

CHAPTER 1

Theory of neutrino physics

Neutrinos were first theoretically proposed by Wolfgang Pauli [1, 2] as very light electrically neutral particles with a half-spin and a possible magnetic moment [3]. They formed a crucial part of Enrico Fermi's successful theory of β decays [4, 5], which solidified their importance in particle physics even before their first experimental detection. Fermi's theory developed into the Standard Model (SM) of particle physics [6–8], which in its current form contains three generations of fermions. Each generation consists of two leptons, one charged lepton and one neutrino, which has no mass, nor magnetic moment.

The SM is mathematically described by a Lagrangian, in which neutrinos are represented by a two-component left-handed chiral fields $\nu_{\alpha L}$, where $\alpha = e, \mu, \tau$ denotes the three neutrino generations, also called flavours [9–11]. Neutrino fields form weak isospin doublets $L_\alpha = \begin{pmatrix} \nu_{\alpha L} \\ \alpha_L \end{pmatrix}$ with their associated left-handed charged lepton fields α_L . Unlike for the charged leptons, there is no right-handed chiral neutrino singlet field in the SM. This means that neutrinos cannot obtain a mass term, since the fermion mass terms arise from the Higgs mechanism [12–14] via the Yukawa coupling of the fermion and the Higgs fields¹ [15], which requires a combination of left-handed and right-handed chiral fields [16]. Additionally, since neutrinos are massless in the SM, all the neutrinos are left-handed helicity particles, and all the antineutrinos ($\bar{\nu}$) are right-handed helicity antiparticles. Neutrinos and antineutrinos are mutually related by Charge conjugation - Parity (CP) symmetry: $\nu \xrightarrow{CP} \bar{\nu}$.

The interaction terms for neutrinos can be separated into two parts, describing the Charged current (CC) interactions with the W_μ gauge field and the Neutral Current (NC) interaction with the Z_μ gauge field, which create/annihilate the W^\pm and Z^0 gauge bosons respectively. Neglecting the non-neutrino components, the two neu-

¹Further discussion about possible neutrino mass terms in Sec. 1.4

trino interaction terms are [16]

$$\mathcal{L}_{\text{CC}}^{\text{SM}} = -\frac{g_w}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \bar{\nu}_{\alpha L} \gamma^\mu \alpha_L W_\mu^+ + \text{h.c.} \quad \text{and} \quad (1.1)$$

$$\mathcal{L}_{\text{NC}}^{\text{SM}} = -\frac{g_w}{2 \cos(\theta_W)} \sum_{\alpha=e,\mu,\tau} \bar{\nu}_{\alpha L} \gamma^\mu \nu_{\alpha L} Z_\mu^0. \quad (1.2)$$

Here g_w is the weak coupling constant, θ_W is the Weinberg angle and γ^μ ($\mu = 0, 1, 2, 3$) are the four Dirac gamma matrices.

These two terms describe all the possible **SM** neutrino interaction vertices, as shown in Fig. 1.1. These diagrams show the **CC** and the **NC** interaction of neutrinos and antineutrinos and, in case of the **CC** diagram, can also be flipped around the vertical axis to show the production of neutrinos from the weak interaction (or decays) of leptons. They can also be rotated 90° to either show the annihilation, or the production of the neutrino-lepton (for **CC**), or neutrino-antineutrino (for **NC**) pairs.

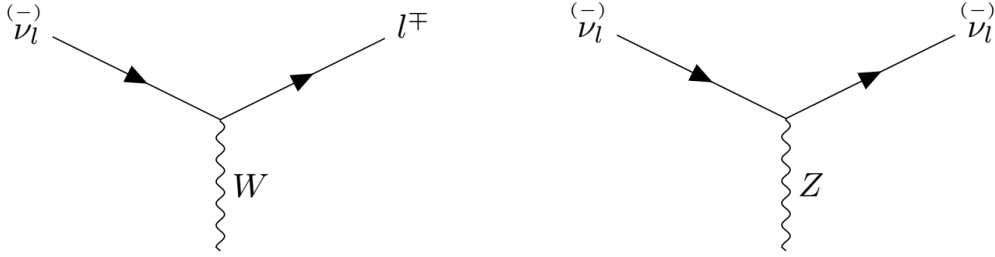


Figure 1.1: Neutrino interaction vertices in the **SM** via the weak charged currents (left) and the neutral currents (right).

1.1 Neutrino Production

Some of the most common neutrino and antineutrino production channels include nucleon transitions via **CC** weak interactions. Specifically, the transition of a neutron into a proton, either as the decay of a free neutron, or as the β^- decay for neutrons bound in nucleus, produces an electron and an electron antineutrino:

$$n \rightarrow p + e^- + \bar{\nu}_e. \quad (1.3)$$

The study of the electron spectrum from β^- decay was the reason Pauli proposed the existence of the neutrino [1]. Additionally, this channel is an abundant source of $\bar{\nu}_e$ from nuclear reactors, which were the first artificial sources of neutrinos, increasing the neutrino flux by about 100 million compared to the naturally occurring sources, thus enabling the first ever detection of a neutrino [17–19].

Similarly, the production of an electron neutrino together with a positron via the transition of a proton into a neutron can occur inside the nucleus either as the β^+ decay:

$$p \rightarrow n + e^+ + \nu_e, \quad (1.4)$$

or via the electron capture:

$$p + e^- \rightarrow n + \nu_e. \quad (1.5)$$

This channel occurs in stars and in the first phase of supernovae [16].

However, most supernovae neutrinos are created via a thermal pair production via NC interaction

$$e^- + e^+ \rightarrow \nu_\alpha + \bar{\nu}_\alpha \quad (1.6)$$

producing neutrinos and antineutrinos of all flavours. Neutrino pair production via the decay of Z^0 was studied in great detail [20], since the magnitude of the Z^0 decay width depends on the number of neutrino flavours which can couple to Z^0 , with the current best fit $N_\nu = 2.984$ [21]. Therefore, there should be exactly three light active neutrino flavours.

An abundant source of ν_μ and $\bar{\nu}_\mu$ is the decay of pions and muons

$$p + X \rightarrow \pi^\pm \rightarrow \mu^\pm + \nu_\mu (\bar{\nu}_\mu) \quad (1.7)$$

$$\mu^\pm \rightarrow e^\pm + \nu_\mu (\bar{\nu}_\mu) + \nu_e (\bar{\nu}_e), \quad (1.8)$$

which naturally occurs in Earth's atmosphere from the interaction of cosmic ray protons. It is notable, that if all the muons decay by the time they reach Earth's surface, the ratio of $\nu_\mu : \nu_e$ should be exactly 2:1. The same process is also used in the modern accelerator-based neutrino sources, which use protons from accelerators with desired energies, impinge them onto a fixed target, and focus the resulting hadrons to achieve a highly pure and precise source of ν_μ or $\bar{\nu}_\mu$ [22, 23].

Similarly to π , heavier hadrons, such as kaons and charmed particles, can be produced from accelerated protons and other particles, either from natural or artificial origins. These hadrons then also produce neutrinos, including ν_τ and $\bar{\nu}_\tau$ if their energies are high enough [24, 25].

1.2 Neutrino Interactions

The interaction of neutrinos with matter can either be categorized based on the target, which is generally either an atomic electron or a nucleus, or the neutrino energy.

Neutrino-electron interactions occur either via elastic scattering, which result in a neutrino and an electron, or via the inverse muon (or tau) decay, which contains a muon (or tau) in the final state. Both of these interactions at the lowest order involve only free leptons and are very well theoretically understood. The elastic scattering has no energy threshold and can occur for any neutrino. On the other hand, due to the large difference between m_e and m_μ/m_τ , the inverse muon decay has an energy threshold of $E_{\nu_\mu} > 10.92 \text{ GeV}$, and the inverse tau decay $E_{\nu_\tau} > 3 \text{ TeV}$ [16, 26].

Neutrino-nucleus interactions can be to an extent approximated by an interaction of neutrino with its quasi-free nucleon constituents [27]. These interactions can be separated into different interaction channels based on what happens to the nucleon and therefore on the resulting particles. The interaction channels depend on the neutrino incident energy, as illustrated on the case of ν_μ CC interactions in Fig. 1.2.

At lower energies, neutrino-nucleon interactions result in the production of either a nucleon together with a neutrino in the case of NC elastic scattering or a nucleon coupled with a charged lepton in the case of CC Quasi-Elastic (QE) interactions. The CCQE interaction of an antineutrino on a proton

$$\bar{\nu}_\alpha + p \rightarrow n + \alpha^+ \quad (1.9)$$

is called the inverse β decay and was used for the first ever detection of neutrinos (specifically $\bar{\nu}_e$ from a nuclear reactor) [17, 18]. Together with the interaction of a neutrino on a neutron

$$\nu_\alpha + n \rightarrow p + \alpha^- \quad (1.10)$$

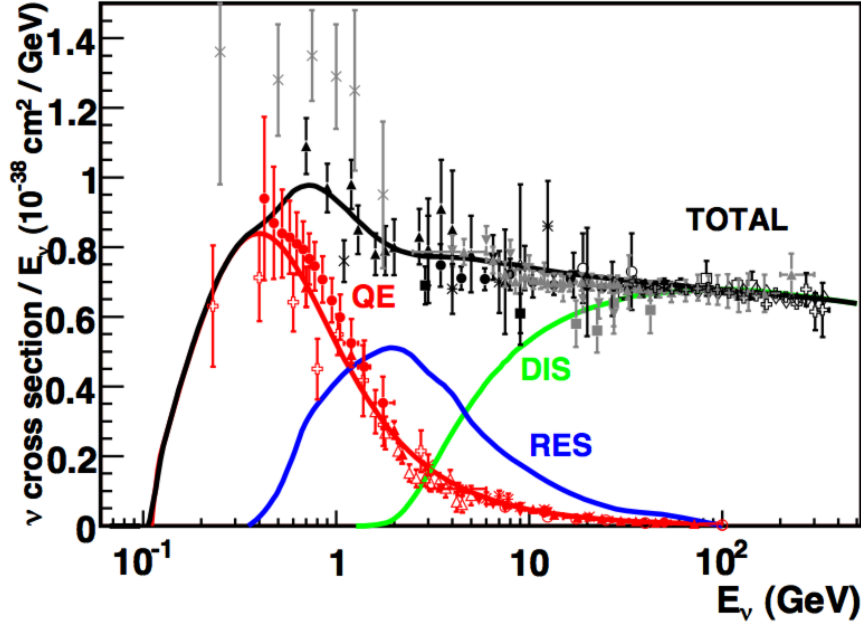


Figure 1.2: Neutrino CC cross sections on an isolated nucleon divided by the neutrino energy based on the interaction types: **Quasi-Elastic (QE)**, **Resonant baryon production (RES)** and **Deep Inelastic Scattering (DIS)**. Figure is from [28] and compares the measured data [27] and the prediction provided by the NUANCE generator [29].

they serve as fundamental processes for neutrino detection [24, 30, 31]. The low energy threshold is based on the mass of the produced charged lepton, with no threshold for ν_e and thresholds of $E_{\bar{\nu}_e} \gtrsim 1.8 \text{ MeV}$, $E_{\nu_\mu} \gtrsim 110 \text{ MeV}$ and $E_{\nu_\tau} \gtrsim 3.5 \text{ GeV}$.

At higher energies, neutrinos can transfer enough energy to the outgoing nucleon to excite it into a resonant baryon, which then decays back into the nucleon and into one or more additional particles. This Resonant baryon production (RES) has a threshold of about 270 MeV for ν_μ and can be distinguished by the presence of an additional π on top of the CCQE products, or, at higher energies, even of multiple additional π 's or other hadrons. Increasing the neutrino incident energy even higher means that neutrinos can start probing the quark contents of the individual nucleons in the Deep Inelastic Scattering (DIS), as can be seen in Fig. 1.2.

Even though the approximation of nuclei as collections of quasi-free nucleons is useful, it has been shown [32] that there are important nuclear effects that have to be considered. For example the Fermi motion of nucleons and their binding inside the nucleus, or Pauli's exclusion principle resulting in nucleon energy levels [33]. Another important example is the two particle - two hole (2p2h) interaction [34–36], which occurs when neutrinos interact with a correlated pair of nucleons and can sig-

nificantly increase the QE cross section [33]. The 2p2h interaction often occurs via Meson Exchange Current (MEC), where the meson effectively propagates the interaction between the two correlated nucleons. Furthermore, the products of all of the aforementioned interactions can re-interact inside of the nucleus in Final State Interactions (FSIs), which can alter the particle content actually seen in the detector.

Additionally, if the total energy transferred to the nucleus is small relative to its size, neutrinos can interact with the entire nucleus coherently, where the contributions from each individual nucleon are added together. At low energies, neutrinos can interact via the Coherent Elastic ν -Nucleus Scattering (CEvNS) [37], which results in the excitation of the nucleus. At higher energies, neutrinos can interact via the Coherent π (COH π) production, which produces a single π without transferring much momentum to the nucleus. In case of the CCCOH π production, there is an additional charged lepton and the produced π is positive (negative) for (anti)neutrinos, and for the NCCOH π production the produced π is neutral. As the π receives most of the transferred momentum, it generally travels in the direction of the initial neutrino and can be difficult to distinguish especially from e and γ signal in a detector [33].

1.3 Neutrino Oscillation

The idea that neutrinos can oscillate originates as a possibility of transitions between neutrinos and antineutrinos [38, 39], analogously to the already known oscillations of $K^0 \leftrightarrow \bar{K}^0$. This was adapted to the oscillations between different neutrino flavours [40, 41], by considering that the flavour neutrino states ν_α , which are the eigenstates of weak interactions described in Eq. 1.1 and 1.2, are not identical to the mass neutrino states ν_k , which are the eigenstates of the vacuum Hamiltonian \mathcal{H}_0 :

$$\mathcal{H}_0 |\nu_k\rangle = E_k |\nu_k\rangle, k = 1, 2, 3, \dots, \quad (1.11)$$

with energy E_k . Instead, the neutrino flavour and mass eigenstates are related as

$$|\nu_\alpha\rangle = \sum_k U_{\alpha k}^* |\nu_k\rangle, \quad (1.12)$$

where U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix, named after the authors of neutrino oscillations [16, 42]. U is defined as unitary, which makes the inverse relation simply

$$|\nu_k\rangle = \sum_{\alpha} U_{\alpha k} |\nu_{\alpha}\rangle. \quad (1.13)$$

Using the Schrödinger equation

$$i \frac{d}{dt} |\nu_k(t)\rangle = \mathcal{H} |\nu_k(t)\rangle, \quad (1.14)$$

the evolution of massive neutrino states in vacuum ($\mathcal{H} = \mathcal{H}_0$) can be described by plane waves

$$|\nu_k(t)\rangle = e^{-iE_k t} |\nu_k\rangle. \quad (1.15)$$

The energy of a neutrino state with mass m_k and momentum \vec{p}

$$E_k = \sqrt{\vec{p}^2 + m_k^2} \quad (1.16)$$

can be approximated as

$$E_k \xrightarrow{m^2 \ll p^2 \approx E^2} E + \frac{m_k^2}{2E}, \quad (1.17)$$

assuming small neutrinos masses and for ultra-relativistic neutrinos [16]. Additionally, as it is generally easier to measure the distance neutrinos travel (L), rather than the time (t), and given the notation $c \equiv 1$, where c is the speed of light in vacuum, it is common to interchange $L \leftrightarrow t$.

Given the orthogonality of neutrino states, $\langle \nu_k | \nu_j \rangle = \delta_{kj}$ and $\langle \nu_{\alpha} | \nu_{\beta} \rangle = \delta_{\alpha\beta}$, and using Eq. 1.15, 1.12 and 1.13, the amplitude of the oscillation (transition) from $\nu_{\alpha} \rightarrow \nu_{\beta}$ over the ‘baseline’ L can be written as

$$A_{\nu_{\alpha} \rightarrow \nu_{\beta}}(L) \equiv \langle \nu_{\beta} | \nu_{\alpha}(L) \rangle = \sum_k U_{\alpha k}^* U_{\beta k} e^{-iE_k L} \quad (1.18)$$

and the probability as

$$P_{\nu_{\alpha} \rightarrow \nu_{\beta}}(L) = |A_{\nu_{\alpha} \rightarrow \nu_{\beta}}(L)|^2 = \sum_{k,j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* e^{-i(E_k - E_j)L}. \quad (1.19)$$

Using Eq. 1.17 and by defining the neutrino mass splitting (also called the mass

squared difference) as

$$\Delta m_{kj}^2 \equiv m_k^2 - m_j^2, \quad (1.20)$$

the oscillation probability can be expressed as

$$P_{\nu_\alpha \rightarrow \nu_\beta}(L) = \sum_{k,j} U_{\alpha k}^* U_{\beta j} U_{\alpha j} U_{\beta k} e^{-i \frac{\Delta m_{kj}^2 L}{2E}}. \quad (1.21)$$

So far no assumption has been made as to the specific number of neutrino mass or flavour states. However, as was described above, there are exactly three active neutrino flavour states, ν_e , ν_μ and ν_τ . Consequently, it is common to also consider exactly three neutrino mass states. This is often called the three neutrino paradigm. Therefore, the [PMNS](#) matrix has size 3×3 and can be written as [\[16\]](#):

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (1.22)$$

where $c_{ij} \equiv \cos \theta_{ij}$ and $s_{ij} \equiv \sin \theta_{ij}$. The matrix is parametrized using three mixing angles θ_{12} , θ_{13} , and θ_{23} and one phase, often denoted δ_{CP} . The phase describes a possible [CP](#) symmetry violation in neutrino oscillations, which would result in a difference between the neutrino and antineutrino oscillation probabilities.

When neutrinos pass through matter, their evolution changes due to coherent elastic [CC](#) and [NC](#) scattering. However, since the [NC](#) scattering affects all neutrino flavours equivalently, it does not have any effect on neutrino oscillations. Additionally, as electrons are the only charged leptons present in matter, only the relative difference between the [CC](#) interactions of ν_e and of other flavours needs to be considered. The effective interaction potential of neutrinos passing through matter with an electron density N_e can be written as

$$V_{CC} = \pm \sqrt{2} G_F N_e. \quad (1.23)$$

Here G_F is the Fermi coupling constant and the plus or minus sign is for neutrinos or antineutrinos respectively. The electron density (and therefore the interaction potential) can change along the neutrino path, as it does in the Sun, which can resonantly increase the probability of oscillations, as described by the Mikheyev-Smirnov-Wolfenstein (MSW) effect [43, 44]. However, in accelerator based experiments, where neutrinos only pass through the surface of the Earth, the N_e can be described as approximately constant.

The effect of neutrinos passing through matter on oscillation probabilities can be expressed as shifts to mixing angles and to mass squared differences, proportional to the V_{CC} . Since the matter effect differs for neutrinos and antineutrinos, it needs to be carefully considered especially for the δ_{CP} measurement, which relies on the comparison of neutrino to antineutrino oscillations [16].

The first experimental signs of neutrino oscillations appeared as an apparent deficit of solar neutrinos compared to their predicted flux [30]. However, due to low confidence in the prediction of the solar neutrino flux, no conclusion could have been drawn. Similarly, experiments measuring atmospheric neutrinos [45–48] saw a disagreement between the measurement and the prediction for the $\nu_\mu : \nu_e$ fraction of the atmospheric neutrino flux. This *atmospheric neutrino anomaly* was finally resolved by the Super-Kamiokande (SK) experiment [49], reporting the first experimental evidence for neutrino oscillations. The *solar neutrino anomaly* was resolved shortly after by the Sudbury Neutrino Observatory (SNO) experiment [50], which compared the NC rate, unaffected by neutrino oscillations, to the rate of CC neutrino interactions. This was proof that solar neutrinos oscillate without reliance on the model of the Sun. This result also confirmed the importance of accounting for the matter effect in neutrino oscillations, especially for the oscillation of solar neutrinos, due to the large matter density in the Sun.

The difference between the frequency of solar neutrino oscillations and that observed in atmospheric neutrinos proves that there are at least two mass splittings governing neutrino oscillations. As a result, there must be at least three separate neutrino mass states, with at least two of them possessing non-zero masses. This is in direct contradiction to the SM and is to-date the only laboratory-based observation of physics Beyond Standard Model (BSM) [51].

Currently, the three neutrino paradigm of oscillations between three neutrino flavour states via three neutrino mass states is well established [52, 53]. The magnitudes of both the neutrino mass splittings and of two mixing angles, θ_{12} and θ_{13} , are measured within 3%. The third mixing angle θ_{23} is measured to be close to the maximum mixing value of 45° . However, there are three main questions yet to be determined for neutrino oscillations [51]:

1. What is the sign of the larger neutrino mass splitting? Is the electron neutrino made up of the lightest neutrino mass states (normal ordering), or the heaviest (inverted ordering)?
2. Is $\theta_{23} < 45^\circ$ or $\theta_{23} > 45^\circ$? These determine the $\nu_\mu : \nu_\tau$ relative contributions to the neutrino mass states and are also referred to as the upper and the lower octant respectively.
3. Is there CP violation in neutrino oscillations? What is the value of δ_{CP} ? If neutrinos oscillate differently than antineutrinos, this could be an important part of the matter-antimatter asymmetry in the Universe.

All three of these questions are jointly investigated in the current Long Baseline (LBL) accelerator neutrino oscillation experiments, namely the NuMI Off-axis ν_e Appearance (NOvA) [54] and the Tokai to Kamioka (T2K) [55] experiments. Both use precise $\bar{\nu}_\mu^{(-)}$ beams, affected by the matter effect, and compare the rates of $\bar{\nu}_\mu^{(-)}$ disappearance and $\bar{\nu}_e^{(-)}$ appearance to constrain neutrino oscillation parameters [51]. The same methods will be used in the next generation LBL neutrino oscillation experiments, the Deep Underground Neutrino Experiment (DUNE) [56] and the Hyper-Kamiokande (HK) [57], which should give the final answers to all three neutrino oscillation questions [51].

1.4 Neutrino Mass

The absolute values of neutrino masses are currently not known and cannot be directly measured in neutrino oscillation experiments. Results from experiments measuring the kinematic distribution of β decays [58], or from cosmology [59], set a limit for each neutrino mass to < 1 eV. This is several orders of magnitude smaller than

the charged fermion masses, suggesting that theories [BSM](#) which introduce neutrino masses should have a different mechanism for their generation than the one used for the other fermions [52]. Furthermore, in order to introduce neutrino masses into the [SM](#), it is necessary to either add new fields, break the renormalizability of the [SM](#) Lagrangian, or do both [42].

The most straight-forward solution, often called the *minimally extended SM*, is to add the missing right-handed chiral neutrino fields, which would enable neutrino mass generation through Yukawa couplings with the Higgs field. These right-handed fields would however be singlets under all the [SM](#) gauge symmetries and would therefore not participate in any of the [SM](#) interactions. Neutrinos created by these fields are called *sterile* and could potentially mix with the *active* neutrinos via neutrino oscillations. There are however a few issues with the minimally extended [SM](#). Since the mass generation mechanism is the same as for the charged fermions, there is no theoretical explanation for the relative smallness of neutrino masses. There is also currently no experimental confirmation of oscillations between active and sterile neutrinos [52], although there are some possible indications [60, 61]. Additionally, having to add new fields by hand makes the [SM](#) an incorrect description of reality even at low energies. This is an issue, as it is generally believed that [SM](#) is at least a good low energy effective theory of a more complex general theory and only breaks down at some New Physics (NP) threshold value Λ_{NP} [62].

Adding new non-renormalizable terms to the [SM](#) Lagrangian which are suppressed by this NP scale as $1/\Lambda_{NP}$, would maintain the renormalizability (and validity) of the [SM](#) at energies well below Λ_{NP} [62]. It is possible to create such a term using only the existing [SM](#) fields and preserving the [SM](#) gauge symmetries, which after spontaneous symmetry breaking generates neutrino mass terms. Additionally, three of the newly generated masses are also suppressed as $1/\Lambda_{NP}$ and belong to mostly left-handed (active) fields, while the rest are very large ($\sim \Lambda_{NP}$) and belong to mostly sterile neutrinos, which are therefore also called Heavy Neutral Leptons (HNLs). This is called the see-saw mechanism [63] and provides a natural explanation for the smallness of neutrino masses. Furthermore, the heavy masses of [HNL](#) make them more likely to avoid experimental detection. However, neutrinos with masses produced by this mechanism are all Majorana particles [64]. This means that neutrinos are their

own antiparticles

$$\nu \equiv \nu^C, \tag{1.24}$$

where C denotes charge conjugation. This is possible for neutrinos, since they are electrically neutral, unlike the other fermions. It also means that neutrinos can effectively annihilate with each other, violating the total lepton number by two units. The antineutrinos that were described in the previous sections are still different particles to neutrinos, but in this case only differ by parity transformation.

A sure way of finding out whether neutrinos are Majorana particles or not is an observation of a neutrino-less double β decay [62]. This is currently a subject of an extensive experimental investigation without a concrete conclusion [52]. Neutrinos being Majorana particles does not affect neutrino oscillations, however, other measurement could probe the nature of neutrinos, or possible theories [BSM](#), such as the measurements of the possible neutrino magnetic moment [51].

Acronyms

2P2H two particle - two hole. [5](#), [6](#)

BSM Beyond Standard Model. [9](#), [11](#), [12](#)

CC Charged current. [1](#), [2](#), [4–6](#), [8](#), [9](#)

CEvNS Coherent Elastic ν -Nucleus Scattering. [6](#)

COH π Coherent π (production). [6](#)

CP Charge conjugation - Parity (symmetry). [1](#), [8](#), [10](#)

DIS Deep Inelastic Scattering. [5](#)

DUNE Deep Underground Neutrino Experiment. [10](#)

FSI Final State Interaction. [6](#)

HK Hyper-Kamiokande. [10](#)

HNL Heavy Neutral Lepton. [11](#)

LBL Long Baseline. [10](#)

MEC Meson Exchange Current. [6](#)

MSW Mikheyev-Smirnov-Wolfenstein. [9](#)

NC Neutral Current. [1–4](#), [6](#), [8](#), [9](#)

NOvA NuMI Off-axis ν_e Appearance (experiment). [10](#)

NP New Physics. [11](#)

PMNS Pontecorvo-Maki-Nakagawa-Sakata. [7](#), [8](#)

QE Quasi Elastic (interaction). [4–6](#)

RES Resonant baryon production. [5](#)

SK Super-Kamiokande. [9](#)

SM Standard Model. [1](#), [2](#), [9](#), [11](#)

SNO Sudbury Neutrino Observatory. [9](#)

T2K Tokai to Kamioka (experiment). [10](#)

Bibliography

- [1] Wolfgang Pauli. Pauli letter collection: letter to Lise Meitner. Typed copy. URL <http://cds.cern.ch/record/83282>.
- [2] L. M. Brown. The idea of the neutrino. *Physics Today*, 31(9):23–28, September 1978. doi:[10.1063/1.2995181](https://doi.org/10.1063/1.2995181). (Including translation of W. Pauli, Aufsdtze und Vortrdge u’ber Physik und Erkenntnistheorie, Braunschweig (1961)).
- [3] H. A. Bethe. Ionization power of a neutrino with magnetic moment. *Mathematical Proceedings of the Cambridge Philosophical Society*, 31(1):108–115, 1935. doi:[10.1017/S0305004100012998](https://doi.org/10.1017/S0305004100012998).
- [4] Enrico Fermi. Tentativo di una teoria dei raggi β . 11(1):1–19. ISSN 1827-6121. doi:[10.1007/BF02959820](https://doi.org/10.1007/BF02959820).
- [5] Fred L. Wilson. Fermi’s theory of beta decay. *American Journal of Physics*, 36(12):1150–1160, 1968. doi:[10.1119/1.1974382](https://doi.org/10.1119/1.1974382). (A complete English translation of E.Fermi, Zeitschrift fur Physik 88, 161 (1934)).
- [6] Sheldon L. Glashow. Partial-symmetries of weak interactions. 22(4):579–588. ISSN 0029-5582. doi:[10.1016/0029-5582\(61\)90469-2](https://doi.org/10.1016/0029-5582(61)90469-2). URL <https://www.sciencedirect.com/science/article/pii/0029558261904692>.
- [7] Steven Weinberg. A model of leptons. *Phys. Rev. Lett.*, 19:1264–1266, Nov 1967. doi:[10.1103/PhysRevLett.19.1264](https://doi.org/10.1103/PhysRevLett.19.1264).
- [8] Abdus Salam. *Weak and electromagnetic interactions*, pages 244–254. doi:[10.1142/9789812795915_0034](https://doi.org/10.1142/9789812795915_0034).
- [9] L. Landau. On the conservation laws for weak interactions. 3(1):127–131. ISSN 0029-5582. doi:[10.1016/0029-5582\(57\)90061-5](https://doi.org/10.1016/0029-5582(57)90061-5). URL <https://www.sciencedirect.com/science/article/pii/0029558257900615>.
- [10] T. D. Lee and C. N. Yang. Parity nonconservation and a two-component theory of the neutrino. *Phys. Rev.*, 105:1671–1675, Mar 1957. doi:[10.1103/PhysRev.105.1671](https://doi.org/10.1103/PhysRev.105.1671).

- [11] Abdus Salam. On parity conservation and neutrino mass. *Nuovo Cim.*, 5:299–301, 1957. doi:[10.1007/BF02812841](https://doi.org/10.1007/BF02812841).
- [12] Peter W. Higgs. Broken symmetries and the masses of gauge bosons. *Phys. Rev. Lett.*, 13:508–509, Oct 1964. doi:[10.1103/PhysRevLett.13.508](https://doi.org/10.1103/PhysRevLett.13.508).
- [13] F. Englert and R. Brout. Broken symmetry and the mass of gauge vector mesons. *Phys. Rev. Lett.*, 13:321–323, Aug 1964. doi:[10.1103/PhysRevLett.13.321](https://doi.org/10.1103/PhysRevLett.13.321).
- [14] G. S. Guralnik, C. R. Hagen, and T. W. B. Kibble. Global conservation laws and massless particles. *Phys. Rev. Lett.*, 13:585–587, Nov 1964. doi:[10.1103/PhysRevLett.13.585](https://doi.org/10.1103/PhysRevLett.13.585).
- [15] Steven Weinberg. A model of leptons. *Phys. Rev. Lett.*, 19:1264–1266, Nov 1967. doi:[10.1103/PhysRevLett.19.1264](https://doi.org/10.1103/PhysRevLett.19.1264).
- [16] Carlo Giunti and Chung W. Kim. *Fundamentals of Neutrino Physics and Astrophysics*. 2007. ISBN 978-0-19-850871-7.
- [17] F. Reines and C. L. Cowan. Detection of the free neutrino. *Phys. Rev.*, 92:830–831, Nov 1953. doi:[10.1103/PhysRev.92.830](https://doi.org/10.1103/PhysRev.92.830).
- [18] Cowan Jr. C.L., Reines F., Harrison F.B., Kruse H.W., and McGuire A.D. Detection of the free neutrino: A confirmation. *Science*, 124(3212):103–104, July 1956. doi:[10.1126/science.124.3212.103](https://doi.org/10.1126/science.124.3212.103).
- [19] F. Reines and C.L. Cowan. Neutrino physics. *Physics Today*, 10(8):12–18, 1957. doi:[10.1063/1.3060455](https://doi.org/10.1063/1.3060455).
- [20] B. Adeva et al. Measurement of Z^0 decays to hadrons and a precise determination of the number of neutrino species. *Phys. Lett. B*, 237:136–146, 1990. doi:[10.1016/0370-2693\(90\)90476-M](https://doi.org/10.1016/0370-2693(90)90476-M).
- [21] S. Schael et al. Precision electroweak measurements on the Z resonance. *Phys. Rept.*, 427:257–454, 2006. doi:[10.1016/j.physrep.2005.12.006](https://doi.org/10.1016/j.physrep.2005.12.006).
- [22] M. C. Goodman. Resource letter anp-1: Advances in neutrino physics. *American Journal of Physics*, 84:309–319, 2016. doi:[10.1119/1.4962228](https://doi.org/10.1119/1.4962228).

- [23] M. Schwartz. Feasibility of using high-energy neutrinos to study the weak interactions. *Phys. Rev. Lett.*, 4:306–307, Mar 1960. doi:[10.1103/PhysRevLett.4.306](https://doi.org/10.1103/PhysRevLett.4.306).
- [24] K. Kodama et al. Observation of tau neutrino interactions. *Phys. Lett. B*, 504: 218–224, 2001. doi:[10.1016/S0370-2693\(01\)00307-0](https://doi.org/10.1016/S0370-2693(01)00307-0).
- [25] K. Kodama et al. Final tau-neutrino results from the DONuT experiment. *Phys. Rev. D*, 78:052002, 2008. doi:[10.1103/PhysRevD.78.052002](https://doi.org/10.1103/PhysRevD.78.052002).
- [26] William J Marciano and Zohreh Parsa. Neutrino–electron scattering theory*. 29 (11):2629. doi:[10.1088/0954-3899/29/11/013](https://doi.org/10.1088/0954-3899/29/11/013).
- [27] J. A. Formaggio and G. P. Zeller. From ν_e to $\bar{\nu}_e$: Neutrino cross sections across energy scales. *Rev. Mod. Phys.*, 84:1307–1341, Sep 2012. doi:[10.1103/RevModPhys.84.1307](https://doi.org/10.1103/RevModPhys.84.1307).
- [28] *Fundamental Physics at the Intensity Frontier*, 5 2012. doi:[10.2172/1042577](https://doi.org/10.2172/1042577).
- [29] D. Casper. The Nuance neutrino physics simulation, and the future. *Nucl. Phys. B Proc. Suppl.*, 112:161–170, 2002. doi:[10.1016/S0920-5632\(02\)01756-5](https://doi.org/10.1016/S0920-5632(02)01756-5).
- [30] Jr. Davis, Raymond, Don S. Harmer, and Kenneth C. Hoffman. Search for neutrinos from the sun. *Phys. Rev. Lett.*, 20:1205–1209, 1968. doi:[10.1103/PhysRevLett.20.1205](https://doi.org/10.1103/PhysRevLett.20.1205).
- [31] G. Danby, J-M. Gaillard, K. Goulianos, L. M. Lederman, N. Mistry, M. Schwartz, and J. Steinberger. Observation of high-energy neutrino reactions and the existence of two kinds of neutrinos. *Phys. Rev. Lett.*, 9:36–44, Jul 1962. doi:[10.1103/PhysRevLett.9.36](https://doi.org/10.1103/PhysRevLett.9.36).
- [32] A. A. Aguilar-Arevalo et al. First measurement of the muon neutrino charged current quasielastic double differential cross section. *Phys. Rev. D*, 81:092005, May 2010. doi:[10.1103/PhysRevD.81.092005](https://doi.org/10.1103/PhysRevD.81.092005).
- [33] M. Sajjad Athar, A. Fatima, and S. K. Singh. Neutrinos and their interactions with matter. *Prog. Part. Nucl. Phys.*, 129:104019, 2023. doi:[10.1016/j.pnpnp.2022.104019](https://doi.org/10.1016/j.pnpnp.2022.104019).
- [34] M. Martini, M. Ericson, G. Chanfray, and J. Marteau. Unified approach for nucleon knock-out and coherent and incoherent pion production in

- neutrino interactions with nuclei. *Phys. Rev. C*, 80:065501, Dec 2009.
doi:[10.1103/PhysRevC.80.065501](https://doi.org/10.1103/PhysRevC.80.065501).
- [35] M. Martini, M. Ericson, G. Chanfray, and J. Marteau. Neutrino and antineutrino quasielastic interactions with nuclei. *Phys. Rev. C*, 81:045502, Apr 2010.
doi:[10.1103/PhysRevC.81.045502](https://doi.org/10.1103/PhysRevC.81.045502).
- [36] M. Martini, M. Ericson, and G. Chanfray. Neutrino quasielastic interaction and nuclear dynamics. *Phys. Rev. C*, 84:055502, Nov 2011.
doi:[10.1103/PhysRevC.84.055502](https://doi.org/10.1103/PhysRevC.84.055502).
- [37] D. Akimov et al. Observation of coherent elastic neutrino-nucleus scattering. 357(6356):1123–1126. doi:[10.1126/science.aao0990](https://doi.org/10.1126/science.aao0990).
- [38] B Pontecorvo. Mesonium and antimesonium. *Sov. Phys. JETP*, 33:549–551, 8 1957.
- [39] B. Pontecorvo. Inverse beta processes and nonconservation of lepton charge. *Sov. Phys. JETP*, 7:172–173, 1958.
- [40] Ziro Maki, Masami Nakagawa, and Shoichi Sakata. Remarks on the unified model of elementary particles. *Prog. Theor. Phys.*, 28:870–880, 1962.
doi:[10.1143/PTP.28.870](https://doi.org/10.1143/PTP.28.870).
- [41] V. Gribov and B. Pontecorvo. Neutrino astronomy and lepton charge. 28(7): 493–496. ISSN 0370-2693. doi:[10.1016/0370-2693\(69\)90525-5](https://doi.org/10.1016/0370-2693(69)90525-5). URL <https://www.sciencedirect.com/science/article/pii/0370269369905255>.
- [42] M.C. Gonzalez-Garcia and Yosef Nir. Neutrino Masses and Mixing: Evidence and Implications. *Rev. Mod. Phys.*, 75:345–402, 2003.
doi:[10.1103/RevModPhys.75.345](https://doi.org/10.1103/RevModPhys.75.345).
- [43] L. Wolfenstein. Neutrino oscillations in matter. *Phys. Rev. D*, 17:2369–2374, May 1978. doi:[10.1103/PhysRevD.17.2369](https://doi.org/10.1103/PhysRevD.17.2369).
- [44] S.P. Mikheyev and A.Yu. Smirnov. Resonance Amplification of Oscillations in Matter and Spectroscopy of Solar Neutrinos. *Sov. J. Nucl. Phys.*, 42:913–917, 1985.

- [45] M. Aglietta et al. Experimental study of atmospheric neutrino flux in the NUSEX experiment. *Europhysics Letters (EPL)*, 8(7):611–614, apr 1989. doi:[10.1209/0295-5075/8/7/005](https://doi.org/10.1209/0295-5075/8/7/005).
- [46] K. Daum et al. Determination of the atmospheric neutrino spectra with the Fréjus detector. *Zeitschrift für Physik C Particles and Fields*, 66(3):417–428, 1995. ISSN 1431-5858. doi:[10.1007/BF01556368](https://doi.org/10.1007/BF01556368).
- [47] R. Becker-Szendy et al. Electron- and muon-neutrino content of the atmospheric flux. *Phys. Rev. D*, 46:3720–3724, Nov 1992. doi:[10.1103/PhysRevD.46.3720](https://doi.org/10.1103/PhysRevD.46.3720).
- [48] Y. Fukuda et al. Atmospheric muon-neutrino / electron-neutrino ratio in the multiGeV energy range. *Phys. Lett. B*, 335:237–245, 1994. doi:[10.1016/0370-2693\(94\)91420-6](https://doi.org/10.1016/0370-2693(94)91420-6).
- [49] Y. Fukuda et al. Evidence for oscillation of atmospheric neutrinos. *Phys. Rev. Lett.*, 81:1562–1567, 1998. doi:[10.1103/PhysRevLett.81.1562](https://doi.org/10.1103/PhysRevLett.81.1562).
- [50] Q.R. Ahmad et al. Direct evidence for neutrino flavor transformation from neutral current interactions in the Sudbury Neutrino Observatory. *Phys. Rev. Lett.*, 89:011301, 2002. doi:[10.1103/PhysRevLett.89.011301](https://doi.org/10.1103/PhysRevLett.89.011301).
- [51] Patrick Huber et al. Snowmass Neutrino Frontier Report. In *Snowmass 2021*, 11 2022.
- [52] R. L. Workman and Others. Review of Particle Physics. *PTEP*, 2022:083C01, 2022. doi:[10.1093/ptep/ptac097](https://doi.org/10.1093/ptep/ptac097).
- [53] Ivan Esteban, M. C. Gonzalez-Garcia, Michele Maltoni, Thomas Schwetz, and Albert Zhou. The fate of hints: updated global analysis of three-flavor neutrino oscillations. *JHEP*, 09:178, 2020. doi:[10.1007/JHEP09\(2020\)178](https://doi.org/10.1007/JHEP09(2020)178).
- [54] M. A. Acero et al. Improved measurement of neutrino oscillation parameters by the NOvA experiment. *Phys. Rev. D*, 106(3):032004, 2022. doi:[10.1103/PhysRevD.106.032004](https://doi.org/10.1103/PhysRevD.106.032004).
- [55] K. Abe et al. Measurements of neutrino oscillation parameters from the T2K experiment using 3.6×10^{21} protons on target. *Eur. Phys. J. C*, 83(9):782, 2023. doi:[10.1140/epjc/s10052-023-11819-x](https://doi.org/10.1140/epjc/s10052-023-11819-x).

- [56] B. Abi et al. Volume I. Introduction to DUNE. 15(08):T08008. doi:[10.1088/1748-0221/15/08/T08008](https://doi.org/10.1088/1748-0221/15/08/T08008).
- [57] K. Abe et al. Hyper-Kamiokande Design Report. 5 2018.
- [58] M. Aker et al. Direct neutrino-mass measurement with sub-electronvolt sensitivity. *Nature Phys.*, 18(2):160–166, 2022. doi:[10.1038/s41567-021-01463-1](https://doi.org/10.1038/s41567-021-01463-1).
- [59] Aghanim, N. et al. Planck 2018 results - vi. cosmological parameters. 641:A6. doi:[10.1051/0004-6361/201833910](https://doi.org/10.1051/0004-6361/201833910).
- [60] A. Aguilar et al. Evidence for neutrino oscillations from the observation of $\bar{\nu}_e$ appearance in a $\bar{\nu}_\mu$ beam. *Phys. Rev. D*, 64:112007, Nov 2001. doi:[10.1103/PhysRevD.64.112007](https://doi.org/10.1103/PhysRevD.64.112007). URL <https://link.aps.org/doi/10.1103/PhysRevD.64.112007>.
- [61] A.A. Aguilar-Arevalo et al. Significant Excess of ElectronLike Events in the Mini-BooNE Short-Baseline Neutrino Experiment. *Phys. Rev. Lett.*, 121(22):221801, 2018. doi:[10.1103/PhysRevLett.121.221801](https://doi.org/10.1103/PhysRevLett.121.221801).
- [62] M.C. Gonzalez-Garcia and Michele Maltoni. Phenomenology with Massive Neutrinos. *Phys. Rept.*, 460:1–129, 2008. doi:[10.1016/j.physrep.2007.12.004](https://doi.org/10.1016/j.physrep.2007.12.004).
- [63] Rabindra N. Mohapatra and Goran Senjanovic. Neutrino Mass and Spontaneous Parity Nonconservation. *Phys. Rev. Lett.*, 44:912, 1980. doi:[10.1103/PhysRevLett.44.912](https://doi.org/10.1103/PhysRevLett.44.912).
- [64] Ettore Majorana. Teoria simmetrica dell’elettrone e del positrone. *Nuovo Cim.*, 14:171–184, 1937. doi:[10.1007/BF02961314](https://doi.org/10.1007/BF02961314). Translated by Luciano Maiani in “Soryushiron Kenkyu”, 63 (1981) 149-462.