'} \cos\theta = E_\nu E_{e'} - E_{\nu'} m_e = E_\nu (T_e + m_e) - (E_\nu - T_e) m_e = T_e (E_\nu + m_e)\,, \tag{22}$$

$$\cos\theta = \frac{E_\nu + m_e}{E_\nu}\sqrt{\frac{T_e^2}{E_{e'}^2 - m_e^2}} = \frac{E_\nu + m_e}{E_\nu}\sqrt{\frac{T_e^2}{T_e^2 + 2T_e m_e}}. \tag{23}$$

And finally we get

$$\cos\theta = \frac{E_\nu + m_e}{E_\nu}\sqrt{\frac{T_e}{T_e + 2m_e}}. \tag{24}$$

Electron's kinetic energy is kinematically constrained by

$$T_e \leq \frac{2E_\nu^2}{2E_\nu + m_e}. \tag{25}$$

Considering $E_\nu \sim \mathsf{GeV}$, we can approximate $\frac{m_e^2}{E_\nu^2} \to 0$ and in the small angle approximation we get from eq.24

$$T\theta^2 \cong 2m_e \left(1 - \frac{T_e}{E_\nu}\right) < 2m_e. \tag{26}$$

In the ultrarelativistic limit, the neutrino magnetic moment changes the neutrino helicity, turning active neutrinos into sterile (TO DO: *this is a very strong statement and it probably need* *a bit more backing up*). Since the SM weak interaction conserves helicity we can add the two contribution to the neutrino on electron cross section incoherently [2]:

$$\frac{d\sigma_{\nu_l e^-}}{dT_e} = \left(\frac{d\sigma_{\nu_l e^-}}{dT_e}\right)_{\mathsf{SM}} + \left(\frac{d\sigma_{\nu_l e^-}}{dT_e}\right)_{\mathsf{MAG}}. \tag{27}$$

The standard model contribution can be expressed as [2]:

$$\left(\frac{d\sigma_{\nu_l e^-}}{dT_e}\right)_{\mathsf{SM}} = \frac{G_F^2 m_e}{2\pi}\left\{\left(g_V^{\nu_l}+g_A^{\nu_l}\right)^2 + \left(g_V^{\nu_l}-g_A^{\nu_l}\right)^2\left(1-\frac{T_e}{E_\nu}\right)^2 + \left(\left(g_A^{\nu_l}\right)^2-\left(g_V^{\nu_l}\right)^2\right)\frac{m_e T_e}{E_\nu^2}\right\}, \tag{28}$$

where the coupling constants g_V and g_A are different for different neutrino flavours and for antineutrinos. Their values are:

$$g_V^{\nu_e} = 2\sin^2\theta_W + 1/2, \qquad g_A^{\nu_e} = 1/2, \tag{29}$$

$$g_V^{\nu_{\mu,\tau}} = 2\sin^2\theta_W - 1/2, \qquad g_A^{\nu_{\mu,\tau}} = -1/2. \tag{30}$$

For antineutrinos $g_A \to -g_A$.

Using expressions 24 and 26 we can also derive [4] cross sections with respect to $\cos\theta$, θ^2 and $T\theta^2$: (TO DO: *I don't think these equations are actually valid. We've been using some approximations and therefore I don't think these equations work for any theta dependence*)

$$\left(\frac{d\sigma_{\nu_l e^-}}{d\cos\theta}\right)_{\mathsf{SM}} = \frac{2G_F^2 E_\nu^2 m_e^2 \cos\theta\,(E_\nu+m_e)^2}{\pi\left((E_\nu+m_e)^2 - E_\nu^2\cos^2\theta\right)^2}\left\{\left(g_V^{\nu_l}+g_A^{\nu_l}\right)^2 + \left(g_V^{\nu_l}-g_A^{\nu_l}\right)^2\left(1-\frac{2m_e E_\nu\cos^2\theta}{(E_\nu+m_e)^2-E_\nu^2\cos^2\theta}\right)^2 + \left(\left(g_A^{\nu_l}\right)^2-\left(g_V^{\nu_l}\right)^2\right)\frac{2m_e^2\cos^2\theta}{\left((E_\nu+m_e)^2-E_\nu^2\cos^2\theta\right)}\right\}, \tag{31}$$

$$\left(\frac{d\sigma_{\nu_l e^-}}{d\theta^2}\right)_{\mathsf{SM}} = \frac{G_F^2 m_e^2}{\pi\left(\theta^2+\frac{2m_e}{E_\nu}\right)^2}\left\{\left(g_V^{\nu_l}+g_A^{\nu_l}\right)^2 + \left(g_V^{\nu_l}-g_A^{\nu_l}\right)^2\left(\frac{\theta^2}{\theta^2+\frac{2m_e}{e_\nu}}\right)^2 + \left(\left(g_A^{\nu_l}\right)^2-\left(g_V^{\nu_l}\right)^2\right)\frac{2m_e^2}{E_\nu^2\left(\theta^2+\frac{2m_e}{E_\nu}\right)}\right\}, \tag{32}$$

$$\left(\frac{d\sigma_{\nu_l e^-}}{dT\theta^2}\right)_{\mathsf{SM}} = \frac{G_F^2 E_\nu}{4\pi}\left\{\left(g_V^{\nu_l}+g_A^{\nu_l}\right)^2+\left(g_V^{\nu_l}-g_A^{\nu_l}\right)^2\left(\frac{T\theta^2}{2m_e}\right)^2+\right.$$
$$\left.\left(\left(g_A^{\nu_l}\right)^2-\left(g_V^{\nu_l}\right)^2\right)\frac{m_e}{E_\nu}\left(1-\frac{T\theta^2}{2m_e}\right)\right\}. \quad (33)$$

The neutrino magnetic moment contribution is (TO DO: *include derivation from [5]*) [2]:

$$\left(\frac{d\sigma_{\nu_l e^-}}{dT_e}\right)_{\mathsf{MAG}} = \frac{\pi\alpha^2}{m_e^2}\left(\frac{1}{T_e}-\frac{1}{E_\nu}\right)\left(\frac{\mu_{\nu_l}}{\mu_B}\right)^2, \quad (34)$$

where α is the fine structure constant.

Analogically to previous, we can also express this cross section in $\cos\theta$, θ^2 and $T\theta^2$:

$$\left(\frac{d\sigma_{\nu_l e^-}}{d\cos\theta}\right)_{\mathsf{MAG}} = \frac{2\pi\alpha^2\left(E_\nu+m_e\right)^2}{m_e^2\cos\theta}\frac{\left(E_\nu+m_e\right)^2-E_\nu^2\cos^2\theta-2m_eE_\nu\cos^2\theta}{\left(\left(E_\nu+m_e\right)^2-E_\nu^2\cos^2\theta\right)^2}\left(\frac{\mu_{\nu_l}}{\mu_B}\right)^2, \quad (35)$$

$$\left(\frac{d\sigma_{\nu_l e^-}}{d\theta^2}\right)_{\mathsf{MAG}} = \frac{\pi\alpha^2}{m_e^2}\frac{\theta^2}{\left(\theta^2+\frac{2m_e}{E_\nu}\right)}\left(\frac{\mu_{\nu_l}}{\mu_B}\right)^2, \quad (36)$$

$$\left(\frac{d\sigma_{\nu_l e^-}}{dT\theta^2}\right)_{\mathsf{MAG}} = \frac{\pi\alpha^2}{4m_e^4}\frac{T\theta^2}{\left(1-\frac{T\theta^2}{2m_e}\right)}\left(\frac{\mu_{\nu_l}}{\mu_B}\right)^2. \quad (37)$$

The magnetic moment contribution exceeds the standard model contribution for low enough T_e [2]:

$$T_e \lesssim \frac{\pi^2\alpha^2}{G_F^2 m_e^3}\left(\frac{\mu_\nu}{\mu_B}\right)^2 \simeq 2.9\times10^{19}\left(\frac{\mu_\nu}{\mu_B}\right)^2[\mathsf{MeV}], \quad (38)$$

which does not depend on the neutrino energy and makes neutrino experiment sensitive to lower energetic neutrinos more sensitive to the neutrino magnetic moment. (TO DO: *this is probably not about sensitivity to lower energetic neutrinos but to lower energetic electron, isn't it?*)

2.3.3 Neutrino on nucleus scattering

2.3.4 Cosmological effects

[NuMMBasicsAndAstro_2022.pdf] One of the most important astrophysical consequences of neutrino non-zero effective magnetic moments is the neutrino helicity change $\nu_l \to \nu_R$ with the appearance of nearly sterile right-handed neutrinos ν_R. In general, this phenomena can proceed in three different mechanisms:

1. the helicity change in the neutrino magnetic moment scattering on electrons (or protons and neutrons),

2. the neutrino spin and spin-flavour precession in an external magnetic field, and
3. the neutrino spin and spin-flavour precession in the transversally moving matter currents or in the transversally polarized matter at rest

For completeness note that the important astrophysical consequence of nonzero neutrino millicharges is the neutrino deviation from the rectilinear trajectory. (TO DO: *find out if this is the same thing that IceCube describes in their paper*)

3 Conclusion

(TO DO: *Report the limit with its uncertainty*)

(TO DO: *Very briefly discuss differences with current world limit and how the techniques differ*)

(TO DO: *Very briefly summarise expectations for future measurements.*)

References

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