Chapter 1

Theory of neutrino physics

Neutrinos were first theoretically proposed by Wolfgang Pauli [1, 2] as very light electrically neutral particles with a half-spin and a possible magnetic moment [3]. They formed a crucial part of Enrico Fermi's successful theory of β decays [4, 5], which solidified their importance in particle physics even before their first experimental detection. Fermi's theory developed into the Standard Model (SM) of particle physics [6–8], which in its current form contains three generations of fermions. Each generation consists of two quarks, one charged lepton and one neutrino, which has no mass, nor magnetic moment.

The SM is mathematically described by a Lagrangian, in which neutrinos are created by a two-component left-handed chiral neutrino fields $\nu_{\alpha L}$, where $\alpha = e, \mu, \tau$ denotes the three neutrino generations, also called flavours [9–11]. Neutrino fields form weak isospin doublets $L_\alpha = \begin{pmatrix} \nu_{\alpha L} \\ \alpha_L \end{pmatrix}$ with their associated left-handed charged lepton fields α_L. Unlike for the charged leptons, there is no right-handed chiral neutrino singlet field in the SM. This means that neutrinos cannot obtain a (Dirac[1]) mass term, since the mass terms for fermions arise from the Higgs mechanism [12–14] via the Yukawa coupling of the fermion and the Higgs fields [15], which requires a combination of left-handed and right-handed chiral fields [16]. Additionally, since neutrinos are massless in the SM, all the neutrinos are left-handed helicity particles, and all the antineutrinos $\overline{\nu}$ are right-handed helicity antiparticles. Neutrinos and antineutrinos are mutually related by the Charge conjugation - Parity (CP) symmetry: $\nu \xleftrightarrow{CP} \overline{\nu}$.

The interaction terms for neutrinos can be separated into two parts, describing the Charged current (CC) and the Neutral Current (NC) interactions, corresponding to interactions with the W_μ and Z_μ massive gauge fields, which create the $W^\pm$ and

[1]Discussion of Dirac or Majorana nature of neutrinos is in Sec. 1.4

Z^0 bosons respectively. Neglecting the non-neutrino components, the two neutrinos interaction terms are [16]

$$\mathcal{L}_{\mathrm{CC}}^{\mathrm{SM}} = -\frac{g_w}{2\sqrt{2}} j_W^\mu W_\mu^+ + \text{h.c.}, \quad \text{and} \quad \mathcal{L}_{\mathrm{NC}}^{\mathrm{SM}} = -\frac{g_w}{2\cos(\theta_W)} j_Z^\mu Z_\mu^0. \tag{1.1}$$

Here g_w is the weak coupling constant, θ_W is the Weinberg angle and j_W^μ and j_Z^μ are the weak currents expressed as

$$j_W^\mu = 2 \sum_{\alpha=e,\mu,\tau} \overline{\nu}_{\alpha L} \gamma^\mu \alpha_L, \tag{1.2}$$

$$j_Z^\mu = \sum_{\alpha=e,\mu,\tau} \overline{\nu}_{\alpha L} \gamma^\mu \nu_{\alpha L}, \tag{1.3}$$

where γ^μ, $\mu = 0, 1, 2, 3$, are the four Dirac gamma matrices.

The two terms of the interaction Lagrangian from Eq. 1.1 describe the possible neutrino interaction vertices shown in Fig. 1.1. These diagrams show the CC and the NC interaction of neutrinos and antineutrinos and, in case of the CC diagram, can also be flipped around the vertical axis to show the production of neutrinos from the weak interaction (or decays) of leptons. They can also be rotated $90°$ to either show the annihilation, or the production of the neutrino-lepton (for CC), or neutrino-antineutrino (for NC) pairs.

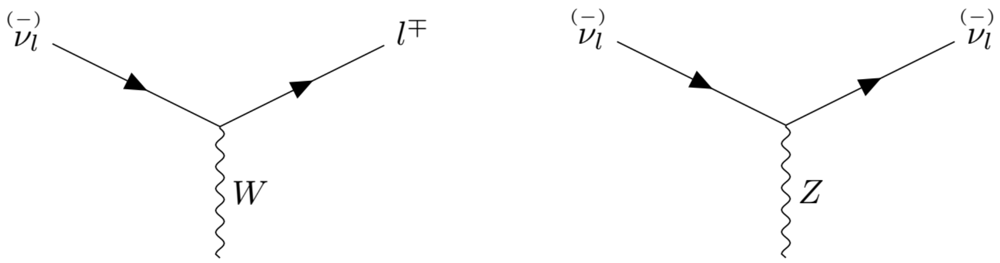

Figure 1.1: Neutrino interaction vertices in the SM via the weak charged currents (left) and the neutral currents (right).

1.1 Neutrino Production

Some of the most common neutrino and antineutrino production channels include nucleon transitions via CC weak interactions. Specifically, the transition of a neutron into a proton, either as a decay of a free neutron, or as a β^- decay for neutrons bound in a nucleus, produces an electron and an electron antineutrino:

$$n \rightarrow p + e^- + \overline{\nu}_e. \tag{1.4}$$

The study of the shape of the electron spectrum from β^- decay was the reason Pauli proposed the existence of the neutrino [1]. Additionally, this channel is an abundant source of $\overline{\nu}_e$ from nuclear reactors, which were the first artificial sources of neutrinos, increasing the neutrino flux by about 100 million compared to the naturally occurring sources, enabling the first ever detection of a neutrino [17–19].

Similarly, the production of an electron neutrino via the transition of a proton into a neutron can occur inside the nucleus either as the β^+ decay:

$$p \rightarrow n + e^+ + \nu_e, \tag{1.5}$$

or via the electron capture:

$$p + e^- \rightarrow n + \nu_e. \tag{1.6}$$

This channel occurs in stars and in the first phase of supernovae [16].

However, most supernovae neutrinos are created via a thermal pair production via NC interaction

$$e^- + e^+ \rightarrow \nu_\alpha + \overline{\nu}_\alpha \tag{1.7}$$

producing neutrinos and antineutrinos of all flavours. Neutrino pair production via the decay of Z^0 was studied in great detail [20], since the magnitude of the decay width depends on the number of light active neutrino flavours, with the current best fit $N_\nu = 2.984$ [21].

An abundant source of ν_μ and $\overline{\nu}_\mu$ is the decay of pions and muons

$$p + X \rightarrow \pi^\pm \rightarrow \mu^\pm + \nu_\mu\,(\overline{\nu}_\mu) \tag{1.8}$$

$$\mu^\pm \rightarrow e^\pm + \nu_\mu\,(\overline{\nu}_\mu) + \nu_e\,(\overline{\nu}_e)\,, \tag{1.9}$$

which naturally occurs in Earth's atmosphere from the interaction of cosmic ray protons. It is notable, that if all the muons decay by the time they reach Earth's surface, the ratio of $\nu_\mu : \nu_e$ should be exactly 2:1. This processed is also used in the modern accelerator-based sources of neutrinos, which accelerate protons to the desired energies, impinge them onto a fixed target, and focus the resulting hadrons to achieve a highly pure and precise source of ν_μ or $\overline{\nu}_\mu$ [22, 23]. Similarly, decays of heavier hadrons, such as kaons and charmed particles, also produce neutrinos, including ν_τ and $\overline{\nu}_\tau$ [24, 25].

1.2 Neutrino Interactions

The interaction of neutrinos can either be categorized based on the target, which is generally either electron or a nucleus, or on the neutrino energy.

Neutrino-electron interactions occur either via elastic scattering, which result in a neutrino and an electron in the final state, or via the inverse muon (or tau) decay, which contains a muon (or tau) in the final state. Both of these interactions are purely governed by Quantum Electro Dynamics (QED) and are theoretically very well understood, and we are currently using their measurements to provide constraints on the parameters within the QED theory. Additionally, while the neutrino-on-electron elastic scattering does not have a threshold energy and can occur for any neutrinos, the inverse muon decay has a threshold for the ν_μ energy of $10.92\,\mathrm{GeV}$, and the inverse tau decay $E_{\nu_\tau} > 3\,\mathrm{TeV}$ [16, 26].

The interaction of neutrinos with a nucleus can be to an extent approximated by an interaction of neutrinos on quasi-free nucleons [27]. The interaction of neutrinos on nucleons can be further classified based on the products it generates. We illustrate these categories on a case of ν_μ CC cross section in Fig. 1.2. At lower energies neutrinos interact via the elastic interactions in the NC channel, simply kicking the nucleon out of the nucleus, and via the Quasi Elastic (QE) interactions in the CC channel, transforming the neutron or proton into its nucleon counterpart. For example, the QE interaction of an antineutrino on a proton

$$\overline{\nu}_l + p \rightarrow n + l^+ \tag{1.10}$$

is often called the inverse β decay and it was used for the first ever detection of neutrinos (specifically electron antineutrinos from a nuclear reactor) by Cowan and

Reines [17, 18]. Analogically, the interaction of neutrinos on a neutron

$$\nu_l + n \rightarrow p + l^- \tag{1.11}$$

is interesting especially due to having no low energy threshold for ν_e and it is commonly used for detection of solar neutrinos [28]. For ν_μ and ν_τ there is a low energy threshold for the QE interactions on a free nucleon. For ν_μ this threshold is about $110\,\mathrm{GeV}$, which can be clearly seen in Fig. 1.2 as the drop off cross section at low energies. The ν_μ CCQE channel was used for the first detection of ν_μ [29] from an accelerator and is commonly used for the detection of atmospheric neutrinos [30–32]. The threshold for the CCQE interaction of ν_τ is about $3.5\,\mathrm{GEV}$, which meant that it was only discovered only in 2000 by the DONUT Collaboration at Fermilab [24].

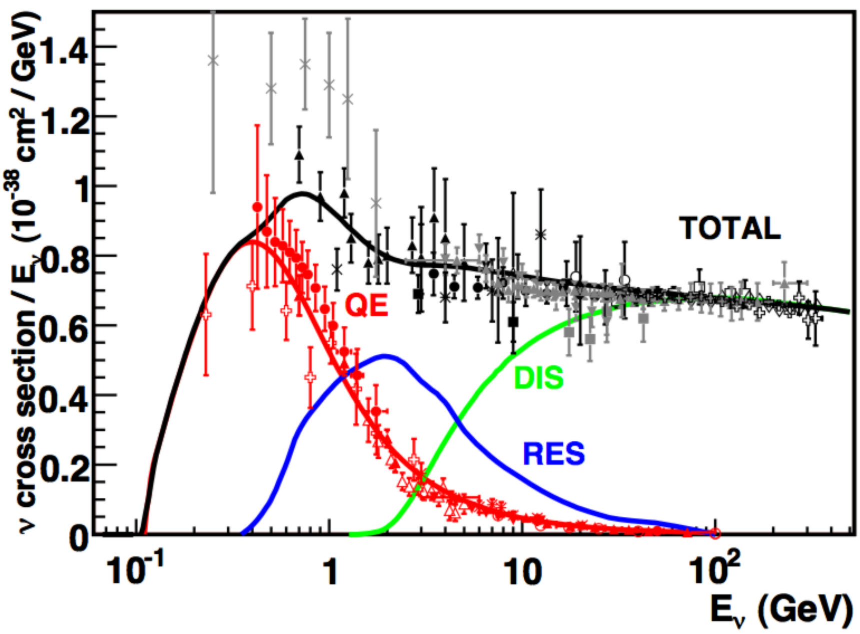

Figure 1.2: Neutrino CC cross sections on an isolated nucleon divided by the neutrino energy based on the interaction types. Figure is from [33] and compares the measured data [27] and the prediction provided by the NUANCE generator [34].

With an increase in neutrino energy above a certain threshold (about $270\,\mathrm{MeV}$ for ν_μ), neutrinos start the Resonant baryon production (Res), which commonly decay into a nucleon and an additional hadron. These hadrons are initially pions at lower energies, but by increasing the neutrino incident energy, they start producing multiple pions and other mesons and hyperons. With even higher incident energies, neutrinos start probing the quark contents of the individual nucleons in the Deep

Inelastic Scattering (DIS) interaction, as can be seen in Fig. 1.2.

Even though the approximation of nuclei as collections of quasi-free nucleons is useful, it has been shown [35] that there are important nuclear effects that have to be considered. Example of these are: Fermi motion of nucleons and their binding inside the nucleus, Pauli's exclusion principle and nucleon energy levels, short and long range nucleon-nucleon correlations, Meson Exchange Current (MEC), or Final State Interaction (FSI) [36].

Additionally, if the total energy transferred to the nucleus is small relative to the size of the nucleus, neutrinos can interact with the entire nucleus coherently. At low energies, neutrinos can interact via the Coherent Elastic ν Nucleus Scattering (CEvNS), where the contributions from each individual nucleon are simply added together coherently [37], leaving behind an excited nucleus. At higher energies, neutrinos can interact via the Coherent pion production (COHπ) without transferring much momentum to the nucleus. Therefore, there is only a single pion produced (charged through CCCOHπ and neutral through NCCOHπ), which receives most of the transferred momentum and generally travels in the direction of the initial neutrino. This means that the COHπ can mimic real $\mu^{\pm}$, e^{-}, or γ signal in a detector [36].

Another notable example is the two particle - two hole (2p2h) interaction [38–40], which occurs when neutrinos interact with a correlated pair of nucleons, resulting in two nucleons leaving the nucleus, which now has two holes. This interaction can significantly increase the QE cross section [36]. Most frequently, this interaction occurs via the MEC, where the meson effectively propagates the interaction between the two bound nucleons.

The products of each of the aforementioned interactions can re-interact inside the nucleus in the so-called FSI. The final products of the interaction will therefore make it appear as a different interaction inside the detector.

1.3 Neutrino Oscillation

The idea that neutrinos can oscillate originates as a possible transition between neutrinos and antineutrinos [41, 42], analogically to the already known oscillations of $K^0 \leftrightarrow \overline{K^0}$. This was adapted to the oscillations between different neutrino flavours

[43, 44] by considering that the flavour neutrino states ν_α, which are the eigenstates of the weak interactions described in Eq. 1.1-1.3, are not identical to the mass neutrino states ν_k, which are the eigenstates of the vacuum hamiltonian

$$\mathcal{H}_0 \left|\nu_k\right\rangle = E_k \left|\nu_k\right\rangle . \tag{1.12}$$

Instead, the neutrino flavour and mass eigenstates are related as

$$\left|\nu_\alpha\right\rangle = \sum_k U^*_{\alpha k} \left|\nu_k\right\rangle , \tag{1.13}$$

where U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix, named after the authors of neutrino oscillations [16, 45]. U is defined as unitary, which makes the inverse relation simply

$$\left|\nu_k\right\rangle = \sum_\alpha U_{\alpha k} \left|\nu_\alpha\right\rangle . \tag{1.14}$$

From the Schrödinger equation

$$i\frac{d}{dt} \left|\nu_k\left(t\right)\right\rangle = \mathcal{H} \left|\nu_k\left(t\right)\right\rangle \tag{1.15}$$

we get that in vacuum ($\mathcal{H} = \mathcal{H}_0$), the massive neutrino states evolve as plane waves

$$\left|\nu_k\left(t\right)\right\rangle = e^{-iE_k t} \left|\nu_k\right\rangle , \tag{1.16}$$

where the energy of the neutrino state with mass m_k and momentum $\overrightarrow{p}$ is

$$E_k = \sqrt{\overrightarrow{p}^2 + m_k^2}. \tag{1.17}$$

Assuming small neutrinos masses, E_k can be approximated as

$$E_k \xrightarrow{m^2 \ll p^2 \approx E^2} E + \frac{m_k^2}{2E} \tag{1.18}$$

for ultra-relativistic neutrinos [16]. Additionally, given the notation $c \equiv 1$, we can approximate $L \approx t$, where L is the distance neutrino travelled in time t and is easier to measure.

We are interested in the oscillation (transition) of $\nu_\alpha \to \nu_\beta$ over some experimental baseline L. Given that $\langle \nu_k | \nu_j \rangle = \delta_{kj}$ and $\langle \nu_\alpha | \nu_\beta \rangle = \delta_{\alpha\beta}$ and using Eq. 1.16,1.13 and 1.14, we can write the amplitude of the oscillation as

$$A_{\nu_\alpha \to \nu_\beta}(L) \equiv \langle \nu_\beta | \nu_\alpha (L) \rangle = \sum_k U^*_{\alpha k} U_{\beta k} e^{-iE_k L} \tag{1.19}$$

and the probability as

$$P_{\nu_\alpha \to \nu_\beta}(L) = \left| A_{\nu_\alpha \to \nu_\beta}(L) \right|^2 = \sum_{k,j} U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j} e^{-i(E_k - E_j)L}. \tag{1.20}$$

Using Eq. 1.18 and by defining the neutrino mass splitting (also called the mass squared difference) as

$$\Delta m^2_{kj} \equiv m^2_k - m^2_j, \tag{1.21}$$

we get

$$P_{\nu_\alpha \to \nu_\beta}(L) = \sum_{k,j} U^*_{\alpha k} U_{\beta j} U_{\alpha j} U^*_{\beta j} e^{-i \frac{\Delta m^2_{kj} L}{2E}}. \tag{1.22}$$

So far we haven't assumed the specific number of neutrino mass and flavour states. However, since we currently know three neutrino flavour states, ν_e, ν_μ and ν_τ, for simplicity we also consider three mass states. This is often called the three neutrino paradigm. Therefore, the PMNS matrix has size 3×3 and can be written as [16]:

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{1.23}$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$. The matrix is parametrized using three mixing angles θ_{12}, θ_{13} ,and θ_{23} and one phase, often denoted δ_{CP}. The phase describes the possible CP symmetry violation in neutrino oscillations, which would result in a difference between neutrino and antineutrino oscillation probabilities.

When neutrinos pass through matter, their evolution changes due to coherent elastic CC and NC scattering. However, since the NC scattering affects all neutrino flavours equivalently, it does not have any effect on neutrino oscillations. Additionally, we only need to consider the effect of CC interactions for ν_e, as electrons are the only charged leptons present in matter. This is described by the Mikheyev-Smirnov-Wolfenstein (MSW) effect [46, 47], which uses an effective potential

$$V_{\mathrm{CC}} = \pm\sqrt{2}G_F N_e \tag{1.24}$$

for neutrinos passing through matter with an electron density N_e. G_F is the Fermi coupling constant and the plus or minus sign is for neutrinos or antineutrinos respectively.

The effect of matter on the oscillation probabilities can be expressed as a shift to the values of the mixing angles and the mass squared differences, proportional to the V_{CC}. Since the MSW effect differs for neutrinos and antineutrinos, it needs to be carefully considered especially for the measurement of the δ_{CP}, which rely on the comparison of neutrino to antineutrino oscillations [16].

The first experimental signs of neutrino oscillations appeared as an apparent deficit of solar neutrinos compared to their predicted flux [28]. However, due to low confidence in the prediction of the solar neutrino flux, no conclusion could have been drawn. Similarly, experiments [48–51] measuring atmospheric neutrinos saw a disagreement between the measurement and the prediction for the $\nu_\mu : \nu_e$ fraction of the atmospheric neutrino flux. This *atmospheric neutrino anomaly* was finally resolved by the Super-Kamiokande (SK) experiment [52], reporting the first experimental evidence for neutrino oscillations. The *solar neutrino anomaly* was resolved shortly after by the Sudbury Neutrino Observatory (SNO) experiment [53], which compared the NC rate, unaffected by neutrino oscillations, to the rate of CC neutrino interactions. This was a prove that solar neutrinos oscillate without a reliance on the model of the Sun. This results also confirmed the importance of accounting for the MSW effect in neutrino oscillations, especially for the oscillation of solar neutrinos, due to the large matter density in the Sun.

The frequency of the oscillations of solar neutrinos, compared to the atmospheric neutrinos, proved that there are at least two mass splittings driving neutrino oscillations. Therefore, there must be at least three different neutrino mass states, which implies an existence of at least two neutrino mass states with non-zero masses. This is in a direct contradiction to the SM and is to-date the only laboratory-based observation of physics Beyond Standard Model (BSM) [54].

Currently, oscillations between three neutrino flavour states via and three neutrino mass states are well established [55, 56]. The magnitude of both the neutrino mass splittings and of two mixing angles, θ_{12} and θ_{13}, are measured within 3%. In case of the third mixing angle, θ_{23}, we know it is close to the maximum mixing value of $45°$. However, there are three main questions yet to be determined for neutrino oscillations [54]:

1. What is the sign of the larger neutrino mass splitting? Is the electron neutrino made up of the lightest neutrino mass states (normal ordering), or the heaviest (inverted ordering)?

2. Is $\theta_{23} < 45°$ or $\theta_{23} > 45°$? These determine the $\nu_\mu : \nu_\tau$ relative contributions to the neutrino mass states and are also referred to as the upper and lower octant respectively.

3. Is there CP violation in neutrino oscillations? What is the value of δ_{CP}? If neutrinos oscillate differently than antineutrinos, this could be an important part of the matter-antimatter asymmetry in the Universe.

All three of these questions are jointly investigated in the current Long Baseline (LBL) accelerator neutrino oscillation experiments, namely the NuMI Off-axis ν_e Appearance (NOvA) [57] and the Tokai to Kamioka (T2K) [58] experiments. Both use precise $\overset{(-)}{\nu}_\mu$ beams, affected by the matter effect, and compare the rates of $\overset{(-)}{\nu}_\mu$ disappearance and $\overset{(-)}{\nu}_e$ appearance to constrain neutrino oscillation parameters [54]. The same methods will be used in the next generation LBL neutrino oscillation experiments, Deep Underground Neutrino Experiment (DUNE) [59] and Hyper-Kamiokande (HK) [60], which should give the final answers to the three neutrino oscillation questions [54]. COMMENT: *Should I mention JUNO here? Wanted to mention DUNE as I want to discuss it in the mag. moment chapter's discussion*

1.4 Neutrino Mass

[55] it is not possible to construct a renormalizable mass term for the neutrinos with

the fermionic content and gauge symmetry of the SM. The obvious consequence is that in order to introduce a neutrino mass in the theory one must extend the particle content of the model, depart from gauge invariance and/or renormalizability, or do both.

[**?**] [for Dirac neutrinos only] Let us stress that in this case both the low-energy matter content and the assumed symmetries are different from those of the SM. Consequently, the SM is not even a good low-energy effective theory. Furthermore, this scenario does not explain the fact that neutrinos are much lighter than the corresponding charged fermions, because all of them acquire their mass via the same mechanism.

Existence of neutrino oscillations proves that neutrinos must have mass, but do not tell us what that mass is, only the mass squared differences. They also give no information on the Dirac/Majorana nature of neutrinos. The only model-independent information on the neutrino masses, rather than mass differences, can be extracted from energy-momentum conservation relation in reactions in which a neutrino or an antineutrino is involved. [55]

The recent measurements of neutrino mass put a limit of , that sets them clearly apart from other leptons as the lightest known massive particles. The theoretical mechanism that generates neutrino masses is currently not known and is directly connected to the question on the nature of neutrinos. So far we have only considered neutrinos to be Dirac particles. This means that... And the most trivial neutrino mass generation mechanism, also called the minimally extended SM, add three right handed dirac neutrinos. This is not used as it doesn't explain the smallness of neutrino masses.

How do Majorana neutrinos fit in with CP violation and the discrepancy between neutrino and antineutrino oscillations... ? How can there be a difference if they are the same particles?

[NeutrinoMassesOverview2003.pdf] Sterile neutrinos have none of the SM gauge interactions and they are singlets of the full SM gauge group. As discussed above, with the fermionic content and gauge symmetry of the SM one cannot construct a renormalizable mass term for the neutrinos. So in order to introduce a neutrino mass one must either extend the particle contents of the model or abandon gauge invariance and/or renormalizability.

The existence of non-zero neutrino mass is, to date, the only laboratory-based observation of BSM physics. The disparity of mass scales between neutrinos and other fundamental particles suggests a high-energy scale for neutrino-mass generation. In models where this mechanism lies at or below the TeV scale, the physics of neutrino mass may be accompanied by complementary signatures at colliders. In models where the scale is higher, experiments probing the nature of neutrino mass are the only feasible way of exploring this new physics. The observation of neutrinoless double beta decay would provide direct evidence that lepton number is violated, opening a path to baryogenesis via leptogenesis in the early universe. As such, direct tests of the scale or nature of neutrino mass target some of the most central open questions in fundamental physics today. Other neutrino properties may be connected to extensions of the standard model, yet are not observable via oscillations. Neutrino electromagnetic properties are of fundamental interest, [54].

Experiments for their values? Theoretical predictions for how they obtained them The best and most recent model independent measurement comes from KATRIN as $m_{\nu_e} < 0.8\,\mathrm{eV/c^2}$ [61]

[Fundamentals of neutrino physics and astrophysics] The only extension of the SM that is needed is the introduction of right-handed components $\nu_{\alpha R}$ of the neutrino fields. Such a model is sometimes called the *minimally extended Standard Model.* The right handed neutrino fields are fundamentally different from the other elementary fermion fields because they are invariant under the symmetries of the SM: they are **singlets** of $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L$ and have hypercharge $Y = 0$. The right handed neutrino fields are called sterile [883 = B. Pontecorvo, Sov. Phys. JETP, 26, 984-988, 1968] because they do not participate in weak interactions and their only interaction is gravitational. their right handedness is not required though! could also be left handed but have to be singlets and therefore sterile!

In the minimally extended standard model with three right handed neutrino fields, the SM Higgs-lepton Yukawa Lagrangian is extended by adding a lepton term with the same structure as the second term on the right handed side, which generates the masses of up-type quarks

$$\mathcal{L}_Y = -\sum_{\alpha,\beta=e,\mu,\tau} Y'^{l}_{\alpha\beta}\overline{L}_{\alpha L}\Phi l'_{\beta R} - \sum_{\alpha,\beta=e,\mu,\tau} Y'^{\nu}_{\alpha\beta}\overline{L}_{\alpha L}\tilde{\Phi}\nu'_{\beta R} + \text{h.c.}, \tag{1.25}$$

where Y'^{ν} is a new matrix of Yukawa couplings.

Using the unitary gauge we can diagonalize the Yukawa couplings we obtain

$$\mathcal{L}_Y = -\sum_{\alpha=e,\mu,\tau} \frac{y^l_\alpha v}{\sqrt{2}}\bar{l}_\alpha l_\alpha - \sum_{k=1}^{N} \frac{y^\nu_k v}{\sqrt{2}}\overline{\nu}_k\nu_k - \sum_{\alpha=e,\mu,\tau} \frac{y^l_\alpha}{\sqrt{2}}\bar{l}_\alpha l_\alpha H - \sum_{k=1}^{N} \frac{y^\nu_k}{\sqrt{2}}\overline{\nu}_k\nu_k H \tag{1.26}$$

Therefore the neutrino masses are given by

$$m_k = \frac{y^\nu_k v}{\sqrt{2}} \quad (k = 1, ..., N)\,, \tag{1.27}$$

and massive Dirac neutrinos couple to the Higgs field through the last term. Note that the neutrinos masses are proportional to the Higgs VEV v, as the masses of charged leptons and quarks. However, it is known that the masses of neutrinos are much smaller than those of charged leptons and quarks, but there is no explanations here of the very small values of the eigenvalues Y^ν_k of the Higgs-neutrino Yukawa coupling matrix that are needed. The lagrangian defined this way does not conserve the lepton flavour number, which leads to neutrino oscillations. The Dirac character of massive neutrinos is closely related to the invariance of the total Lagrangian under the global U(1) gauge transformations.

The sterile neutrino fields do not participate in weak interaction with both their left and right components, but can couple with the ordinary neutrinos through the mass therm, generating a complicated mixing between active and sterile degrees of freedom.Since at present there is no indication of the existence of such additional sterile Dirac neutrino fields, ockham's razor suggests to ignore them...

1.4.1 Majorana neutrinos

Detection of neutrinoless double beta decay is the only known method with plausible sensitivity to the Majorana nature of the neutrino, one of the most important open questions in particle physics. [54]

[Fundamentals of neutrinos physics and astrophysics,p.190] The majorana condition can be written as $\nu = \nu^C$, which implies the equality of particle and antiparticle. As noted already by Majorana [765 = E. Majorana, Nuovo Cim., 14, 171-184, 1937], since a Majorana spinor has only two independent components, the Majorana theory is simpler and more economical than the Dirac theory. Hence, the Majorana nature of massive neutrinos may be more natural than the Dirac nature. In fact, neutrinos are majorana particles in most theories beyond the SM. If the neutrino is massless, since the left handed chiral component of the neutrino field obeys the Weyl equation in both the Dirac and Majorana descriptions and the right handed chiral component is irrelevant for neutrino interactions, the Dirac and Majorana theories are physically equivalent. From this it is clear that in practice one can distinguish a Dirac from a Majorana neutrino only by measuring some effect due to the neutrino mass. Moreover, the mass effect must not be of kinematical nature, because the kinematical effects of Dirac and Majorana masses are the same. For example, the Dirac and Majorana nature of neutrinos cannot be revealed through neutrino oscillations! The most promising way to find if neutrinos are Majorana particles is the search for neutrinoless double beta decay.

[OverviewOfNeutrinoPhysicsPheno2024.pdf] In contrast, the Majorana phases do not enter the flavour neutrino oscillation probabilities [22, 85], but contribute to the $\beta\beta_{0\nu}$ decay rate

[62] With the particle contents of the SM and the addition of an arbitrary m number of sterile neutrinos one can construct two types mass terms that arise from gauge invariant **renormalizable** operators:

$$-\mathcal{L}_{M_\nu} = M_{D_{ij}} \overline{\nu}_{si} \nu_{Lj} + \frac{1}{2} M_{N_{ij}} \overline{\nu}_{si} \nu^C_{sj} + \text{h.c.}, \tag{1.28}$$

where ν^C indicated a charge conjugated field, $\nu^C = C\overline{\nu}^T$ and C is the charge conjugation matrix. M_D is a complex $m \times 3$ matrix and M_N is a symmetric matrix of dimension $m \times m$. First term is the Dirac term generated from Yukawa interactions after spontaneous electroweak symmetry breaking. The second term in Eq. (10) is a Majorana mass term. It is different from the Dirac mass terms in many important aspects. It is a singlet of the SM gauge group. Therefore, it can appear as a bare mass term. Furthermore, since it involves two neutrino fields, it breaks lepton number by two units.

In case of only Dirac neutrinos ($M_N = 0$) Let's point out that in this case the SM is not even a good low-energy effective theory since both the matter content and the assumed symmetries are different. Furthermore there is no explanation to the fact that neutrino masses happen to be much lighter than the corresponding charged fermion masses as in this case all acquire their mass via the same mechanism.

In case of $M_N \gg M_D$ (see-saw model), The diagonalization of M_ν leads to three light, ν_α , and m heavy, N, neutrinos, where the heavy states are mostly right-handed while the light ones are mostly left-handed. Both the light and the heavy neutrinos are Majorana particles. Two well-known examples of extensions of the SM that lead to a see-saw mechanism for neutrino masses are SO(10) GUTs [25–27] and left-right symmetry [28]. This is the see-saw mechanism [24–28]. In this case the SM is a good effective low energy theory. Indeed the see-saw mechanism is a particular realization of the general case of a full theory which leads to the SM with three light Majorana neutrinos as its low energy effective realization as we discuss next.

In general, if the SM is an effective low energy theory valid up to the scale Λ_{NP} , the gauge group, the fermionic spectrum, and the pattern of spontaneous symmetry breaking of the SM are still valid ingredients to describe Nature at energies $E \ll \Lambda_{NP}$. We need to consider also non-renormalizable higher dimensional terms in the lagrangian. In this approach the largest effects at low energy are expected to come from dim= 5 operators. Indeed, there is a single set of dimension-five terms that is made of SM fields and is consistent with the gauge symmetry, and this set violates the spontaneous symmetries of the SM. It is given by

$$\mathcal{O}_5 = \frac{Z^\nu_{ij}}{\Lambda_{NP}} \left(\overline{L}_{Li}\tilde{\Phi}\right)\left(\tilde{\Phi}^T L^C_{Lj}\right) + \text{h.c.}, \tag{1.29}$$

which violate total lepton number by two units and leads, upon spontaneous symmetry breaking, to:

$$-\mathcal{L}_{M_\nu} = \frac{Z^\nu_{ij}}{2}\frac{v^2}{\Lambda_{NP}}\overline{\nu}_{Li}\nu^C_{Lj} + \text{h.c.}. \tag{1.30}$$

we see that this is a Majorana mass term built with the left-handed neutrino fields and with:

$$(M_\nu)_{ij} = Z^\nu_{ij}\frac{v^2}{\Lambda_{NP}} \tag{1.31}$$

Since Eq. (29) would arise in a generic extension of the SM, we learn that neutrino masses are very likely to appear if there is NP. As mentioned above, a theory with SM plus m heavy sterile neutrinos leads to three light mass eigenstates and an effective low energy interaction of the form (27). In particular, the scale Λ_{NP} is identified with the mass scale of the heavy sterile neutrinos, that is the typical scale of the eigenvalues of M_N. ...the scale of neutrino masses is suppressed by v/Λ_{NP} when compared to the scale of charged fermion masses providing an explanation not only for the existence of neutrino masses but also for their smallness. Finally, this supports lepton mixing and CP violation unless additional symmetries are imposed on the coefficients Z_{ij}.

Chapter 2

Muon Neutrino Magnetic Moment Measurement

Also check out NeutrinoMassesPheno2007.pdf, sec 6.4

In the standard model, neutrinos have small charge radii induced by radiative corrections. The predicted values of the electron and muon neutrino charge radii are less than an order of magnitude smaller than the current experimental upper limits and can be tested in the next generation of accelerator and reactor experiments through the observation of neutrino-electron elastic scattering and CEvNS. Precision measurements of the neutrino charge radii would either be an important confirmation of the standard model, or would discover new physics. The same types of experimental measurements are also sensitive to more exotic neutrino electromagnetic properties: magnetic moments and millicharges, which would be certainly due to new BSM physics. The discovery of millicharges or anomalously large neutrino magnetic moments would have also important implications for astrophysics and cosmology.[54]

2.1 Theory of neutrino magnetic moment

TO DO: *Re-read the three main theory papers and double check the theoretical overview*

In the Standard Model (SM), neutrinos are massless and electrically neutral particles. However, even in the SM neutrinos can have electromagnetic interaction through loop diagrams involving the charged leptons and the W boson. These interactions are described by the neutrino charge radius, described in section 2.1.2 [**?**].

To include neutrino masses required by neutrino oscillations, we must go Beyond the Standard Model (BSM), where neutrinos can acquire other electromagnetic properties [**?**]. In the most general case, considering interactions with a single photon as shown on Fig. 2.1, neutrino electromagnetic interactions can be described by an *effective* interaction Hamiltonian [?]

$$\mathcal{H}_{em}^{(\nu)}(x) = \sum_{k,j=1}^{N} \overline{\nu}_k(x)\,\Lambda_\mu^{kj}\nu_j(x)\,A^\mu(x)\,. \tag{2.1}$$

Here $\nu_k(x)\,, k \in \{1, ..., N\}$ are neutrino fields in the mass basis with N neutrino mass states. Λ_μ^{kj} is a general vertex function and $A^\mu(x)$ is the electromagnetic field.

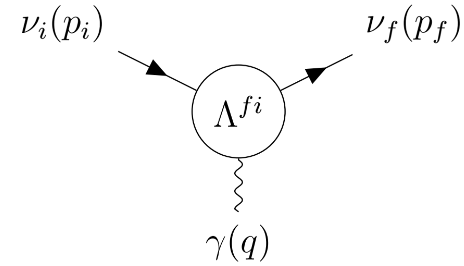

Figure 2.1: Effective coupling of neutrinos with one photon electromagnetic field 2.

The vertex function $\Lambda_\mu^{fi}(q)$ is generally a matrix and in the most general case can be written in terms of linearly independent products of Dirac matrices (γ) and only depends on the square of the four momentum of the photon $(q = p_f - p_i)$:

$$\begin{aligned}\Lambda_\mu^{fi}(q) =& \mathbb{F}_1^{fi}\left(q^2\right)q_\mu + \mathbb{F}_2^{fi}\left(q^2\right)q_\mu\gamma_5 + \mathbb{F}_3^{fi}\left(q^2\right)\gamma_\mu + \mathbb{F}_4^{fi}\left(q^2\right)\gamma_\mu\gamma_5 + \\ & \mathbb{F}_5^{fi}\left(q^2\right)\sigma_{\mu\nu}q^\nu + \mathbb{F}_6^{fi}\left(q^2\right)\epsilon_{\mu\nu\rho\gamma}q^\nu\sigma^{\rho\gamma},\end{aligned} \tag{2.2}$$

where $\mathbb{F}_i^{fi}(q^2)$ are six Lorentz invariant form factors [?].

Applying conditions of hermiticity $\left(\mathcal{H}_{em}^{(\nu)\dagger} = \mathcal{H}_{em}^{(\nu)}\right)$ and of the gauge invariance of the electromagnetic field, we can rewrite the vertex function as

$$\Lambda_\mu^{fi}(q) = \left(\gamma_\mu - q_\mu \not{q}/q^2\right)\left[\mathbb{F}_Q^{fi}\left(q^2\right) + \mathbb{F}_A^{fi}\left(q^2\right)q^2\gamma_5\right] - i\sigma_{\mu\nu}q^\nu\left[\mathbb{F}_M^{fi}\left(q^2\right) + i\mathbb{F}_E^{fi}\left(q^2\right)\gamma_5\right], \tag{2.3}$$

where $\mathbb{F}_Q^{fi}, \mathbb{F}_M^{fi}, \mathbb{F}_E^{fi}$ and $\mathbb{F}_A^{fi}$ are hermitian matrices representing the charge, dipole magnetic, dipole electric and anapole neutrino form factors. In coupling with a real photon $(q^2 = 0)$ these become the neutrino charge and magnetic, electric and anapole moments. The neutrino charge radius corresponds to the second term in the expansion of the charge form factor [?].

We can simplify the above expression as [?]

$$\Lambda_{\mu}^{fi}(q) = \gamma_{\mu}\left(Q_{\nu_{fi}} + \frac{q^2}{6}\langle r^2\rangle_{\nu_{fi}}\right) - i\sigma_{\mu\nu}q^{\nu}\mu_{\nu_{fi}}, \tag{2.4}$$

where $Q_{\nu_{fi}}$, $\langle r^2\rangle_{\nu_{fi}}$, and $\mu_{\nu_{fi}}$ are the neutrino charge, effective charge radius (also containing anapole moment), and an effective magnetic moment (also containing electric moment) respectively. This is possible thanks to the proportional effect of the neutrino charge radius and the anapole moment, or the neutrino magnetic and electric moment respectively [?]. These quantities (charge, charge radius and magnetic moment) are the three neutrino electromagnetic properties measured in experiments.

2.1.1 Neutrino electric and magnetic dipole moments

The size and effect of the neutrino electromagnetic properties depends on the specific theory beyond the standard model.

Evaluating the one loop diagrams in the minimal extension of the standard model with three right handed Dirac neutrinos gives us the first approximation of the electric and magnetic moments:

$$\left.\begin{matrix}\mu_{kj}^{D}\\ i\epsilon_{kj}^{D}\end{matrix}\right\} \simeq \frac{3eG_F}{16\sqrt{2}\pi^2}\left(m_k \pm m_j\right)\left(\delta_{kj} - \frac{1}{2}\sum_{l=e,\mu,\tau} U_{lk}^{\star}U_{lj}\frac{m_l^2}{m_W^2}\right), \tag{2.5}$$

where m_k, m_j are the neutrino masses and m_l are the masses of charged leptons which appear in the loop diagrams [?]. e is the electron charge, G_F is the Fermi coupling constant, and U is the PMNS neutrino oscillation matrix. Higher order electromagnetic corrections were neglected, but those can also have a significant contribution, depending on the theory.

It can be seen that there dirac neutrinos have no diagonal electric moments $\left(\epsilon_{kk}^{D} = 0\right)$ and their diagonal magnetic moments are approximately

$$\mu_{kk}^{D} \simeq \frac{3eG_F m_k}{8\sqrt{2}\pi^2} \simeq 3.2 \times 10^{-19}\left(\frac{m_k}{\text{eV}}\right)\mu_B, \tag{2.6}$$

where μ_B is the Bohr magneton [?].

The transition magnetic moments are suppressed with respect to the largest of the diagonal magnetic moments by at least a factor of 10^{-4} due to the m_W^2 in the denominator. The transition electric moments are even smaller due to the mass difference in Eq.2.5. Therefore an experimental observation of a magnetic moment larger than in Eq.2.6 would indicate physics beyond the minimally extended standard model [? ?].

Majorana neutrinos in a minimal extension can be obtained by either adding a SU $(2)_L$ Higgs triplet, or right handed neutrinos together with a SU $(2)_L$ Higgs singlet [?]. If we neglect the Feynman diagrams which depend on the model of the scalar sector, the magnetic and electric dipole moments are

$$\mu_{kj}^M \simeq -\frac{3ieG_F}{16\sqrt{2}\pi^2}\left(m_k + m_j\right) \sum_{l=e,\mu,\tau} \mathrm{Im}\left[U_{lk}^\star U_{lj}\right] \frac{m_l^2}{m_W^2}, \tag{2.7}$$

$$\epsilon_{kj}^M \simeq \frac{3ieG_F}{16\sqrt{2}\pi^2}\left(m_k - m_j\right) \sum_{l=e,\mu,\tau} \mathrm{Re}\left[U_{lk}^\star U_{lj}\right] \frac{m_l^2}{m_W^2}. \tag{2.8}$$

These are difficult to compare to the Dirac case, due to possible presence of Majorana phases in the PMNS matrices, but it is clear that they have the same order of magnitude as Dirac transition dipole moments. However, the neglected model dependent contributions can enhance the transition dipole moments [?].

It is possible [?] to obtain a "natural" upper limits on the size of neutrino magnetic moment by calculating its contribution to the neutrino mass by standard model radiative corrections. For Dirac neutrinos, the radiative correction induced by neutrino magnetic moment, generated at an energy scale Λ, to the neutrino mass is generically

$$m_\nu^D \sim \frac{\mu_\nu^D}{3\times 10^{-15}\mu_B}\left[\Lambda\left(\text{TeV}\right)\right]^2 \text{eV}. \tag{2.9}$$

So for $\Lambda \simeq 1$TeV TO DO: *figure out what exactly does this energy scale actually relate to and explain it here?* and $m_\nu \lesssim 0.3$eV the limit becomes $\mu_\nu^D \lesssim 10^{-15}\mu_B$. This applies only if the new physics is well above the electroweak scale ($\Lambda_{EW} \sim 100$GeV). It is possible to get Dirac neutrino magnetic moment higher than this limit, for example in frameworks of minimal super-symmetric standard model, by adding more Higgs doublets, or by considering large extra dimensions [?].

A similar limit for Majorana neutrino magnetic moment would be less stringent due to the antisymmetry of the Majorana neutrino magnetic moment form factors.

Considering $m_\nu \lesssim 0.3$eV, the limit can be expressed as

$$\mu_{\tau\mu}, \mu_{\tau e} \lesssim 10^{-9} \left[\Lambda\,(\text{TeV})\right]^{-2} \tag{2.10}$$

$$\mu_{\mu e} \lesssim 3 \times 10^{-7} \left[\Lambda\,(\text{TeV})\right]^{-2} \tag{2.11}$$

which is shown in the flavour basis [?]. This expression relates to the framework used previously as

$$\mu_{ij} = \sum_{\alpha\beta} \mu_{\alpha\beta} U^{\star}_{\alpha i} U_{\beta j}, \quad \alpha, \beta \in \{e, \mu, \tau\}\,. \tag{2.12}$$

These considerations imply, that if a magnetic moment $\mu \gtrsim 10^{-15}\mu_B$ would be measured, it is more plausible that neutrinos are Majorana fermions and that the scale of lepton violation would be well below the conventional see-saw scale [?] TO DO: *double check this claim*.

Effective neutrino magnetic moment

Since experiments detect neutrino flavour states, not the mass states, what we measure in experiments is an effective "flavour" magnetic moment μ_{eff}. μ_{eff} is influenced by mixing of the neutrino magnetic moments (and electric moments) expressed in the mass basis (as described above) and neutrino oscillations. In the ultra-relativistic limit, the (anti)neutrino effective magnetic moment is

$$\mu^2_{\nu_l}(L, E_\nu) = \sum_j \left| \sum_k U^{\star}_{lk} e^{\mp i \Delta m^2_{kj} L / 2E_\nu} \left(\mu_{jk} - i\epsilon_{jk}\right) \right|^2, \tag{2.13}$$

where L is the distance the neutrino travelled, E_ν is the neutrino energy and δm^2 is the neutrino mass squared difference [?]. The minus sign in the exponent is for neutrinos and the plus sign for antineutrinos, therefore the only difference is in the phase induced by neutrino oscillations.

For experiments with baselines short enough that neutrino oscillations would not have time to develop ($\Delta m^2 L/2E_\nu \ll\sim 1$), such as the NOvA Near Detector, the effective magnetic moment can be expressed as

$$\mu_{\nu_l}^2 = \mu_{\overline{\nu}_l}^2 \simeq \sum_j \left| \sum_k U_{lk}^\star \left(\mu_{jk} - i\epsilon_{jk}\right) \right|^2 = \left[U\left(\mu^2 + \epsilon^2\right) U^\dagger + 2\,\mathrm{Im}\left(U\mu\epsilon U^\dagger\right)\right]_{ll'}, \tag{2.14}$$

which is independent of the neutrino energy and of the source to detector distance.

It is important to mention, that since the effective magnetic moment depends on the flavour of the studied neutrino, it is different (but related) for neutrino experiment studying neutrinos from different sources. Additionally some experiments, namely solar neutrino experiments, need to include matter effects on the neutrino oscillations. Therefore the reports on the value (or upper limit) of the effective neutrino magnetic moment are not directly comparable between different types of neutrino experiments. Theorists publish papers trying to extrapolate the measured effective magnetic moments to each neutrino flavour, but necessarily apply assumptions that might not hold in all BSM theories.

2.1.2 Other neutrino electromagnetic properties

TO DO: *This section is not finished, most of this text is just copied from some theory papers for now*

Neutrino electric charge is heavily constraint by the measurements on the neutrality of matter (since generally neutrinos having an electric charge would also mean that neutrons have charge which would affect all heavier nuclei). It is also constrained by the SN1987A, since neutrino having an effective charge would lengthen its path through the extragalactic magnetic fields and would arrive on earth later. It can also be obtained from nu-on-e scatter from the relationship between neutrino millicharge and magnetic moment. [nuElmagInt2015.pdf - sec. VIIA]

The neutrino charge radius is determined by the second term in the expansion of the neutrino charge form factor and can be interpreted using the Fourier transform of a spherically symmetric charge distribution. It can also be negative since the charge density is not a positively defined quantity. In the SM the charge radius has the form of (possible other definitions exist)

$$\langle r_{\nu_l}^2 \rangle_{\mathsf{SM}} = \frac{G_{\mathsf{F}}}{4\sqrt{2}\pi^2}\left[3 - 2\log\left(\frac{m_l^2}{m_W^2}\right)\right]. \tag{2.15}$$

This corresponds to $\langle r^2_{\nu_\mu} \langle_{\text{SM}} = 2.4 \times 10^{-33}\ \text{cm}^2$ and similar scale for other neutrino flavours. [nuElmagInt2015.pdf - sec. VIIB]

[nuElmagInt2015.pdf - sec. VIIB] The effect of the neutrino charge radius on the neutrino-on-electron scattering cross section is through the following shift of the vector coupling constant (Grau and Grifols, 1986; Degrassi, Sirlin, and VMarciano, 1989; Vogel and Engel, 1989; Hagiwara et al., 1994):

$$g_V^{\nu_l} \to g_V^{\nu_l} + \frac{2}{3} m_W^2 \langle r^2_{\nu_l} \rangle \sin^2 \theta_W \tag{2.16}$$

[nuElmagInt2015.pdf - sec. VIIB] The current experimental limits for muon neutrinos are from TO DO: *check the current exp. limits* Hirsch, Nardi, and Restrepo (2003) who obtained the following 90% C.L. bounds on $\langle r^2_{\nu_\mu} \rangle$ from a reanalysis of CHARM-II (Vilain et al., 1995) and CCFR (McFarland et al.,1998) data:

$$-0.52 \times 10^{-32} < \langle r^2_{\nu_\mu} \rangle < 0.68 \times 10^{-32}\ \text{cm}^2 \tag{2.17}$$

In the Standard Model, the neutrino anapole moment is somehow coupled with the neutrino charge radii and is functionally identical. the phenomenology of neutrino anapole moments is similar to that of neutrino charge radii. Hence, the limits on the neutrino charge radii discussed in Sec. VII.B also apply to the neutrino anapole moments multiplied by 6. in the standard model the neutrino charge radius and the anapole moment are not defined separately and one can interpret arbitrarily the charge form factor as a charge radius or as an anapole moment. Therefore, the standard model values for the neutrino charge radii in Eqs. (7.35)–(7.38) can be interpreted also as values of the corresponding neutrino anapole moments. [nuElmagInt2015.pdf - sec. VIIC]

It is possible to consider the toroidal dipole moment as a characteristic of the neutrino which is more convenient and transparent than the anapole moment for the description of T-invariant interactions with nonconservation of the P and C symmetries. the toroidal and anapole moments coincide in the static limit when the masses of the initial and final neutrino states are equal to each other. The toroidal (anapole) interactions of a Majorana as well as a Dirac neutrino are expected to contribute to the total cross section of neutrino elastic scattering off electrons, quarks, and nuclei.

Because of the fact that the toroidal (anapole) interactions contribute to the helicity preserving part of the scattering of neutrinos on electrons, quarks, and nuclei, its contributions to cross sections are similar to those of the neutrino charge radius. In principle, these contributions can be probed and information about toroidal moments can be extracted in low-energy scattering experiments in the future. Different effects of the neutrino toroidal moment are discussed by Ginzburg and Tsytovich (1985), Bukina, Dubovik, and Kuznetsov (1998a, 1998b), and Dubovik and Kuznetsov (1998). In particular, it has been shown that the neutrino toroidal electromagnetic interactions can produce Cherenkov radiation of neutrinos propagating in a medium. [nuElmagInt2015.pdf - sec. VIIC]

2.1.3 Measuring neutrino magnetic moment

The most sensitive method to measure neutrino magnetic moment is the low energy elastic scattering of (anti)neutrinos on electrons [?]. The diagram for this interaction is shown on Fig.2.2 showing the two observables, the recoil electron's kinetic energy ($T_e = E_{e\prime} - m_e$) and the recoil angle with respect to the incoming neutrino beam (θ).

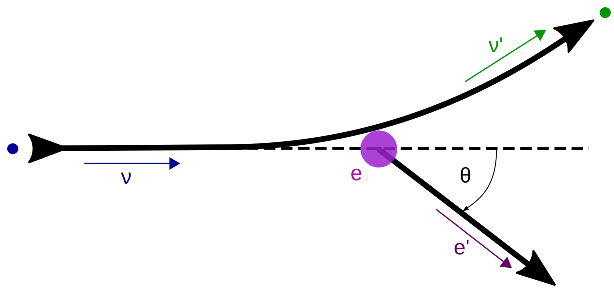

Figure 2.2: Neutrino-on-electron elastic scattering diagram

From simple $2 \to 2$ kinematics we can calculate

$$(P_\nu - P_{e'})^2 = (P_{\nu'} - P_e)^2 \,, \tag{2.18}$$

$$m_\nu^2 + m_e^2 - 2E_\nu E_{e'} + 2E_\nu p_{e'} \cos\theta = m_\nu^2 + m_e^2 - 2E_{\nu'} m_e. \tag{2.19}$$

Using the energy conservation

$$E_\nu + m_e = E_{\nu'} + E_{e'} = E_{\nu'} + T_e + m_e \Rightarrow E_{\nu'} = E_\nu - T_e \tag{2.20}$$

we get

$$E_\nu p_{e'} \cos\theta = E_\nu E_{e'} - E_{\nu'} m_e = E_\nu \left(T_e + m_e\right) - \left(E_\nu - T_e\right) m_e = T_e \left(E_\nu + m_e\right), \tag{2.21}$$

$$\cos\theta = \frac{E_\nu + m_e}{E_\nu} \sqrt{\frac{T_e^2}{E_{e'}^2 - m_e^2}} = \frac{E_\nu + m_e}{E_\nu} \sqrt{\frac{T_e^2}{T_e^2 + 2T_e m_e}}. \tag{2.22}$$

And finally we get

$$\cos\theta = \frac{E_\nu + m_e}{E_\nu} \sqrt{\frac{T_e}{T_e + 2m_e}}. \tag{2.23}$$

We can rearrange the Eq. 2.23 to get

$$T_e = \frac{2m_e E_\nu^2 \cos^2\theta}{\left(E_\nu + m_e\right)^2 - E_\nu^2 \cos^2\theta}. \tag{2.24}$$

Electron's kinetic energy is therefore kinematically constrained by the energy conservation as

$$T_e \leq \frac{2E_\nu^2}{2E_\nu + m_e}, \tag{2.25}$$

which corresponds to the $\cos\theta \to 1$ when the recoil electron goes exactly forward in the incident neutrino direction.

Considering $E_\nu \sim$ GeV, we can approximate $\frac{m_e^2}{E_\nu^2} \to 0$ and from Fig.2.3 we can see that we can approximate all recoil angles to be very small, therefore $\theta^2 \cong (1 - \cos^2\theta)$. Using Eq.2.23 we get

$$T_e\theta^2 \cong T_e \left(1 - \left(\frac{E_\nu + m_e}{E_\nu}\right)^2 \frac{T_e}{T_e + 2m_e}\right) = T_e \left(1 - \left(1 + \frac{2m_e}{E_\nu}\right) \frac{T_e}{T_e + 2m_e}\right), \tag{2.26}$$

therefore

$$T_e\theta^2 \cong \frac{2m_e T_e}{T_e + 2m_e} \left(1 - \frac{T_e}{E_\nu}\right) = 2m_e \left(\frac{1}{1 + \frac{2m_e}{T_e}}\right) \left(1 - \frac{T_e}{E_\nu}\right), \tag{2.27}$$

and finally

$$T_e\theta^2 \cong 2m_e \left(1 - \frac{T_e}{E_\nu}\right) < 2m_e. \tag{2.28}$$

This is a strong limit that clearly distinguishes the neutrino-on-electron elastic scattering events from other similar interaction involving single electron (mainly the ν_e Charged Current interaction).

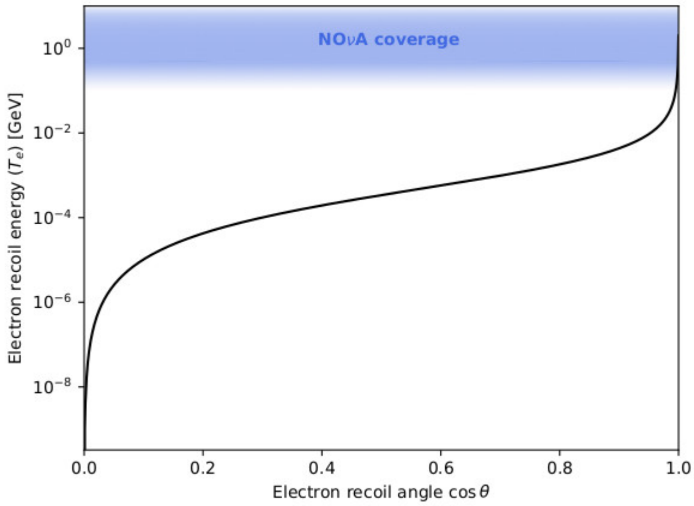

Figure 2.3: Relation between the recoil electron's kinetic energy and angle for neutrino-on-electron elastic scattering. The coverage of the NOvA detectors for measuring the electron recoil energy is shown in blue. Only very forwards electron's are recorded in NOvA.

Neutrino magnetic moment cross section

In the ultrarelativistic limit, the neutrino magnetic moment changes the neutrino helicity, turning active neutrinos into sterile TO DO: *this is a very strong statement and it probably need a bit more backing up*. Since the SM weak interaction conserves helicity we can simply add the two contribution to the neutrino-on-electron cross section incoherently [?]:

$$\frac{d\sigma_{\nu_l e^-}}{dT_e} = \left(\frac{d\sigma_{\nu_l e^-}}{dT_e}\right)_{\text{SM}} + \left(\frac{d\sigma_{\nu_l e^-}}{dT_e}\right)_{\text{MAG}} . \tag{2.29}$$

The standard model contribution can be expressed as [?]:

$$\left(\frac{d\sigma_{\nu_l e^-}}{dT_e}\right)_{\text{SM}} = \frac{G_F^2 m_e}{2\pi}\left\{(g_V^{\nu_l} + g_A^{\nu_l})^2 + (g_V^{\nu_l} - g_A^{\nu_l})^2\left(1 - \frac{T_e}{E_\nu}\right)^2 + \left((g_A^{\nu_l})^2 - (g_V^{\nu_l})^2\right)\frac{m_e T_e}{E_\nu^2}\right\} , \tag{2.30}$$

where the coupling constants g_V and g_A are different for different neutrino flavours ht

Table 2.1: Neutrino-on-electron elastic scattering total cross sections

Process	Total cross section
$\nu_e + e^-$	$\simeq 93 \times 10^{-43} E_\nu \mathrm{cm}^2\mathrm{GeV}^{-1}$
$\overline{\nu}_e + e^-$	$\simeq 39 \times 10^{-43} E_\nu \,\mathrm{cm}^2\mathrm{GeV}^{-1}$
$\nu_{\mu,\tau} + e^-$	$\simeq 15 \times 10^{-43} E_\nu \,\mathrm{cm}^2\mathrm{GeV}^{-1}$
$\overline{\nu}_{\mu,\tau} + e^-$	$\simeq 13 \times 10^{-43} E_\nu \,\mathrm{cm}^2\mathrm{GeV}^{-1}$

and for antineutrinos. Their values are:

$$g_V^{\nu_e} = 2\sin^2\theta_W + 1/2, \qquad g_A^{\nu_e} = 1/2, \tag{2.31}$$

$$g_V^{\nu_{\mu,\tau}} = 2\sin^2\theta_W - 1/2, \qquad g_A^{\nu_{\mu,\tau}} = -1/2. \tag{2.32}$$

For antineutrinos $g_A \to -g_A$.

Using Eq. 2.24 to get the differential cross section on $\cos\theta$:

$$dT_e = \frac{4m_e E_\nu^2 \left(m_e + E_\nu\right)^2}{\left[\left(m_e + E_\nu\right)^2 - E_\nu^2\cos^2\theta\right]^2} \cos\theta d\cos\theta \tag{2.33}$$

We can also express this as

$$\begin{aligned}\left(\frac{d\sigma_{\nu_l e^-}}{d\cos\theta}\right)_{\mathrm{SM}} &= \frac{2G_F^2 E_\nu^2 m_e^2 \cos\theta \left(E_\nu + m_e\right)^2}{\pi\left(\left(E_\nu + m_e\right)^2 - E_\nu^2\cos^2\theta\right)^2} \\ &\left\{ (g_V^{\nu_l} + g_A^{\nu_l})^2 + (g_V^{\nu_l} - g_A^{\nu_l})^2 \left(1 - \frac{2m_e E_\nu \cos^2\theta}{\left(E_\nu + m_e\right)^2 - E_\nu^2\cos^2\theta}\right)^2 + \right. \\ &\left. \left((g_A^{\nu_l})^2 - (g_V^{\nu_l})^2\right) \frac{2m_e^2\cos^2\theta}{\left(\left(E_\nu + m_e\right)^2 - E_\nu^2\cos^2\theta\right)} \right\}, \end{aligned} \tag{2.34}$$

[Fundamentals of neutrino Physics and Astrophysics, p.139] The total neutrino-electron elastic scattering cross section for large energies is

The neutrino magnetic moment contribution is TO DO: *include derivation from [?]* [?]:

$$\left(\frac{d\sigma_{\nu_l e^-}}{dT_e}\right)_{\mathrm{MAG}} = \frac{\pi\alpha^2}{m_e^2}\left(\frac{1}{T_e} - \frac{1}{E_\nu}\right)\left(\frac{\mu_{\nu_l}}{\mu_B}\right)^2, \tag{2.35}$$

where α is the fine structure constant.

Comparison of the Standard Model and the neutrino magnetic moment cross sections is shown on Fig.2.4. Whereas the SM cross section is flat with $T_e \to 0$, the νMM cross section keeps increasing to infinity. However, this reach is limited by the experimental capabilities of detecting such low energetic neutrinos. Possible NOvA coverage is shown in a shaded blue and it is uncertain we could actually reach as low as $100\ \mathrm{MeV}$.

TO DO: *Reference the colours on the figures to the origins of the values (LSND and Biao)*

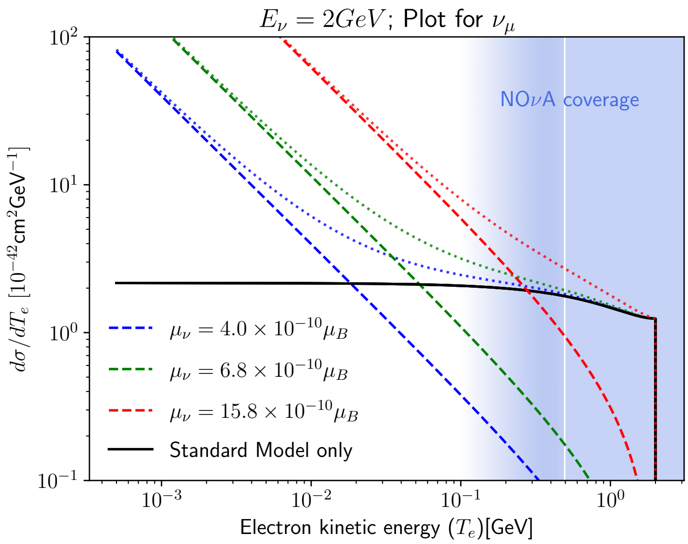

Figure 2.4: Comparison of the neutrino magnetic moment (coloured) and Standard Model (black) cross sections for the neutrino-on-electron elastic scattering. Different colours depict different values of the neutrino magnetic moment. Dashed lines are the individual cross sections and dotted lines are the added total cross section with the standard model contribution. NOvA coverage of electron recoil energies is shown in shaded blue.

As can be seen on Fig.2.4 and Fig.2.5, the magnetic moment contribution exceeds the standard model contribution for low enough T_e. This can be approximated as [?]:

$$T_e \lesssim \frac{\pi^2\alpha^2}{G_F^2 m_e^3}\left(\frac{\mu_\nu}{\mu_B}\right)^2 \simeq 2.9 \times 10^{19}\left(\frac{\mu_\nu}{\mu_B}\right)^2 [\mathrm{MeV}]\,, \tag{2.36}$$

which does not depend on the neutrino energy and makes experiments sensitive to lower energetic electrons more sensitive to the neutrino magnetic moment. This is especially true for the recent dark matter experiments which put stringent limits on the solar neutrino effective magnetic moment, as described in the following section.

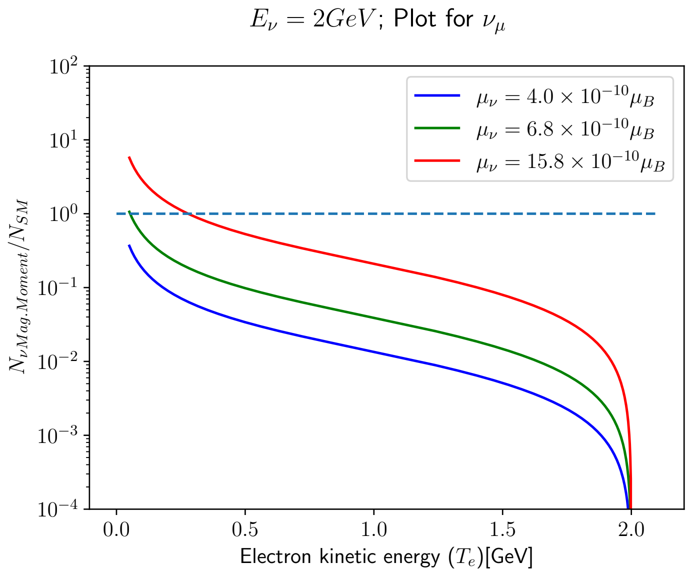

Figure 2.5: Ratio of the netrino magnetic moment cross section to the standard model cross section for the neutrino-on-electron elastic scattering. Different colours depict different effective muon neutrino magnetic moment values.

Acronyms

2p2h two particle - two hole. 6

BSM Beyond Standard Model. 10

CC Charged current. 1, 2, 4–6, 8, 9

CEvNS Coherent Elastic ν Nucleus Scattering. 6

COHπ Coherent pion production. 6

CP Charge conjugation - Parity (symmetry). 1, 8, 10

DIS Deep Inelastic Scattering. 5

DUNE Deep Underground Neutrino Experiment. 10

FSI Final State Interaction. 6

HK Hyper-Kamiokande. 10

LBL Long Baseline. 10

MEC Meson Exchange Current. 6

MSW Mikheyev-Smirnov-Wolfenstein. 9

NC Neutral Current. 1–4, 6, 8, 9

NOvA NuMI Off-axis ν_e Appearance (experiment). 10

PMNS Pontecorvo-Maki-Nakagawa-Sakata. 7, 8

QE Quasi Elastic (interaction). 4–6

QED Quantum Electro Dynamics. 4

Res Resonant baryon production. 5

SK Super-Kamiokande. 9

SM Standard Model. 1, 2, 10, 11

SNO Sudbury Neutrino Observatory. 9

T2K Tokai to Kamioka (experiment). 10

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