Problem Set 2

Time Series Econometrics

CEU Spring 2022

Please upload your work to Moodle by May 17, 8pm. Please work on your own; you can ask me for help if you get stuck. Each problem is worth 20 points.

1. Let Y be a scalar random variable and let $X = (X_1, X_2, ...)$ be a random vector (which may even be infinite dimensional). Let $\hat{E}(Y \mid X) = \alpha_0^* + \alpha_1^* X_1 + \alpha_2^* X_2 + \ldots$ denote the linear projection of Y on X and a constant.

   a) Show that the linear projection coefficients solve the problem

   $$\min_{a_0, a_1, a_2, \ldots} E\Big\{\big[Y - (a_0 + a_1 X_1 + a_2 X_2 + \ldots)\big]^2\Big\}.$$

   Hint: Subtract and add $\hat{E}(Y|\,X)$ inside the parenthesis, expand the square appropriately and use the definition of linear projection.

   b) Suppose that X is a scalar and write down an explicit expression for α_0^* and α_1^*.

   c) Given an example in which $\hat{E}(Y|\,X) \neq E(Y|X)$

2. Let Y_t be a unit root process; specifically, suppose that $\Delta Y_t \sim AR(1)$, i.e., $\Delta Y_t = c + \phi \Delta Y_{t-1} + \epsilon_t$, where ϵ_t is w.n.$(0, \sigma^2)$ and $|\phi| < 1$. Information available at time t contains the history Y up to time t.

   a) What is the drift δ of the process?

   b) Derive $\hat{Y}_{t+h|t}$ $h \geq 1$. (As usual, $\hat{Y}_{t+h|t}$ denotes the h step ahead optimal linear forecast made at time t.) You can take the class notes as given – you don't need to rederive everything from scratch.

   c) Find $\lim_{h\to\infty}(\hat{Y}_{t+h|t} - Y_t - \delta h)$. Make a rough sketch of the forecast path $\hat{Y}_{t+h|t}$, $h = 1, 2, \ldots$ starting from the last data point. Assume $\delta > 0$.

3. Suppose an econometrician fits an ARMA(1,1) model to the process Y_t. The estimated model is given by

$$Y_t = 0.20Y_{t-1} + \epsilon_t + 0.10\epsilon_{t-1},$$

and the estimated variance of ϵ_t is 1.00. Another econometrician instead fits an AR(2) model using the same data and obtains

$$Y_t = 0.32Y_{t-1} - 0.03Y_{t-2} + \eta_t,$$

with the estimated error variance also very close to 1. They both use their own model to forecast Y_t several steps ahead (treating ϵ_t and η_t as white noise) and are surprised to see that the resulting point forecasts are very similar. Argue that there is nothing to be surprised about.

4. The data file `arma.txt` contains 500 observations from an ARMA(p, q) process ordered as a column vector (the first entry is the earliest observation). Identify the underlying model, i.e. find $p \geq 0$ and $q \geq 0$ that adequately describes the data. I am also willing to let you know that $p \leq 4$ and $q \leq 4$. You can work with any software package, but Eviews is particularly well suited for this problem.

   Use the first 475 observations for points a) and b):

   a) Graph the sample autocorrelation function along with the approximate 95% confidence bands. Graph the sample partial autocorrelation function along with the approximate 95% confidence bands. Make an educated guess about the values of p and q based on these graphs.

   b) Use the Bayesian information criterion (BIC) to find the values of p and q that fit the data best. You can summarize the candidate models in a table. Did your initial guess hold up? (Eviews calls BIC the "Scwhartz criterion".)

   Now consider the last 25 observations as well:

   c) Using the estimated model, forecast the process 25 periods into the future. Produce a graph with the point forecast along with the ± 2 standard error bounds as well as the realizations. Comment on the accuracy and qualitative properties of the forecast.

5. The file `SPXD.csv` contains daily observations on the S&P500 stock price index P_t from 1/4/1999 to 11/10/2008 (closing values). The following exercises are also easiest to do with Eviews.

   a) Plot $\log(P_t)$ and its sample ACF. Test for the presence of a unit root using an appropriate case of the (A)DF test.

   b) Compute the corresponding return series as $r_t = \log(P_t) - \log(P_{t-1})$. Plot r_t and its sample ACF. Test for the presence of unit roots in r_t using an appropriate case of the (A)DF test.