Scientific Computing Lab 1

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Problem 1:

In this problem we have recursion $x_{n+1} = \dfrac{x_n(x_n^2 + 3a)}{3x_n^2 + a}$

Which can be rewritten in the following form

$$x_{n+1} - \sqrt{a} = \frac{x_n(x_n^2 + 3a) - \sqrt{a}(3x_n^2 + a)}{3x_n^2 + a}$$

$$\implies x_{n+1} - \sqrt{a} = \frac{(x_n - \sqrt{a})^3}{3x_n^2 + a}$$

So, theoretically we should get $\lim_{n\to\infty} \dfrac{x_{n+1} - \sqrt{a}}{(x_n - \sqrt{a})^3} = \dfrac{1}{4a}$, that is order of convergence is 3.

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Problem 2:

For n = 100000 we have root = 2.0287542875428755;

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Problem 3:

$$f(x) = \frac{x}{2} - sin(x)$$

Coded with bisection method:

**root = 1.8960002538266523 in 7 iterations with tolerance = 0.001**
**root = 1.8954969163806275 in 15 iterations with tolerance = 1e-05**
**root = 1.8954942948314295 in 21 iterations with tolerance = 1e-07**

For Newton's method we have $f'(x) = \dfrac{1}{2} - cos(x)$

Thus, $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)} = x_n - \dfrac{x_n - 2sin(x_n)}{1 - 2cos(x_n)} = \dfrac{2(sin(x_n) - cos(x_n))}{1 - 2cos(x_n)}$

Coded with Newton's method:

**root = 1.8954942672087132 in 3 iterations with tolerance = 0.001**
**root = 1.895494267033981 in 7 iterations with tolerance = 1e-05**
**root = 1.895494267033981 in 11 iterations with tolerance = 1e-07**

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Problem 4:

We now formulate $f(x) = \dfrac{x}{2} - sin(x) = 0$ in the form of $x = g(x)$

That is, $x = 2sin(x)$

Hence recursion becomes $x_{n+1} = 2sin(x_n)$

Coded with Fixed Point Iteration:

```
root = 1.8951742279274637 in 13 iterations with tolerance = 0.001
Order of Convergence in this case = 1.002413376758368
root = 1.8954906894347723 in 36 iterations with tolerance = 1e-05
Order of Convergence in this case = 1.0000269944883338
root = 1.8954942925583511 in 70 iterations with tolerance = 1e-07
Order of Convergence in this case = 0.9999998066195256
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Problem 5:

Secant method: $x_{n+1} = x_n - f(x_n)\dfrac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})}$

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For the given function:
root = -0.5791589060508088 in 6 iterations!
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Problem 6:

$$\text{Order of Convergence} = \lim_{n\to\infty} \frac{\log|(x_{n+1}-r)/(x_n-r)|}{\log|(x_n-r)/(x_{n-1}-r)|}$$

```
Bisection Method:
root = -0.5791015625 in 78 iterations with tolerance = 0.001
approximation of root = -0.5791015625
Order of Convergence of 2nd part = 0.8644352421324487
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Problem 7:

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Bisection Method:
root = 1.8954250110311952 in 12 iterations with tolerance = 0.0001
approximation of root = 1.8954250110311952
Order of Convergence of 2nd part = 0.9156364708429194
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Problem 8:

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Bisection Method:
root = -0.5791015625 in 118 iterations with tolerance = 0.001
root = -0.5791587829589844 in 126 iterations with tolerance = 1e-05
root = -0.5791589021682739 in 131 iterations with tolerance = 1e-07
approximation of root = -0.5791589021682739
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 Let us iterate with x0 = 0:
Order of Convergence of 2nd part = 2.0066544803768083
```