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EIGEN VALUES AND EIGEN VECTORS

A $_{nxn}$ is a square matrix then we are going to have 'n' eigen values and 'n' eigen vectors.

$X1^1 = X^Te1 \implies AX = \lambda X$

'X' is called Eigen vector of 'A', λ is called Eigen value of 'A'

So, For what Matrix I am multiplying with Matrix A. I am getting the same X that X is called Eigen vector of 'A'.

Can we write the equation as $AX - \lambda X = 0$

$\implies (A - \lambda) X = 0$

$\implies |A - \lambda| X = 0$

$\implies |A - \lambda| = 0$ --> Characteristic equation.

Ex: Find the eigen values and eigen vectors of A = $\begin{bmatrix} 1 & -5 \\ -5 & 1 \end{bmatrix}_{2\times 2}$

We know $|A - \lambda I| = 0$

$$\Rightarrow \begin{bmatrix} 1 & -5 \\ -5 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0 \Rightarrow \begin{bmatrix} 1-\lambda & -5 \\ -5 & 1-\lambda \end{bmatrix} = 0$$

$$\Rightarrow (1-\lambda)^2 - 5^2 = 0 \Rightarrow (1-\lambda-5)(1-\lambda+5) = 0$$

$$\Rightarrow \lambda = -4, \quad \lambda = 6$$

We have now λ has two values.

$$Ax = \lambda x$$

$$\Rightarrow \begin{bmatrix} 1 & -5 \\ -5 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 6 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \Rightarrow \begin{bmatrix} x_1 - 5x_2 \\ -5x_1 + x_2 \end{bmatrix} = \begin{bmatrix} 6x_1 \\ 6x_2 \end{bmatrix}$$

$\Rightarrow x_1 - 5x_2 = 6x_1$; $-5x_1 + x_2 = 6x_2$

$\Rightarrow -5x_1 = 5x_2$; $\Rightarrow -5x_1 = 5x_2$

$\Rightarrow x_1 = -x_2$; $\Rightarrow x_1 = -x_2$

we know our 'x' vector is $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \Rightarrow x = \begin{bmatrix} x_1 \\ -x_1 \end{bmatrix}$

Similarly if I put $\lambda = 4 \Rightarrow x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \Rightarrow x = \begin{bmatrix} x_1 \\ x_1 \end{bmatrix}$

$\hookrightarrow$ we can see $x_1 = x_2$

If I take x_1 outside $\Rightarrow x = x_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix}$; $x = x_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Convert these in to unit vector $= e_1$

$$\Rightarrow \text{we know } x = \begin{bmatrix} x_1 \\ -x_1 \end{bmatrix} = \begin{bmatrix} \dfrac{x_1}{\sqrt{x_1^2 + (-x_1)^2}} \\ \dfrac{-x_1}{\sqrt{x_1^2 + (-x_1)^2}} \end{bmatrix} = \begin{bmatrix} \dfrac{x_1}{x_1\sqrt{2}} \\ \dfrac{-x_1}{x_1\sqrt{2}} \end{bmatrix}$$

$$e_1 = \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix} \Rightarrow \text{Similarly for } e_2 = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

$\Rightarrow$ These e_1 & e_2 are eigen vectors