Ohm's and Power Law

Arthur Li, Reeshaav Kumar

January 8, 2025

Abstract

The purpose of this experiment was to confirm that Ohm's law is appropriate to model the Voltage - Current relationship of a circuit containing an ohmic resistor and that Power Law can be used to model the same relationship for a blackbody (lightbulb). The experiment was performed using two series circuits, one for the resistor and the other for the lightbulb. The voltage and current relationship data was analyzed using a python program. The main result obtained from part one of this experiment was that the resistor value obtained using the model ($476 \pm 15\ \Omega$) was within the error range for the 470 ohm resistor proving that Ohm's Law holds for ohmic resistor. The exponent value obtained in the second part of the experiment was 0.68 ± 0.01 which is higher than expected ideal value of 0.6 suggesting that the model used to calculate the value is not ideal. The graphs plotted for the data prove that Power Law holds for the blackbody.

Introduction

The objective of this experiment was to analyse the relationship between voltage (V), current (I) and resistance (R) with Ohm's Law and Power Law. The experiment was done in two parts; the first part examines the data of voltage and current taken from a simple circuit with a resistor in between the reading devices. We tested two different resistors. The second part was done with replacing the resistor with a light-bulb.

$$I = V/R \tag{1}$$

Ohm's law (1) tells that Current and Voltage have a linear relationship with resistance being the constant of proportionality[1].

$$I \propto V^d \tag{2}$$

When one replaces the resistor with a light-bulb, the Power law (2) is applicable due to the lightbulb being a blackbody, it converts electrical energy into light which radiates energy away, and decreases the current afterwards.The ideal black body will have d=3/5 [1] .

To graph the predictions and to make a best fit for the data requires two linear models (3) and the power model (5):

$$y = ax + b \tag{3}$$

$$y = ax \tag{4}$$

$$y = cx^d \tag{5}$$

a,b,c and d are proportionality contants that will shape the graph. We will attempt to curve fit the linear model in the second part as well with the power model, so we will take the logarithm of both x and y.

$$log(y) = alog(x) + b \tag{6}$$

$$log(y) = dlog(x) + log(c) \tag{7}$$

And finally, the criterion's by which one can judge the experiment data to compare to the Ohm's law formulas;

$$\sigma_a = \sqrt{Var(a)} \tag{8}$$

$$\chi_r{}^2 = \frac{1}{N-m}\sum_{i}^{N}\frac{(y_i - f(x_i))^2}{\sigma_i^2} \tag{9}$$

σ_a is the standard deviation of a, where a is a parameter of the models used in fitting the data.$var(a)$ is the variance of the parameter. χ_r^2 is the chi squared test, which is used to determine the correlation of data to the model.[1] The following equation was used to calculate uncertainty in resistance given slope and it's uncertainty of a V-I graph

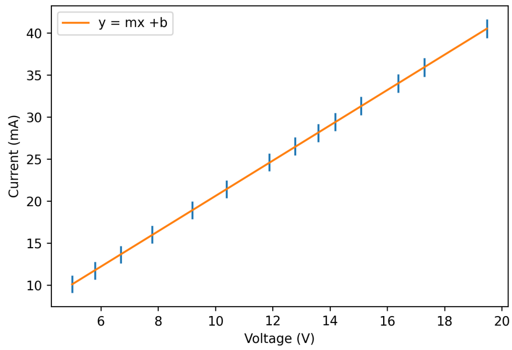
$$u(y) = \frac{u(x)}{x^2} \tag{10}$$

where y is the resistance value and x is the input value.

Methods and Procedures

For the first part of the experiment a 470 ohm resistor connected to a DC power supply was used. The voltage and current across the resistor was measured using two multi-meters, one of which acted as a Voltmeter and another as an Ammeter. The Keysight 1272A multi-meter, Keysight E36103B DC power supply and the UofT - Dept. of Physics LCR breakout box were used for this experiment.[1] The circuit was connected according to the circuit diagram in Figure 13. Fifteen voltage and current measurements were taken starting with the voltage of 5.00V from the power supply and gradually increasing it to 19.5 V the raw data with uncertainties for which is displayed in Table 1. Figure 1 and Figure 3 displays the data fitted to two different linear models. The uncertainty for each measurement was calculated using the given specifications for the multi-meter(See Appendix: Figure 14). Then the source of resistance was switched to be the potentiometer and measurements for voltages ranging from 3.96V - 14.5V were taken as shown in Table 2. Figure 5 and Figure 7 display this data curve fitted to the two linear models. For the second part of the experiment the same equipment was used but the resistor was switched out for a tungsten filament bulb. Measurements were taken for voltages ranging from 6.59V - 20.1V. The raw data for the lightbulb is displayed in Table 3. Figure 9 displays the data fitted to Log-Linear model and Power model The data was recorded to CSV files and plotted as V-I graphs for all the three data sets. Python's "matplotlib" library was used to plot the data and "scipy" library was used to optimize and curve fit the data to different models (i.e linear or exponential). The x-axis for all the plots generated is the Voltage and the y-axis displays current. The output parameters from the curve fit function were used to plot curves for the data. Residual plots were also made for each model and $\chi_r{}^2$ values associated with each fit was also calculated. The voltage reading was measured in Volts and Current in Amperes. When translating the data to python the current was converted from Amperes to mili-Amperes. From the models the value for the resistance was obtained in the first part of the experiment and values for the exponent and the proportionality constant were obtained in the second part. To plot a theoretical model the ideal exponent value of 3/5 was used. Uncertainty in the output resistance values was measured using Equation 10 with the y value being the slope of the graph and x value being the slope of the graph.

Results



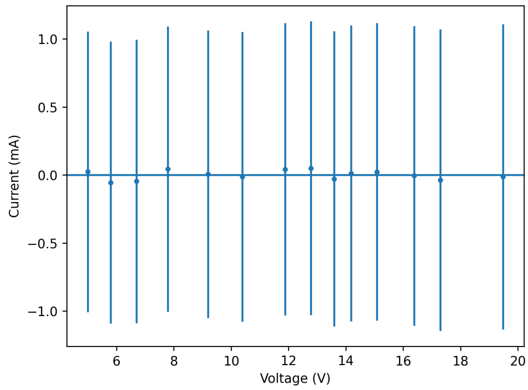
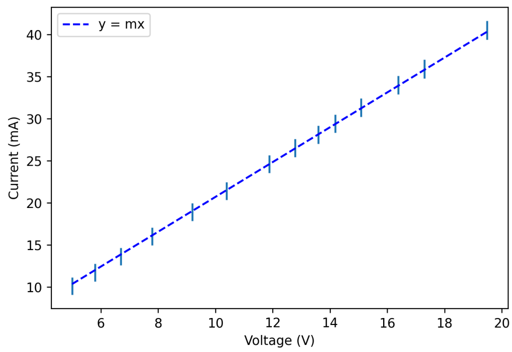


Figure 1: Resistor Measurements Fitted to linear model (Equation 3)

Figure 2: Residual plot for linear model (Equation 3)

Figure 1 shows the V-I graph for the first experiment with the measured data, calculated uncertainties and curve fitting using the linear model (Equation 3) is shown in. The measured data points are plotted with uncertainties. The curve fits the measured points quite well and from the curve fit function the slope of the graph is obtained to be $2.10 \pm 0.07\ \Omega^{-1}$. Reciprocal of this value gives resistance which comes out to $476 \pm 15\ \Omega$. Plotting the residuals (Figure 2) for the above fit, it is evident that all the values calculated for current using the resistance from the linear model (Equation 3) are within the uncertainty range of the measured values. The above model also gives a minuscule vertical shift of 0.43 ± 0.82 mA. When the curve is forced to pass through origin using Equation 4 to model the data the following plots are obtained.



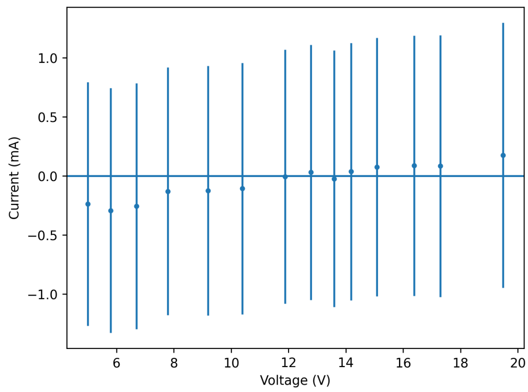


Figure 3: Resistor measurements fitted to linear model (Equation 4)

Figure 4: Residual plot for linear model (Equation 4)

The slope of Figure 3 comes out to $2.06 \pm 0.02\ \Omega^{-1}$ which gives the resistance value of $485 \pm 5.0\ \Omega$. The residual plot of this model Figure 4 shows that the calculated values are further away from the measured values when compared to the previous model. Switching from the resistor to the potentiometer gives the following plots for the linear model Equation 3.

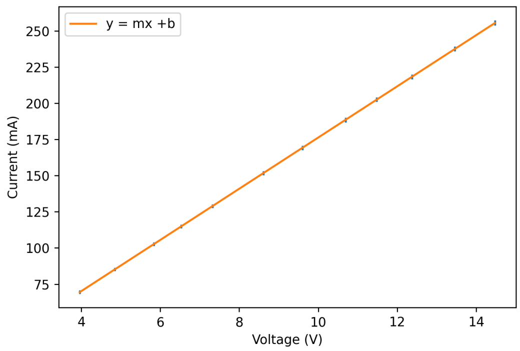


Figure 5: Potentiometer measurements Fitted to linear model (Equation 3)

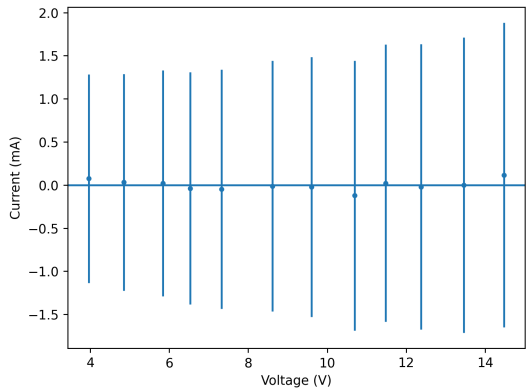


Figure 6: Residual plot for (Figure 5)

Again, scanning Figure 5 shows that the curve fits the data quite well and the residual plot Figure 6 confirms this. The slope of the graph Figure 5 is $17.7 \pm 0.1\ \Omega^{-1}$ which gives us a resistance of $56.5 \pm 0.4\ \Omega$. This graph also has a vertical shift of 0.54 ± 1.0 mA. Force fitting the data to Equation 4 gives the curve as depicted in Figure 7 and the residual plot for this fit is shown in Figure 8

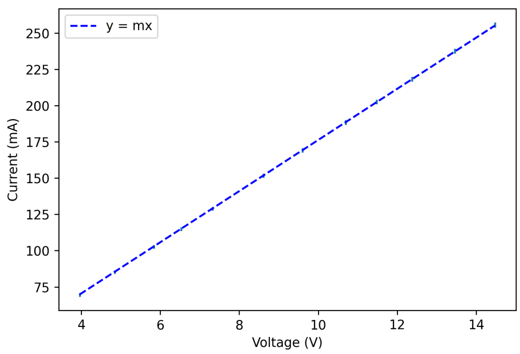


Figure 7: Measurements Fitted to linear model (Equation 4)

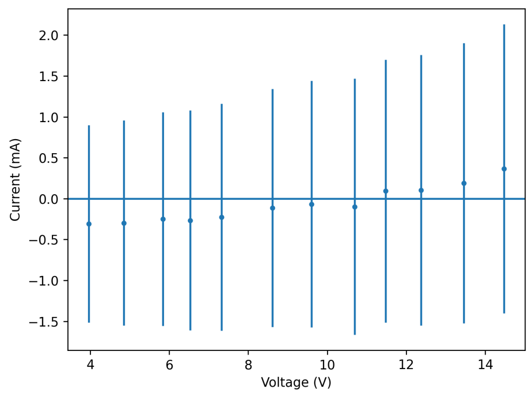


Figure 8: Residual plot for linear model (Equation 4)

The slope of Figure 7 is $17.6 \pm 0.05\ \Omega^{-1}$ which gives us a resistance of $56.7 \pm 0.1\ \Omega$.
Experiment two data modelled using Equation 6, gives the curve in Figure 9 and it's residuals are plotted in Figure 10

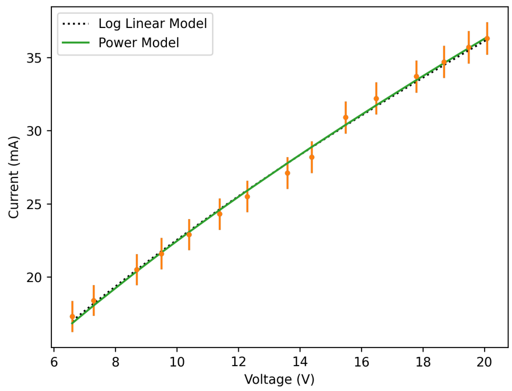


Figure 9: Light bulb measurements Fitted to Log Linear model (Equation 6) and Power Model Equation 5

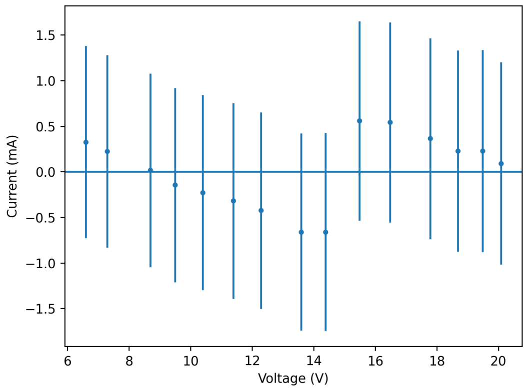


Figure 10: Residual plot for Log model and power model

When the data was modelled using Equation 5, the curve "Power Model" in Figure 9 is obtained. A residual plot that is exactly the same as the log model. The exponent value obtained from the log model is 0.68 ± 0.01 and the exponent value obtained from the power model was 0.69 ± 0.01. The residual plot for either model shows that the calculated values falls within the uncertainty range for the measured current values so either fit is reasonable. The obtained value for the exponent is higher than expected. The plot for the theoretical model Figure 11 was obtained setting the exponent value (d) in Equation 5 to be the ideal value 0.6. The residual for this plot is given by Figure 12.

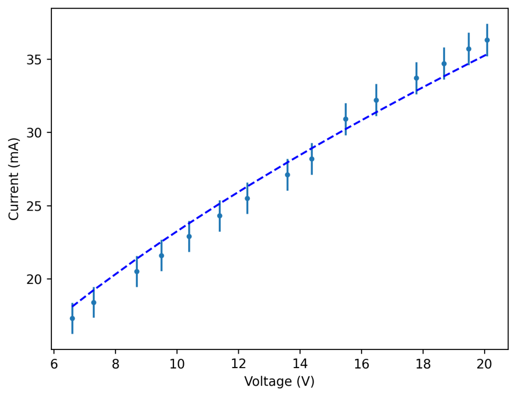


Figure 11: Measurements Fitted to Theoretical model

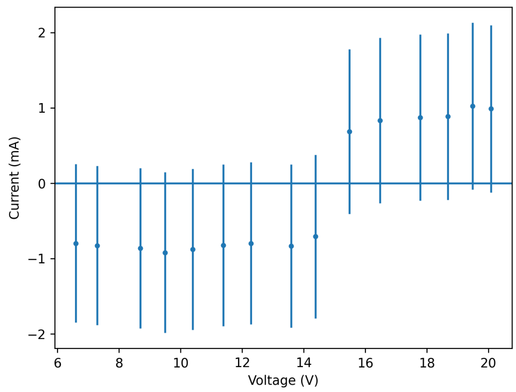


Figure 12: Residual plot for Theoretical model

Analysis

Our biggest source of uncertainty in both the experiments was the device uncertainty provided by the manufacturer. Another source of uncertainty was the heating of the resistor or the light bulb but this uncertainty was very small compared to the device uncertainty so it was ignored.

In the first part of the experiment the resistance value obtained ($476 \pm 15\ \Omega$) from the Equation 3 model was extremely close to the measured value of the resistor. The measured value of the resistor was $470 \pm 4.0\ \Omega$ which was measured using the multi-meter. The resistance value obtained using the Equation 4 ($485 \pm 5.0\ \Omega$) was much further away from the measured value suggesting that Equation 3 is a better model for the data. The reduced ${\chi_r}^2$ value obtained for the first model was very small (0.001), which suggests that the data points on average were very close to the curve. The residual plot for this model Figure 2 supports the obtained ${\chi_r}^2$ value. Similar results were obtained when the data was fitted to Equation 4, (Figure 3). The ${\chi_r}^2$ value in this case was 0.02 suggesting that in this this fit the data points were further away from the curve than the previous model Figure 4. Comparing the two models it can be deduced that Equation 3 is clearly a better fit for the measurements. It gives resistance values closer to the measured values as compared to Equation 4. Equation 3 model also gives a small vertical shift which can be explained as residual voltage that is present in the circuit with the power supply turned off. Also the value obtained from both models is a little higher than the measured resistance of the resistor. This is because the model takes into account the tiny resistance of the connecting wires. In case of the potentiometer very similar results were observed in both cases. For Equation 3 model the resistance value obtained was $56.5 \pm 0.04\ \Omega$ and the ${\chi_r}^2$ value for the fit was 0.002. For Equation 4 model the resistance value was $56.8\ \Omega$ and ${\chi_r}^2$ value was 0.03. Again proving that Equation 3 is a better model for the measured data. It can also be concluded that the resistor used for this experiment follows ohm's law. The small ${\chi_r}^2$ values suggest that the uncertainty for the multi-meters provided by the manufacturer is much larger than the actual uncertainty.

For the light-bulb part of the experiment the value of the exponent obtained from the Equation 6 model was 0.68 ± 0.01 and the value obtained from Equation 5 model was 0.67 ± 0.01. The exponent value for an ideal blackbody is supposed to be 0.6. The values obtained from the curves suggest that the given light-bulb is not ideal. The ${\chi_r}^2$ value obtained for Equation 6 curve was 0.15 which is backed up by the residual plot in Figure 10. In the case of Equation 5 model in the ${\chi_r}^2$ value was 0.14. Just like before, ${\chi_r}^2$ values obtained here were very small and for the same reason. Equation 6 and Equation 5 are just different versions of the same equation so the values obtained from both models were almost identical. The minuscule difference in values can be explained by the log and anti-log function used in python program could have introduced small uncertainty in the calculations. The residual plots for both models are identical but the data points are spread apart more than expected. Also there is a pattern that can be seen in the residual plots which at the moment is unexplained. Because of the scatter of values in the residual plot it can be inferred that either of these equations are not very good at modelling the lightbulb data. For the purposes of this experiment these models were reasonable. This experiment proves that the lightbulb follows the power law.
The theoretical model used an ideal constant exponent value of 0.6. The ${\chi_r}^2$ value for this model was 0.72 which is much higher than the previous two plots but is backed up by the fact that the data points on the plot are much further away from the curve. The theoretical curve is probably the worst equation to use to model the data because it assumes that the lightbulb is an ideal black-body which it is not.

Conclusion

In conclusion part 1 of this experiment proves that the resistor used to perform the experiment follows ohm's law and Equation 3 model is better to model the data because it accounts for residual voltage. The resistance value for the resistor obtained using Equation 3 was $476 \pm 15\ \Omega$ and the value obtained using Equation 4 was $485 \pm 5.0\ \Omega$. The value obtained using the first model was closer to the actual resistor value and accounts for resiudal current in the circuit.
For part 2, it can be concluded that the voltage-current relationship in a circuit with a blackbody is best modelled by the power law. Thew exponent value obtained using log linear model Equation 6 0.68 ± 0.01 and the value from the power model was 0.69 ± 0.01. Both these values are much higher than the expected ideal value of 0.6 suggesting that the model used was not the best for the lightbulb data.

References

[1] T. Vahabi, E. Horsley, A. Harlick, C Lee, J. Ladan, & R. M. Serbanescu. Ohm and Power Laws. PyLab - Ohm and Power Law.
https://q.utoronto.ca/courses/337705/assignments/1213477?module_item_id=5344991
2023, September 28.

Appendix

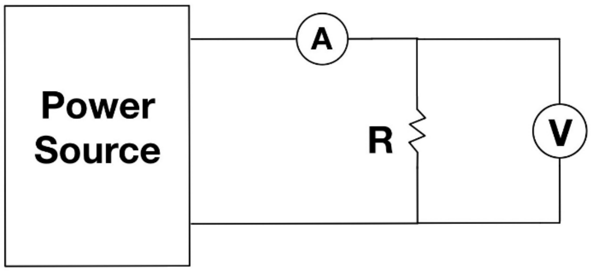


Figure 13: circuit diagram of experimental setup.[1]

Voltage (V)	Current (A)	Voltage error	Current error
4.994	0.0101	0.2697	0.00252
5.793	0.0117	0.30965	0.00284
6.693	0.0136	0.35465	0.00322
7.792	0.016	0.4096	0.0037
9.19	0.0189	0.4795	0.00428
10.389	0.0214	0.53945	0.00478
11.886	0.0246	0.6143	0.00542
12.786	0.0265	0.6593	0.0058
13.585	0.0281	0.69925	0.00612
14.184	0.0294	0.7292	0.00638
15.083	0.0313	0.77415	0.00676
16.382	0.034	0.8391	0.0073
17.301	0.0359	0.88505	0.00768
19.479	0.0405	0.99395	0.0086

Table 1: Resistor Data

Voltage (V)	Current (A)	Voltage error	Current error
3.96	0.0695	0.2000	0.0144
4.85	0.0852	0.2445	0.01754
5.84	0.1027	0.294	0.02104
6.533	0.1149	0.32865	0.02348
7.325	0.1289	0.36825	0.02628
8.612	0.1517	0.4326	0.03084
9.602	0.1692	0.4821	0.03434
10.693	0.1884	0.53665	0.03818
11.482	0.2025	0.5761	0.041
12.372	0.2182	0.6206	0.04414
13.462	0.2375	0.6751	0.048
14.473	0.2555	0.72565	0.0516

Table 2: Potentiometer Data

Voltage (V)	Current (A)	Voltage error	Current error
6.591	0.0173	0.33155	0.00396
7.29	0.0184	0.3665	0.00418
8.689	0.0205	0.43645	0.0046
9.488	0.0216	0.4764	0.00482
10.387	0.0229	0.52135	0.00508
11.387	0.0243	0.57135	0.00536
12.286	0.0255	0.6163	0.0056
13.586	0.0271	0.6813	0.00592
14.385	0.0282	0.72125	0.00614
15.484	0.0309	0.7762	0.00668
16.482	0.0322	0.8261	0.00694
17.782	0.0337	0.8911	0.00724
18.681	0.0347	0.93605	0.00744
19.481	0.0357	0.97605	0.00764
20.08	0.0363	1.006	0.00776

Table 3: Lightbulb Data

Equations used to calculate data uncertainties:

voltage uncertainty:

$$= (Measured\_Voltage * 0.05) + (0.01 * 2) \tag{11}$$

Current uncertainty:

$$= (Measured\_Current * 0.2) + (0.0001 * 5) \tag{12}$$

DC specifications for U1271A, U1272A, U1273A and U1273AX

Function	Range	Resolution	Accuracy ± (% of reading + counts of least significant digit)			Test current / Burden voltage
			U1271A	U1272A	U1273A / U1273AX	
Voltage[1]	30 mV	0.001 mV	—	0.05 + 20	0.05 + 20	—
	300 mV	0.01 mV	0.05 + 5	0.05 + 5	0.05 + 5	—
	3 V	0.0001 V	0.05 + 5	0.05 + 5	0.05 + 5	—
	30 V	0.001 V	0.05 + 2	0.05 + 2	0.05 + 2	—
	300 V	0.01 V	0.05 + 2	0.05 + 2	0.05 + 2	—
	1000 V	0.1 V	0.05 + 2	0.05 + 2	0.05 + 2	—
	Z_{LOW} (low impedance) enabled, applicable for 1000 V range and resolution only	0.1 V	—	1 + 20	1 + 20	—
Resistance[2]	30 Ω	0.001 Ω	—	0.2 + 10	0.2 + 10	0.65 mA
	300 Ω	0.01 Ω	0.2 + 5	0.2 + 5	0.2 + 5	0.65 mA
	3 kΩ	0.0001 kΩ	0.2 + 5	0.2 + 5	0.2 + 5	65 µA
	30 kΩ	0.001 kΩ	0.2 + 5	0.2 + 5	0.2 + 5	6.5 µA
	300 kΩ	0.01 kΩ	0.2 + 5	0.2 + 5	0.2 + 5	0.65 µA
	3 MΩ	0.0001 MΩ	0.6 + 5	0.6 + 5	0.6 + 5	93 nA/10 MΩ
	30 MΩ	0.001 MΩ	1.2 + 5	1.2 + 5	1.2 + 5	93 nA/10 MΩ
	100 MΩ	0.01 MΩ	2.0 +10	—	—	93 nA/10 MΩ
	300 MΩ	0.01 MΩ	—	2.0 + 10 @ < 100 MΩ 8.0 + 10 @ > 100 MΩ	2.0 + 10 @ < 100 MΩ 8.0 + 10 @ > 100 MΩ	93 nA/10 MΩ
	300 nS	0.01 nS	1 + 10	1 + 10	1 + 10	93 nA/10 MΩ
Current[3]	300 µA	0.01 µA	0.2 + 5	0.2 + 5	0.2 + 5	< 0.04 V/100Ω
	3000 µA	0.1 µA	0.2 + 5	0.2 + 5	0.2 + 5	< 0.4 V/100 Ω
	30 mA	0.001 mA	0.2 + 5	0.2 + 5	0.2 + 5	< 0.08 V/1 Ω
	300 mA	0.01 mA	0.2 + 5	0.2 + 5	0.2 + 5	< 1.00 V/1 Ω
	3 A	0.0001 A	0.3 + 10	0.3 + 10	0.3 + 10	< 0.1 V/0.01 Ω
	10 A	0.001 A	0.3 + 10	0.3 + 10	0.3 + 10	< 0.3 V/0.01 Ω
Diode Test[4]	3 V	0.0001 V	0.5 + 5	0.5 + 5	0.5 + 5	Approximately 1 to 2 mA
	Auto	0.0001 V	—	0.5 + 5	0.5 + 5	Approximately 1 to 2 mA

Figure 14: Multi-meter specifications setup.