Chapter 1

Theory of neutrino physics

Neutrinos were first theoretically proposed by Wolfgang Pauli [1, 2] as very light electrically neutral particles with a half-spin and a possible magnetic moment [3]. They formed a crucial part of Enrico Fermi's successful theory of β decays [4, 5], which solidified their importance in particle physics even before their first experimental detection. Fermi's theory developed into the Standard Model (SM) of particle physics [6–8], which in its current form contains three generations of fermions. Each generation consists of two quarks, one charged lepton and one neutrino, which has no mass, nor magnetic moment.

The SM is mathematically described by a Lagrangian, in which neutrinos are represented by a two-component left-handed chiral neutrino fields $\nu_{\alpha L}$, where $\alpha = e, \mu, \tau$ denotes the three neutrino generations, also called flavours [9–11]. Neutrino fields form weak isospin doublets $L_\alpha = \begin{pmatrix} \nu_{\alpha_L} \\ \alpha_L \end{pmatrix}$ with their associated left-handed charged lepton fields α_L. Unlike for the charged leptons, there is no right-handed chiral neutrino singlet field in the SM. This means that neutrinos cannot obtain a (Dirac[1]) mass term, since the mass terms for fermions arise from the Higgs mechanism [12–14] via the Yukawa coupling of the fermion and the Higgs fields [15], which requires a combination of left-handed and right-handed chiral fields [16]. Additionally, since neutrinos are massless in the SM, all the neutrinos are left-handed helicity particles, and all the antineutrinos $\overline{\nu}$ are right-handed helicity antiparticles. Neutrinos and antineutrinos are mutually related by the Charge conjugation - Parity (CP) symmetry: $\nu \overset{CP}{\longleftrightarrow} \overline{\nu}$.

The interaction terms for neutrinos can be separated into two parts, describing the Charged current (CC) and the Neutral Current (NC) interactions, corresponding to interactions with the W_μ and Z_μ massive gauge fields, which create the $W^\pm$ and

[1] Discussion of Dirac or Majorana nature of neutrinos is in Sec. 1.4

Z^0 bosons respectively. Neglecting the non-neutrino components, the two neutrinos interaction terms are [16]

$$\mathcal{L}_{\text{CC}}^{\text{SM}} = -\frac{g_w}{2\sqrt{2}} j_W^\mu W_\mu^+ + \text{h.c.}, \quad \text{and} \quad \mathcal{L}_{\text{NC}}^{\text{SM}} = -\frac{g_w}{2\cos(\theta_W)} j_Z^\mu Z_\mu^0. \tag{1.1}$$

Here g_w is the weak coupling constant, θ_W is the Weinberg angle and j_W^μ and j_Z^μ are the weak currents expressed as

$$j_W^\mu = 2 \sum_{\alpha=e,\mu,\tau} \overline{\nu}_{\alpha L} \gamma^\mu \alpha_L, \tag{1.2}$$

$$j_Z^\mu = \sum_{\alpha=e,\mu,\tau} \overline{\nu}_{\alpha L} \gamma^\mu \nu_{\alpha L}, \tag{1.3}$$

where γ^μ, $\mu = 0, 1, 2, 3$, are the four Dirac gamma matrices.

The two terms of the interaction Lagrangian from Eq. 1.1 describe the possible neutrino interaction vertices shown in Fig. 1.1. These diagrams show the CC and the NC interaction of neutrinos and antineutrinos and, in case of the CC diagram, can also be flipped around the vertical axis to show the production of neutrinos from the weak interaction (or decays) of leptons. They can also be rotated $90°$ to either show the annihilation, or the production of the neutrino-lepton (for CC), or neutrino-antineutrino (for NC) pairs.

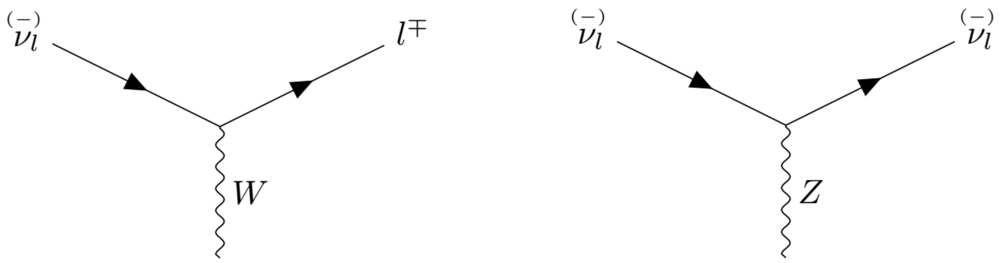

Figure 1.1: Neutrino interaction vertices in the SM via the weak charged currents (left) and the neutral currents (right).

1.1 Neutrino Production

Some of the most common neutrino and antineutrino production channels include nucleon transitions via CC weak interactions. Specifically, the transition of a neutron into a proton, either as a decay of a free neutron, or as a β^- decay for neutrons bound in a nucleus, produces an electron and an electron antineutrino:

$$n \to p + e^- + \overline{\nu}_e. \tag{1.4}$$

The study of the shape of the electron spectrum from β^- decay was the reason Pauli proposed the existence of the neutrino [1]. Additionally, this channel is an abundant source of $\overline{\nu}_e$ from nuclear reactors, which were the first artificial sources of neutrinos, increasing the neutrino flux by about 100 million compared to the naturally occurring sources, enabling the first ever detection of a neutrino [17–19].

Similarly, the production of an electron neutrino via the transition of a proton into a neutron can occur inside the nucleus either as the β^+ decay:

$$p \to n + e^+ + \nu_e, \tag{1.5}$$

or via the electron capture:

$$p + e^- \to n + \nu_e. \tag{1.6}$$

This channel occurs in stars and in the first phase of supernovae [16].

However, most supernovae neutrinos are created via a thermal pair production via NC interaction

$$e^- + e^+ \to \nu_\alpha + \overline{\nu}_\alpha \tag{1.7}$$

producing neutrinos and antineutrinos of all flavours. Neutrino pair production via the decay of Z^0 was studied in great detail [20], since the magnitude of the decay width depends on the number of light active neutrino flavours, with the current best fit $N_\nu = 2.984$ [21].

An abundant source of ν_μ and $\overline{\nu}_\mu$ is the decay of pions and muons

$$p + X \to \pi^\pm \to \mu^\pm + \nu_\mu \left(\overline{\nu}_\mu\right) \tag{1.8}$$

$$\mu^\pm \to e^\pm + \nu_\mu \left(\overline{\nu}_\mu\right) + \nu_e \left(\overline{\nu}_e\right), \tag{1.9}$$

which naturally occurs in Earth's atmosphere from the interaction of cosmic ray protons. It is notable, that if all the muons decay by the time they reach Earth's surface, the ratio of $\nu_\mu : \nu_e$ should be exactly 2:1. This process is also used in the modern accelerator-based sources of neutrinos, which accelerate protons to the desired energies, impinge them onto a fixed target, and focus the resulting hadrons to achieve a highly pure and precise source of ν_μ or $\overline{\nu}_\mu$ [22, 23]. Similarly, decays of heavier hadrons, such as kaons and charmed particles, also produce neutrinos, including ν_τ and $\overline{\nu}_\tau$ [24, 25].

1.2 Neutrino Interactions

The interaction of neutrinos can either be categorized based on the target, which is generally either an atomic electron or a nucleus, or on the neutrino energy.

Neutrino-electron interactions occur either via elastic scattering, which result in a neutrino and an electron in the final state, or via the inverse muon (or tau) decay, which contains a muon (or tau) in the final state. Both of these interactions at the lowest order only involve free leptons and are theoretically very well understood. The neutrino-on-electron elastic scattering does not have a threshold energy and can occur for any neutrinos. However, due to the energy conservation and the large difference between m_e and m_μ/m_τ, the inverse muon decay has an energy threshold of $E_{\nu_\mu} > 10.92\,\mathrm{GeV}$, and the inverse tau decay $E_{\nu_\tau} > 3\,\mathrm{TeV}$ [16, 26].

The interaction of neutrinos with a nucleus can be to an extent approximated by an interaction of neutrinos on quasi-free nucleons [27]. The interaction of neutrinos on nucleons can be further classified based on the products it generates. We illustrate these categories on a case of ν_μ CC cross section in Fig. 1.2. At lower energies neutrinos interact via the elastic interactions in the NC channel, simply kicking the nucleon out of the nucleus, and via the Quasi Elastic (QE) interactions in the CC channel, transforming the neutron or proton into its nucleon counterpart. For example, the QE interaction of an antineutrino on a proton

$$\overline{\nu}_l + p \to n + l^+ \tag{1.10}$$

is often called the inverse β decay and it was used for the first ever detection of neutrinos (specifically electron antineutrinos from a nuclear reactor) by Cowan and

Reines [17, 18]. Analogically, the interaction of neutrinos on a neutron

$$\nu_l + n \rightarrow p + l^- \tag{1.11}$$

is interesting especially due to having no low energy threshold for ν_e and it is commonly used for detection of solar neutrinos [28]. For ν_μ and ν_τ there is a low energy threshold for the QE interactions on a free nucleon. For ν_μ this threshold is about $110\,\mathrm{MeV}$, which can be clearly seen in Fig. 1.2 as the drop in the cross section at low energies. The ν_μ CCQE channel was used for the first detection of ν_μ [29] from an accelerator and is commonly used for the detection of atmospheric neutrinos [30–32]. The threshold for the CCQE interaction of ν_τ is about $3.5\,\mathrm{GeV}$, which meant that it was only discovered only in 2000 by the DONUT Collaboration at Fermilab [24].

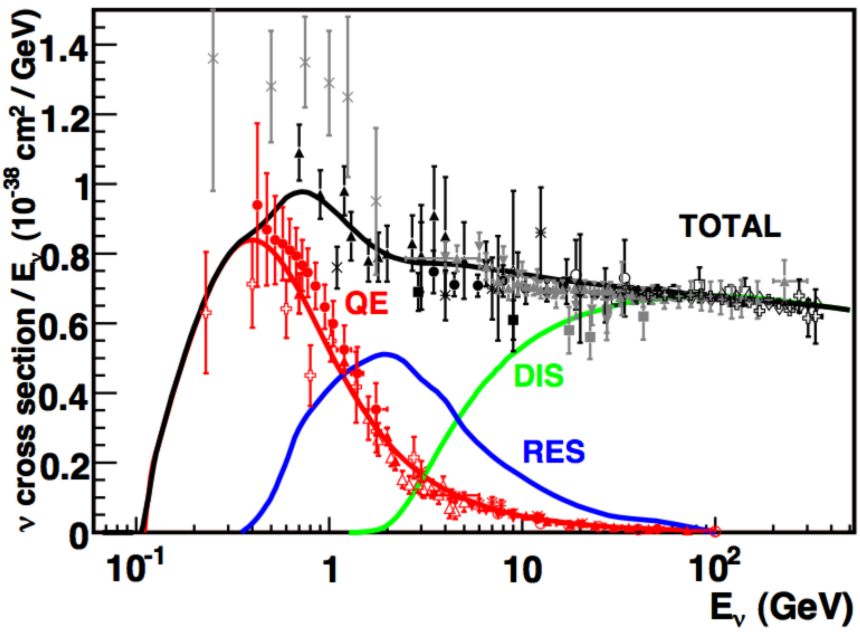

Figure 1.2: Neutrino CC cross sections on an isolated nucleon divided by the neutrino energy based on the interaction types: Quasi Elastic (QE), Resonant baryon production (Res) and Deep Inelastic Scattering (DIS). Figure is from [33] and compares the measured data [27] and the prediction provided by the NUANCE generator [34].

With an increase in neutrino energy above a certain threshold (about $270\,\mathrm{MeV}$ for ν_μ), neutrinos start the Resonant baryon production (Res). These baryons then commonly decay into a nucleon, same as for the QE interaction, however now with additional hadrons. These hadrons are initially single pions at lower neutrino incident energies, but become multiple pions and other hadrons as energy increases. With even higher incident energies, neutrinos start probing the quark contents of the individual nucleons in the Deep Inelastic Scattering (DIS) interaction, as can be seen in Fig. 1.2.

Even though the approximation of nuclei as collections of quasi-free nucleons is useful, it has been shown [35] that there are important nuclear effects that have to be considered. Examples of these are: Fermi motion of nucleons and their binding inside the nucleus, Pauli's exclusion principle and nucleon energy levels, short and long range nucleon-nucleon correlations, Meson Exchange Current (MEC), or Final State Interaction (FSI) [36].

Additionally, if the total energy transferred to the nucleus is small relative to the size of the nucleus, neutrinos can interact with the entire nucleus coherently. At low energies, neutrinos can interact via the Coherent Elastic ν Nucleus Scattering (CEvNS), where the contributions from each individual nucleon are simply added together coherently [37], leaving behind an excited nucleus. At higher energies, neutrinos can interact via the Coherent pion production (COHπ) without transferring much momentum to the nucleus. Therefore, there is only a single pion produced (charged through CCCOHπ and neutral through NCCOHπ), which receives most of the transferred momentum and generally travels in the direction of the initial neutrino. This means that the COHπ can mimic real $\mu^{\pm}$, e^{-}, or γ signal in a detector [36].

Another notable example is the two particle - two hole (2p2h) interaction [38–40], which occurs when neutrinos interact with a correlated pair of nucleons, resulting in two nucleons leaving the nucleus, which now has two holes. This interaction can significantly increase the QE cross section [36]. Most frequently, this interaction occurs via the MEC, where the meson effectively propagates the interaction between the two bound nucleons.

The products of each of the aforementioned interactions can re-interact inside the nucleus in the so-called FSI. The final products of the interaction will therefore make it appear as a different interaction inside the detector.

1.3 Neutrino Oscillation

The idea that neutrinos can oscillate originates as a possible transition between neutrinos and antineutrinos [41, 42], analogically to the already known oscillations of

$K^0 \leftrightarrow \overline{K^0}$. This was adapted to the oscillations between different neutrino flavours [43, 44] by considering that the flavour neutrino states ν_α, which are the eigenstates of the weak interactions described in Eq. 1.1-1.3, are not identical to the mass neutrino states ν_k, which are the eigenstates of the vacuum Hamiltonian

$$\mathcal{H}_0 \left|\nu_k\right\rangle = E_k \left|\nu_k\right\rangle , \tag{1.12}$$

with energy E_k.

Instead, the neutrino flavour and mass eigenstates are related as

$$\left|\nu_\alpha\right\rangle = \sum_k U^*_{\alpha k} \left|\nu_k\right\rangle , \tag{1.13}$$

where U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix, named after the authors of neutrino oscillations [16, 45]. U is defined as unitary, which makes the inverse relation simply

$$\left|\nu_k\right\rangle = \sum_\alpha U_{\alpha k} \left|\nu_\alpha\right\rangle . \tag{1.14}$$

From the Schrödinger equation

$$i\frac{d}{dt} \left|\nu_k\left(t\right)\right\rangle = \mathcal{H} \left|\nu_k\left(t\right)\right\rangle \tag{1.15}$$

we get that in vacuum ($\mathcal{H} = \mathcal{H}_0$), the massive neutrino states evolve as plane waves

$$\left|\nu_k\left(t\right)\right\rangle = e^{-iE_k t} \left|\nu_k\right\rangle , \tag{1.16}$$

where the energy of the neutrino state with mass m_k and momentum $\overrightarrow{p}$ is

$$E_k = \sqrt{\overrightarrow{p}^2 + m_k^2}. \tag{1.17}$$

Assuming small neutrinos masses, E_k can be approximated as

$$E_k \xrightarrow{m^2 \ll p^2 \approx E^2} E + \frac{m_k^2}{2E} \tag{1.18}$$

for ultra-relativistic neutrinos [16]. Additionally, given the notation $c \equiv 1$, we can approximate $L \approx t$, where L is the distance neutrino travelled in time t and is easier to measure.

We are interested in the oscillation (transition) of $\nu_\alpha \to \nu_\beta$ over some experimental baseline L. Given that $\langle \nu_k | \nu_j \rangle = \delta_{kj}$ and $\langle \nu_\alpha | \nu_\beta \rangle = \delta_{\alpha\beta}$ and using Eq. 1.16, 1.13 and 1.14, we can write the amplitude of the oscillation as

$$A_{\nu_\alpha \to \nu_\beta}(L) \equiv \langle \nu_\beta | \nu_\alpha(L) \rangle = \sum_k U^*_{\alpha k} U_{\beta k} e^{-iE_k L} \tag{1.19}$$

and the probability as

$$P_{\nu_\alpha \to \nu_\beta}(L) = \left| A_{\nu_\alpha \to \nu_\beta}(L) \right|^2 = \sum_{k,j} U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j} e^{-i(E_k - E_j)L}. \tag{1.20}$$

Using Eq. 1.18 and by defining the neutrino mass splitting (also called the mass squared difference) as

$$\Delta m^2_{kj} \equiv m^2_k - m^2_j, \tag{1.21}$$

we get

$$P_{\nu_\alpha \to \nu_\beta}(L) = \sum_{k,j} U^*_{\alpha k} U_{\beta j} U_{\alpha j} U^*_{\beta j} e^{-i\frac{\Delta m^2_{kj} L}{2E}}. \tag{1.22}$$

So far we haven't assumed the specific number of neutrino mass and flavour states. However, since we currently know three neutrino flavour states, ν_e, ν_μ and ν_τ, for simplicity we also consider three mass states. This is often called the three neutrino paradigm. Therefore, the PMNS matrix has size 3×3 and can be written as [16]:

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{1.23}$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$. The matrix is parametrized using three mixing angles θ_{12}, θ_{13} ,and θ_{23} and one phase, often denoted δ_{CP}. The phase describes the possible CP symmetry violation in neutrino oscillations, which would result in a difference between neutrino and antineutrino oscillation probabilities.

When neutrinos pass through matter, their evolution changes due to coherent elastic CC and NC scattering. However, since the NC scattering affects all neutrino flavours equivalently, it does not have any effect on neutrino oscillations. Additionally, we only need to consider the effect of CC interactions for ν_e, as electrons are the only charged leptons present in matter. The effective interaction potential of neutrinos passing through matter with an electron density N_e can be written as

$$V_{\mathrm{CC}} = \pm\sqrt{2}G_F N_e. \tag{1.24}$$

G_F is the Fermi coupling constant and the plus or minus sign is for neutrinos or antineutrinos respectively. The electron density (and therefore the potential) can change along the neutrino path, such as in the Sun, described by the Mikheyev-Smirnov-Wolfenstein (MSW) effect [46, 47]. N_e can also be described as approximately constant, simplifying the description, such as in the accelerator based experiments, where neutrinos pass through the surface of the Earth.

The effect of matter on the oscillation probabilities can be expressed as a shift to the values of the mixing angles and the mass squared differences, proportional to the V_{CC}. Since the MSW effect differs for neutrinos and antineutrinos, it needs to be carefully considered especially for the measurement of the δ_{CP}, which rely on the comparison of neutrino to antineutrino oscillations [16].

The first experimental signs of neutrino oscillations appeared as an apparent deficit of solar neutrinos compared to their predicted flux [28]. However, due to low confidence in the prediction of the solar neutrino flux, no conclusion could have been drawn. Similarly, experiments [48–51] measuring atmospheric neutrinos saw a disagreement between the measurement and the prediction for the $\nu_\mu : \nu_e$ fraction of the atmospheric neutrino flux. This *atmospheric neutrino anomaly* was finally resolved by the Super-Kamiokande (SK) experiment [52], reporting the first experimental evidence for neutrino oscillations. The *solar neutrino anomaly* was resolved shortly after by the Sudbury Neutrino Observatory (SNO) experiment [53], which compared the NC rate, unaffected by neutrino oscillations, to the rate of CC neutrino interactions. This was proof that solar neutrinos oscillate without a reliance on the model of the Sun. This results also confirmed the importance of accounting for the MSW effect in neutrino oscillations, especially for the oscillation of solar neutrinos, due to the large matter density in the Sun.

The frequency of the oscillations of solar neutrinos, compared to the atmospheric neutrinos, proved that there are at least two mass splittings driving neutrino oscillations. Therefore, there must be at least three different neutrino mass states, with at least two of them with non-zero masses. This is in direct contradiction to the SM and is to-date the only laboratory-based observation of physics Beyond Standard Model (BSM) [54].

Currently, oscillations between three neutrino flavour states via three neutrino mass states are well established [55, 56]. The magnitude of both the neutrino mass splittings and of two mixing angles, θ_{12} and θ_{13}, are measured within 3%. In case of the third mixing angle, θ_{23}, we know it is close to the maximum mixing value of $45°$. However, there are three main questions yet to be determined for neutrino oscillations [54]:

1. What is the sign of the larger neutrino mass splitting? Is the electron neutrino made up of the lightest neutrino mass states (normal ordering), or the heaviest (inverted ordering)?

2. Is $\theta_{23} < 45°$ or $\theta_{23} > 45°$? These determine the $\nu_\mu : \nu_\tau$ relative contributions to the neutrino mass states and are also referred to as the upper and lower octant respectively.

3. Is there CP violation in neutrino oscillations? What is the value of δ_{CP}? If neutrinos oscillate differently than antineutrinos, this could be an important part of the matter-antimatter asymmetry in the Universe.

All three of these questions are jointly investigated in the current Long Baseline (LBL) accelerator neutrino oscillation experiments, namely the NuMI Off-axis ν_e Appearance (NOvA) [57] and the Tokai to Kamioka (T2K) [58] experiments. Both use precise $\overset{(-)}{\nu}_\mu$ beams, affected by the matter effect, and compare the rates of $\overset{(-)}{\nu}_\mu$ disappearance and $\overset{(-)}{\nu}_e$ appearance to constrain neutrino oscillation parameters [54]. The same methods will be used in the next generation LBL neutrino oscillation experiments, Deep Underground Neutrino Experiment (DUNE) [59] and Hyper-Kamiokande (HK) [60], which should give the final answers to the three neutrino oscillation questions [54].

1.4 Neutrino Mass

The absolute value of neutrino masses is not known and cannot be measured in neutrino oscillation experiment. The only model-independent information on the size of neutrino masses comes from measurements of missing energy in reactions involving a neutrino. The current best limit on the effective mass of ν_e (a combination of neutrino mass states via neutrino oscillation parameters) is $m_{\nu_e} < 0.8\,\mathrm{eV/c^2}$ at 90% C.L. [61]. The overall best limit on neutrino masses comes from cosmology, placing a limit on the sum of neutrino masses $\sum_a m_a < (0.12 - 0.69)\,\mathrm{eV/c^2}$ at 95% C.L. [62]. This limit however depends on the cosmological model [55]. These are significantly smaller than the mass scales of other fermions, suggesting a different theoretical mechanism for their generation.

To introduce neutrino masses into the SM, one must either add new fields, or break the gauge invariance, or renormalizability of the SM Lagrangian, or do both [45]. The easiest solution is to add new right-handed chiral neutrino fields, which would be singlets under all the SM gauge symmetries. Neutrinos created by these fields would not interact with any of the SM forces and are therefore called *sterile neutrinos*. Sterile neutrinos could however mix with *active neutrinos* via neutrino oscillations. This is also called the *minimally extended SM*. Neutrinos would obtain masses in the same way as the other fermions, via the Higgs mechanism through Yukawa couplings. This is however one of the largest limitations of this simple extension of the SM. Since the neutrino masses are so different from other fermions, it is unlikely their mass generation is the same as for the other fermions. The minimally extended SM gives no explanation about the smallness of neutrino masses. Furthermore, this simple extension would render SM incorrect even at low energies, even though it is generally believed that SM is an effective low-energy extrapolation of a generally correct theory, that break at some threshold value of New Physics (NP) Λ_{NP} [63]. Additionally, there was no oscillation between active and light sterile neutrinos detected to date.

Since neutrinos have mass they can no longer be theoretically describe by Weyl spinors. They can be described by either Dirac of Majorana spinors. The question of neutrino mass is intrinsically connected to the question of the theoretical nature of neutrinos. This is in turn related to the question of the conservation of the total lepton number, which is preserved in the SM. if neutrinos are majorana particles then they can also have majorana masses.

So far we only considered neutrinos to be Dirac particles, for which particles and antiparticles are different, same as all the other fermions.

[Fundamentals of neutrinos physics and astrophysics,p.190] The majorana condition can be written as $\nu = \nu^C$, which implies the equality of particle and antiparticle. As noted already by Majorana [765 = E. Majorana, Nuovo Cim., 14, 171-184, 1937], since a Majorana spinor has only two independent components, the Majorana theory is simpler and more economical than the Dirac theory. Hence, the Majorana nature of massive neutrinos may be more natural than the Dirac nature. In fact, neutrinos are majorana particles in most theories beyond the SM. If the neutrino is massless, since the left handed chiral component of the neutrino field obeys the Weyl equation in both the Dirac and Majorana descriptions and the right handed chiral component is irrelevant for neutrino interactions, the Dirac and Majorana theories are physically equivalent. From this it is clear that in practice one can distinguish a Dirac from a Majorana neutrino only by measuring some effect due to the neutrino mass. Moreover, the mass effect must not be of kinematical nature, because the kinematical effects of Dirac and Majorana masses are the same. For example, the Dirac and Majorana nature of neutrinos cannot be revealed through neutrino oscillations! The most promising way to find if neutrinos are Majorana particles is the search for neutrinoless double beta decay.

[Master thesis] A new type of neutrino was suggested by Ettore Majorana in his paper on "Symmetrical Theory of Electron and Positron" [352], published in 1937, but most probably already developed a few years before [144]...Majorana suggested in his paper that the neutron or the neutrino, or both, were particles of this type, namely neutral particles which identify themselves with the corresponding antiparticles. (FromNeutronToNuclearFission.pdf)

[Master thesis] In 1937, however, probably after being invited by Fermi to compete for a full professorship, Majorana published what was to become his most famous paper: "Symmetric theory of electrons and positrons," in which he introduced the so-called Majorana neutrino hypothesis. A consequence of this theory is that a neutral fermion can coincide with its antiparticle: This hypothesis was revolutionary because it argued that the antimatter partner of a given matter particle could be the particle itself. And this was in direct contradiction to what Dirac had successfully assumed in order to solve the problem of negative energy states in quantum field theory (i.e., the existence of the positron).But Majorana was just interested among the others in eliminating the “Dirac sea” postulate…Anyway, with unprecedented farsightedness Majorana suggested that the neutrino, which had just been postulated, as we know, by Pauli and Fermi, could be such a particle. This would make the neutrino unique among the elementary particles and, moreover, enable it to have mass: a property that favoured the possibility of neutrino oscillations (a phenomenon, predicted by B.Pontecorvo, and later on experimentally verified). After that Fermi made neutrinos the basis of an impressive, quantitative theory of beta decay,it became interesting to reconsider whether one could have spinning particles that are their own antiparticles. Could one, specifically, have a version of the Dirac equation that involved real fields? This was a mathematical question asked, and answered, by Majorana.The exceptional theory of neutrinos by Majorana would lead us too far, and we are glad to be able to confine ourselves to refer the reader to the excellent presentations of it in works like [14,31], and refs. therein. Let us only stress that, even if the important experiments presently performed all over the world (e.g., in USA, Japan, Italy,…) haven’t revealed yet if neutrinos are Dirac’s or Majorana’s, nevertheless, in condendsed matter physics, structures have been found several times that are “Majorana fermions”: see Refs.[34-36]. (MajoranaNeutronEtNeutrino.pdf)

If neutrinos can have both Dirac and Majorana masses, then we can have mass terms in the Lagrangian in which the additional sterile neutrinos interact with with the active neutrinos (this is the case for purely Dirac neutrinos), but can also interact with their own antineutrinos, generating Majorana masses. If we add m sterile neutrinos, we get m heavy sterile neutrinos and 3 light active neutrinos, which are mostly left handed. Both the light and the heavy neutrinos are Majorana particles. Furthermore, if neutrinos are Majorana, we do not even have to add any new fields into the SM. If we add a new non-renormalizable term into the SM Lagrangian, that is suppressed with respect to Λ_{NP}, after spontaneous symmetry breaking leads to the exact same case with Majorana sterile neutrinos, with the masses of the light active neutrinos being suppressed by Λ_{NP} with respect to the masses of the other fermions. This would explain the observed smallness of neutrino masses and would also mean that SM is an effective theory of more full theory that breaks down at Λ_{NP}.

[63] With the particle contents of the SM and the addition of an arbitrary m number of sterile neutrinos one can construct two types mass terms that arise from gauge invariant **renormalizable** operators:

$$-\mathcal{L}_{M_\nu} = M_{D_{ij}} \overline{\nu}_{si} \nu_{Lj} + \frac{1}{2} M_{N_{ij}} \overline{\nu}_{si} \nu^C_{sj} + \text{h.c.}, \tag{1.25}$$

where ν^C indicated a charge conjugated field, $\nu^C = C\overline{\nu}^T$ and C is the charge conjugation matrix. M_D is a complex $m \times 3$ matrix and M_N is a symmetric matrix of dimension $m \times m$. First term is the Dirac term generated from Yukawa interactions after spontaneous electroweak symmetry breaking. The second term in Eq. (10) is a Majorana mass term. It is different from the Dirac mass terms in many important aspects. It is a singlet of the SM gauge group. Therefore, it can appear as a bare mass term. Furthermore, since it involves two neutrino fields, it breaks lepton number by two units.

[63] In case of $M_N \gg M_D$ (see-saw model), The diagonalization of M_ν leads to three light, ν_α , and m heavy, N, neutrinos, where the heavy states are mostly right-handed while the light ones are mostly left-handed. Both the light and the heavy neutrinos are Majorana particles. Two well-known examples of extensions of the SM that lead to a see-saw mechanism for neutrino masses are SO(10) GUTs [25–27] and left-right symmetry [28]. This is the see-saw mechanism [24–28]. In this case the SM is a good effective low energy theory. Indeed the see-saw mechanism is a particular realization of the general case of a full theory which leads to the SM with three light Majorana neutrinos as its low energy effective realization as we discuss next.

[63] In general, if the SM is an effective low energy theory valid up to the scale Λ_{NP} , the gauge group, the fermionic spectrum, and the pattern of spontaneous symmetry breaking of the SM are still valid ingredients to describe Nature at energies $E \ll \Lambda_{NP}$. We need to consider also non-renormalizable higher dimensional terms in the lagrangian. In this approach the largest effects at low energy are expected to come from dim= 5 operators. Indeed, there is a single set of dimension-five terms that is made of SM fields and is consistent with the gauge symmetry, and this set violates

the spontaneous symmetries of the SM. It is given by

$$\mathcal{O}_5 = \frac{Z^{\nu}_{ij}}{\Lambda_{NP}} \left(\overline{L}_{Li}\tilde{\Phi}\right)\left(\tilde{\Phi}^T L^C_{Lj}\right) + \text{h.c.}, \tag{1.26}$$

which violate total lepton number by two units and leads, upon spontaneous symmetry breaking, to:

$$-\mathcal{L}_{M_\nu} = \frac{Z^{\nu}_{ij}}{2}\frac{v^2}{\Lambda_{NP}}\overline{\nu}_{Li}\nu^C_{Lj} + \text{h.c.}. \tag{1.27}$$

we see that this is a Majorana mass term built with the left-handed neutrino fields and with:

$$(M_\nu)_{ij} = Z^{\nu}_{ij}\frac{v^2}{\Lambda_{NP}} \tag{1.28}$$

[63] Since Eq. (29) would arise in a generic extension of the SM, we learn that neutrino masses are very likely to appear if there is NP. As mentioned above, a theory with SM plus m heavy sterile neutrinos leads to three light mass eigenstates and an effective low energy interaction of the form (27). In particular, the scale Λ_{NP} is identified with the mass scale of the heavy sterile neutrinos, that is the typical scale of the eigenvalues of M_N. ...the scale of neutrino masses is suppressed by v/Λ_{NP} when compared to the scale of charged fermion masses providing an explanation not only for the existence of neutrino masses but also for their smallness. Finally, this supports lepton mixing and CP violation unless additional symmetries are imposed on the coefficients Z_{ij}.

Detection of neutrinoless double beta decay is the only known method with plausible sensitivity to the Majorana nature of the neutrino, one of the most important open questions in particle physics. [54]

[OverviewOfNeutrinoPhysicsPheno2024.pdf] In contrast, the Majorana phases do not enter the flavour neutrino oscillation probabilities [22, 85], but contribute to the $\beta\beta_{0\nu}$ decay rate

[Master thesis] Another question which was studied was whether the neutrino was truly a Dirac particle in the same sense as in an electron. The formalism for ‘single’ beta decay is indifferent to the answer, but in the case of ‘double’ beta decay in which the simultaneous emission of two electrons occurs the situation was somewhat altered. Here the requirement that the neutrino must obey the Dirac electron equations as for ‘electrons’ but with zero charge, and that there exist particle (ν)-

antiparticle ($\overline{\nu}$) pairs, resulted in very long half lives for double beta decay, whereas if the neutrinos emitted in positron decay and negative electron decay are identical, the lifetimes should be short and observable. The original work of Firemen (1948) on double beta decay stimulated many subsequent searches with the conclusion based on the presently unobservably great lifetimes that the neutrino can best be described as a 'true' Dirac particle, i.e., by the Dirac equation, and that the neutrino is distinct from the antineutrino. (NeutrinoPhysicsCowanReines.pdf)

The disparity of mass scales between neutrinos and other fundamental particles suggests a high-energy scale for neutrino-mass generation. In models where this mechanism lies at or below the TeV scale, the physics of neutrino mass may be accompanied by complementary signatures at colliders. In models where the scale is higher, experiments probing the nature of neutrino mass are the only feasible way of exploring this new physics. The observation of neutrino-less double beta decay would provide direct evidence that lepton number is violated, opening a path to baryogenesis via leptogenesis in the early universe. As such, direct tests of the scale or nature of neutrino mass target some of the most central open questions in fundamental physics today. Other neutrino properties may be connected to extensions of the standard model, yet are not observable via oscillations. Neutrino electromagnetic properties are of fundamental interest, [54].

Acronyms

2P2H two particle - two hole. 6

BSM Beyond Standard Model. 10

CC Charged current. 1, 2, 4–6, 9

CEvNS Coherent Elastic ν Nucleus Scattering. 6

COHπ Coherent pion production. 6

CP Charge conjugation - Parity (symmetry). 1, 8, 10

DIS Deep Inelastic Scattering. 5, 6

DUNE Deep Underground Neutrino Experiment. 10

FSI Final State Interaction. 6

HK Hyper-Kamiokande. 10

LBL Long Baseline. 10

MEC Meson Exchange Current. 6

MSW Mikheyev-Smirnov-Wolfenstein. 9

NC Neutral Current. 1–4, 6, 9

NOvA NuMI Off-axis ν_e Appearance (experiment). 10

NP New Physics. 11

PMNS Pontecorvo-Maki-Nakagawa-Sakata. 7, 8

QE Quasi Elastic (interaction). 4–6

RES Resonant baryon production. 5

SK Super-Kamiokande. 9

SM Standard Model. 1, 2, 10, 11, 13, 14

SNO Sudbury Neutrino Observatory. 9

T2K Tokai to Kamioka (experiment). 10

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