Chapter 1

Measuring the Muon Neutrino Magnetic Moment

TO DO: *Also check out NeutrinoMassesPheno2007.pdf, sec 6.4*

TO DO: *Write an introduction to the NuMM* "In the standard model, neutrinos have small charge radii induced by radiative corrections. The predicted values of the electron and muon neutrino charge radii are less than an order of magnitude smaller than the current experimental upper limits and can be tested in the next generation of accelerator and reactor experiments through the observation of neutrino-electron elastic scattering and CEvNS. Precision measurements of the neutrino charge radii would either be an important confirmation of the standard model, or would discover new physics. The same types of experimental measurements are also sensitive to more exotic neutrino electromagnetic properties: magnetic moments and millicharges, which would be certainly due to new BSM physics. The discovery of millicharges or anomalously large neutrino magnetic moments would have also important implications for astrophysics and cosmology."[1]

1.1 Theory of neutrino magnetic moment

As was describe in Sec. **??**, neutrinos in the Standard Model (SM) are massless and electrically neutral particles. However, even SM neutrinos can have electromagnetic interaction through loop diagrams involving charged leptons and the W boson. These interactions are described by the neutrino charge radius, described in section 1.1.2 TO DO: *Re-write this since I'm not going to include the other elmag properties section* [1].

In general Beyond Standard Model (BSM) theories, considering interactions with a single photon as shown on Fig. 1.1, neutrino electromagnetic interactions can be described by an *effective* interaction Hamiltonian [2]

$$\mathcal{H}_{em}^{(\nu)}(x) = \sum_{k,j=1}^{N} \overline{\nu}_k(x)\, \Lambda_{\mu}^{kj} \nu_j(x)\, A^{\mu}(x)\,. \tag{1.1}$$

Here $\nu_k(x)\,, k = 1, ..., N$, are neutrino fields in the mass basis with N neutrino mass states and x denotes the position. Λ_{μ}^{kj} is a general vertex function and $A^{\mu}(x)$ is the electromagnetic field.

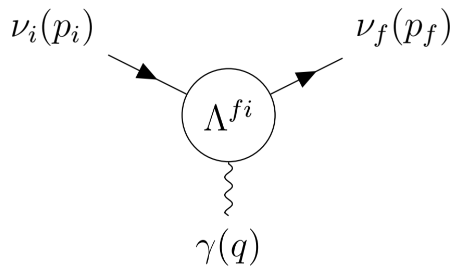

Figure 1.1: Effective coupling of neutrinos with one photon electromagnetic field.

The vertex function $\Lambda_{\mu}^{fi}(q)$ is generally a matrix and in the most general case consistent with the SM gauge invariance [3, 4] can be written in terms of linearly independent products of Dirac matrices (γ) and only depends on the four momentum of the photon $(q = p_f - p_i)$:

$$\begin{aligned}\Lambda_{\mu}^{fi}(q) =& \mathbb{F}_1^{fi}(q^2)\, q_{\mu} + \mathbb{F}_2^{fi}(q^2)\, q_{\mu}\gamma_5 + \mathbb{F}_3^{fi}(q^2)\, \gamma_{\mu} + \mathbb{F}_4^{fi}(q^2)\, \gamma_{\mu}\gamma_5 + \\ & \mathbb{F}_5^{fi}(q^2)\, \sigma_{\mu\nu} q^{\nu} + \mathbb{F}_6^{fi}(q^2)\, \epsilon_{\mu\nu\rho\gamma} q^{\nu} \sigma^{\rho\gamma},\end{aligned} \tag{1.2}$$

where $\mathbb{F}_i^{fi}(q^2)$ are six Lorentz invariant form factors and δ and ϵ are the Dirac delta and the Levi-Civita symbols respectively.

Applying conditions of hermiticity $\left(\mathcal{H}_{em}^{(\nu)\dagger} = \mathcal{H}_{em}^{(\nu)}\right)$ and of the gauge invariance of the electromagnetic field, the vertex function can be rewritten as

$$\Lambda_{\mu}^{fi}(q) = \left(\gamma_{\mu} - q_{\mu}\not{q}/q^2\right)\left[\mathbb{F}_Q^{fi}(q^2) + \mathbb{F}_A^{fi}(q^2)\, q^2\gamma_5\right] - i\sigma_{\mu\nu}q^{\nu}\left[\mathbb{F}_M^{fi}(q^2) + i\mathbb{F}_E^{fi}(q^2)\,\gamma_5\right], \tag{1.3}$$

where $\mathbb{F}_Q^{fi}, \mathbb{F}_M^{fi}, \mathbb{F}_E^{fi}$ and $\mathbb{F}_A^{fi}$ are hermitian matrices representing the charge, dipole magnetic, dipole electric and anapole neutrino form factors respectively. It is clear that the vertex function only depends on the square of the four momentum of the photon q^2. In coupling with a real photon $(q^2 = 0)$ these form factors become the neutrino charge and magnetic, electric and anapole moments. The neutrino charge radius corresponds to the second term in the expansion of the charge form factor [2].

The above expression can be simplified as [5]

$$\Lambda_{\mu}^{fi}(q) = \gamma_{\mu}\left(Q_{\nu_{fi}} + \frac{q^2}{6}\langle r^2\rangle_{\nu_{fi}}\right) - i\sigma_{\mu\nu}q^{\nu}\mu_{\nu_{fi}}, \tag{1.4}$$

where $Q_{\nu_{fi}}$, $\langle r^2\rangle_{\nu_{fi}}$, and $\mu_{\nu_{fi}}$ are the neutrino charge, effective charge radius (also containing anapole moment), and an effective magnetic moment (also containing electric moment) respectively. This is possible thanks to the similar effect of the neutrino charge radius and the anapole moment, or of the neutrino magnetic and electric moment respectively [2]. These are the three neutrino electromagnetic properties (charge, charge radius and magnetic moment) measured in the experiments.

TO DO: *Add a note briefly describing the other elmag properties and mentioning that they could be measured as well, but not describe here. Maybe refer reader to the theoretical overview paper*

1.1.1 Neutrino electric and magnetic dipole moments

The size and effect of neutrino electromagnetic properties depend on the specific BSM theory. Evaluating the one loop diagrams in the minimally extended SM with three right-handed Dirac neutrinos as described in Sec. ?? gives the first approximation of the electric and magnetic moments:

$$\left.\begin{matrix}\mu_{kj}^{D}\\ i\epsilon_{kj}^{D}\end{matrix}\right\} \simeq \frac{3eG_F}{16\sqrt{2}\pi^2}\left(m_k \pm m_j\right)\left(\delta_{kj} - \frac{1}{2}\sum_{l=e,\mu,\tau} U_{lk}^{\star}U_{lj}\frac{m_l^2}{m_W^2}\right), \tag{1.5}$$

where m_k, m_j are the neutrino masses and m_l are the masses of charged leptons which appear in the loop diagrams [2]. Also, D superscript denotes Dirac neutrinos, e is the electron charge, G_F is the Fermi coupling constant, and U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino oscillation matrix. Higher order electromagnetic corrections were neglected, but can also have a significant contribution, depending on the theory.

It can be seen that Dirac neutrinos have no diagonal electric moments $\left(\epsilon_{kk}^{D} = 0\right)$

and their diagonal magnetic moments are approximately

$$\mu_{kk}^{D} \simeq \frac{3eG_F m_k}{8\sqrt{2}\pi^2} \simeq 3.2 \times 10^{-19} \left(\frac{m_k}{\mathrm{eV}}\right) \mu_B, \tag{1.6}$$

where μ_B is the Bohr magneton which represents the value of the electron magnetic moment [2]. Neutrino magnetic moments are therefore strongly suppressed by the smallness of neutrino masses, with theoretical predictions in Eq. 1.6 several orders of magnitude below the reach of current experiments [5].

The transition magnetic moments from Eq. 1.5 are suppressed with respect to the largest of the diagonal magnetic moments by at least a factor of 10^{-4} due to the m_W^2 in the denominator. The transition electric moments are even smaller due to the mass difference in Eq. 1.5. Therefore an experimental observation of a magnetic moment larger than in Eq. 1.6 would indicate physics beyond the minimally extended SM [2, 6].

TO DO: *Actually write why these values are different for Majorana neutrinos than for Dirac neutrinos* Majorana neutrinos in a minimal extension can be obtained by either adding a $\mathrm{SU}(2)_L$ Higgs triplet, or right handed neutrinos together with a $\mathrm{SU}(2)_L$ Higgs singlet [2]. If we neglect the Feynman diagrams which depend on the model of the scalar sector, the magnetic and electric dipole moments are

$$\mu_{kj}^{M} \simeq -\frac{3ieG_F}{16\sqrt{2}\pi^2}\left(m_k + m_j\right) \sum_{l=e,\mu,\tau} \mathrm{Im}\left[U_{lk}^{\star} U_{lj}\right] \frac{m_l^2}{m_W^2}, \tag{1.7}$$

$$\epsilon_{kj}^{M} \simeq \frac{3ieG_F}{16\sqrt{2}\pi^2}\left(m_k - m_j\right) \sum_{l=e,\mu,\tau} \mathrm{Re}\left[U_{lk}^{\star} U_{lj}\right] \frac{m_l^2}{m_W^2}. \tag{1.8}$$

These are difficult to compare to the Dirac case, due to possible presence of Majorana phases in the PMNS matrices, but it is clear that they have the same order of magnitude as Dirac transition dipole moments. However, the neglected model dependent contributions can enhance the transition dipole moments [2].

TO DO: *Re-read the natural upper bounds paper* It is possible [6] to obtain a 'natural' upper limits on the size of the neutrino magnetic moment by calculating its contribution to the neutrino mass by standard model radiative corrections. TO DO: *I don't think this is clear enough, how is this done* For Dirac neutrinos, the radiative correction induced by neutrino magnetic moment, generated at an energy scale Λ_{NP}, to the neutrino mass is generically

$$m_\nu^D \sim \frac{\mu_\nu^D}{3\times 10^{-15}\mu_B}\left[\Lambda\,(\mathsf{TeV})\right]^2 \mathsf{eV}. \tag{1.9}$$

So for $\Lambda_{NP} \simeq 1\text{TeV}$ and $m_\nu \lesssim 0.3\text{eV}$ the limit becomes $\mu_\nu^D \lesssim 10^{-15}\mu_B$. This applies only if New Physics (NP) is well above the electroweak scale ($\Lambda_{EW} \sim 100\text{GeV}$) TO DO: *Finish this sentence*. However, there are theories that contain a Dirac neutrino magnetic moment higher than this limit, for example in frameworks of minimal super-symmetric standard model, by adding more Higgs doublets, or by considering large extra dimensions TO DO: *Add references to the specific theories?* [2].

Similar limit for Majorana neutrino magnetic moment would be less stringent than for Dirac neutrinos due to the antisymmetry of the Majorana neutrino magnetic moment form factors TO DO: *Probably explain here a bit more what does this mean*. Considering $m_\nu \lesssim 0.3\text{eV}$, the limit can be expressed as

$$\mu_{\tau\mu}, \mu_{\tau e} \lesssim 10^{-9}\left[\Lambda\,(\mathsf{TeV})\right]^{-2} \tag{1.10}$$

$$\mu_{\mu e} \lesssim 3\times 10^{-7}\left[\Lambda\,(\mathsf{TeV})\right]^{-2} \tag{1.11}$$

which is shown in the flavour basis TO DO: *Explain here what is the flavour basis*, which relates to the framework used previously via the PMNS matrix as

$$\mu_{ij} = \sum_{\alpha\beta} \mu_{\alpha\beta} U^\star_{\alpha i} U_{\beta j}, \quad \alpha,\beta \in \{e,\mu,\tau\}\,. \tag{1.12}$$

TO DO: *Add a discussion about the triangular inequalities*

These considerations imply, that if a magnetic moment $\mu \gtrsim 10^{-15}\mu_B$ would be measured, it is more plausible that neutrinos are Majorana fermions and that the scale of lepton violation would be well below the conventional see-saw scale [6] TO DO: *double check this claim, also reword this sentence*.

Effective neutrino magnetic moment

Since experiments detect neutrino flavour states, not the mass states, what we measure is an effective ‘flavour’ magnetic moment μ_{eff}. μ_{eff} is influenced by mixing of the neutrino magnetic moments (and electric moments) expressed in the mass basis (as described above) and neutrino oscillations TO DO: *This basis relation was already partly described above, mention that and combine the descriptions*. In the ultra-relativistic limit, the neutrino effective magnetic moment is

$$\mu_{\nu_l}^2 (L, E_\nu) = \sum_j \left| \sum_k U_{lk}^\star e^{\mp i \Delta m_{kj}^2 L / 2E_\nu} \left(\mu_{jk} - i\epsilon_{jk}\right) \right|^2 , \tag{1.13}$$

where the minus sign in the exponent is for neutrinos and the plus sign for antineutrinos [2]. Therefore the only difference between the effective neutrino and antineutrinos magnetic moment is in the phase induced by neutrino oscillations.

For experiments with baselines short enough that neutrino oscillations would not have time to develop ($\Delta m^2 L/2E_\nu \ll\sim 1$), such as the NuMI Off-axis ν_e Appearance (NOvA) Near Detector (ND), the effective magnetic moment can be expressed as

$$\mu_{\nu_l}^2 = \mu_{\overline{\nu}_l}^2 \simeq \sum_j \left| \sum_k U_{lk}^\star \left(\mu_{jk} - i\epsilon_{jk}\right) \right|^2 = \left[U\left(\mu^2 + \epsilon^2\right)U^\dagger + 2\,\mathrm{Im}\left(U\mu\epsilon U^\dagger\right)\right]_{ll'} , \tag{1.14}$$

which is independent of the neutrino energy TO DO: *Figure out how does this relate to the mag moment cross section which does depend on the neutrino energy!*.

TO DO: *Consider if this paragraph is actually important* Since the effective magnetic moment depends on the flavour of the studied neutrino, it is different (but related) for neutrino experiments studying neutrinos from different sources. Additionally some experiments, namely solar neutrino experiments, need to include matter effects on the neutrino oscillations. Therefore the reports on the value (or upper limit) of the effective neutrino magnetic moment are not directly comparable between different types of neutrino experiments. Theorists publish papers trying to extrapolate the measured effective magnetic moments to each neutrino flavour, but necessarily apply assumptions that might not hold in all BSM theories.

1.1.2 Other neutrino electromagnetic properties

COMMENT: *I am not going to report results on these, so should I even mention them here? Maybe it's enough to just mention that they exist in the intro section...* TO DO: *This section is not finished, most of this text is just copied from some theory papers for now*

TO DO: *See also StatusAndPerspectiveOfNuMM2016.pdf*

Neutrino electric charge is heavily constraint by the measurements on the neutrality of matter (since generally neutrinos having an electric charge would also mean that neutrons have charge which would affect all heavier nuclei). It is also constrained by the SN1987A, since neutrino having an effective charge would lengthen its path through the extragalactic magnetic fields and would arrive on earth later. It can also be obtained from nu-on-e scatter from the relationship between neutrino millicharge and magnetic moment. [nuElmagInt2015.pdf - sec. VIIA] TO DO: *Make this description shorter, just a single sentence and combine with the charge radius*

The neutrino charge radius is determined by the second term in the expansion of the neutrino charge form factor and can be interpreted using the Fourier transform of a spherically symmetric charge distribution. It can also be negative since the charge density is not a positively defined quantity. In the SM the charge radius has the form of (possible other definitions exist)

$$\langle r^2_{\nu_l} \rangle_{\mathsf{SM}} = \frac{G_{\mathsf{F}}}{4\sqrt{2}\pi^2} \left[3 - 2 \log \left(\frac{m_l^2}{m_W^2} \right) \right]. \tag{1.15}$$

This corresponds to $\langle r^2_{\nu_\mu} \rangle_{\mathsf{SM}} = 2.4 \times 10^{-33}$ cm^2 and similar scale for other neutrino flavours. [nuElmagInt2015.pdf - sec. VIIB]

[nuElmagInt2015.pdf - sec. VIIB] The effect of the neutrino charge radius on the neutrino-on-electron scattering cross section is through the following shift of the vector coupling constant (Grau and Grifols, 1986; Degrassi, Sirlin, and VMarciano, 1989; Vogel and Engel, 1989; Hagiwara et al., 1994):

$$g_V^{\nu_l} \to g_V^{\nu_l} + \frac{2}{3} m_W^2 \langle r^2_{\nu_l} \rangle \sin^2 \theta_W \tag{1.16}$$

[nuElmagInt2015.pdf - sec. VIIB] The current experimental limits for muon neutrinos are from TO DO: *check the current exp. limits* Hirsch, Nardi, and Restrepo (2003) who obtained the following 90% C.L. bounds on $\langle r^2_{\nu_\mu} \rangle$ from a reanalysis of CHARM-II (Vilain et al., 1995) and CCFR (McFarland et al.,1998) data:

$$-0.52 \times 10^{-32} < \langle r^2_{\nu_\mu} \rangle < 0.68 \times 10^{-32} \text{ cm}^2 \tag{1.17}$$

In the Standard Model, the neutrino anapole moment is somehow coupled with the neutrino charge radii and is functionally identical. the phenomenology of neutrino anapole moments is similar to that of neutrino charge radii. Hence, the limits on the neutrino charge radii discussed in Sec. VII.B also apply to the neutrino anapole moments multiplied by 6. in the standard model the neutrino charge radius and the anapole moment are not defined separately and one can interpret arbitrarily the charge form factor as a charge radius or as an anapole moment. Therefore, the standard model values for the neutrino charge radii in Eqs. (7.35)–(7.38) can be interpreted also as values of the corresponding neutrino anapole moments. [nuElmagInt2015.pdf - sec. VIIC]

It is possible to consider the toroidal dipole moment as a characteristic of the neutrino which is more convenient and transparent than the anapole moment for the description of T-invariant interactions with nonconservation of the P and C symmetries. the toroidal and anapole moments coincide in the static limit when the masses of the initial and final neutrino states are equal to each other. The toroidal (anapole) interactions of a Majorana as well as a Dirac neutrino are expected to contribute to the total cross section of neutrino elastic scattering off electrons, quarks, and nuclei. Because of the fact that the toroidal (anapole) interactions contribute to the helicity preserving part of the scattering of neutrinos on electrons, quarks, and nuclei, its contributions to cross sections are similar to those of the neutrino charge radius. In principle, these contributions can be probed and information about toroidal moments can be extracted in low-energy scattering experiments in the future. Different effects of the neutrino toroidal moment are discussed by Ginzburg and Tsytovich (1985), Bukina, Dubovik, and Kuznetsov (1998a, 1998b), and Dubovik and Kuznetsov (1998). In particular, it has been shown that the neutrino toroidal electromagnetic interactions can produce Cherenkov radiation of neutrinos propagating in a medium. [nuElmagInt2015.pdf - sec. VIIC]

1.1.3 Measuring neutrino magnetic moment

The most sensitive method to measure neutrino magnetic moment is the low energy elastic scattering of (anti)neutrinos on electrons [2]. The diagram for this interaction is shown in Fig. 1.2 displaying the two observables, the recoil electron's kinetic energy

$(T_e = E_{e'} - m_e)$ and the recoil angle with respect to the incoming neutrino beam (θ).

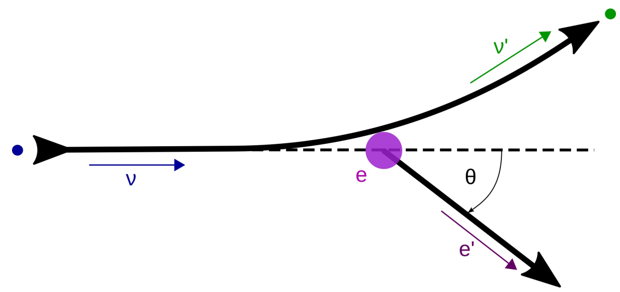

Figure 1.2: Neutrino-on-electron elastic scattering diagram

COMMENT: *Is this derivation too trivial to mention in a thesis? Should I just mention the results? I wanted to have this in the technote, but probably too detailed for a thesis...* TO DO: *Also change all we to passive voice - or should I keep we here?* From simple $2 \rightarrow 2$ kinematics we can calculate

$$(P_\nu - P_{e'})^2 = (P_{\nu'} - P_e)^2 \,, \tag{1.18}$$

$$m_\nu^2 + m_e^2 - 2E_\nu E_{e'} + 2E_\nu p_{e'} \cos\theta = m_\nu^2 + m_e^2 - 2E_{\nu'} m_e . \tag{1.19}$$

Using the energy conservation

$$E_\nu + m_e = E_{\nu'} + E_{e'} = E_{\nu'} + T_e + m_e \Rightarrow E_{\nu'} = E_\nu - T_e \tag{1.20}$$

we get

$$E_\nu p_{e'} \cos\theta = E_\nu E_{e'} - E_{\nu'} m_e = E_\nu (T_e + m_e) - (E_\nu - T_e) m_e = T_e (E_\nu + m_e) \,, \tag{1.21}$$

$$\cos\theta = \frac{E_\nu + m_e}{E_\nu} \sqrt{\frac{T_e^2}{E_{e'}^2 - m_e^2}} = \frac{E_\nu + m_e}{E_\nu} \sqrt{\frac{T_e^2}{T_e^2 + 2T_e m_e}} . \tag{1.22}$$

And finally we get

$$\cos\theta = \frac{E_\nu + m_e}{E_\nu} \sqrt{\frac{T_e}{T_e + 2m_e}} . \tag{1.23}$$

We can rearrange the Eq. 1.23 to get

$$T_e = \frac{2m_e E_\nu^2 \cos^2\theta}{(E_\nu + m_e)^2 - E_\nu^2 \cos^2\theta} . \tag{1.24}$$

Electron's kinetic energy is therefore kinematically constrained by the energy conservation as

$$T_e \leq \frac{2E_\nu^2}{2E_\nu + m_e}, \tag{1.25}$$

which corresponds to the $\cos\theta \to 1$ when the recoil electron goes exactly forward in the incident neutrino direction.

Considering $E_\nu \sim$ GeV, we can approximate $\frac{m_e^2}{E_\nu^2} \to 0$ and from Fig.1.3 we can see that we can approximate all recoil angles to be very small, therefore $\theta^2 \cong (1 - \cos^2\theta)$. Using Eq.1.23 we get

$$T_e\theta^2 \cong T_e \left(1 - \left(\frac{E_\nu + m_e}{E_\nu}\right)^2 \frac{T_e}{T_e + 2m_e}\right) = T_e \left(1 - \left(1 + \frac{2m_e}{E_\nu}\right) \frac{T_e}{T_e + 2m_e}\right), \tag{1.26}$$

therefore

$$T_e\theta^2 \cong \frac{2m_e T_e}{T_e + 2m_e}\left(1 - \frac{T_e}{E_\nu}\right) = 2m_e \left(\frac{1}{1 + \frac{2m_e}{T_e}}\right)\left(1 - \frac{T_e}{E_\nu}\right), \tag{1.27}$$

and finally

$$T_e\theta^2 \cong 2m_e \left(1 - \frac{T_e}{E_\nu}\right) < 2m_e. \tag{1.28}$$

This is a strong limit that clearly distinguishes the neutrino-on-electron (ν-on-e) elastic scattering events from other similar interaction involving single electron (mainly the ν_eCharged current (CC) interaction).

Neutrino magnetic moment cross section

COMMENT: *Should this only be a subsubsection?* In the ultra-relativistic limit, the neutrino magnetic moment changes the neutrino helicity, turning active neutrinos into sterile TO DO: *cite this properly*. Since the SM weak interaction conserves helicity we can simply add the two contribution to the ν-on-e cross section incoherently [2]:

$$\frac{d\sigma_{\nu_l e^-}}{dT_e} = \left(\frac{d\sigma_{\nu_l e^-}}{dT_e}\right)_{\text{SM}} + \left(\frac{d\sigma_{\nu_l e^-}}{dT_e}\right)_{\text{MAG}}. \tag{1.29}$$

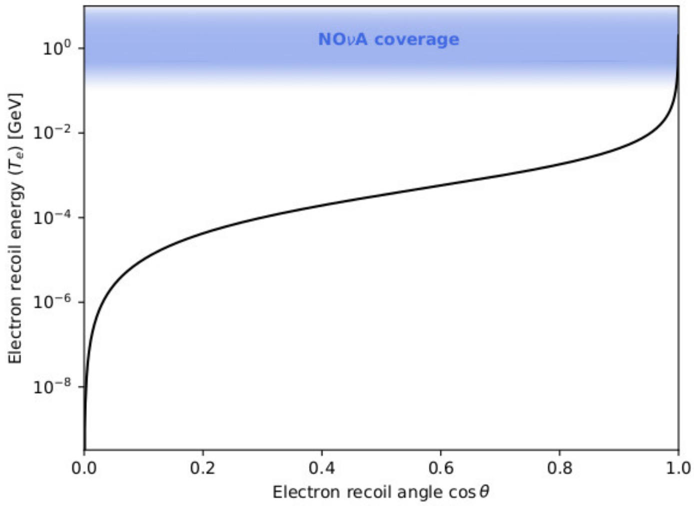

Figure 1.3: Relation between the recoil electron's kinetic energy and angle for ν-ON-E elastic scattering. The coverage of the NOvA detectors for measuring the electron recoil energy is shown in blue. Only very forwards electron's are recorded in NOvA.

The SM contribution can be expressed as [2]:

$$\left(\frac{d\sigma_{\nu_l e^-}}{dT_e}\right)_{\mathrm{SM}} = \frac{G_F^2 m_e}{2\pi}\left\{(g_V^{\nu_l}+g_A^{\nu_l})^2 + (g_V^{\nu_l}-g_A^{\nu_l})^2\left(1-\frac{T_e}{E_\nu}\right)^2 + \left((g_A^{\nu_l})^2-(g_V^{\nu_l})^2\right)\frac{m_e T_e}{E_\nu^2}\right\}, \quad (1.30)$$

where the coupling constants g_V and g_A are different for different neutrino flavours and for antineutrinos. Their values are:

$$g_V^{\nu_e} = 2\sin^2\theta_W + 1/2, \qquad g_A^{\nu_e} = 1/2, \quad (1.31)$$

$$g_V^{\nu_{\mu,\tau}} = 2\sin^2\theta_W - 1/2, \qquad g_A^{\nu_{\mu,\tau}} = -1/2. \quad (1.32)$$

For antineutrinos $g_A \to -g_A$.

TO DO: *Decide if this is actually useful or not* Using Eq. 1.24 it is possible to get the differential cross section for $\cos\theta$:

$$dT_e = \frac{4m_e E_\nu^2 (m_e+E_\nu)^2}{\left[(m_e+E_\nu)^2 - E_\nu^2\cos^2\theta\right]^2}\cos\theta d\cos\theta \quad (1.33)$$

ht

Table 1.1: Neutrino-on-electron elastic scattering total cross sections. TO DO: *Move units to title and add cross sections with thresholds. Also reference this somewhere in text* from Fundamentals of neutrino Physics and Astrophysics, p.139

Process	Total cross section
$\nu_e + e^-$	$\simeq 93 \times 10^{-43} E_\nu \mathrm{cm}^2 \mathrm{GeV}^{-1}$
$\overline{\nu}_e + e^-$	$\simeq 39 \times 10^{-43} E_\nu \, \mathrm{cm}^2 \mathrm{GeV}^{-1}$
$\nu_{\mu,\tau} + e^-$	$\simeq 15 \times 10^{-43} E_\nu \, \mathrm{cm}^2 \mathrm{GeV}^{-1}$
$\overline{\nu}_{\mu,\tau} + e^-$	$\simeq 13 \times 10^{-43} E_\nu \, \mathrm{cm}^2 \mathrm{GeV}^{-1}$

as

$$\left(\frac{d\sigma_{\nu_l e^-}}{d\cos\theta}\right)_{\text{SM}} = \frac{2G_F^2 E_\nu^2 m_e^2 \cos\theta \left(E_\nu + m_e\right)^2}{\pi \left(\left(E_\nu + m_e\right)^2 - E_\nu^2 \cos^2\theta\right)^2}$$
$$\left\{ (g_V^{\nu_l} + g_A^{\nu_l})^2 + (g_V^{\nu_l} - g_A^{\nu_l})^2 \left(1 - \frac{2m_e E_\nu \cos^2\theta}{\left(E_\nu + m_e\right)^2 - E_\nu^2 \cos^2\theta}\right)^2 + \right.$$
$$\left. \left((g_A^{\nu_l})^2 - (g_V^{\nu_l})^2\right) \frac{2m_e^2 \cos^2\theta}{\left(\left(E_\nu + m_e\right)^2 - E_\nu^2 \cos^2\theta\right)} \right\}, \quad (1.34)$$

The neutrino magnetic moment contribution is TO DO: *include derivation from [?]* [2]:

$$\left(\frac{d\sigma_{\nu_l e^-}}{dT_e}\right)_{\text{MAG}} = \frac{\pi\alpha^2}{m_e^2}\left(\frac{1}{T_e} - \frac{1}{E_\nu}\right)\left(\frac{\mu_{\nu_l}}{\mu_B}\right)^2, \quad (1.35)$$

where α is the fine structure constant TO DO: *Calculate the total mag moment cross sections*.

Comparison of the SM and the neutrino magnetic moment cross sections is shown on Fig.1.4. Whereas the SM cross section is flat with $T_e \to 0$, the neutrino magnetic moment cross section keeps increasing to infinity. However, this reach is limited by the experimental capabilities of detecting such low energetic neutrinos. Possible NOvA coverage is shown in a shaded blue and it is uncertain we could actually reach as low as 100 MeV TO DO: *Change this claims a little bit*.

As can be seen in Fig. 1.4 and Fig. 1.5, the magnetic moment contribution exceeds the SM contribution for low enough T_e. This can be approximated as [2]:

$$T_e \lesssim \frac{\pi^2\alpha^2}{G_F^2 m_e^3}\left(\frac{\mu_\nu}{\mu_B}\right)^2 \simeq 2.9 \times 10^{19} \left(\frac{\mu_\nu}{\mu_B}\right)^2 [\text{MeV}], \quad (1.36)$$

which does not depend on the neutrino energy and makes experiments sensitive to

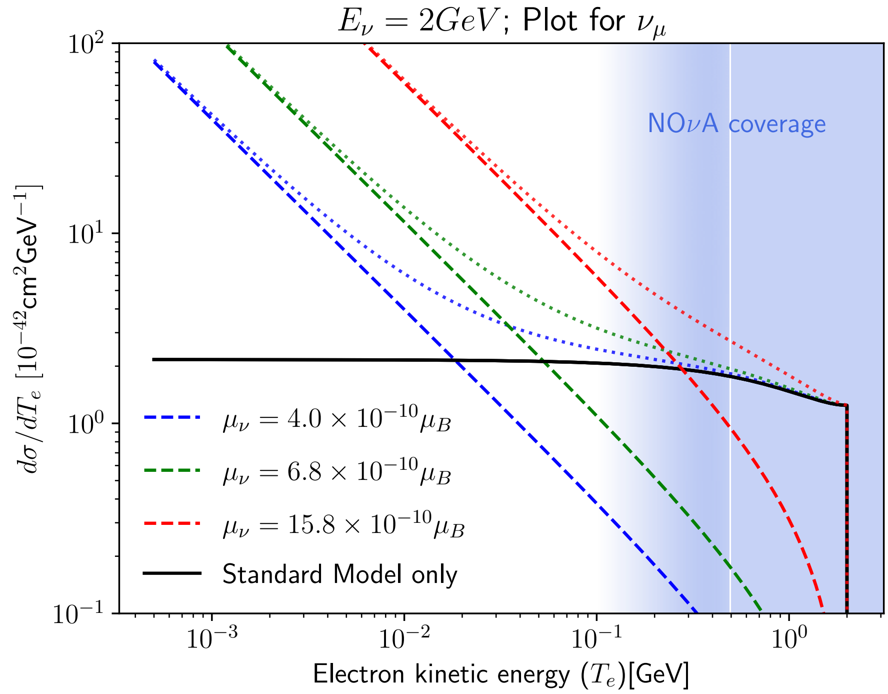

Figure 1.4: Comparison of the neutrino magnetic moment (coloured) and the SM (black) cross sections for the ν-ON-E elastic scattering. Different colours depict different values of the neutrino magnetic moment. Dashed lines are the individual cross sections and dotted lines are the added total cross section with the standard model contribution. NOvA coverage of electron recoil energies is shown in shaded blue TO DO: *Reference the colours on the figures to the origins of the values (LSND and Biao)*.

lower energetic electrons more sensitive to the neutrino magnetic moment. This is especially true for the recent dark matter experiments which put stringent limits on the solar neutrino effective magnetic moment, as described in the following section.

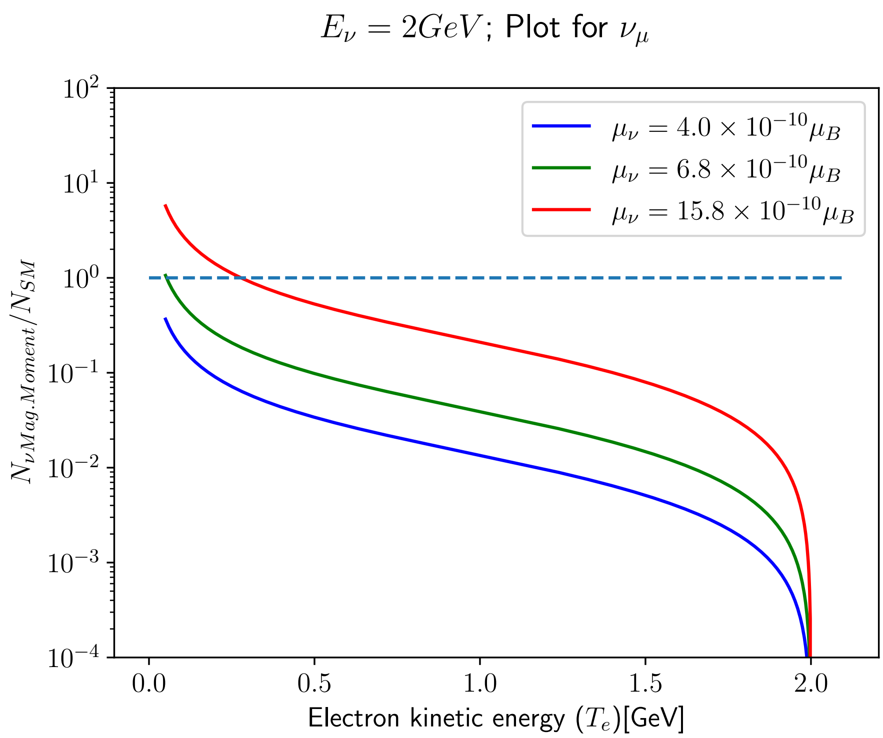

Figure 1.5: Ratio of the neutrino magnetic moment cross section to the SM cross section for the ν-ON-E elastic scattering. Different colours depict different effective muon neutrino magnetic moment values.

Acronyms

ν-ON-E neutrino-on-electron. 10, 11, 13, 14

BSM Beyond Standard Model. 1, 3, 6

CC Charged current. 10

ND Near Detector. 6

NOvA NuMI Off-axis ν_e Appearance (experiment). 6, 11–13

NP New Physics. 5

PMNS Pontecorvo-Maki-Nakagawa-Sakata. 3–5

SM Standard Model. 1–4, 10–14

Bibliography

[1] Patrick Huber et al. Snowmass Neutrino Frontier Report. In *Snowmass 2021*, 11 2022.

[2] Carlo Giunti and Alexander Studenikin. Neutrino electromagnetic interactions: A window to new physics. *Rev. Mod. Phys.*, 87:531–591, Jun 2015. doi:10.1103/RevModPhys.87.531.

[3] Boris Kayser. Majorana neutrinos and their electromagnetic properties. *Phys. Rev. D*, 26:1662–1670, Oct 1982. doi:10.1103/PhysRevD.26.1662. URL https://link.aps.org/doi/10.1103/PhysRevD.26.1662.

[4] José F. Nieves. Electromagnetic properties of majorana neutrinos. *Phys. Rev. D*, 26:3152–3158, Dec 1982. doi:10.1103/PhysRevD.26.3152. URL https://link.aps.org/doi/10.1103/PhysRevD.26.3152.

[5] Carlo Giunti, Julieta Gruszko, Benjamin Jones, Lisa Kaufman, Diana Parno, and Andrea Pocar. Report of the Topical Group on Neutrino Properties for Snowmass 2021. 9 2022.

[6] Nicole F. Bell, Mikhail Gorchtein, Michael J. Ramsey-Musolf, Petr Vogel, and Peng Wang. Model independent bounds on magnetic moments of Majorana neutrinos. *Phys. Lett. B*, 642:377–383, 2006. doi:10.1016/j.physletb.2006.09.055.