Chapter 1

Theory of neutrino physics

Just give a short overview of the historical context, but mainly focusing on the actual description of the neutrino theory, mostly stating the fact rather than giving a large background 1.neutrino interactions (neutrinos in the SM) - detection of neutrinos? 2.neutrino oscillations (including possible sterile oscillation) - maybe current status? 3.neutrino masses (theoretical prediction for their origins and their measurements)

I should discuss everything that is even briefly mentioned in the neutrino magnetic moment theory section.

- Dirac vs Majorana neutrinos
- Neutrino masses
- Neutrino interactions with electrons and nuclei
- Neutrino oscillations and their implications

The story:

1. Brief history up to neutrinos being in the SM
2. Description of neutrinos in the SM
3. Interactions of neutrinos and their detection
4. Production of neutrinos
5. Solar and atmospheric neutrino anomalies and neutrino oscillations
6. Detail of neutrino oscillations for three flavours
7. Current state of neutrino oscillation measurements
8. Mass ordering, octant, delta CP

9. Neutrino masses - generation and measurements

10. Dirac V Majorana neutrinos

Neutrinos were first introduced [1, 2] as very light, half-spin electrically neutral particles with possible magnetic moments [3]. Being a crucial part of the successful theory of weak interactions [4, 5], neutrinos solidified their important in particle physics even before they were first experimentally detected. Neutrinos eventually developed into the two-component left-handed chiral fields $\nu_{\alpha L}$, with three generations $\alpha = e, \mu, \tau$ denoting the three know neutrino flavours [6–8], with no mass or magnetic moment, we use today in the Standard Model (SM) of particle physics [9–11]. They form weak isospin doublets together with their associated left handed charged lepton fields and unlike the charged leptons, neutrinos do not have an associated right handed singlet in the SM. Therefore, they do not obtain their masses via the Higgs mechanism through the Yukawa coupling, which requires the both left handed and right handed fields, therefore there is no mass term in the SM lagrangian [12]. The neutrino interaction terms of the SM lagrangian can be separated into two, Charged current (CC) or Neutral Current (NC) based on the massive gauge field they interact with. They can be written as

$$\mathcal{L}_{\text{CC}}^{\text{SM}} = -\frac{g_w}{2\sqrt{2}} j_W^\mu W_\mu + \text{h.c.}, \quad \mathcal{L}_{\text{NC}}^{\text{SM}} = -\frac{g_w}{2\cos(\theta_W)} j_Z^\mu Z_\mu, \tag{1.1}$$

where g_w is the weak coupling constant, θ_W is the Weinberg angle, W_μ and Z_μ are the 4-component vector gauge fields describing the $W^\pm$ and Z^0 weak bosons and j_W^μ and j_Z^μ are the weak currents. The currents are expressed as as

$$j_W^\mu = 2 \sum_{\alpha=e,\mu,\tau} \overline{\nu}_{\alpha L} \gamma^\mu \alpha_L, \tag{1.2}$$

$$j_Z^\mu = 2 \sum_{\alpha=e,\mu,\tau} g_L^\nu \overline{\nu}_{\alpha L} \gamma^\mu \nu_{\alpha L}, \tag{1.3}$$

where $\gamma^\mu, \mu = 0, 1, 2, 3$, are the four Dirac gamma matrices, α_R is the right handed chiral field for the charged lepton α and g_L^ν is the coupling term.

The interaction lagrangian described in Eq. 1.1 describes two possible vertices for the neutrino interaction, as shown on Fig. 1.1.

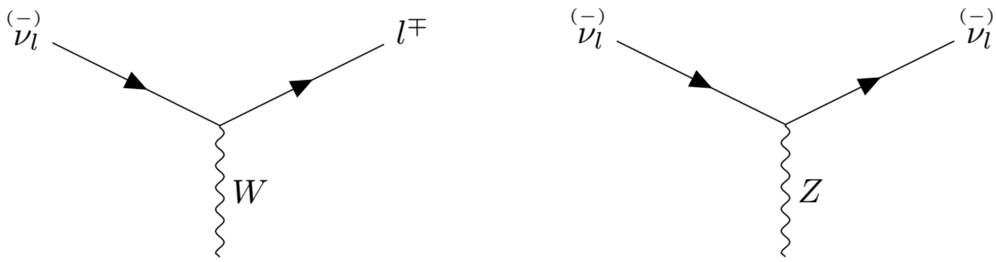

Figure 1.1: Neutrino interaction vertices in the SM

The main neutrino sources [12] are the β^- decay, which is essentially a neutron decay, happening in radioactive materials, or nuclear reactors, and can be described as

$$n \rightarrow p + e^- + \overline{\nu}_e. \tag{1.4}$$

The source of neutrinos from nuclear reactor was the first artificial source of neutrinos which brought increase of neutrino rate of about 10^7, as well as higher neutrino energies and enabled the first detection of neutrinos [13] Similarly is the β^+ decay

$$p \rightarrow n + e^+ + \nu_e \tag{1.5}$$

or the electron capture

$$p + e^- \rightarrow n + \nu_e, \tag{1.6}$$

which occurs in the stars and for Earth especially in the Sun. Source of ν_μ is especially pion, muon and kaon decay, occurring in the atmosphere and the accelerators

$$p + X \rightarrow \pi^\pm \rightarrow \mu^\pm + \nu_\mu \, (\overline{\nu}_\mu) \tag{1.7}$$

$$\mu^\pm \rightarrow e^\pm + \nu_\mu \, (\overline{\nu}_\mu) + \nu_e \, (\overline{\nu}_e) \tag{1.8}$$

Notice that if all muons decay by the time they reach Earth's surface, then there should be almost exactly 2:1 ratio of $\nu_\mu : \nu_e$ for atmospheric neutrinos. This is also used in the modern accelerator based source of neutrinos which have accelerated protons strike a fixed target [14][15].

For supernovas its partly electron capture but 90% through thermal pair production

$$e^- + e^+ \rightarrow \nu_\alpha + \overline{\nu}_\alpha, \alpha = e, \mu, \tau \tag{1.9}$$

There is also cosmic neutrino background that decoupled from the rest of matter shortly after the Big Bang.

Inversely, the detection of neutrinos is usually done by reverting the above mentioned processes.

For example the inverse beta decay

$$\nu + p \rightarrow n + e^+ \tag{1.10}$$

was used for the first detection of neutrinos by Cowan and Reines [16, 17].

Leon Lederman, Jack Steinberger and others joined Schwartz and using a spark chamber detector in 1962 observed[18] a different kind of neutrino, which we now call the muon neutrino (ν_μ), produced in pion decay ($\pi^\pm \rightarrow \mu^\pm + \nu$), while the previously known neutrino, produced in beta decays, was dubbed the electron neutrino (ν_e).

In 1990 the L3 Collaboration studied properties of the Z^0 boson and fitted to its peak cross-section and decay width to determine the total number of active (interacting with Z^0) light ($m_\nu < m_Z/2$) neutrino flavours (N_ν). They found the best fit integer value to be 3 and ruled out the possibility of four or more active light neutrino flavours at 4σ [19]. Latest most precise results put the fitted value to $N_\nu = 2.9840 \pm 0.0082$ [20]. After this result it was only a matter of time, before the third neutrino, the tau neutrino (ν_τ) was discovered. Evidence for that were shown in 2000 from the DONUT Collaboration at Fermilab [21].

I think I should mention here the basic neutrino interactions and their corresponding cross section. For neutrino on nucleons, the total cross section per neutrino energy is around $0.7 \times 10^{-38}\,\mathrm{cm}^2\mathrm{GeV}^{-1}$ for neutrinos and half that for antineutrinos. For neutrino on electron interactions, the total cross section per neutrino energy is more similar to $10^-41 - 10^{-42}\,\mathrm{cm}^2\mathrm{GeV}^{-1}$.

The main neutrino interactions are

$$\nu_l + n \rightarrow p + l^- \tag{1.11}$$

$$\overline{\nu}_l + p \rightarrow n + l^+ \tag{1.12}$$

$$\nu_l + N \rightarrow \nu_l + N \tag{1.13}$$

$$\overline{\nu}_l + N \rightarrow \overline{\nu}_l + N \tag{1.14}$$

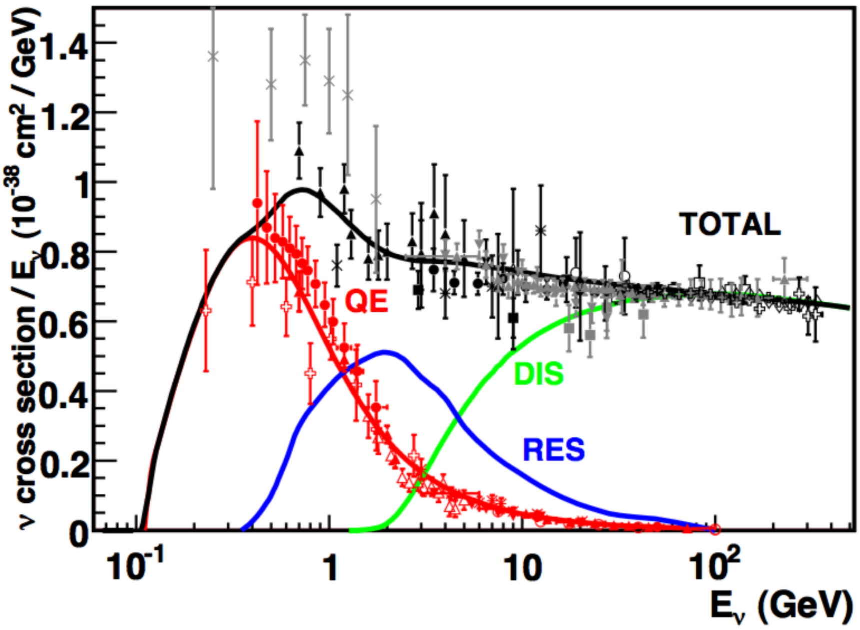

Figure 1.2: Neutrino CC cross sections based on the interaction types. Figure from [22] compares the measured data [23] and the prediction [24]

Problems of the SM

TO DO: *Describe neutrino interactions, CC, NC elastic. QE, Res., DIS. Also nuclear effects - MEC, FSI,...*

1.1 Neutrino oscillation

Neutrino oscillate and therefore have mass. Describe neutrino oscillations and the current status of their measurements. Maybe also when they were discovered and how?

This led B. Pontecorvo, inspired by already known $K^0 \leftrightarrow \overline{K^0}$ oscillations, to consider $\nu \leftrightarrow \overline{\nu}$ transitions (oscillations), in case the conservation of neutrino charge does not apply[25]. Pontecorvo later built upon this statement in 1958 considering that oscillations between ν and $\overline{\nu}$ are due to them being combinations of particles ν_1 and ν_2 and that the transformation lifetime is related to the mass difference between ν_1 and ν_2 [26], laying foundation for neutrino oscillations as we know them today.

In 1962 Z. Maki, M. Nakagawa and S. Sakata applied Pontecorvo's idea of neutrino oscillations to *weak neutrino* eigenstates ν_α (ν_e, ν_μ) produced in weak interactions.

They assumed that oscillation $\nu_\alpha \leftrightarrow \nu_\beta$ are driven by a non-zero mass difference (therefore if true implying at least one neutrino has a non-zero mass) between *true neutrinos* (= mass neutrino eigenstates ν_i (ν_1, ν_2)), which are related to weak eigenstates via a linear combination. This relation in general case looks like [27]

$$|\nu_\alpha\rangle = \sum_{i=1}^{n} U^*_{\alpha i} |\nu_i\rangle\,, \tag{1.15}$$

where U is a unitary matrix now know as Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix and n is the (general) number of light neutrino species [28].

In the contemporary description of neutrino interactions it is common to treat neutrino as a plane wave in a relativistic approximation, which after a distance L evolves as [28]

$$|\nu_\alpha(L)\rangle = \sum_{\alpha'} \sum_{i} U^*_{\alpha i} U_{\alpha' i} e^{-im_i^2 L/2E} |\nu_{\alpha'}\rangle\,. \tag{1.16}$$

Neutrino ν_α can oscillate and therefore be detected as a different neutrino flavour ν_β with a probability

$$P_{\nu_\alpha \to \nu_\beta}(L) = |\langle \nu_\beta | \nu_\alpha(L)\rangle|^2 = \sum_{i,j} U_{\beta i} U^*_{\alpha i} U^*_{\beta j} U_{\alpha j} e^{-i\left(m_i^2 - m_j^2\right) L/2E}, \tag{1.17}$$

where the difference of the masses squared is usually denoted as

$$\Delta m_{ij}^2 = m_i^2 - m_j^2. \tag{1.18}$$

Oscillation probability can be also expressed as

$$\begin{aligned} P_{\nu_\alpha \to \nu_\beta}(L) = \delta_{\alpha\beta} &- 4 \sum_{i>j} \mathrm{Re}\left(U_{\beta i} U^*_{\alpha i} U^*_{\beta j} U_{\alpha j}\right) \sin^2 \Delta_{ij} \\ &+ 2 \sum_{i>j} \mathrm{Im}\left(U_{\beta i} U^*_{\alpha i} U^*_{\beta j} U_{\alpha j}\right) \sin 2\Delta_{ij}, \end{aligned} \tag{1.19}$$

where[28]

$$\Delta_{ij} \equiv \Delta m_{ij}^2 \frac{L}{4E} = 1.267 \frac{\Delta m_{ij}^2}{\mathrm{eV}^2} \frac{L/E}{\mathrm{m/MeV}}.$$

Since real neutrino beams are not monochromatic, what is measured in experiments is an **average** oscillation probability with $\langle \sin^2 \Delta_{ij} \rangle$ and $\langle \sin 2\Delta_{ij} \rangle$ in eq.1.19.

We can notice that if $E/L \gg \Delta m_{ij}^2$ the oscillation does not show any effect yet and if $E/L \ll \Delta m_{ij}^2$ the oscillating phase goes through many cycles and is averaged to $\langle \sin^2 \Delta_{ij} \rangle = 1/2$. Therefore different experimental settings can measure different oscillation parameters [29].

Several experimental indications for neutrino oscillations were found shortly after its theoretical predictions. Davis continued looking for ^{37}Ar in a detector with ^{37}Cl, but now moving to Homestake Gold Mine with Harmer and Hoffman focusing on detecting the first electron neutrinos from the Sun [30]. Already in 1968 their Homestake Solar Neutrino Observatory saw a solar neutrino flux less than 3 Solar Neutrino Units (SNU = one interaction per 10^{36} target atoms s^{-1}), well below the solar model prediction of the time [31]. This discrepancy became the *"solar neutrino problem"*, which is in line with neutrino oscillations, but no direct implications could have been drawn since it might have been caused by a lack of understanding of nuclear physics, astrophysics of the Sun, or particle physics of the neutrino[14].

In 2002, Raymond Davis was together with Masatoshi Koshiba awarded the Nobel prize "for pioneering contributions to astrophysics, in particular for the detection of cosmic neutrinos" [32]. Koshiba was part of the Kamiokande experiment, which confirmed the results from Homestake [33].

Possible explanation of the Solar neutrino problem was proposed in 1978 by L. Wolfenstein, who considered the effect of matter on neutrino oscillations [34]. His modification of neutrino oscillations when passing through matter arises from the coherent forward scattering of electron neutrinos, as a result of their charged current (CC) interaction with electrons, which are abundant in matter, as opposed to other lepton flavours, muons and tauons, resulting in an imbalance between ν_e and ν_μ/ν_τ. This manifests as an effective potential, which depends on the density and composition of the matter [34]. This idea was later further developed for neutrinos passing through the Sun by Mikheyev and Smirnov in 1985 [35][28] and we now call this effect the Mikheyev-Smirnov-Wolfenstein (MSW) effect.

To showcase this effect we consider only two neutrino flavours, ν_e and ν_X, where X denotes a combination of all other non-electron flavours. Vacuum oscillations are in this two-neutrino approximation driven by a single mass splitting Δm^2 and the corresponding PMNS matrix is a rotational matrix parametrized by one angle θ:

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}. \tag{1.20}$$

The MSW effect can be described as the presence of an Effective Potential [36]

$$V = \pm\sqrt{2}G_F N_e = \pm 3.8 \times 10^{-14} \left(\frac{\rho}{\text{g cm}^{-3}}\right)\left(\frac{Y_e}{0.5}\right) \text{eV}, \tag{1.21}$$

where G_F is the Fermi coupling constant, N_e is the electron density, Y_e is the electron number per nucleon and plus or minus sign is for neutrinos or antineutrinos respectively.

This potential can be seen as having the effect of modifying the Δm^2 and θ of the neutrino oscillations: [36]

$$\sin^2 2\theta_m = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta \mp \xi)^2} \tag{1.22}$$

$$\left(\Delta m^2\right)_{\mathit{eff}} = \Delta m^2 \times \sqrt{\sin^2 2\Theta + (\cos 2\theta \mp \xi)^2}, \tag{1.23}$$

where

$$\xi = \frac{2\sqrt{2}G_F N_e}{\Delta m^2}. \tag{1.24}$$

Measuring atmospheric neutrinos brought about another neutrino conundrum, the *Atmospheric neutrino anomaly*. It came from the disagreement between experiments such as NUSEX[37] and Fréjus[38], which used iron calorimeters detectors, and experiments IMP[39] and Kamiokande[40], which used water Cherenkov detectors. All of these experiments were looking for a deficit of ν_μ, or an excess of ν_e, compared to prediction. While the first two experiments saw a good agreement between experimental results and predictions, the latter two did not and suggested the possibility of neutrino oscillations, which could explain their disagreement.

Solution to the Atmospheric neutrino anomaly came in 1998, when the Super-Kamiokande (SK) experiment showed for the first time the experimental evidence for neutrino oscillations [41]. SK has however also disfavoured the two neutrino hypothesis, with regards to the existence of an additional neutrino flavour.

The Solar neutrino anomaly was also resolved, when the Sudbury Neutrino Observatory (SNO) provided $> 5\sigma$ evidence for solar ν_e oscillations in 2002, independent on the solar model [42]. While other solar neutrino experiments measured solar ν_e only via the charged current (CC) interactions

$$\nu_e + n \to p + e^- \qquad (CC), \tag{1.25}$$

SNO had an ability to also detect neutrinos via the neutral current (NC) interaction

$$\nu + X \to \nu + X' \qquad (NC), \tag{1.26}$$

which are equally sensitive to all active neutrino flavours and their rate is therefore unaffected by standard neutrino oscillations. SNO could compare CC and NC event rates and conclude that ν_e from the Sun oscillate into other neutrino flavours along the way [42].

Takaaki Kajita from SK and Arthur B. McDonald from SNO were jointly awarded the Nobel Prize in 2015 "for the discovery of neutrino oscillations, which shows that neutrinos have mass" [32].

In 1990 the L3 Collaboration studied properties of the Z^0 boson and fitted to its peak cross-section and decay width to determine the total number of active (interacting with Z^0) light ($m_\nu < m_Z/2$) neutrino flavours (N_ν). They found the best fit integer value to be 3 and ruled out the possibility of four or more active light neutrino flavours at 4σ [19]. Latest most precise results put the fitted value to $N_\nu = 2.9840 \pm 0.0082$ [20]. After this result it was only a matter of time, before the third neutrino, the tau neutrino (ν_τ) was discovered. Evidence for that were shown in 2000 from the DONUT Collaboration at Fermilab [21].

The PMNS matrix describing neutrino oscillations in the so called 3ν para-digm depends on six independent parameters: 3 mixing angles (θ_{12}, θ_{13}, θ_{23}) and 3 phases. One of the phases is δ_{CP}, which, if different from 0 or π, implies CP violation, and the other two are α and β, so called Majorana phases, which are non zero only if neutrinos are Majorana (neutrinos and antineutrinos are described by just one field, i.e. neutrinos are the same particle as antineutrinos). Majorana phases play no role in neutrino oscillations, so they are usually left out in the description [29]. The PMNS matrix in this case can be parametrized as

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}, \tag{1.27}$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$.

Other than the PMNS matrix, neutrino oscillations depend on the mass squared differences (eq.1.18). In case of 3 neutrinos, those are Δm^2_{21} and Δm^2_{31}. Δm^2_{21} mainly drives oscillations of solar neutrinos and is therefore often denoted as $\Delta m^2_{\odot}$ or Δm^2_{sol}, while Δm^2_{31} drives oscillations on the scale for atmospheric neutrinos and is often written as Δm^2_{atm} [28]. There can only be two independent mass squared differences for oscillation of three neutrinos, since

$$\Delta m^2_{21} + \Delta m^2_{32} + \Delta m^2_{13} = 0. \tag{1.28}$$

1.2 Neutrino masses

Experiments for their values? Theoretical predictions for how they obtained them

Theories of neutrino mass generation

[Fundamentals of neutrinos physics and astrophysics] The only extension of the SM that is needed is the introduction of right-handed components $\nu_{\alpha R}$ of the neutrino fields. Such a model is sometimes called the *minimally extended Standard Model*. The right handed neutrino fields are fundamentally different from the other elementary fermion fields because they are invariant under the symmetries of the SM: they are **singlets** of $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L$ and have hypercharge $Y = 0$. The right handed neutrino fields are called sterile [883] because they do not participate in weak interactions and their only interaction is gravitational. their right handedness is not required though! could also be left handed but have to be singlets and therefore sterile!

In the minimally extended standard model with three right handed neutrino fields, the SM Higgs-lepton Yukawa Lagrangian is extended by adding a lepton term with the same structure as the second term on the right handed side, which generates the masses of up-type quarks

$$\mathcal{L}_Y = -\sum_{\alpha,\beta=e,\mu,\tau} Y'^{l}_{\alpha\beta}\overline{L}_{\alpha L}\Phi l'_{\beta R} - \sum_{\alpha,\beta=e,\mu,\tau} Y'^{\nu}_{\alpha\beta}\overline{L}_{\alpha L}\tilde{\Phi}\nu'_{\beta R} + \text{h.c.}, \tag{1.29}$$

where Y'^{ν} is a new matrix of Yukawa couplings.

Using the unitary gauge we can diagonalize the Yukawa couplings we obtain

$$\mathcal{L}_Y = -\sum_{\alpha=e,\mu,\tau} \frac{y^l_\alpha v}{\sqrt{2}}\bar{l}_\alpha l_\alpha - \sum_{k=1}^{N} \frac{y^\nu_k v}{\sqrt{2}}\overline{\nu}_k\nu_k - \sum_{\alpha=e,\mu,\tau} \frac{y^l_\alpha}{\sqrt{2}}\bar{l}_\alpha l_\alpha H - \sum_{k=1}^{N} \frac{y^\nu_k}{\sqrt{2}}\overline{\nu}_k\nu_k H \tag{1.30}$$

Therefore the neutrino masses are given by

$$m_k = \frac{y^\nu_k v}{\sqrt{2}} \quad (k = 1, ..., N)\,, \tag{1.31}$$

and massive Dirac neutrinos couple to the Higgs field through the last term. Note that the neutrinos masses are proportional to the Higgs VEV v, as the masses of charged leptons and quarks. However, it is known that the masses of neutrinos are much smaller than those of charged leptons and quarks, but there is no explanations here of the very small values of the eigenvalues Y^ν_k of the Higgs-neutrino Yukawa coupling matrix that are needed. The lagrangian defined this way does not conserve the lepton flavour number, which leads to neutrino oscillations. The Dirac character of massive neutrinos is closely related to the invariance of the total Lagrangian under the global U(1) gauge transformations.

The sterile neutrino fields do not participate in weak interaction with both their left and right components, but can couple with the ordinary neutrinos through the mass therm, generating a complicated mixing between active and sterile degrees of freedom.Since at present there is no indication of the existence of such additional sterile Dirac neutrino fields, ockham's razor suggests to ignore them...

1.2.1 Majorana neutrinos

[Fundamentals of neutrinos physics and astrophysics,p.190] If the neutrino is massless, since the left handed chiral component of the neutrino field obeys the Weyl equation in both the Dirac and Majorana descriptions and the right handed chiral component is irrelevant for neutrino interactions, the Dirac and Majorana theories are physically equivalent. From this it is clear that in practice one can distinguish a Dirac from a Majorana neutrino only by measuring some effect due to the neutrino mass. Moreover, the mass effect must not be of kinematical nature, because the kinematical effects of Dirac and Majorana masses are the same. For example, the Dirac and Majorana nature of neutrinos cannot be revealed through neutrino oscillations! The most promising way to find if neutrinos are Majorana particles is the search for neutrinoless double beta decay.

[OverviewOfNeutrinoPhysicsPheno2024.pdf] In contrast, the Majorana phases do not enter the flavour neutrino oscillation probabilities [22, 85], but contribute to the $\beta\beta_{0\nu}$ decay rate

Chapter 2

Contraining neutrino magnetic moment in the NOvA near detector

2.1 Theory of neutrino magnetic moment

TO DO: *Re-read the three main theory papers and double check the theoretical overview*

In the Standard Model (SM), neutrinos are massless and electrically neutral particles. However, even in the SM neutrinos can have electromagnetic interaction through loop diagrams involving the charged leptons and the W boson. These interactions are described by the neutrino charge radius, described in section 2.1.2 [?].

To include neutrino masses required by neutrino oscillations, we must go Beyond the Standard Model (BSM), where neutrinos can acquire other electromagnetic properties [?]. In the most general case, considering interactions with a single photon as shown on Fig. 2.1, neutrino electromagnetic interactions can be described by an *effective* interaction Hamiltonian [?]

$$\mathcal{H}_{em}^{(\nu)}(x) = \sum_{k,j=1}^{N} \overline{\nu}_k(x)\, \Lambda_{\mu}^{kj} \nu_j(x)\, A^{\mu}(x)\,. \tag{2.1}$$

Here $\nu_k(x), k \in \{1, ..., N\}$ are neutrino fields in the mass basis with N neutrino mass states. Λ_μ^{kj} is a general vertex function and $A^\mu(x)$ is the electromagnetic field.

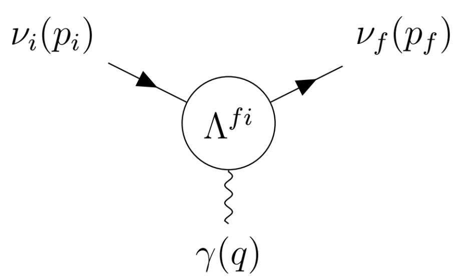

Figure 2.1: Effective coupling of neutrinos with one photon electromagnetic field 2.

The vertex function $\Lambda_\mu^{fi}(q)$ is generally a matrix and in the most general case can be written in terms of linearly independent products of Dirac matrices (γ) and only depends on the square of the four momentum of the photon ($q = p_f - p_i$):

$$\begin{aligned}\Lambda_\mu^{fi}(q) =& \mathbb{F}_1^{fi}\left(q^2\right) q_\mu + \mathbb{F}_2^{fi}\left(q^2\right) q_\mu \gamma_5 + \mathbb{F}_3^{fi}\left(q^2\right) \gamma_\mu + \mathbb{F}_4^{fi}\left(q^2\right) \gamma_\mu \gamma_5 + \\ & \mathbb{F}_5^{fi}\left(q^2\right) \sigma_{\mu\nu} q^\nu + \mathbb{F}_6^{fi}\left(q^2\right) \epsilon_{\mu\nu\rho\gamma} q^\nu \sigma^{\rho\gamma},\end{aligned} \tag{2.2}$$

where $\mathbb{F}_i^{fi}\left(q^2\right)$ are six Lorentz invariant form factors [?].

Applying conditions of hermiticity $\left(\mathcal{H}_{em}^{(\nu)\dagger} = \mathcal{H}_{em}^{(\nu)}\right)$ and of the gauge invariance of the electromagnetic field, we can rewrite the vertex function as

$$\Lambda_\mu^{fi}(q) = \left(\gamma_\mu - q_\mu \not{q}/q^2\right)\left[\mathbb{F}_Q^{fi}\left(q^2\right) + \mathbb{F}_A^{fi}\left(q^2\right) q^2 \gamma_5\right] - i\sigma_{\mu\nu} q^\nu \left[\mathbb{F}_M^{fi}\left(q^2\right) + i\mathbb{F}_E^{fi}\left(q^2\right)\gamma_5\right], \tag{2.3}$$

where $\mathbb{F}_Q^{fi}, \mathbb{F}_M^{fi}, \mathbb{F}_E^{fi}$ and $\mathbb{F}_A^{fi}$ are hermitian matrices representing the charge, dipole magnetic, dipole electric and anapole neutrino form factors. In coupling with a real photon ($q^2 = 0$) these become the neutrino charge and magnetic, electric and anapole moments. The neutrino charge radius corresponds to the second term in the expansion of the charge form factor [?].

We can simplify the above expression as [?]

$$\Lambda_\mu^{fi}(q) = \gamma_\mu \left(Q_{\nu_{fi}} + \frac{q^2}{6}\langle r^2\rangle_{\nu_{fi}}\right) - i\sigma_{\mu\nu} q^\nu \mu_{\nu_{fi}}, \tag{2.4}$$

where $Q_{\nu_{fi}}$, $\langle r^2\rangle_{\nu_{fi}}$, and $\mu_{\nu_{fi}}$ are the neutrino charge, effective charge radius (also containing anapole moment), and an effective magnetic moment (also containing electric moment) respectively. This is possible thanks to the proportional effect of the neutrino charge radius and the anapole moment, or the neutrino magnetic and electric moment respectively [?]. These quantities (charge, charge radius and magnetic moment) are the three neutrino electromagnetic properties measured in experiments.

2.1.1 Neutrino electric and magnetic dipole moments

The size and effect of the neutrino electromagnetic properties depends on the specific theory beyond the standard model.

Evaluating the one loop diagrams in the minimal extension of the standard model with three right handed Dirac neutrinos gives us the first approximation of the electric and magnetic moments:

$$\left.\begin{matrix} \mu_{kj}^{D} \\ i\epsilon_{kj}^{D} \end{matrix}\right\} \simeq \frac{3eG_F}{16\sqrt{2}\pi^2}\left(m_k \pm m_j\right)\left(\delta_{kj} - \frac{1}{2}\sum_{l=e,\mu,\tau} U_{lk}^{\star} U_{lj} \frac{m_l^2}{m_W^2}\right), \tag{2.5}$$

where m_k, m_j are the neutrino masses and m_l are the masses of charged leptons which appear in the loop diagrams [?]. e is the electron charge, G_F is the Fermi coupling constant, and U is the PMNS neutrino oscillation matrix. Higher order electromagnetic corrections were neglected, but those can also have a significant contribution, depending on the theory.

It can be seen that there dirac neutrinos have no diagonal electric moments $\left(\epsilon_{kk}^{D} = 0\right)$ and their diagonal magnetic moments are approximately

$$\mu_{kk}^{D} \simeq \frac{3eG_F m_k}{8\sqrt{2}\pi^2} \simeq 3.2 \times 10^{-19}\left(\frac{m_k}{\text{eV}}\right)\mu_B, \tag{2.6}$$

where μ_B is the Bohr magneton [?].

The transition magnetic moments are suppressed with respect to the largest of the diagonal magnetic moments by at least a factor of 10^{-4} due to the m_W^2 in the denominator. The transition electric moments are even smaller due to the mass difference in Eq.2.5. Therefore an experimental observation of a magnetic moment larger than in Eq.2.6 would indicate physics beyond the minimally extended standard model [? ?].

Majorana neutrinos in a minimal extension can be obtained by either adding a SU $(2)_L$ Higgs triplet, or right handed neutrinos together with a SU $(2)_L$ Higgs singlet [?]. If we neglect the Feynman diagrams which depend on the model of the scalar sector, the magnetic and electric dipole moments are

$$\mu_{kj}^{M} \simeq -\frac{3ieG_F}{16\sqrt{2}\pi^2}\left(m_k + m_j\right)\sum_{l=e,\mu,\tau} \text{Im}\left[U_{lk}^{\star} U_{lj}\right]\frac{m_l^2}{m_W^2}, \tag{2.7}$$

$$\epsilon_{kj}^{M} \simeq \frac{3ieG_F}{16\sqrt{2}\pi^2}\left(m_k - m_j\right)\sum_{l=e,\mu,\tau} \text{Re}\left[U_{lk}^{\star} U_{lj}\right]\frac{m_l^2}{m_W^2}. \tag{2.8}$$

These are difficult to compare to the Dirac case, due to possible presence of Majorana phases in the PMNS matrices, but it is clear that they have the same order of magnitude as Dirac transition dipole moments. However, the neglected model dependent contributions can enhance the transition dipole moments [?].

It is possible [?] to obtain a "natural" upper limits on the size of neutrino magnetic moment by calculating its contribution to the neutrino mass by standard model radiative corrections. For Dirac neutrinos, the radiative correction induced by neutrino magnetic moment, generated at an energy scale Λ, to the neutrino mass is generically

$$m_\nu^D \sim \frac{\mu_\nu^D}{3 \times 10^{-15} \mu_B} \left[\Lambda\,(\mathrm{TeV})\right]^2 \mathrm{eV}. \tag{2.9}$$

So for $\Lambda \simeq 1$TeV TO DO: *figure out what exactly does this energy scale actually relate to and explain it here?* and $m_\nu \lesssim 0.3$eV the limit becomes $\mu_\nu^D \lesssim 10^{-15}\mu_B$. This applies only if the new physics is well above the electroweak scale ($\Lambda_{EW} \sim 100$GeV). It is possible to get Dirac neutrino magnetic moment higher than this limit, for example in frameworks of minimal super-symmetric standard model, by adding more Higgs doublets, or by considering large extra dimensions [?].

A similar limit for Majorana neutrino magnetic moment would be less stringent due to the antisymmetry of the Majorana neutrino magnetic moment form factors. Considering $m_\nu \lesssim 0.3$eV, the limit can be expressed as

$$\mu_{\tau\mu}, \mu_{\tau e} \lesssim 10^{-9} \left[\Lambda\,(\mathrm{TeV})\right]^{-2} \tag{2.10}$$

$$\mu_{\mu e} \lesssim 3 \times 10^{-7} \left[\Lambda\,(\mathrm{TeV})\right]^{-2} \tag{2.11}$$

which is shown in the flavour basis [?]. This expression relates to the framework used previously as

$$\mu_{ij} = \sum_{\alpha\beta} \mu_{\alpha\beta} U^{\star}_{\alpha i} U_{\beta j}, \quad \alpha, \beta \in \{e, \mu, \tau\}\,. \tag{2.12}$$

These considerations imply, that if a magnetic moment $\mu \gtrsim 10^{-15}\mu_B$ would be measured, it is more plausible that neutrinos are Majorana fermions and that the scale of lepton violation would be well below the conventional see-saw scale [?] TO DO: *double check this claim*.

Effective neutrino magnetic moment

Since experiments detect neutrino flavour states, not the mass states, what we measure in experiments is an effective "flavour" magnetic moment μ_{eff}. μ_{eff} is influenced by mixing of the neutrino magnetic moments (and electric moments) expressed in the mass basis (as described above) and neutrino oscillations. In the ultra-relativistic limit, the (anti)neutrino effective magnetic moment is

$$\mu^2_{\nu_l}\left(L, E_\nu\right) = \sum_j \left| \sum_k U^\star_{lk} e^{\mp i \Delta m^2_{kj} L/2E_\nu} \left(\mu_{jk} - i\epsilon_{jk}\right) \right|^2 , \tag{2.13}$$

where L is the distance the neutrino travelled, E_ν is the neutrino energy and δm^2 is the neutrino mass squared difference [?]. The minus sign in the exponent is for neutrinos and the plus sign for antineutrinos, therefore the only difference is in the phase induced by neutrino oscillations.

For experiments with baselines short enough that neutrino oscillations would not have time to develop ($\Delta m^2 L/2E_\nu \ll\sim 1$), such as the NOvA Near Detector, the effective magnetic moment can be expressed as

$$\mu^2_{\nu_l} = \mu^2_{\bar{\nu}_l} \simeq \sum_j \left| \sum_k U^\star_{lk} \left(\mu_{jk} - i\epsilon_{jk}\right) \right|^2 = \left[U\left(\mu^2 + \epsilon^2\right)U^\dagger + 2\,\mathrm{Im}\left(U\mu\epsilon U^\dagger\right)\right]_{ll'} , \tag{2.14}$$

which is independent of the neutrino energy and of the source to detector distance.

It is important to mention, that since the effective magnetic moment depends on the flavour of the studied neutrino, it is different (but related) for neutrino experiment studying neutrinos from different sources. Additionally some experiments, namely solar neutrino experiments, need to include matter effects on the neutrino oscillations. Therefore the reports on the value (or upper limit) of the effective neutrino magnetic moment are not directly comparable between different types of neutrino experiments. Theorists publish papers trying to extrapolate the measured effective magnetic moments to each neutrino flavour, but necessarily apply assumptions that might not hold in all BSM theories.

2.1.2 Other neutrino electromagnetic properties

TO DO: *This section is not finished, most of this text is just copied from some theory papers for now*

Neutrino electric charge is heavily constraint by the measurements on the neutrality of matter (since generally neutrinos having an electric charge would also mean that neutrons have charge which would affect all heavier nuclei). It is also constrained by the SN1987A, since neutrino having an effective charge would lengthen its path through the extragalactic magnetic fields and would arrive on earth later. It can also be obtained from nu-on-e scatter from the relationship between neutrino millicharge and magnetic moment. [nuElmagInt2015.pdf - sec. VIIA]

The neutrino charge radius is determined by the second term in the expansion of the neutrino charge form factor and can be interpreted using the Fourier transform of a spherically symmetric charge distribution. It can also be negative since the charge density is not a positively defined quantity. In the SM the charge radius has the form of (possible other definitions exist)

$$\langle r^2_{\nu_l} \rangle_{\mathsf{SM}} = \frac{G_{\mathsf{F}}}{4\sqrt{2}\pi^2}\left[3 - 2\log\left(\frac{m_l^2}{m_W^2}\right)\right]. \tag{2.15}$$

This corresponds to $\langle r^2_{\nu_\mu} \langle_{\mathsf{SM}} = 2.4 \times 10^{-33}$ cm^2 and similar scale for other neutrino flavours. [nuElmagInt2015.pdf - sec. VIIB]

[nuElmagInt2015.pdf - sec. VIIB] The effect of the neutrino charge radius on the neutrino-on-electron scattering cross section is through the following shift of the vector coupling constant (Grau and Grifols, 1986; Degrassi, Sirlin, and VMarciano, 1989; Vogel and Engel, 1989; Hagiwara et al., 1994):

$$g_V^{\nu_l} \to g_V^{\nu_l} + \frac{2}{3} m_W^2 \langle r^2_{\nu_l} \rangle \sin^2\theta_W \tag{2.16}$$

[nuElmagInt2015.pdf - sec. VIIB] The current experimental limits for muon neutrinos are from TO DO: *check the current exp. limits* Hirsch, Nardi, and Restrepo (2003) who obtained the following 90% C.L. bounds on $\langle r^2_{\nu_\mu} \rangle$ from a reanalysis of CHARM-II (Vilain et al., 1995) and CCFR (McFarland et al.,1998) data:

$$-0.52 \times 10^{-32} < \langle r^2_{\nu_\mu} \rangle < 0.68 \times 10^{-32} \text{ cm}^2 \tag{2.17}$$

In the Standard Model, the neutrino anapole moment is somehow coupled with the neutrino charge radii and is functionally identical. the phenomenology of neutrino anapole moments is similar to that of neutrino charge radii. Hence, the limits on the neutrino charge radii discussed in Sec. VII.B also apply to the neutrino anapole moments multiplied by 6. in the standard model the neutrino charge radius and the anapole moment are not defined separately and one can interpret arbitrarily the charge form factor as a charge radius or as an anapole moment. Therefore, the standard model values for the neutrino charge radii in Eqs. (7.35)–(7.38) can be interpreted also as values of the corresponding neutrino anapole moments. [nuElmagInt2015.pdf - sec. VIIC]

It is possible to consider the toroidal dipole moment as a characteristic of the neutrino which is more convenient and transparent than the anapole moment for the description of T-invariant interactions with nonconservation of the P and C symmetries. the toroidal and anapole moments coincide in the static limit when the masses of the initial and final neutrino states are equal to each other. The toroidal (anapole) interactions of a Majorana as well as a Dirac neutrino are expected to contribute to the total cross section of neutrino elastic scattering off electrons, quarks, and nuclei. Because of the fact that the toroidal (anapole) interactions contribute to the helicity preserving part of the scattering of neutrinos on electrons, quarks, and nuclei, its contributions to cross sections are similar to those of the neutrino charge radius. In principle, these contributions can be probed and information about toroidal moments can be extracted in low-energy scattering experiments in the future. Different effects of the neutrino toroidal moment are discussed by Ginzburg and Tsytovich (1985), Bukina, Dubovik, and Kuznetsov (1998a, 1998b), and Dubovik and Kuznetsov (1998). In particular, it has been shown that the neutrino toroidal electromagnetic interactions can produce Cherenkov radiation of neutrinos propagating in a medium. [nuElmagInt2015.pdf - sec. VIIC]

2.1.3 Measuring neutrino magnetic moment

The most sensitive method to measure neutrino magnetic moment is the low energy elastic scattering of (anti)neutrinos on electrons [**?**]. The diagram for this interaction is shown on Fig.2.2 showing the two observables, the recoil electron's kinetic energy

$(T_e = E_{e\prime} - m_e)$ and the recoil angle with respect to the incoming neutrino beam (θ).

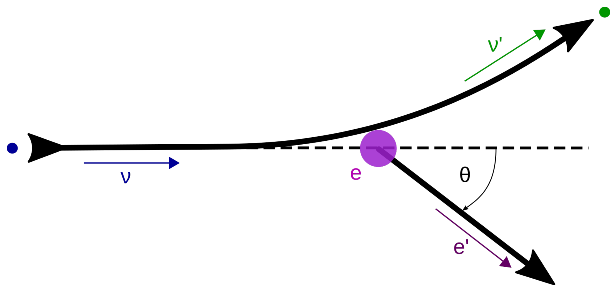

Figure 2.2: Neutrino-on-electron elastic scattering diagram

From simple $2 \to 2$ kinematics we can calculate

$$(P_\nu - P_{e'})^2 = (P_{\nu'} - P_e)^2 \,, \tag{2.18}$$

$$m_\nu^2 + m_e^2 - 2E_\nu E_{e'} + 2E_\nu p_{e'} \cos\theta = m_\nu^2 + m_e^2 - 2E_{\nu'} m_e. \tag{2.19}$$

Using the energy conservation

$$E_\nu + m_e = E_{\nu'} + E_{e'} = E_{\nu'} + T_e + m_e \Rightarrow E_{\nu'} = E_\nu - T_e \tag{2.20}$$

we get

$$E_\nu p_{e'} \cos\theta = E_\nu E_{e'} - E_{\nu'} m_e = E_\nu \left(T_e + m_e\right) - \left(E_\nu - T_e\right) m_e = T_e \left(E_\nu + m_e\right), \tag{2.21}$$

$$\cos\theta = \frac{E_\nu + m_e}{E_\nu} \sqrt{\frac{T_e^2}{E_{e'}^2 - m_e^2}} = \frac{E_\nu + m_e}{E_\nu} \sqrt{\frac{T_e^2}{T_e^2 + 2T_e m_e}}. \tag{2.22}$$

And finally we get

$$\cos\theta = \frac{E_\nu + m_e}{E_\nu} \sqrt{\frac{T_e}{T_e + 2m_e}}. \tag{2.23}$$

Electron's kinetic energy is kinematically constrained by the energy conservation as

$$T_e \leq \frac{2E_\nu^2}{2E_\nu + m_e}. \tag{2.24}$$

Considering $E_\nu \sim$ GeV, we can approximate $\frac{m_e^2}{E_\nu^2} \to 0$ and from Fig.2.3 we can see that we can approximate all recoil angles to be very small, therefore $\theta^2 \cong (1 - \cos^2\theta)$.

Using Eq.2.23 we get

$$T_e\theta^2 \cong T_e\left(1-\left(\frac{E_\nu+m_e}{E_\nu}\right)^2\frac{T_e}{T_e+2m_e}\right)=T_e\left(1-\left(1+\frac{2m_e}{E_\nu}\right)\frac{T_e}{T_e+2m_e}\right), \tag{2.25}$$

therefore

$$T_e\theta^2 \cong \frac{2m_eT_e}{T_e+2m_e}\left(1-\frac{T_e}{E_\nu}\right)=2m_e\left(\frac{1}{1+\frac{2m_e}{T_e}}\right)\left(1-\frac{T_e}{E_\nu}\right), \tag{2.26}$$

and finally

$$T_e\theta^2 \cong 2m_e\left(1-\frac{T_e}{E_\nu}\right)<2m_e. \tag{2.27}$$

This is a strong limit that clearly distinguishes the neutrino-on-electron elastic scattering events from other similar interaction involving single electron (mainly the ν_e Charged Current interaction).

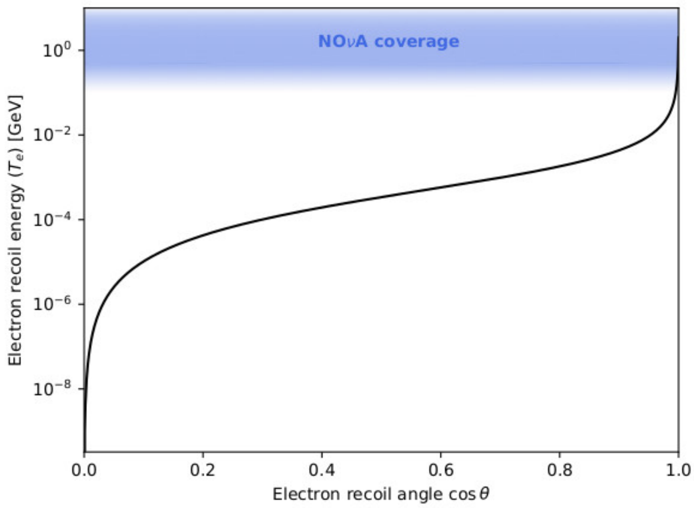

Figure 2.3: Relation between the recoil electron's kinetic energy and angle for neutrino-on-electron elastic scattering. The coverage of the NOvA detectors for measuring the electron recoil energy is shown in blue. Only very forwards electron's are recorded in NOvA.

[Fundamentals of neutrino Physics and Astrophysics, p.139] The total neutrino-electron elastic scattering cross section for large energies is ht

Table 2.1: Neutrino-on-electron elastic scattering total cross sections

Process	Total cross section
$\nu_e + e^-$	$\simeq 93 \times 10^{-43} E_\nu \mathrm{cm}^2\mathrm{GeV}^{-1}$
$\overline{\nu}_e + e^-$	$\simeq 39 \times 10^{-43} E_\nu\, \mathrm{cm}^2\mathrm{GeV}^{-1}$
$\nu_{\mu,\tau} + e^-$	$\simeq 15 \times 10^{-43} E_\nu\, \mathrm{cm}^2\mathrm{GeV}^{-1}$
$\overline{\nu}_{\mu,\tau} + e^-$	$\simeq 13 \times 10^{-43} E_\nu\, \mathrm{cm}^2\mathrm{GeV}^{-1}$

Neutrino magnetic moment cross section

In the ultrarelativistic limit, the neutrino magnetic moment changes the neutrino helicity, turning active neutrinos into sterile TO DO: *this is a very strong statement and it probably need a bit more backing up*. Since the SM weak interaction conserves helicity we can simply add the two contribution to the neutrino-on-electron cross section incoherently [?]:

$$\frac{d\sigma_{\nu_l e^-}}{dT_e} = \left(\frac{d\sigma_{\nu_l e^-}}{dT_e}\right)_{\mathsf{SM}} + \left(\frac{d\sigma_{\nu_l e^-}}{dT_e}\right)_{\mathsf{MAG}}. \tag{2.28}$$

The standard model contribution can be expressed as [?]:

$$\left(\frac{d\sigma_{\nu_l e^-}}{dT_e}\right)_{\mathsf{SM}} = \frac{G_F^2 m_e}{2\pi}\left\{(g_V^{\nu_l} + g_A^{\nu_l})^2 + (g_V^{\nu_l} - g_A^{\nu_l})^2\left(1 - \frac{T_e}{E_\nu}\right)^2 + \left((g_A^{\nu_l})^2 - (g_V^{\nu_l})^2\right)\frac{m_e T_e}{E_\nu^2}\right\}, \tag{2.29}$$

where the coupling constants g_V and g_A are different for different neutrino flavours and for antineutrinos. Their values are:

$$g_V^{\nu_e} = 2\sin^2\theta_W + 1/2, \qquad g_A^{\nu_e} = 1/2, \tag{2.30}$$

$$g_V^{\nu_{\mu,\tau}} = 2\sin^2\theta_W - 1/2, \qquad g_A^{\nu_{\mu,\tau}} = -1/2. \tag{2.31}$$

For antineutrinos $g_A \to -g_A$.

The neutrino magnetic moment contribution is TO DO: *include derivation from [?]* [?]:

$$\left(\frac{d\sigma_{\nu_l e^-}}{dT_e}\right)_{\mathsf{MAG}} = \frac{\pi\alpha^2}{m_e^2}\left(\frac{1}{T_e} - \frac{1}{E_\nu}\right)\left(\frac{\mu_{\nu_l}}{\mu_B}\right)^2, \tag{2.32}$$

where α is the fine structure constant.

Comparison of the Standard Model and the neutrino magnetic moment cross sections is shown on Fig.2.4. Whereas the SM cross section is flat with $T_e \to 0$, the

νMM cross section keeps increasing to infinity. However, this reach is limited by the experimental capabilities of detecting such low energetic neutrinos. Possible NOvA coverage is shown in a shaded blue and it is uncertain we could actually reach as low as 100 MeV.

TO DO: *Reference the colours on the figures to the origins of the values (LSND and Biao)*

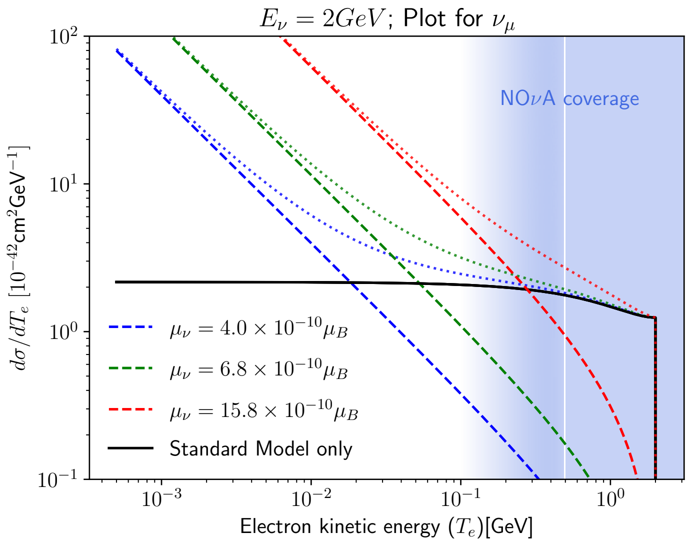

Figure 2.4: Comparison of the neutrino magnetic moment (coloured) and Standard Model (black) cross sections for the neutrino-on-electron elastic scattering. Different colours depict different values of the neutrino magnetic moment. Dashed lines are the individual cross sections and dotted lines are the added total cross section with the standard model contribution. NOvA coverage of electron recoil energies is shown in shaded blue.

As can be seen on Fig.2.4 and Fig.2.5, the magnetic moment contribution exceeds the standard model contribution for low enough T_e. This can be approximated as [?]:

$$T_e \lesssim \frac{\pi^2\alpha^2}{G_F^2 m_e^3}\left(\frac{\mu_\nu}{\mu_B}\right)^2 \simeq 2.9 \times 10^{19}\left(\frac{\mu_\nu}{\mu_B}\right)^2 [\text{MeV}]\,, \tag{2.33}$$

which does not depend on the neutrino energy and makes experiments sensitive to lower energetic electrons more sensitive to the neutrino magnetic moment. This is especially true for the recent dark matter experiments which put stringent limits on the solar neutrino effective magnetic moment, as described in the following section.

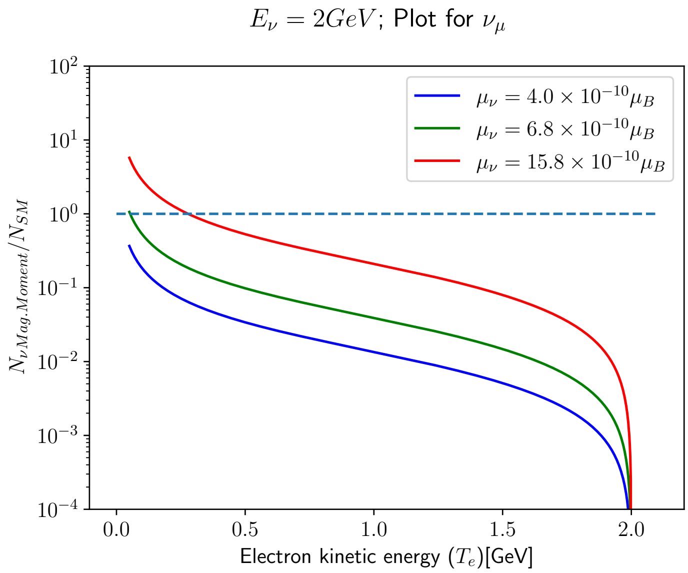

Figure 2.5: Ratio of the netrino magnetic moment cross section to the standard model cross section for the neutrino-on-electron elastic scattering. Different colours depict different effective muon neutrino magnetic moment values.

2.2 Experimental overview

2.3 Event selection

2.4 Fitting and hypothesis testing, parameter estimation

How do we find the value of or limit for the effective neutrino magnetic moment?

Large section on statistics in the PDG.

Maximum likelihood with binned data:

N bins with a vector of data $n = (n1, ..., n_N)$ with expectation values $\mu = E[n]$ and probabilities $f(n; \mu)$. Suppose the mean values μ can be determined as a function of a set of parameters θ (I assume for us there's either only one parameter - magnetic moment, or three parameters - mag. moment, scale of SM signal and scale of SM background). Then one may maximize the likelihood function based on the contents of the bins.

If the n_i is regarded as independent and Poisson distributed (which I'd say is the case for us), then the data are instead described by a product of Poisson probabilities,

$$f_p\left(n;\theta\right)=\prod_{i=1}^{N}\frac{\mu_i^{n_i}}{n_i!}e^{-\mu_i}, \tag{2.34}$$

where the mean values μ_i are given functions of θ. The total number of events n_{tot} thus follows a Poisson distribution with mean $\mu_{tot}=\sum_i \mu_i$.

When using maximum likelihood with binned data, one can find the maximum likelihood estimators and at the same time obtain a statistic usable for a test of goodness-of-fit. Maximizing the likelihood $L\left(\theta\right)=f_P\left(n;\theta\right)$ is equivalent to maximizing the likelihood ratio $\lambda\left(\theta\right)=f_P\left(n;\theta\right)/f\left(n;\hat{\mu}\right)$, where in the denominator $f\left(n;\hat{\mu}\right)$ is a model with an adjustable parameter for each bin, $\mu=\left(\mu_1,...,\mu_N\right)$, and the corresponding estimators are $\hat{\mu}=\left(n_1,...,n_N\right)$ (called the "saturated model").

Equivalently one often minimizes the quantity $-2\ln\lambda\left(\theta\right)$. For independent Poisson distributed n_i this is

$$-2\ln\lambda\left(\theta\right)=2\sum_{i=1}^{N}\left[\mu_i\left(\theta\right)-n_i+n_i\ln\frac{n_i}{\mu_i\left(\theta\right)}\right], \tag{2.35}$$

where for bins with $n_i=0$, the last term is zero. In our term $\mu_i\left(\theta\right)$ is the **expected number of events in bin i if magnetic moment is** θ and n_i is the observed (measured) number of events in that bin.

A smaller value of $-2\ln\lambda\left(\hat{\theta}\right)$ corresponds to better agreement between the data and the hypothesized form of $\mu\left(\theta\right)$. The value of $-2\ln\lambda\left(\hat{\theta}\right)$ can thus be translated into a **p-value as a measure of goodness-of-fit**. Assuming the model is correct, then according to **Wilk's theorem**, for **sufficiently large** μ_i and provided certain regularity conditions are met, **the minimum of** $-2\ln\lambda$ **follows a** χ^2 **distribution.** If there are N bins and M fitter parameters, then the number of degrees of freedom for the χ^2 distribution is $N-M$ if the data are threated as Poisson distributed - which they are for us.

The method of least squares coincides with the method of maximum likelihood in a special case where the independent variables are Gaussian distributed - so I suppose this means that if I have enough events in each single bin, then I could equate the method of log likelihood and the method of least squares...

2.4.1 Nuisance parameters

In general the model is not perfect, which is to say it cannot provide an accurate description of the data even at the most optimal point of its parameter space. As a result, the estimated parameters can have a systematic bias. One can improve the model by including in it additional parameters. That is, $P(x|\theta)$ is replaced by a more general model $P(x|\theta,\nu)$, which depends on parameters of interest θ and *nuisance parameters* ν. The additional parameters are not of intrinsic interest but must be included for the model to be sufficiently accurate for some point in the enlarged parameter space.

Although including additional parameters may eliminate or at least reduce the effect of systematic uncertainties, their presence will result in increased statistical uncertainties for the parameters of interest. This occurs because the estimators for the nuisance parameters and those of interest will in general be correlated, which results in an enlargement of the contour.

To reduce the impact of the nuisance parameters one often tries to constrain their values by means of control or calibration measurements, say, having data **y** (I assume for us this would represent a control sample - like they use in the ND group). For example, some components of y could represent estimates of the nuisance parameters, often from separate experiments. Suppose the measurements y are statistically independent from x and are described by a model $P(y|\nu)$. The joint model for both x and y is in this case therefore the product of the probabilities for x and y, and thus the likelihood function for the full set of parameters is

$$L(\theta,\nu) = P(x|\theta,\nu)\,P(y|\nu)\,. \tag{2.36}$$

Note that in this case if one wants to simulate the experiment by means of Monte Carlo, both the primary and control measurements, x and y, must be generated for each repetition under assumption of fixed values for the parameters θ and ν.

Using all of the parameters (θ,ν) to find the statistical errors in the parameters of interest θ is equivalent to using the *profile likelihood*, which depends only on θ. It is defined as

$$L_p(\theta) = L(\theta,\hat{\nu}(\theta))\,, \tag{2.37}$$

This equation is supposed to have double hat for the neutrino on RHS but that throws an error when compiling... where the double-hat notation indicates the profiled values of the parameters ν, defined as values that maximize L for the specified θ.

2.4.2 Unbinned parameter estimation

If the total number of data values is small, the unbinned maximum likelihood method is preferred, since binning can only result in a loss of information, and hence the larger statistical errors for the parameter estimates. Does't this mean that if the number of events for the neutrino magnetic moment analysis is small, it would be better to do a completely unbinned maximum likelihood method, instead of a single bin method?

2.4.3 Subsection

Subsubsection

Acronyms

CC Charged current. 2, 5

NC Neutral Current. 2

SM Standard Model. 2, 3

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