Finding the Radius of Earth using a Gravimeter

Reeshav Kumar, Arthur Li

April 2024

Abstract

The purpose of this experiment is to calculate the radius of the earth using a Sodin 410 Gravimeter. The radius of earth obtained from the first set of data was $6,476 \pm 119$ kilo-meters which is within two percent of the expected value of 6371 kilo-meters. The radius obtained from the second data set was $6,317 \pm 115$ kilo-meters which is within one percent of the expected value.

Introduction

This lab involves using changes in altitude and gravity measurements to calculate the radius of the earth. Acceleration due to gravity (g) follows the inverse square law in relation to distance from the gravity location to the centre of the earth (R). m_{Earth} is the mass of the earth, and G is the Earth's (or any planet for that matter's) gravitational constant.

$$g = G * \frac{m_{Earth}}{R^2} \tag{1}$$

the relation between change in gravitational force (Δg) and change in Radius of the earth (ΔR) is proportional[1].

$$\frac{\Delta g}{g} = -2 * \frac{\Delta R}{R} \tag{2}$$

g is the value of gravity at the first height of where the calculations for (Δg) and (ΔR) take place, and g is proportional to R. In our lab setting, g is known to be 9.804253 $\frac{m}{s^2}$ [1] at the first floor of Burton Tower from where we first start taking the change in gravity and change in Radius data. Burton tower contains 14 floors which can be used to find the data.

To make it accessible in a curve-fit/non-linear regression analysis, we may use the following equation by plotting (ΔR) against (Δg), with

$$\Delta g = -2 * \frac{g}{R} * \Delta R + c \tag{3}$$

by fixing the constant that we know (g), we can calculate the Radius of the earth as a fitting parameter shifted by some constant c.

(Δg) and (ΔR) are found by using a Gravimeter (manufactured by Sodin), which is essentially a very sensitive s balanced by a torsion fiber[1]. The local gravitational value is found by measuring how much the mass stretches the spring. The gravimeter measures gravity in milli-gals. A gal is defined as ($\frac{cm}{s^2}$).

Method and procedure

The only equipment used in this experiment setup is the Gravimeter (Sodin model 410). Our data curation site (Burton Tower) consisted of 14 floor sand the data was recorded at every floor. The gravimeter counter value was recorded at each floor, and the change in distance between each floor was measured using a tape ruler. The gravimeter was placed at the same position on the horizontal plane of each floor. The ground floor was chosen as the reference point. The gravimeter consists of two peep holes. One for leveling the device and other to obtain the reading.

To operate the gravimeter, first centre the two bubble levels in the centre glass aperture (with the gravimeter light turned on) to stabilise the device. Then adjust the slits by rotating the counter until the black slits are centered on the cross-hair of the offset aperture. Record the counter value and multiply the value by 0.10055 mlgs/div to obtain the reading in milli-gals. The data was plotted in python an curve fitted using Equation 3 model to obtain the radius of the earth.

Analysis and Results

The graph for the first set of data (Figure 5) is shown in the figure below.

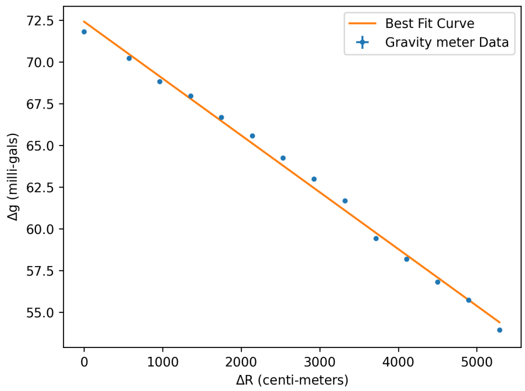

Figure 1: The gravity meter data set 1 plotted and curve fitted using Equation 3 model. The x-axis displays the change in radius (ΔR) in centimeters vs the change in gravitational constant (Δg) in milli-gals. The Orange curve is the best fit line for the data points.

The Radius of the earth obtained from Figure 1 is $6,476 \pm 119$ kilo-meters. The value obtained form this plot is within two percent of the expected value of 6,371 kilo-meters. The residual plot of the above curve is displayed in Figure 2. The reduced χ^2 value for this curve was 17.5 suggesting that the curve was not a great fit for the data. This value is backed by the fact that most data points on the residual plot miss the curve and only one point fully passes the curve. The high reduced χ^2 value is also due to the fact that the gravimeter has very low uncertainty.

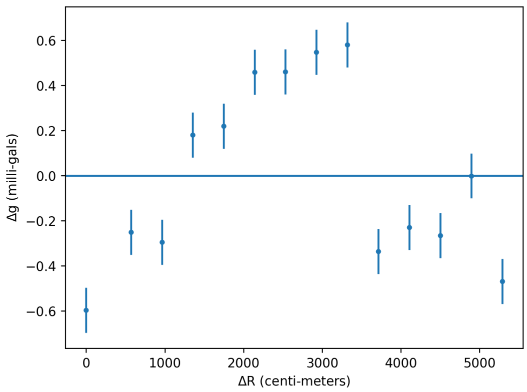

Figure 2: The residual plot for the Figure 1 curve. The x-axis shows the change in radius (ΔR) in centimeters and the y-axis shows how far from the curve the data points are.

The biggest source of uncertainty for this experiment was the rotation of earth which changes the gravitational source exerted by the sun. To account for this variation the ground floor was chosen as a reference point and a reference reading was taken every thirty minutes while the data was being curated. The data was them corrected based on this reference reading. Another source of uncertainty present was the centrifugal force from the rotation of the earth. Since the mass difference when going from floor to floor was very small compared to the mass of the earth, the difference centrifugal force at floor fourteen and ground floor would be very small, hence it was ignored for this experiment. The second set of data (Figure 6) is shown in the graph below.

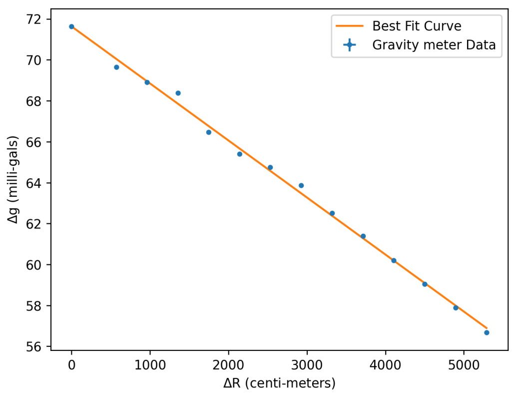

Figure 3: The gravity meter data set 2 plotted and curve fitted using Equation 3 model. The x-axis displays the change in radius (ΔR) in centimeters vs the change in gravitational constant (Δg) in milli-gals. The Orange curve is the best fit line for the data points.

The value of the radius of the earth obtained from Figure 3 curve is 6317 ± 115 kilo-meters which is within one percent of the expected value for the radius of the earth. The residual plot for the best fit curve for this graph is shown in Figure 4. The associated reduced χ^2 value for the curve is 7.2 suggesting a better fit than the previous data set. This value is also high because a lot of the data points completely miss the curve and the uncertainty in the gravimeter is very low.

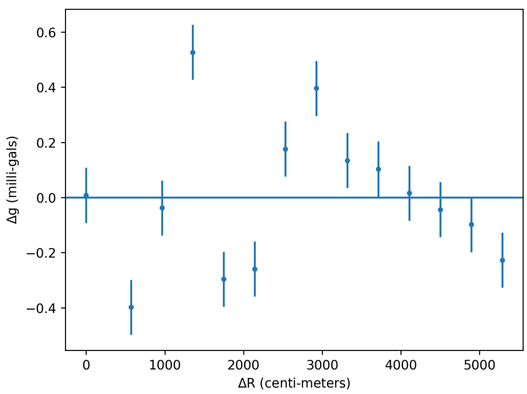

Figure 4: The residual plot for the Figure 3 curve. The x-axis shows the change in radius (ΔR) in centimeters and the y-axis shows how far from the curve the (Δg) data points are in milli-gals.

Conclusion

In conclusion, the calculated radius of the earth values where very close to the expected value. For data 1, the radius obtained is $6,476 \pm 119$ kilo-meters, for data 2 it is $6,317 \pm 115$ kilo-meters. These values are within 1-2 percent of the expected radius of earth value, which is 6371 km. The accompanying χ^2 values of 17.5 and 7.2 for data 1 and 2 respectively show that the fit of the second data was better than the first one hence giving a closer radius value. The respective residual graphs for data 1 and 2 show that most of the data and their error-bar do not overlap with the best-fit curve. Since the values of radius obtained were very close to the expected value, the model was effective but with more data the model will be a better fit.

References

[1] https://www.physics.utoronto.ca/ phy224_324/LabManuals/RadiusOfTheEarth.pdf Radius of the Earth Lab Manual

Appendix

Raw Data

Data Set 1

G-val	delta-g	delta-R	R	delta_g-uncert	delat_R_uncert	R_uncert
714.25	71.81784	0	0	0.1	0.0001	0.0001
698.35	70.21909	570.5	570.5	0.1	0.5	0.5
684.6	68.83653	963	392.5	0.1	1.5	1
676.05	67.97683	1355	392	0.1	2	0.5
663.1	66.67471	1748.5	393.5	0.1	3	1
652.2	65.57871	2140.5	392	0.1	3.5	0.5
638.9	64.2414	2533	392.5	0.1	4.5	1
629.4	63.28617	2927.5	394.5	0.1	5.5	1
616.4	61.97902	3320.5	393	0.1	6.5	1
593.9	59.71665	3715	394.5	0.1	7	0.5
581.65	58.48491	4107.5	392.5	0.1	7.5	0.5
567.95	57.10737	4501	393.5	0.1	8	0.5
557.25	56.03149	4894.5	393.5	0.1	8.5	0.5
539.25	54.22159	5288	393.5	0.1	9	0.5

Figure 5: Data Set 1 that corresponds to the Figure 1

Data Set 2

G-val	delta-g	delta-R	R	delta_g-uncert	delat_R_uncert	R_uncert
712.51	71.64288	0	0	0.1	0	0
692.65	69.64596	570.5	570.5	0.1	0.5	0.5
685.35	68.91194	963	392.5	0.1	1.5	1
680.1	68.38406	1355	392	0.1	2	0.5
661	66.46355	1748.5	393.5	0.1	3	1
650.5	65.40778	2140.5	392	0.1	3.5	0.5
643.95	64.74917	2533	392.5	0.1	4.5	1
631.95	63.54257	2927.5	394.5	0.1	5.5	1
618.45	62.18515	3320.5	393	0.1	6.5	1
607.2	61.05396	3715	394.5	0.1	7	0.5
595.45	59.8725	4107.5	392.5	0.1	7.5	0.5
583.95	58.71617	4501	393.5	0.1	8	0.5
572.5	57.56488	4894.5	393.5	0.1	8.5	0.5
560.3	56.33817	5288	393.5	0.1	9	0.5

Figure 6: Data Set 1 that corresponds to the Figure 3