Scientific Computing 04

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Problem 1:

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Output:
part (a) :
Midpoint approximate = 0.6232355214961439 for number of subintervals = 100
Midpoint approximate = 0.6232253429486613 for number of subintervals = 1000
Midpoint approximate = 0.6232252401505126 for number of subintervals = 100000
Trapezoid approximate = 0.6232046778724095 for number of subintervals = 100
Trapezoid approximate = 0.6232250345234129 for number of subintervals = 1000
Trapezoid approximate = 0.6232252401196731 for number of subintervals = 100000
Simpson approximate = 0.6232252425096133 for number of subintervals = 100
Simpson approximate = 0.6232252401404669 for number of subintervals = 1000
Simpson approximate = 0.6232252401402306 for number of subintervals = 100000
part (b) :
Midpoint approximate = 1.0471975511965976 for number of subintervals = 100
Midpoint approximate = 1.0471975511965974 for number of subintervals = 1000
Midpoint approximate = 1.0471975511965976 for number of subintervals = 100000
Trapezoid approximate = 1.0471975511965979 for number of subintervals = 100
Trapezoid approximate = 1.0471975511965987 for number of subintervals = 1000
Trapezoid approximate = 1.0471975511965965 for number of subintervals = 100000
Simpson approximate = 1.0471975511965976 for number of subintervals = 100
Simpson approximate = 1.0471975511965983 for number of subintervals = 1000
Simpson approximate = 1.0471975511965936 for number of subintervals = 100000
part (c) :
Midpoint approximate = 0.7468271984923199 for number of subintervals = 100
Midpoint approximate = 0.746824163469049 for number of subintervals = 1000
Midpoint approximate = 0.7468241328154887 for number of subintervals = 100000
Trapezoid approximate = 0.7468180014679696 for number of subintervals = 100
Trapezoid approximate = 0.7468240714991843 for number of subintervals = 1000
Trapezoid approximate = 0.7468241328062869 for number of subintervals = 100000
Simpson approximate = 0.7468241328941757 for number of subintervals = 100
Simpson approximate = 0.746824132812436 for number of subintervals = 1000
Simpson approximate = 0.7468241328124278 for number of subintervals = 100000
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Problem 2:

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Output:
Trapezoid approximate = 1.5
Simpson approximate = 1.6666666666666667
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Problem 3:

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Accurate value = 0.5235987755982988
Trapezoid approximate = 0.5235654422650429 with relative error of 1e-4
Simpson approximate = 0.5235654422761937 with relative error of 1e-4
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Problem 4:

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accurate value = 1.0986122886681098
Trapezoid approximate = 1.0919752503144537
Simpson approximate = 1.0986122939305363
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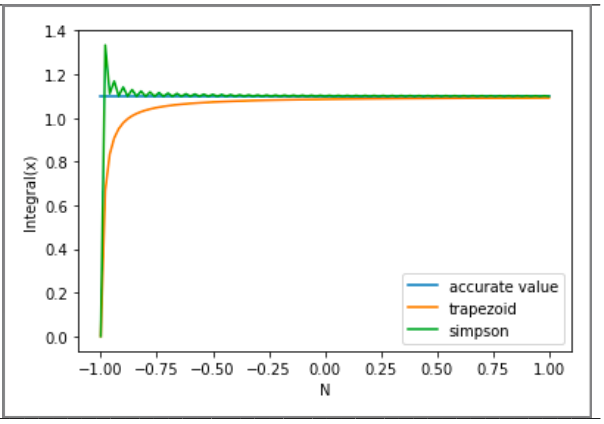

Problem 5:

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Exact value = 0.4054651081081644
 Below are n values for various approximator
midpoint: n=25, h = 0.08 and value = 0.4054558497848948
trapezoid: n=35, h = 0.05714285714285714 and value = 0.40547455605341015
simpson: n=4, h = 0.5 and value = 0.40547138047138054
```

Problem 6:

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part (a):
Accurate value = 1.151292546497023
h/2 :  T(h/2)  :  |T(h)-T(h/2)|/(|T(h/2)|)
1.5 : 0.9173076923076922 : 0.5094339622641509
0.375 : 1.0970043616177638 : 0.16380670451033677
0.1875 : 1.1384585664015612 : 0.03641257223337145
0.09375 : 1.1481180339673183 : 0.008413305322257526
0.046875 : 1.1505008862257964 : 0.002071143348959109
0.0234375 : 1.151094752428111 : 0.0005159142642792098
0.01171875 : 1.1512431055122723 : 0.00012886338554467683
0.005859375 : 1.1512801867211504 : 3.2208674574465474e-05
0.0029296875 : 1.151289456582443 : 8.051720824481838e-06
0.00146484375 : 1.151291774020214 : 2.0129022227990604e-06
0.000732421875 : 1.1512923533779378 : 5.032238093962171e-07
Number of function evaluation = 2049
approximate value = 1.1512923533779378
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part (b):
Accurate value = 2.995732273553991
h/2 :  T(h/2)  :  |T(h)-T(h/2)|/(|T(h/2)|)
0.475 : 5.8922619047619005 : 0.6928982725527828
0.11875 : 4.083693318738755 : 0.4428757119747965
0.059375 : 3.3570758478478937 : 0.2164435669085852
0.0296875 : 3.1017719811934454 : 0.0823090376089531
0.01484375 : 3.024133502616016 : 0.025672966656488105
0.007421875 : 3.0029962433063964 : 0.007038723194121185
0.0037109375 : 2.997559819984561 : 0.001813616290687857
0.00185546875 : 2.996189909008545 : 0.000457217672316779
0.000927734375 : 2.995846729673321 : 0.00011455169979987864
0.0004638671875 : 2.9957608905445365 : 2.8653531413505992e-05
0.00023193359375 : 2.9957394279867833 : 7.164360675949552e-06
0.000115966796875 : 2.9957340621737636 : 1.7911513199516074e-06
5.79833984375e-05 : 2.9957327207096607 : 4.477916516507605e-07
Number of function evaluation = 8193
approximate value = 2.9957327207096607
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part (c):
h/2 :  T(h/2)  :  |T(h)-T(h/2)|/(|T(h/2)|)
0.7853981633974483 : 1.8549591310856286 : 0.022189042422364393
0.19634954084936207 : 1.8540752277673078 : 0.00047673541239487963
0.09817477042468103 : 1.8540746773016663 : 2.96895075615034e-07
Number of function evaluation = 9
approximate value = 1.8540746773016663
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h/2 :  T(h/2)  :  |T(h)-T(h/2)|/(|T(h/2)|)
0.7853981633974483 : 2.2847455920722175 : 0.112422256104431
0.19634954084936207 : 2.2576215269727475 : 0.012014442977003641
0.09817477042468103 : 2.2572054614626076 : 0.0001843277084179799
0.04908738521234052 : 2.257205326820873 : 5.964975053466718e-08
Number of function evaluation = 17
approximate value = 2.257205326820873
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h/2 :  T(h/2)  :  |T(h)-T(h/2)|/(|T(h/2)|)
0.7853981633974483 : 3.2328552103215404 : 0.3294147895013055
0.19634954084936207 : 2.94266734632302 : 0.09861388660227652
0.09817477042468103 : 2.9089732768591396 : 0.01158280474142427
0.04908738521234052 : 2.908337561384487 : 0.0002185837995883022
0.02454369260617026 : 2.9083372484446572 : 1.0760094269421378e-07
Number of function evaluation = 33
approximate value = 2.9083372484446572
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Problem 7:

part(a):

$$| \int_a^b f(x)dx - T(h) | = | \int_a^b f(x)dx - \int_a^b \hat{f}(x)dx + \int_a^b \hat{f}(x)dx - T(h) | =$$

$$\leq | \int_a^b f(x)dx - \int_a^b \hat{f}(x)dx | + | \int_a^b \hat{f}(x)dx - T(h) |$$

$$\leq | \int_a^b \delta(x)dx | + | \int_a^b \hat{f}(x)dx - T(h) |$$

$$\leq (b-a)\delta + | \int_a^b \hat{f}(x)dx - T(h) |$$

$$\leq (b-a)\delta + | \frac{(b-a)^3}{12N^2} \hat{f}''(\xi) |$$

$$\leq (b-a)(\delta + | \frac{(b-a)^2}{12N^2} \hat{f}''(\xi) |)$$

$$\leq (b-a)(\delta + | \frac{h^2}{12} \hat{f}''(\xi) |)$$

$$\leq (b-a)(\delta + \frac{h^2}{12} k)$$

Where $k =$ infinite_norm of f''

part(b):

For this part we will choose h such that $\delta + \frac{h^2}{12}k$ is minimum possible. But since $\delta = 0.01$ in this,
It will be reasonable to use $\frac{h^2}{12}k < 0.001$ as error will be no more than 0.0011.

Thus to achieve this $N \geq 23$ and $h \leq 0.0434$

output:

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part (b):
The reasonable step size would be h = 0.043478260869565216
accurate value = 0.25
trapezoidal value for inexact function = 0.25549734904693117
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