Internal Resistance of a Power Source

Arthur Li, Reeshav Kumar

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Abstract

The purpose of this experiment was to calculate the internal/output resistance of a power source with different voltages and currents using the *terminal voltage* formula. Different load resistances and setups in the circuit were used to calculate the internal resistance. The main result obtained from this experiment was that the internal resistance obtained for the cell battery is very small ($5.7 \times 10^{-4} \pm 2 \times 10^{-4}\Omega$) and the internal resistance of the DC power supply is effectively zero. The resistance of the ammeter used was around $1.8 \times 10^{-3} \pm 2 \times 10^{-4}\Omega$. The current that passes through the voltmeter when the cell battery is connected to the circuit is 0.008 ± 0.003 mA.

Introduction

The goal of this experiment was to calculate the Output/Internal resistance of a power source with different Voltage and Current values using the Terminal Voltage formula (1) :

$$V = V_{\infty} - R_{Int}I \tag{1}$$

Where V is the terminal voltage, V_{∞} is the open circuit voltage, R_{Int} is the internal resistance and I is the current.

The initial graph of V should follow a linear pattern beginning at V_{∞}, and then at higher currents, the behaviour becomes non linear. The can R_{Int} value can be calculated by observing the voltage output in relation to the Current input.

Besides the curve fitting algorithm provided by python, The formula used for the χ^2_R metric for analysing the curve fit in relation to the data. A perfect fit is indicated by χ^2_R value of 1, an over-fit model or lack of data is indicated by χ^2_R value <1, and a poor model is indicated by χ^2_R value > 1

$${\chi_r}^2 = \frac{1}{N-m}\sum_{i}^{N}\frac{(y_i - f(x_i))^2}{\sigma_i^2} \tag{2}$$

The curve fit function used in this experiment is scipy.optimize.curve_fit package used in the Python programming language. The resistance for the voltmeter was calculated using the following formula.

$$I = v(\frac{1}{R_{load}} + \frac{1}{R_v}) \tag{3}$$

Methods and Procedure

The first part of the experiment used a battery (DC), which had an open circuit voltage measured at 6.53 Volts.

Two Keysight 1272A multimeters were used, one functioned as the ammeter, the other served as the voltmeter. The other essential equipment used in this experiment is the UofT - Dept. of Physics LCR breakout box which was used to control the external load resistance R_{Load}.

Resistors of 220, 470, 2.7×10^3, 27×10^3, and 100×10^3 Ohms were each used to measure the terminal voltage and Current. The smaller value resistor was avoided because it got really hot when plugged into the circuit and the risk of blowing out the resistor was higher. The data taking was repeated for the two setups of the circuit. The voltage was measured in Volts and Current measured in mili-Amps.

In the second part, a Keysight E36103B DC power supply was substituted for the battery pack. The voltage of this supplier was dialed to 6.5, 10, 15, and 20 Volts and the experimental procedure was repeated from part 1 for each of the voltages, and the data recorded. Setting the voltage to 6.5 volts the circuit was connected to the potentiometer and the maximum current for the DC power supply was recorded.

The plots had x-axis of current, and y- axis Voltage. The internal resistance is determined by curve fitting the data. Both data and its respective curve fit were plotted onto the same graph. The χ^2_R value was calculated for each fit, and every error associated with the model parameters were solved for with the curve-fit Python function.

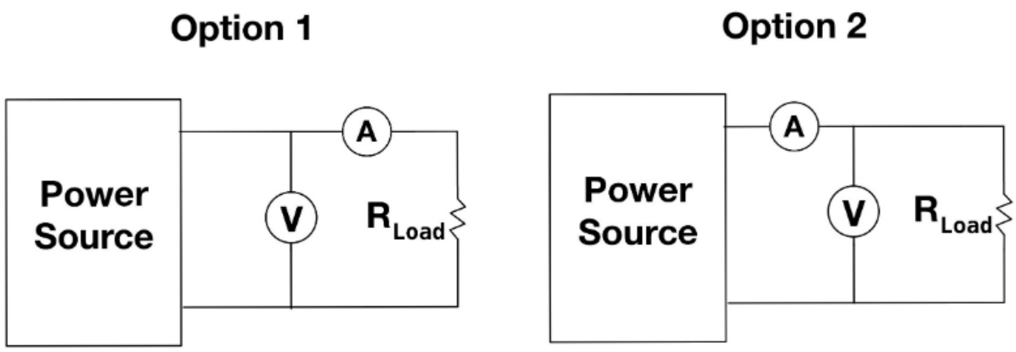

Figure 1: Circuit setup options

Result and Analysis

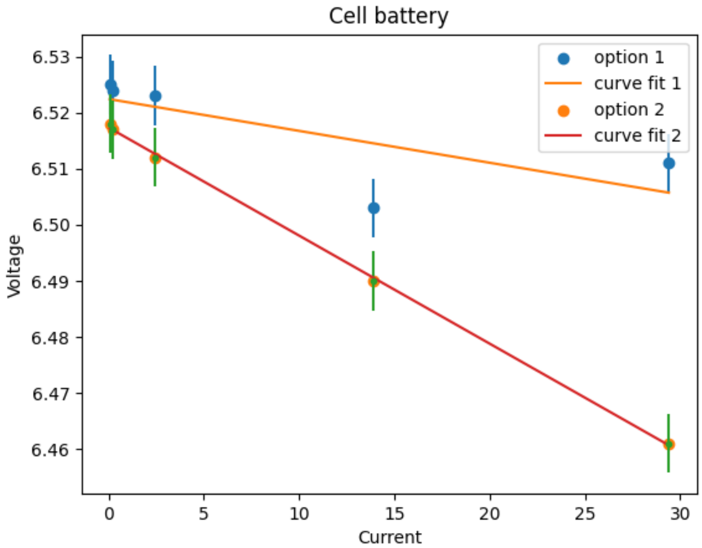

Figure 2: Terminal Voltage and Current measurements of the Battery fitted with Terminal Voltage formula (Equation 1) curve fit line. Current measured (x-axis) in mA and Voltage(y-axis) in Volts

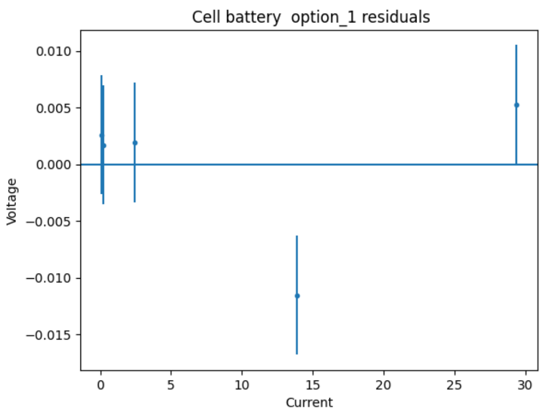

Figure 3: Residual graph of circuit with battery, Setup 1 Figure 1

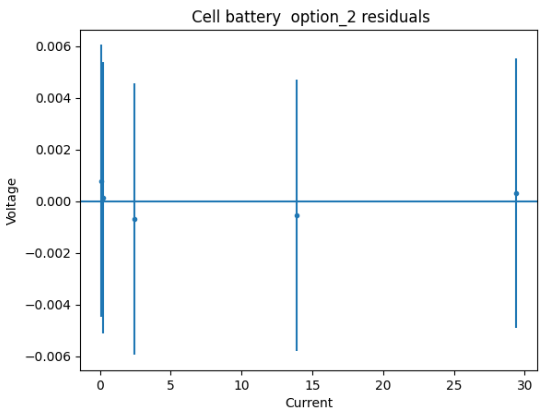

Figure 4: Residual graph of circuit with battery, Setup 1 Figure 1

Figure 2 shows the I-V graph for the part one experiment with the battery using the measured data, vertical errors plotted with the data, and curve fit of the data using the Terminal Voltage formula Equation 1. The appears to be an outlier data at I = 13.88 mA to the otherwise linear curve fit. The curve fit function gives us the slope of the graph to be $(5.7 \times 10^{-4} \pm 2 \times 10^{-4} \Omega)$ for option 1 setupFigure 1 , and $(1.8 \times 10^{-3} \pm 2 \times 10^{-4} \Omega)$ for option 2 setup Figure 1 , which is the resistance. Option 1 setup gives a better estimate for the internal resistance of the battery. In an ideal circuit the current flowing through the Voltmeter is zero i.e the voltmeter has infinite resistance and the resistance of the ammeter should be zero. It is much easier to simulate infinite resistance than zero resistance. In option 1 voltmeter measures the voltage drop only across the battery so it gives a better voltage reading but option 2 gives better current reading of the circuit. If going by the assumption that the voltmeter has very high resistance than the difference in ammeter reading in option 1 and option 2 is very small so choosing to get a better voltage reading from option 1 will give a better estimate of the internal resistance of the battery.

Setup 2 calculates the resistance of ammeter and the cell battery. Subtracting the internal resistance of the battery from setup 2 calculation we get the resistance of the ammeter to be 1.23 $m\Omega$. Looking at the raw data it can be observed that the current values for option 2 are slightly higher than option 1 measurement for the same resistor. This difference is the amount of current flowing through the voltmeter. Taking the difference and averaging the values give the current through the voltmeter to be 0.008 ± 0.003 mA, which is very small compared to the calculated current values.

Plotting the residuals for both setups, we see that the first setup Figure 1 has a data whose calculated fit line does not fall within the uncertainty. The second setup Figure 1 has all the calculated values fall within the uncertainty of the data. The χ_r^2 value of the setup 1 Figure 1 fit is (2.109). For setup 2, χ_r^2 = (0.019) For setup one the value is high because of the outlier data point pulling the curve down. The biggest source of uncertainty was the multimeter specifications provided by the manufacturer that led to small χ_r^2 value for setup 2.

Figure 6, Figure 9, Figure 12 and Figure 15 show the curve fitted data for DC power supply for 6.5V, 10V, 15V and 20V respectively. The internal resistance for the dc power supply was calculated using the same method as above.

For 6.5V input the resistance calculated for option 1 was $(3.9 \times 10^{-5} \pm 2 \times 10^{-4}\Omega)$ which is the internal resistance and the value for option 2 is $(1.8 \times 10^{-3} \pm 2 \times 10^{-4}\Omega)$. This gives the resistance of the ammeter to be $1.7 \pm 0.2 m\Omega$. This suggests that the internal resistance of the power supply is extremely low and can be treated as effectively zero. Similar results were obtained for 10, 15 and 20 V with the internal resistance of the DC power supply to be extremely low.

The results were plotted against the voltage reading as follows:

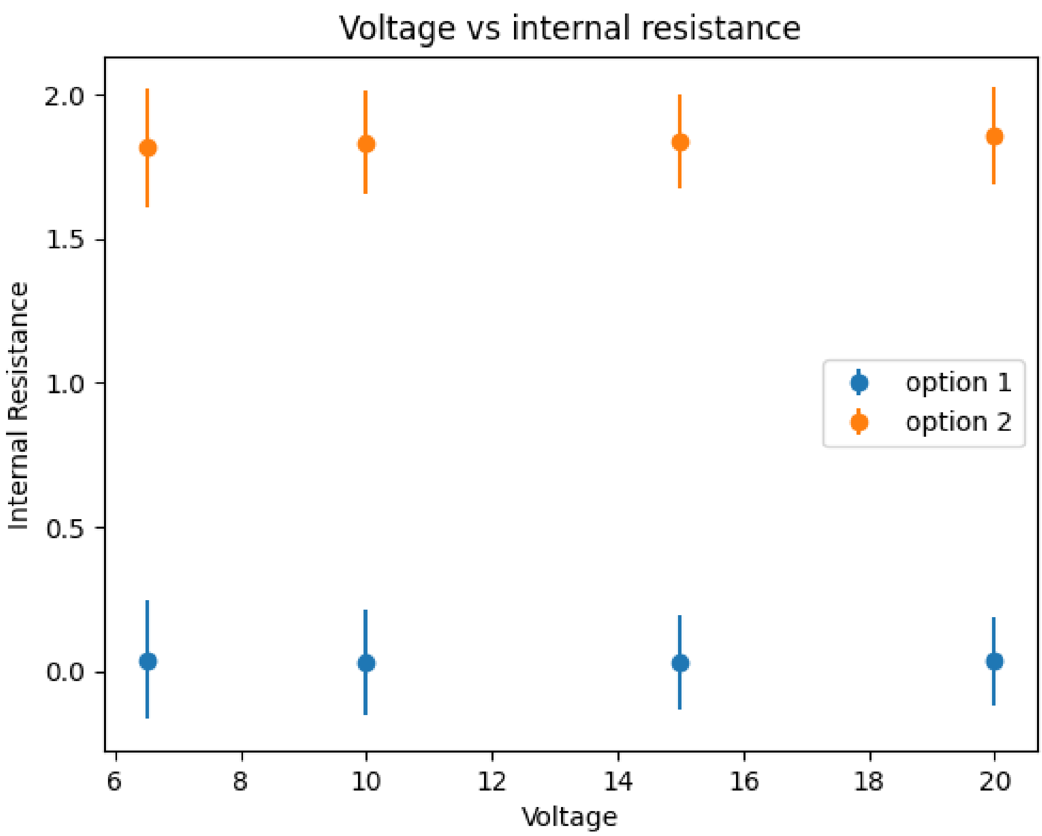

Figure 5: Graph of Voltage (x-axis) and internal resistance values (y-axis) for the DC generator. Voltage is measured in volts and resistance in $m\Omega$

From the graph it can be inferred that the internal resistance of the dc cell battery calculated using option 1 is effectively zero. The resistance value obtained in option 2 is the combined resistance of the dc power supply and the ammeter but since the effective internal resistance is zero the option 2 value is the resistance of the ammeter. The plot also shows that the resistance is constant across different voltages. The maximum current that the DC power supply could output before displaying the OCP tripped error was 0.26 A. This was calculated by connecting the power supply to a potentiometer and turning down the resistance till the power supply tripped. Looking at the residual plots for the DC power supply plots (See: Appendix) it can be observed that the values for all the plots are extremely close to the fit line and these give very small χ_r^2 value because of the high multimeter uncertainty.

Conclusion

In conclusion, the internal resistance for the battery in setup 1 Figure 1 is $(5.7 \times 10^{-4} \pm 2 \times 10^{-4}\Omega)$, for setup 2 it is calculated at $(1.8 \times 10^{-3} \pm 2 \times 10^{-4}\Omega)$.

The internal resistance for the DC generator at 6.5 Voltage output in setup 1 is $(3.9 \times 10^{-5} \pm 2 \times 10^{-4}\Omega)$, with setup two it is $(1.8 \times 10^{-3} \pm 2 \times 10^{-4}\Omega)$.

At 10 and 15 Voltage output in setup 1 the resistance is $(3.0 \times 10^{-5} \pm 2 \times 10^{-4}\Omega)$, with setup 2: $(1.8 \times 10^{-3} \pm 2 \times 10^{-4}\Omega)$.

At 20 Voltage output in setup 1 the resistance is $(3.3 \times 10^{-5} \pm 2 \times 10^{-4}\Omega)$, with setup 2: $(1.9 \times 10^{-3} \pm 2 \times 10^{-4}\Omega)$. The DC power supply has resistance so small that it can be treated as effectively zero and the resistance for the Ammeter is is also very small $1.8 \times 10^{-3} \pm 2 \times 10^{-4}\Omega$. The current that passes through the voltmeter when the cell battery is connected to the circuit is 0.008 ± 0.003 mA. Overall of the two setups option 1 gives a better estimate of the internal resistance of the battery.

References

[1] https://q.utoronto.ca/courses/337705/assignments/1213452?module_item_id=5344998
2023, February 19.

Appendix

Each row of data in the tables (top to bottom) represents the data taken at Resistances 220, 470, 2.7×10^3, 27×10^3, and 100×10^3 Ohms respectively.

Voltage (V)	Current (mA)	Voltage error	Current error
6.511	29.402	0.03275	0.063804
6.503	13.875	0.0052515	0.03275
6.523	2.405	0.0052615	0.00981
6.524	0.241	0.005262	0.005482
6.525	0.06	0.0052625	0.00512

Table 1: Cell battery option 1 data

Voltage (V)	Current (mA)	Voltage error	Current error
6.461	29.413	0.0052305	0.063826
6.49	13.905	0.005245	0.03281
6.512	2.404	0.005256	0.009808
6.517	0.242	0.0052585	0.005484
6.518	0.061	0.005259	0.005122

Table 2: Cell battery option 2 data

Voltage (V)	Current (mA)	Voltage error	Current error
6.499	29.393	0.0052495	0.063786
6.499	13.872	0.0052495	0.032744
6.5	2.398	0.00525	0.009796
6.5	0.241	0.00525	0.005482
6.5	0.061	0.00525	0.005122

Table 3: 6.5V DC power source option 1 data

Voltage (V)	Current (mA)	Voltage error	Current error
6.447	29.357	0.0052235	0.063714
6.475	13.874	0.0052375	0.032748
6.496	2.399	0.005248	0.009798
6.5	0.243	0.00525	0.005486
6.5	0.062	0.00525	0.005124

Table 4: 6.5V DC power source option 2 data

Voltage (V)	Current (mA)	Voltage error	Current error
9.998	45.14	0.006999	0.09528
9.999	21.338	0.0069995	0.047676
9.999	3.689	0.0069995	0.012378
9.999	0.372	0.0069995	0.005744
10	0.094	0.007	0.005188

Table 5: 10V DC power source option 1 data

Voltage (V)	Current (mA)	Voltage error	Current error
9.917	45.15	0.0069585	0.0953
9.961	21.339	0.0069805	0.047678
9.993	3.689	0.0069965	0.012378
9.999	0.372	0.0069995	0.005744
10	0.094	0.007	0.005188

Table 6: 10V DC power source option 2 data

Voltage (V)	Current (mA)	Voltage error	Current error
4.997	67.7	0.0094985	0.1404
14.998	32.003	0.009499	0.069006
14.999	5.533	0.0094995	0.016066
14.999	0.558	0.0094995	0.006116
14.999	0.141	0.0094995	0.005282

Table 7: 15V DC power source option 1 data

Voltage (V)	Current (mA)	Voltage error	Current error
14.875	67.7	0.0094375	0.1404
14.941	32.004	0.0094705	0.069008
14.99	5.534	0.009495	0.016068
14.998	0.559	0.009499	0.006118
14.999	0.143	0.0094995	0.005286

Table 8: 15V DC power source option 2 data

Voltage (V)	Current (mA)	Voltage error	Current error
19.997	90.23	0.0119985	0.18546
19.999	42.66	0.0119995	0.09032
20	7.378	0.012	0.019756
20	0.745	0.012	0.00649
20	0.189	0.012	0.005378

Table 9: 20V DC power source option 1 data

Voltage (V)	Current (mA)	Voltage error	Current error
19.833	90.15	0.0119165	0.1853
19.922	42.66	0.011961	0.09032
19.987	7.379	0.0119935	0.019758
19.999	0.746	0.0119995	0.006492

Table 10: 20V DC power source option 2 data

DC Power Supply Graphs

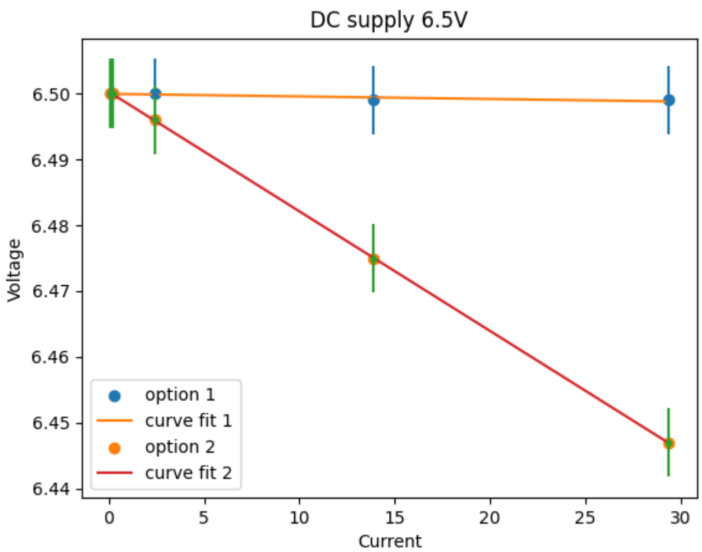

Figure 6: (Terminal Voltage and Current measurements of the DC power supply set at 6.5 Volts fitted with Terminal Voltage formula (Equation 1) curve fit line)

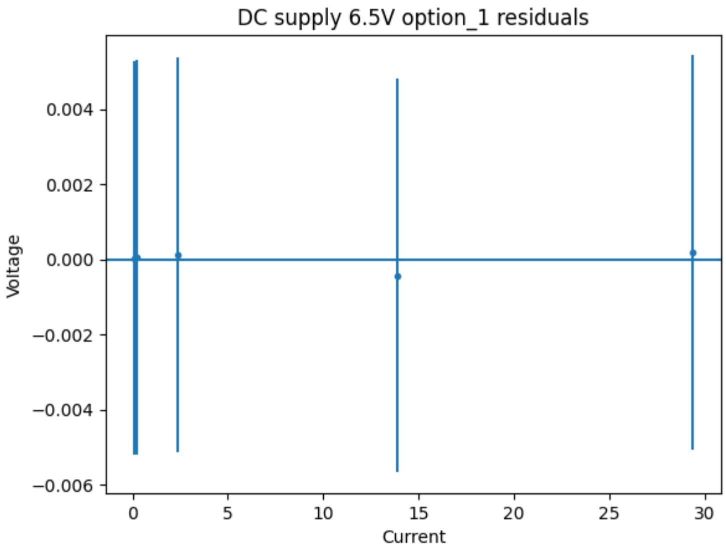

Figure 7: Residual graph of circuit with DC generator at 6.5 Volts, Setup 1 Figure 1

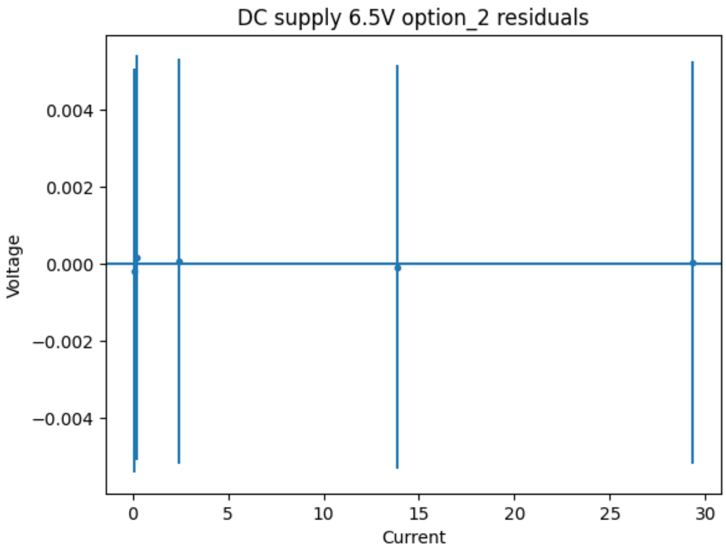

Figure 8: Residual graph of circuit with DC generator at 6.5 Volts, Setup 2 Figure 1

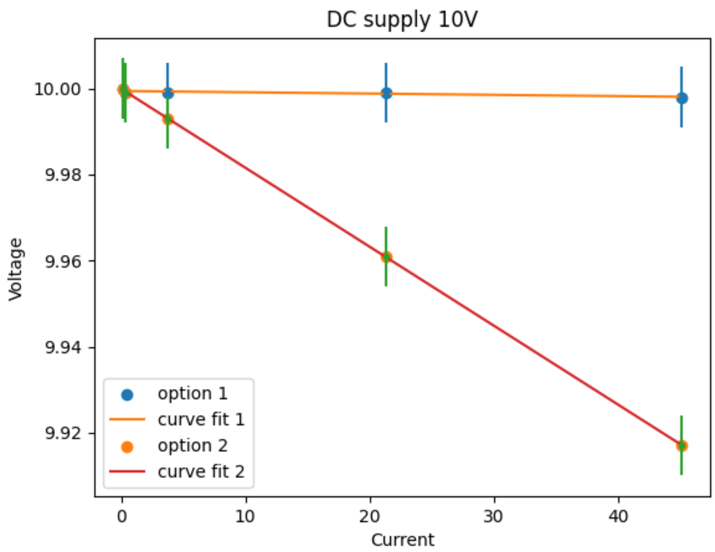

Figure 9: (Terminal Voltage and Current measurements of the DC power supply set at 10 Volts fitted with Terminal Voltage formula (Equation 1) curve fit line)

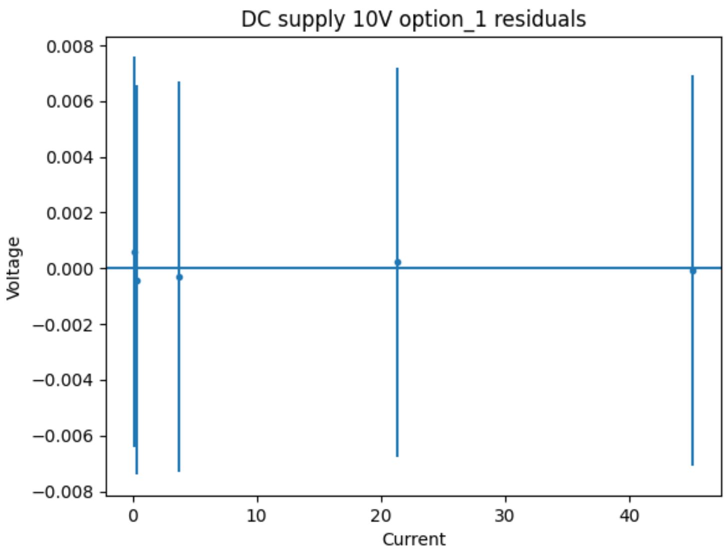

Figure 10: Residual graph of circuit with DC generator at 10 Volts, Setup 1 Figure 1

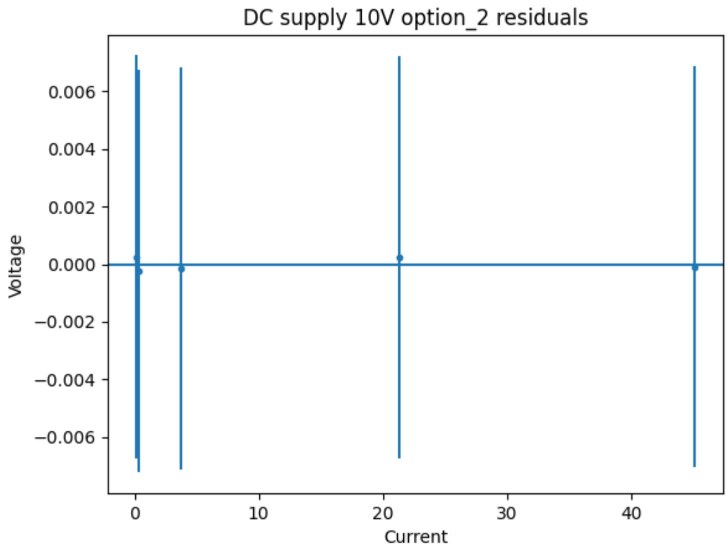

Figure 11: Residual graph of circuit with DC generator at 10 Volts, Setup 2 Figure 1

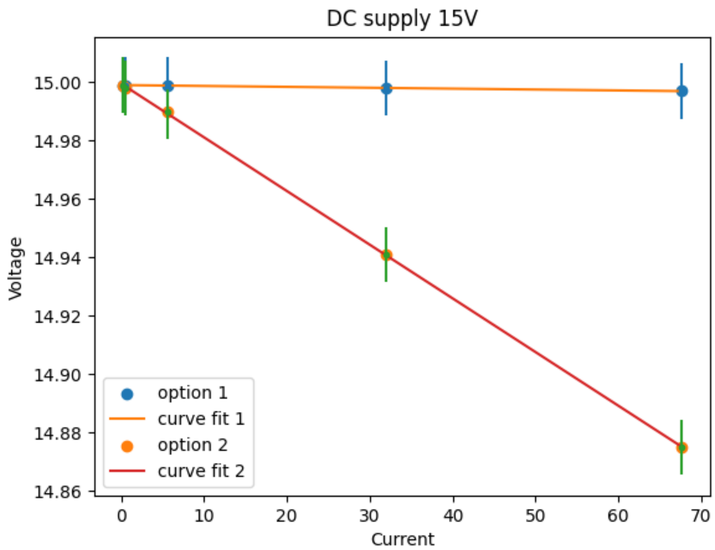

Figure 12: (Terminal Voltage and Current measurements of the DC power supply set at 15 Volts fitted with Terminal Voltage formula (Equation 1) curve fit line)

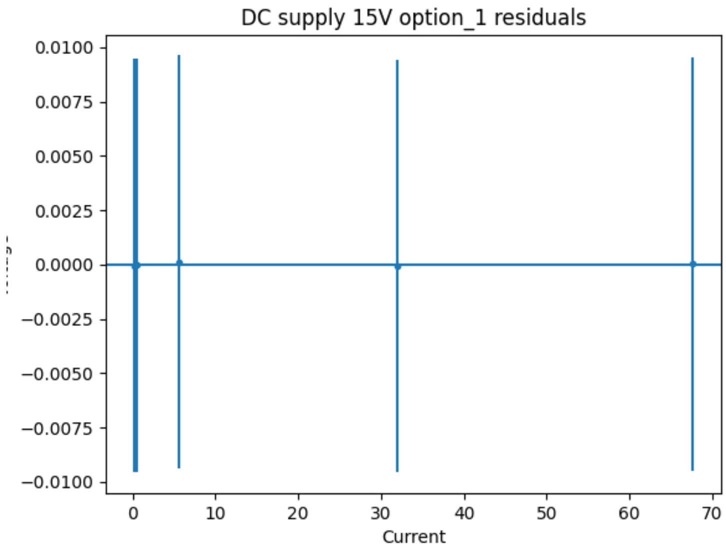

Figure 13: Residual graph of circuit with DC generator at 15 Volts, Setup 1 Figure 1

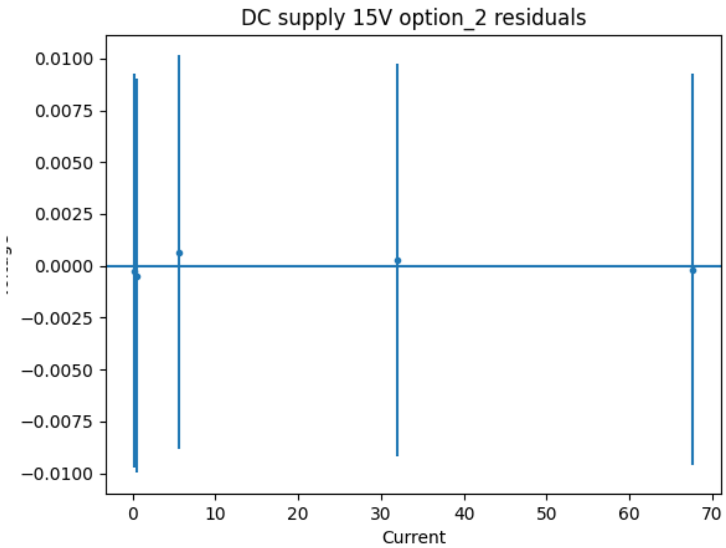

Figure 14: Residual graph of circuit with DC generator at 15 Volts, Setup 2 Figure 1

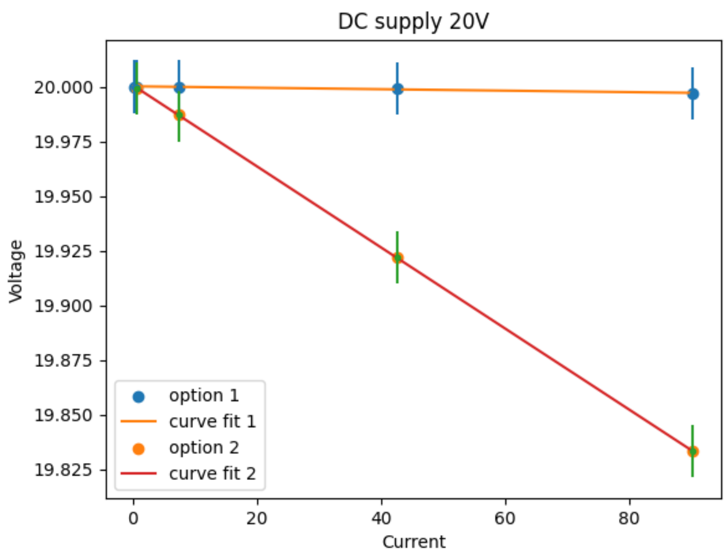

Figure 15: (Terminal Voltage and Current measurements of the DC power supply set at 20 Volts fitted with Terminal Voltage formula (Equation 1) curve fit line)

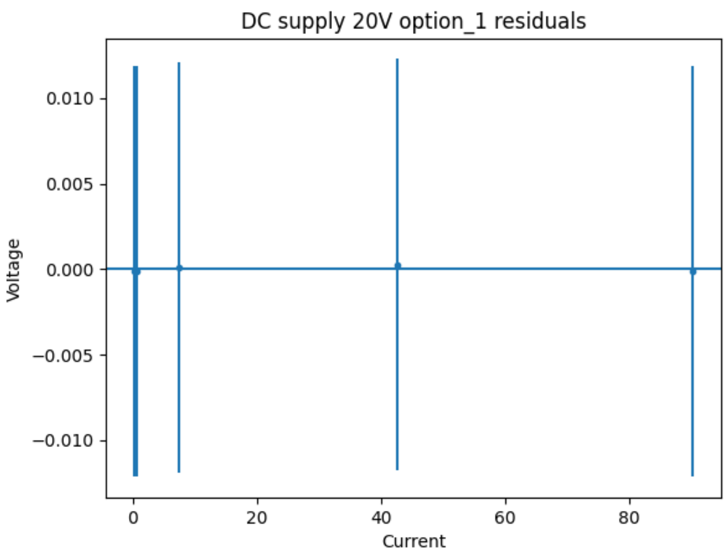

Figure 16: Residual graph of circuit with DC generator at 20 Volts, Setup 1 Figure 1

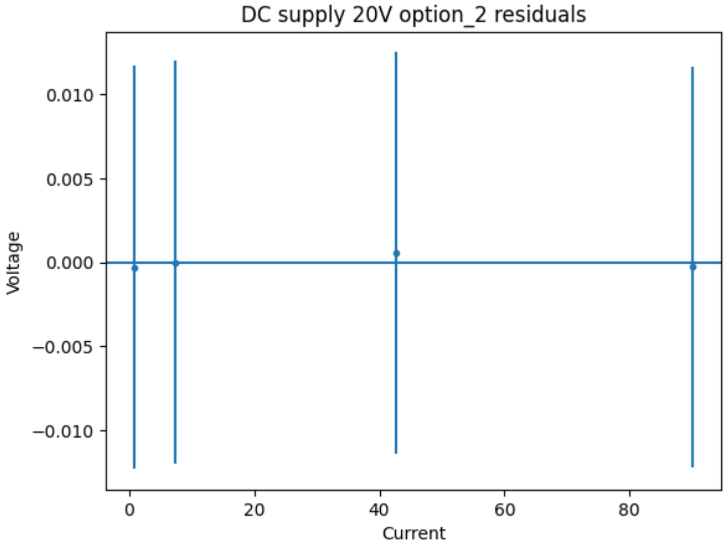

Figure 17: Residual graph of circuit with DC generator at 20 Volts, Setup 2 Figure 1

Equations used to calculate data uncertainties:

voltage uncertainty:

$$= (Measured\_Voltage * 0.05\%) + (0.001 * 2) \tag{4}$$

Current uncertainty:

$$= (Measured\_Current * 0.2\%) + (0.001 * 5) \tag{5}$$

DC specifications for U1271A, U1272A, U1273A and U1273AX

Function	Range	Resolution	Accuracy ± (% of reading + counts of least significant digit)			Test current / Burden voltage
			U1271A	U1272A	U1273A / U1273AX	
Voltage[1]	30 mV	0.001 mV	—	0.05 + 20	0.05 + 20	—
	300 mV	0.01 mV	0.05 + 5	0.05 + 5	0.05 + 5	—
	3 V	0.0001 V	0.05 + 5	0.05 + 5	0.05 + 5	—
	30 V	0.001 V	0.05 + 2	0.05 + 2	0.05 + 2	—
	300 V	0.01 V	0.05 + 2	0.05 + 2	0.05 + 2	—
	1000 V	0.1 V	0.05 + 2	0.05 + 2	0.05 + 2	—
	Z_{LOW} (low impedance) enabled, applicable for 1000 V range and resolution only	0.1 V	—	1 + 20	1 + 20	—
Resistance[2]	30 Ω	0.001 Ω	—	0.2 + 10	0.2 + 10	0.65 mA
	300 Ω	0.01 Ω	0.2 + 5	0.2 + 5	0.2 + 5	0.65 mA
	3 kΩ	0.0001 kΩ	0.2 + 5	0.2 + 5	0.2 + 5	65 µA
	30 kΩ	0.001 kΩ	0.2 + 5	0.2 + 5	0.2 + 5	6.5 µA
	300 kΩ	0.01 kΩ	0.2 + 5	0.2 + 5	0.2 + 5	0.65 µA
	3 MΩ	0.0001 MΩ	0.6 + 5	0.6 + 5	0.6 + 5	93 nA/10 MΩ
	30 MΩ	0.001 MΩ	1.2 + 5	1.2 + 5	1.2 + 5	93 nA/10 MΩ
	100 MΩ	0.01 MΩ	2.0 +10	—	—	93 nA/10 MΩ
	300 MΩ	0.01 MΩ	—	2.0 + 10 @ < 100 MΩ 8.0 + 10 @ > 100 MΩ	2.0 + 10 @ < 100 MΩ 8.0 + 10 @ > 100 MΩ	93 nA/10 MΩ
	300 nS	0.01 nS	1 + 10	1 + 10	1 + 10	93 nA/10 MΩ
Current[3]	300 µA	0.01 µA	0.2 + 5	0.2 + 5	0.2 + 5	< 0.04 V/100Ω
	3000 µA	0.1 µA	0.2 + 5	0.2 + 5	0.2 + 5	< 0.4 V/100 Ω
	30 mA	0.001 mA	0.2 + 5	0.2 + 5	0.2 + 5	< 0.08 V/1 Ω
	300 mA	0.01 mA	0.2 + 5	0.2 + 5	0.2 + 5	< 1.00 V/1 Ω
	3 A	0.0001 A	0.3 + 10	0.3 + 10	0.3 + 10	< 0.1 V/0.01 Ω
	10 A	0.001 A	0.3 + 10	0.3 + 10	0.3 + 10	< 0.3 V/0.01 Ω
Diode Test[4]	3 V	0.0001 V	0.5 + 5	0.5 + 5	0.5 + 5	Approximately 1 to 2 mA
	Auto	0.0001 V	—	0.5 + 5	0.5 + 5	Approximately 1 to 2 mA

Figure 18: Multi-meter specifications setup.