Scientific Computing 06

180123035 - Rahul Krishna

Problem 1:

```
part (a) :
 approx  solution F(0.0) = 1.0
 approx  solution F(0.5) = 1.1839397205857212
 approx  solution F(1.0) = 1.436252215343503
part (b) :
 approx  solution F(1.0) = 2.0
 approx  solution F(1.5) = 2.3333333333333335
 approx  solution F(2.0) = 2.7083333333333335
part (c) :
 approx  solution F(2.0) = 2.0
 approx  solution F(2.25) = 2.2071067811865475
 approx  solution F(2.5) = 2.4909989078432044
 approx  solution F(2.75) = 2.854680348463929
 approx  solution F(3.0) = 3.3025964649736848
part (d) :
 approx  solution F(1.0) = 2.0
 approx  solution F(1.25) = 1.2273243567064205
 approx  solution F(1.5) = 0.8321501570804853
 approx  solution F(1.75) = 0.570446772282531
 approx  solution F(2.0) = 0.37882661467612466
```

Problem 2:

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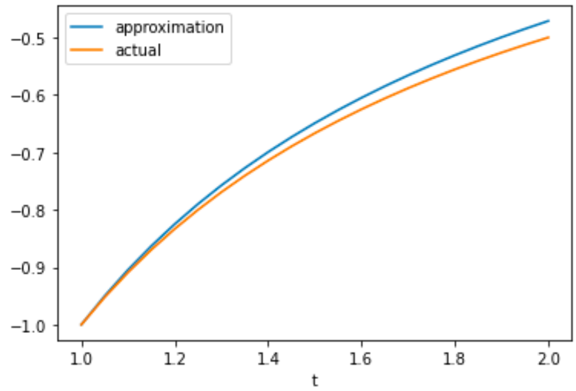
part (a) :
 approx  solution F(0.0) = 1.0, actual = 1.0 and abs-error=0.0
 approx  solution F(0.5) = 1.1839397205857212, actual = 1.2140230606297089 and abs-
error=0.030083340043987716
 approx  solution F(1.0) = 1.436252215343503, actual = 1.4898801256447498 and abs-
error=0.05362791030124692
part (b) :
 approx  solution F(1.0) = 2.0, actual = 2.0 and abs-error=0.0
 approx  solution F(1.5) = 2.3333333333333335, actual = 2.3541019662496847 and abs-
error=0.020768632916351226
 approx  solution F(2.0) = 2.7083333333333335, actual = 2.7416573867739413 and abs-
error=0.03332405344060785
part (c) :
 approx  solution F(2.0) = 2.0, actual = 2.0000000000000004 and abs-
error=4.440892098500626e-16
 approx  solution F(2.25) = 2.2071067811865475, actual = 2.244121110336502 and abs-
error=0.037014329149954506
 approx  solution F(2.5) = 2.4909989078432044, actual = 2.5644519492316347 and abs-
error=0.07345304138843023
 approx  solution F(2.75) = 2.854680348463929, actual = 2.965193834491754 and abs-
error=0.11051348602782518
 approx  solution F(3.0) = 3.3025964649736848, actual = 3.451286652264298 and abs-
error=0.14869018729061345
part (d) :
 approx  solution F(1.0) = 2.0, actual = 2.0 and abs-error=0.0
 approx  solution F(1.25) = 1.2273243567064205, actual = 1.4031989692799332 and abs-
error=0.17587461257351267
 approx  solution F(1.5) = 0.8321501570804853, actual = 1.0164101466785118 and abs-
error=0.18425998959802647
 approx  solution F(1.75) = 0.570446772282531, actual = 0.7380097715499844 and abs-
error=0.16756299926745333
 approx  solution F(2.0) = 0.37882661467612466, actual = 0.5296870980395587 and abs-
error=0.15086048336343405
```

Problem 3:
Part (a)

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part (b)
interpolate y(1.052) = -0.9481814172335601 and actual = -0.9505703422053231 :: abs-error
= -0.002388924971763018
interpolate y(1.555) = -0.6242100522864794 and actual = -0.6430868167202572 :: abs-error
= -0.018876764433777837
interpolate y(1.978) = -0.47730178235981846 and actual = -0.5055611729019212 :: abs-error
= -0.02825939054210269
```

Problem 4:

Part I:

$$e_n = y(x_n) - y_n = y(x_n) - (y_{n-1} + hf(x_{n-1}, y_{n-1}))$$

$$\implies e_n = y(x_n) - y(x_{n-1}) + e_{n-1} - hf(x_{n-1}, y_{n-1})$$

Now using Taylor's Theorem for y around x_{n-1},

$$\implies e_n = e_{n-1} + hy'(x_{n-1}) + \frac{h^2}{2}y''(\tau) - hf(x_{n-1}, y_{n-1}) \text{ , where } \tau \in (x_{n-1}, x_n) \quad \text{..... (eq 1)}$$

Please note that $f(x_{n-1}, y_{n-1})$ may not be same as $f(x_{n-1}, y(x_{n-1})) = y'(x_{n-1})$
Now, let us write the Taylor expansion for f(x,y) in terms of y around y_{n-1}.

$$f(x_{n-1}, y(x_{n-1})) = f(x_{n-1}, y_{n-1}) + fy(x_{n-1}, \eta)[y(x_{n-1}) - y_{n-1}] = f(x_{n-1}, y_{n-1}) + f_y(x_{n-1}, \eta)e_{n-1}$$
$$\text{, where } \eta \in (x_{n-1}, x_n)$$

Thus, now put this as $f(x_{n-1}, y(x_{n-1})) = y'(x_{n-1})$, in eq 1

To get,

$$\implies e_n = [1 + f_y(x_{n-1}, \eta)]e_{n-1} + \frac{h^2}{2}y''(\tau)$$

Now, if $-2 \le f_y(x_{n-1}, \eta) \le 0$, Then we can say $|e_n| \le |e_{n-1}| + \frac{h^2}{2}|y''(\tau)|$

Part II:

Now if we assume, $|e_n| \le |e_{n-1}| + \frac{h^2}{2}|y''(\xi_{n-1})|$, where $\xi_{n-1} \in (x_{n-1}, x_n)$
Adding all the inequalities starting from n=1;
We get

$$|e_n| \le |e_{n-1}| + \frac{h^2}{2}|y''(\xi_{n-1})| \le |e_0| + \frac{h^2}{2}\sum_{k=0}^{n-1}|y''(\xi_k)| \le |e_0| + \frac{nh^2}{2}max_{0\le k\le n-1}|y''(\xi_k)|$$

Problem 5:

There is difference because we can see,
$f_y(x, y) = \lambda = -20 < -2$

And, this can be the reason for this behaviour.
Code part is in .ipynb file.