Problem Set 3

Time Series Econometrics
CEU Spring 2021

Please upload your work to Moodle by 11:59pm on Thursday, June 2. Please work on your own; you can ask me for help if you get stuck.

1. (35 pts) Suppose that the growth rate of GDP (y) and the money supply (m) are related as in the following VAR:

$$\begin{pmatrix} y_t \\ m_t \end{pmatrix} = \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix} + \begin{pmatrix} 0.1 & 0.4 \\ 0.3 & 0.2 \end{pmatrix} \begin{pmatrix} y_{t-1} \\ m_{t-1} \end{pmatrix} + \begin{pmatrix} \epsilon_{yt} \\ \epsilon_{mt} \end{pmatrix}, \tag{1}$$

where $(\epsilon_{yt}, \epsilon_{mt})'$ is vector white noise with $E(\epsilon_{yt}^2) = 1$, $E(\epsilon_{mt}^2) = 1.14$ and $E(\epsilon_{yt}\epsilon_{mt}) = 0.8$.

a) Is this VAR covariance stationary? If yes, calculate the mean vector and write down the VAR representation of the de-meaned process.

b) Invert the model and calculate the first two matrices in the $MA(\infty)$ representation (not counting the leading identity matrix).

c) What is the best linear forecast (linear projection) of ϵ_{yt} given ϵ_{mt}? (Hint: in problem 1b of the previous homework you were asked to derive the linear projection coefficients in this simple case.) Suppose in period $t-1$ both y_{t-1} and m_{t-1} were at their mean values, but in period t you learn that m_t is one unit over its mean. Find the value of ϵ_{mt}. What is the estimated value of y_t (denote it as $\hat{y}_t$)?

d) Calculate $\partial y_{t+2}/\partial \epsilon_{mt}$ and $\partial y_{t+2}/\partial \epsilon_{yt}$. Someone claims that the optimal linear forecast of y_{t+2} given the value of ϵ_{mt} is $\hat{y}_t + (\partial y_{t+2}/\partial \epsilon_{mt})\epsilon_{mt}$. Explain why this claim is false and find the optimal forecast.

Suppose someone puts forth a piece of economic theory that implies that y_t and m_t are determined simultaneously by the following structural model:

$$\begin{pmatrix} 1 & b \\ a & 1 \end{pmatrix} \begin{pmatrix} y_t \\ m_t \end{pmatrix} = c + A \begin{pmatrix} y_{t-1} \\ m_{t-1} \end{pmatrix} + \begin{pmatrix} u_{yt} \\ u_{mt} \end{pmatrix}, \tag{2}$$

where $(u_{yt}, u_{mt})'$ is vector white noise with a diagonal (but otherwise unknown) variance-covariance matrix.

e) Based on additional theory, someone proposes the restriction $b = 0$. Describe verbally the meaning of this condition.

f) Regarding the VAR in (1) as the reduced form of (2), express the reduced form shocks $\epsilon_{mt}, \epsilon_{yt}$ in terms of the structural shocks u_{mt}, u_{yt}. Show that the structural model is identified, i.e. the parameters of the structural VAR can be (uniquely) recovered given the reduced form parameters. In particular, calculate $\text{var}(u_{mt})$, $\text{var}(u_{yt})$ and a.

g) Find the structural impulse responses $\partial y_{t+2}/\partial u_{mt}$ and $\partial y_{t+2}/\partial u_{yt}$.

2. (15 pts) The Eviews workfile `price-dividend-data.wf1` contains monthly observations on the S&P stock price index and the corresponding dividends. The sample period is January 1960 to June 2016.[1] To ensure stationarity both stock prices (sp) and dividends (div) have been transformed to yields by taking logs and first differencing. Real prices and dividends are included as well (the real variables are measured in terms of constant November, 2016 dollars).

   a) Test whether dividends Granger cause prices (use the nominal as well as the real variables). Interpret your results in light of some simple economic theory. To implement the test, estimate bivariate VAR(p) models for $p = 2, 4, 6, 8, 10, 12$. Report all test results in order to check their robustness, but also indicate which choice of p is favored by BIC (known in Eviews as the Schwartz criterion).

   b) Now test whether prices Granger cause dividends (in nominal or real terms). Interpret your results in light of the simple economic theory discussed in class.

3. (15 pts) This problem consists of two unrelated ARMA questions.

[1] The data source is Robert Shiller's web site: http://www.econ.yale.edu/ shiller/data.htm; see the U.S. stock markets link. Feel free to update the data set with the last few years if you feel like (optional).

a) Do you think the AR(2) process

$$X_t = 0.9X_{t-1} + 0.7X_{t-2} + \epsilon_t?$$

could be a realistic model of a macroeconomic variable?

b) Suppose that $Y_t \sim ARMA(1,1)$, i.e. $Y_t = c + \phi Y_{t-1} + \theta\epsilon_{t-1} + \epsilon_t$ for some $|\phi| < 1$, $|\theta| < 1$, and white noise process ϵ_t. Consider the following simple procedure for estimating the model:

(i) Regress Y_t on a constant and $Y_{t-1}, \ldots, Y_{t-K}$ for some large K, but $K << T$; denote the residuals from this regression by e_t.

(ii) Regress Y_t on a constant, Y_{t-1} and e_{t-1}.

Explain the rationale behind this procedure.

4. (20 pts) For $T = 400$, generate two independent iid $N(0,1)$ sequences ϵ_t and η_t, $t = 1, \ldots, T$. Construct two random walk processes $X_t = \sum_{s=1}^{t} \epsilon_s$ and $Y_t = \sum_{s=1}^{t} \eta_s$ with $X_0 = 0$, $Y_0 = 0$. Clearly, these time series are statistically unrelated and are, in particular, not cointegrated.

a) Run a regression of Y_t on X_t and a constant. Report the slope coefficient, the associated t-statistic, the R-squared statistic, the estimated ACF of the residuals, and a unit root test result for the residuals. (Even though it's strictly speaking not appropriate for residuals, you can use the basic ADF test window in Eviews.)

b) Run a regression of Y_t on Y_{t-1}, X_t and a constant. Report the same statistics as in part a).

c) Run a regression of ΔY_t on ΔX_t and a constant. Report the same statistics as in part a).

d) Discuss the results. Based on your findings, what are your general suggestions when modeling the relationships between I(1) time series?

Hints: If you do this in Eviews, you can use the "series e=nrnd" command to generate a time series e with iid $N(0,1)$ observations over the sample period. Then you can set the sample period to $t = 1$ ("smpl 1 1" if you work with integer dates) and put "series y=0". This creates a time series object y and sets the first observation to zero. Next set the sample range to $t = 2, \ldots, 401$ (smpl 2 401) and put y=y(-1)+e to generate a random walk.

5. (15 pts) The file `stock_index.xls` contains weekly observations on two stock indices from Jan. 1998 to Apr. 2008. The two indices are the S&P Europe 350, and the "classic" (U.S.) S&P 500. Both indices are expressed in U.S. dollars. Let $Y_t = \log(\text{S\&P Europe 350})$ and $Z_t = \log(\text{S\&P 500})$.

   a) Use an appropriate case of the ADF test to check Y_t and Z_t for unit roots.

   b) Run a simple linear regression of, say, Y_t on Z_t, and report the regression output. Do you think this regression is spurious or are there (economic) arguments for Y_t and Z_t to be cointegrated?

   c) Explain how you would statistically distinguish between the two possibilities in part b). Conduct a formal test for cointegration between Y_t and Z_t using the Engle-Granger method built into Eviews. State your conclusions based on the output.