Single Slit and Double slit experiment with Heisenberg Uncertainty Verification.

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Abstract

This experiment deals with Single Slit and Double Slit light diffraction experiment. For the single slit experiments three sets of light intensity data from a light sensor was used to calculate the slit width. The slit widths were calculated using curve fitting the data and finding the parameters. The silt widths calculated were $0.023 \pm (2.8 * 10^{-5})$ mm, $0.041 \pm (1.09 * 10^{-5})$ mm and $0.082 \pm 2.01 * (10^{-5})$ mm for the light data from 0.02 mm, 0.04 mm and 0.08 mm slits. The values calculated were extremely close to the expected values.

For the double slit part of the experiment the same procedure was applied to obtain slit width and slit separation. The values calculated for the double slit experiment were also very close to the expected values. From curve fitting, the values calculated for data from slits of width 0.04 mm separated by 0.25 mm were $0.041 \pm (1.76 * 10^{-3})$ mm for the slit width and $0.025 \pm 5.6 * 10^{-4}$ mm for the slit separation. For data from slits of width 0.04 mm separated by 0.50 mm the slit width was calculated to be $0.041 \pm (8.8 * 10^{-4})$ mm and the slit separation was $0.51 \pm (1.4 * 10^{-3})$ mm. In case of the 0.08 mm slits separated by 0.25 mm, the slit width was calculated to be $0.076 \pm (4.2 * 10^{-3})$ mm and the separation was $0.23 \pm (1.8 * 10^{-3})$ mm.

The Heisenberg Uncertainty principle is successfully verified in the third part of the experiment for all the three single slits.

Introduction

Light passing through a slit experiences diffraction, where light slightly bends as it passes around the edges of the slit. When shining a precise laser through a small slit, projecting the passing light onto a blank surface perpendicular to the laser shows a dotted light diffraction pattern.

An important element for analysing light wave patterns is the angle to the minima of a diffraction pattern. Light passing through a single slit only experiences diffraction:

$$a \sin \theta = m' \lambda \tag{1}$$

a is the slit width, θ is the centre of the pattern to the m'th minimum, where m'is the order of diffraction counting from the centre out(m'=1,2,3, zero for the central maximum), and λ is the wavelength of light.

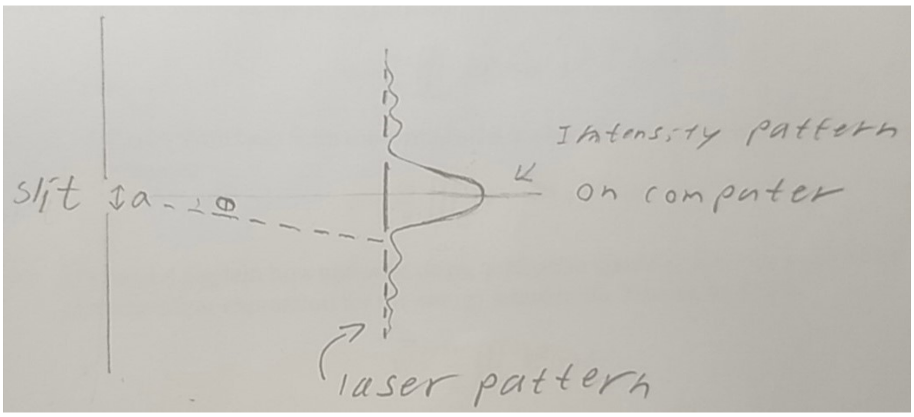


Figure 1: A sketch of the slit (having width a) that diffracts light to produce a diffraction pattern. Theta is the angle from the centre of the diffraction pattern to the first minima of the diffraction pattern.

The other phenomena we will analyse is light interference, where more than one wave of light interacts with each other to alter the combined amplitudes of the waves(This is superposition of waves in action, which states that at the crossing of waves, the displacement is the sum of individual waves displacement). Light passing through a double slit experiences both diffraction and interference. Its angle to the minima is given by the relation:

$$d \sin\theta = m\lambda \tag{2}$$

Where d is the slit separation, θ is the angle to the minima, m is the order from the centre out (m=1,2,3, zero for the central maximum), and λ is the light wavelength. These formulas come from experimental setup geometry. The diffraction pattern for both the Single Slit and Double slit experiments are governed by the same physical quantity which is the slit width. The slit separation in double slit is responsible for determining where the amplitude of interference peaks go to zero.

Since the aim of this experiment is to compare light diffraction and interference, we will need to compare the light intensity pattern of light undergoing diffraction and or interference. This will require the equation that relates light intensity (I) of the light passing through single slits and the sensor position (x) on the screen. The inverse square law of light [1] says that the intensity of the light is inversely proportional to the square of the distance that the light travels (x_{light}).

$$Intensity_{light} \propto \frac{1}{x_{light}^2} \tag{3}$$

In the slit experiments, the light propagates as a sinusoidal pattern, so modulating the inverse square law with sine, the diffraction wave takes the form of $\frac{sin(x_{light})}{x_{light}}$. Thus the intensity of the diffraction pattern as a function of x can be written as

$$I = I_0 * (\frac{sin(bx+p)}{bx+p})^2 + c \tag{4}$$

where b is the angular frequency of the wave, p is the phase, c is the vertical shift and I_0 is the intensity at the centre of the diffraction pattern. From this equation the slit width (a) can be calculated as

$$a = b * \frac{\lambda}{\pi} * D \tag{5}$$

where λ is the wavelength of the light source and D is the distance between the slit and the screen
When the light passes through double slits, the diffraction pattern is modulated by wave interference which takes the form of $cos^2 x$, so the wave amplitude becomes:

$$I_{Double_slit} = I_0 * \left(\frac{\sin(\phi)}{\phi}\cos(\phi)\right)^2 \tag{6}$$

where ϕ is the phase of the wave in general terms. But in case of difficulty with fitting the double slit wave amplitude, one may use a simple cos model, and derive the slit separation of the diffraction pattern with it. b_2 and p_2 are the angular frequency and phase of the interference pattern.

$$I_{interference} = I_0 * \cos^2(b_2 x + p_2) \tag{7}$$

We will also be verifying the Heisenberg Uncertainty Principle in this lab. The principle states that the measurement of both the movement and momentum of a particle involves an limitation on the precision of each measurement. The more precise one of the measurement is, the less precise the other will be.

$$\Delta y * \Delta p \geq \frac{h}{4\pi} \tag{8}$$

Δp is the momentum uncertainty, and Δy is the position uncertainty, h is Planck's constant. Using the de Broglie's relationship ($\frac{h}{\lambda} = p$), $\Delta V_y = c_{Light} \sin\theta_1$, $\Delta P_y = \frac{h}{\lambda}\sin\theta_1$ relationships derived from the experimental geometry (c_{Light} is speed of light),we can rewrite the inequality as

$$\frac{a}{\lambda} \sin\left(\tan^{-1}\frac{l}{b}\right) = 1 \tag{9}$$

a is the slit width Δy, l is the half width of the diffraction maximum envelope, and b is the distance between the light sensor and the slit aperture. Verifying this (approximate) equality is the same as demonstrating Heisenberg's uncertainty principle.

Methods and procedure

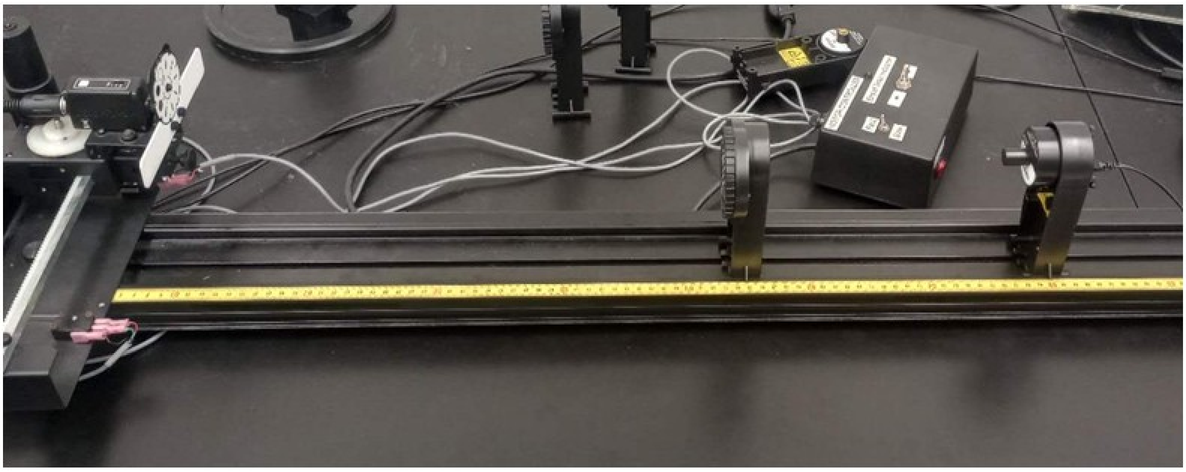


Figure 2: The experimental setup. On the right is the laser (i), in the middle is the slit aperture (ii), and on the left the light sensor on a moving fixture(iii)

The setup of this experiment consists of a laser (i), a vertical single slit aperture (ii), a vertical double slit aperture (ii), and a laser receiver sensor that measures the intensity of the laser. The laser receiver is lined up on the same axis with the slit and the laser aperture; the three are mounted on a rail for ease and precise manipulation, with the slit aperture and the laser being free to slide along the rail. The laser receiver is fixed on a mount that can move perpendicularly to the axis to record the intensity at different locations of the diffraction pattern on the laser receiver. The laser receiver should be attached to a white surface with a hold cut out for the receiver so that the diffraction pattern can be seen clearly around the receiver. The slit should be aligned with the laser so that the diffraction pattern is horizontal, visible left and right of the laser receiver, and vertically centred with the laser receiver so that the receiver can read the laser intensity as it moves horizontally from one side of the diffraction pattern to the other. The moving mount of which the receiver is mounted on takes roughly over a minute to move from one side of the diffraction pattern to the other, for a slit aperture that is roughly half a meter distance from the receiver.

Before recording the data, move the laser receiver mount to one side of the diffraction pattern. Then, record the data of the laser intensity as the receiver moves across the pattern to the other side of the pattern.

The laser model is a Pasco OS-8525A. The slit width's used in the experiment are 0.02, 0.04 and 0.08 mm. The double slit separations were 0.25 and 0.5 mm. The wavelength of light from the source was 515 nm. In the single slit, the slit was 0.526 ± 0.001 m from the sensor; the distance between the laser sensor in the double slit part was 0.706 ± 0.001 m. In all cases, the laser was located at 1m from the sensor, and adjusted so that sliding the laser back and forth on the rail does not affect sensor reading.

The uncertainty in the light sensor was calculated by fixing the location of the sensor and letting the light of constant intensity hit the sensor for some time. The difference between the highest and lowest intensity reading was chosen to be the device uncertainty.

The slit width was calculated using two methods. First method required fitting the data to the Equation 4 model and obtaining the slit width using Equation 5. In the second method the distance between the central maximum and the first minimum of the diffraction pattern is obtained from the data and the slit width is calculated using Equation 1.

Analysis and Results

The first part of the experiment deals with three Single slits of 0.02 mm, 0.04 mm and 0.08 mm and their corresponding intensity data from the sensor. The light source used for this part had wavelength of 515 nm.

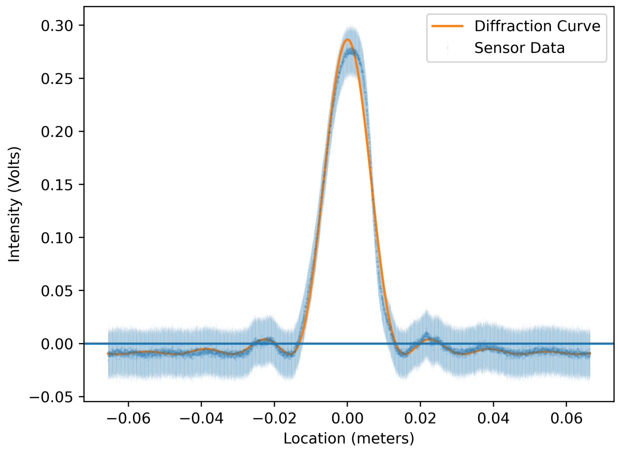
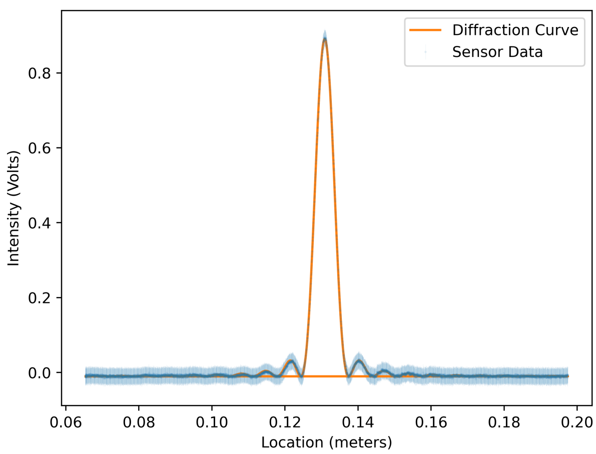


Figure 3: Light Sensor data from the single slit of width 0.02 mm. The x axis shows the location on the sensor screen in meters and the y-axis shows the Light intensity recorded in Volts. The green line is the best fit curve for the data obtained using Equation 4 model.

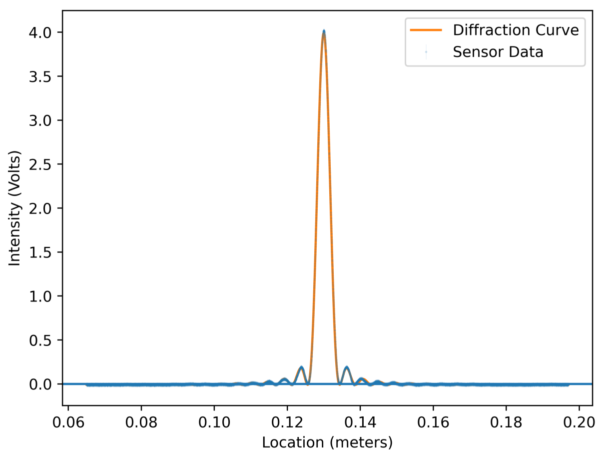
The above figure shows the diffraction pattern obtained by single slit of 0.02 mm. The distance between the Light sensor and the slit was 0.71 m. Fitting this data to Equation 4 model gives us slit width calculated using Equation 5 to be $0.023 \pm 2.8 * 10^{-5}$ mm. The residual plot of the above curve is shown in Figure 7(a). The reduced χ^2 value for the curve is 0.09. The small reduced χ^2 value is due to the high relative uncertainty of the light sensor. Using the second method the half width of the central maximum is recorded to be 0.014 meters. Using this value the slit width is calculated to be 0.026 milli-meters. Between the two methods the value obtained by curve fitting the data is closer to the actual slit width even though the curve in Figure 3 deviates a bit from the data-points near the central peak.

The exact same procedure was applied to data obtained from the 0.04 mm and 0.08 mm slits. The diffraction pattern of both is shown in Figure 5. In case of the data shown in Figure 5(a) the value of slit width obtained from curve fitting the data is $0.041 \pm 1.09 * 10^{-5}$ mm. The residual plot for this curve is shown in Figure 7(b) with the associated reduced χ^2 value to be 0.02 due to high uncertainty of the sensor and the curve passing through the error bars of almost all the data points suggesting a really good fit. The value of the slit width obtained by the second method is exactly the same as the curve fit value up to four decimal figures. The value obtained is 0.041 mm. Both values were almost the exact same as the expected value. In case of the data in Figure 5(b) the value obtained from curve fitting the data is $0.082 \pm 2.01 * 10^{-5}$ mm. The reduced χ^2 value for this curve was 0.21 which is higher that the other two previous curves but still low due to the same reasons. The half width in this case was 0.0034 m giving the slit width as 0.083 mm.

The biggest source of uncertainty in this experiment was the device uncertainty of the Light Sensor which was calculated to be 0.02 Volts. The presence of background light sources is another source of uncertainty for this experiment but was ignored because of it being consistent across data collection for all the experiments.



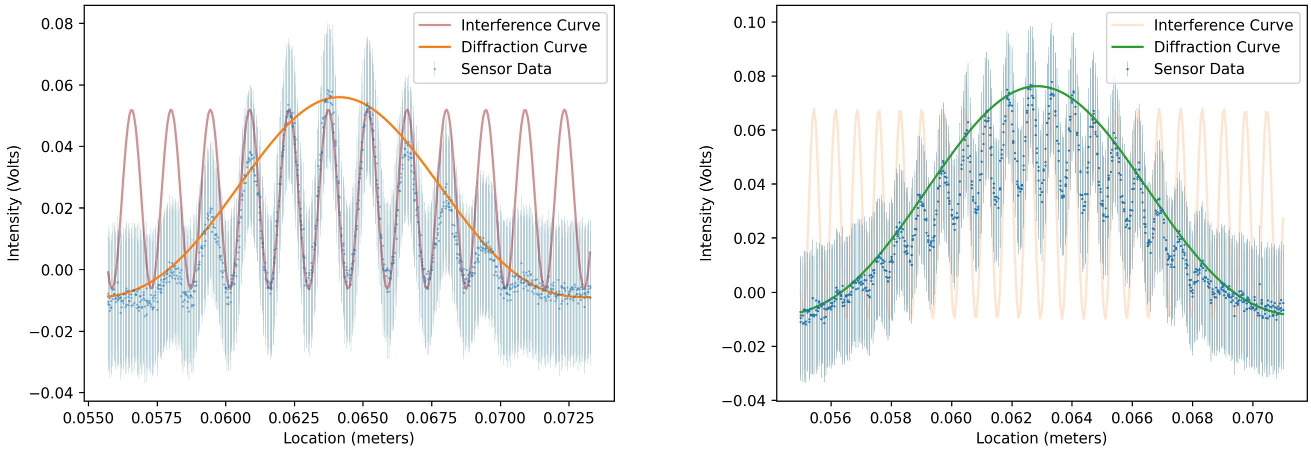
(a) Diffraction pattern of 0.04 mm slit



(b) Diffraction pattern of 0.08 mm slit

Figure 4: The above graphs show the light intensity data for two single slit experiments and their associated best fit curves fitted to Equation 4 model. The x-axis of both curves show the position on the sensor screen and the y-data show the associated intensity recorded at that position.

The second part of the experiment deals with finding the slit width and slit separation from the Light intensity data in Double slit experiment. The intensity data for the central envelope is fitted to two different curves for diffraction and interference pattern to obtain the slit width and slit separation respectively. The Light intensity data from the sensor for the slits with width of 0.04 mm separated by 0.25 mm and 0.50 mm is shown below.



(a) Intensity pattern from slits of width 0.04 mm separated by 0.25 mm

(b) Intensity pattern from slits of width 0.04 mm separated by 0.50 mm

Figure 5: The above graphs show the light intensity data for two double slit experiments and their associated best fit curves fitted to Equation 4 model for diffraction and Equation 7 model for interference. The x-axis of both curves show the position on the sensor screen and the y-data show the associated intensity recorded at that position.

In case of Figure 5(a), fitting the data points to the diffraction equation (Equation 4) gives the slit width to be $0.041 \pm 1.76 * 10^{-3}$ mm and fitting the points to the interference equation (Equation 7) gives the slit separation to be $0.025 \pm 5.6 * 10^{-4}$ mm. Both the values obtained from curve fitting are almost exactly the same as the expected values and have low error. The number of peaks observed in the central envelope were 10 which is less than the expected number of peaks ≈ 12.5 for this combination of slits. The residual plot of the diffraction curve vs the peaks is shown in Figure 8(a). The reduced χ^2 value associated with this residual plot is 0.06 which is low because of the high sensor uncertainty and the fact that the peaks are extremely close to the curve. If the slit width is calculated using method two, the half width of the central envelope is measured to be 0.009 m, which gives the slit width to be $0.042mm$ which is slightly higher than expected value.

For Figure 5(b), the slit width obtained from curve fitting is $0.041 \pm 8.8 * 10^{-4}$mm and the slit separation obtained was $0.51 \pm 1.4 * 10^{-3}$ mm. The residual plot associated with this diffraction pattern is shown in Figure 8(b) with associated reduced χ^2 value to be 0.03. The fact that the sensor has high uncertainty and all the peak values with error bars overlap the curve support this value. The half width of the central envelope for this data is measured to be

The third double slit combination used was with the slit width 0.08 mm separated by 0.25mm. The light sensor data for this combination is shown below.

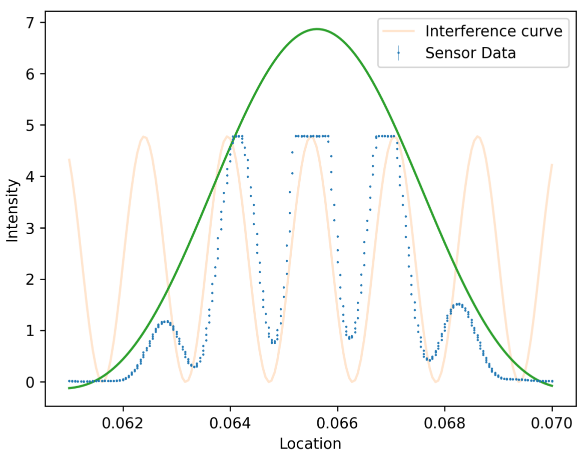


Figure 6: Light Sensor data from double slits of width 0.08 mm and separated by 0.25 mm with associated best fit curves fitted to Equation 4 model for diffraction and Equation 7 model for interference.. The x axis shows the location on the sensor screen in meters and the y-axis shows the Light intensity recorded in Volts.

For the above graph the intensity data has a cutoff at the maximum limit of the light sensor which is 5 Volts. The data was curve fitted to obtain the slit width of $0.076 \pm 4.2 * 10^{-3}$ mm and the slit separation of $0.23 \pm 1.8 * 10^{-3}$ mm. Both these values were smaller than the expected values because of the cutoff in the data. The residual for the peaks for this data is shown in Figure 8(c) which has a very high associated reduced χ^2 value of 10332 which is supported by the fact that none of the peaks actually overlap with the curve and are very far away from the expected peak of ≈ 7 volts.

The third part of the experiment deals with the verification of the Heisenberg uncertainty principle. For the slit with width 0.04 mm the half width of the central envelope was measured to be 0.0066 m and the slit aperture was 0.526 m away from the light sensor. Using these values in LHS of Equation 9 give us the value of 0.99 which is very close to the ideal value of 1. The calculation for this is shown in Appendix Figure 9. For the slit with width 0.02 mm the half width of the central envelope was measured to be 0.014 m giving the LHS value in Equation 9 to be 1.033 again confirming the Heisenberg Uncertainty Principle (Figure 10). In case of the slit width 0.08 mm the LHS value of Equation 9 was obtained to be 0.99 as shown in Figure 11.

Conclusion

In this experiment where we recorded the intensity across the diffraction and interference pattern to find the slit width and slit separations, we found that in the single slit data, the slit width found by curve-fitting the data with equation (4) was very close to the measured value. $0.023 \pm (2.8 {*} 10^{-5})$ mm, $0.041 \pm (1.09 {*} 10^{-5})$ mm and $0.082 \pm (2.01 {*} 10^{-5})$ mm for the 0.02, 0.04 and 0.08 mm slits respectively. Due to the high uncertainty in the light sensor, the reduced χ^2 value for the single slit data were very small, at 0.09, 0.02 and 0.21 for the 0.02, 0.04 and 0.08 mm slits. This also suggests that the curves fit the data very well as evident in the residual plots for the curves.

For the double slits, fitting equations (4) and (7) and using their parameters to calculate the slit width and separation, on the indicated 0.04 mm slit width and 0.25 mm slit separation gave a calculated slit width and slit separation value of $0.041 \pm (1.76 * 10^{-3})$ mm and $0.025 \pm (5.6 \cdot 10^{-4})$ mm. For the 0.04 mm slit width and 0.50 separation, the calculated slit width and separation are $0.041 \pm (1.76 * 10^{-3})$ mm and $0.51 \pm (1.4 * 10^{-3})$ mm. The calculated values are very close to the measured values. The associated χ^2 values are 0.06 and 0.03, which are low. The third double slit combination was 0.08 mm slit width and 0.25 mm separation. calculated slit width was $0.076 \pm (4.2 * 10^{-3})$ mm, and the calculated slit separation was $0.23 \pm (1.8 * 10^{-3})$ mm. These calculated values are less than the expected values, and the associating χ^2 value for the data and the outline fit is very high at 10332.

In applying our results to Heisenberg's uncertainty principle, we found that the our experiment setup values for the single slits with 0.08, 0.04 and 0.02 mm slit width both confirmed Heisenberg's Uncertainty Principle (Figure 9).

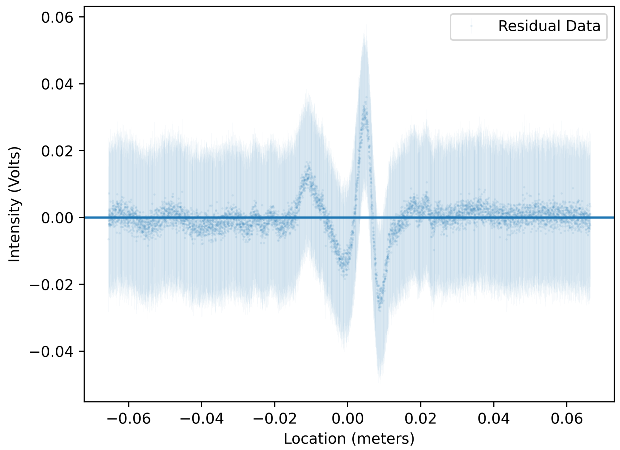
In conclusion, the calculated slit width and separation using the theory was very close to the indicated values for all of the cases except for the 0.08 mm and 0.25 mm slit width and separation case. For all but the last case, the residuals indicated that the fit lies mostly within the uncertainty around the data, with a small χ^2 values that indicates a large uncertainty and a close fit; the large χ^2 value of the last case indicates a poor fit. This means that the main theory we used to analyse this data was sufficient to replicate the expected parameters, barring the last case due to device measurement restraints. Thus the most noticeable shortcoming of this experiment was the inability of the device to record intensity above 5 volts, which in the last double slit case made curve-fitting less accurate and resulted in smaller values than the indicated/expected value on the slit. Therefore to avoid this error, one should give regard to the measurement limits of the device and not go above it, or a light source with lower intensity could be used instead.

References

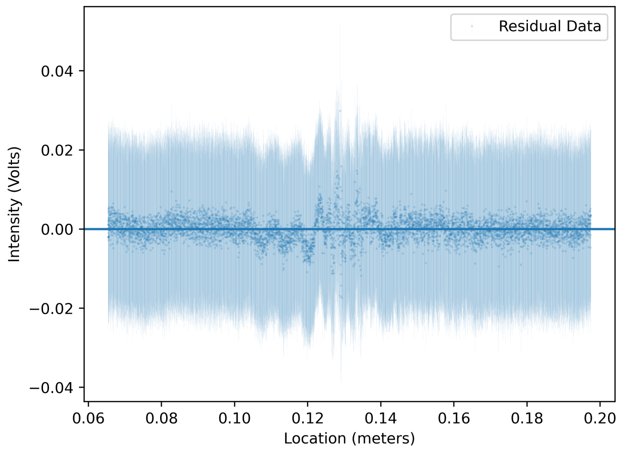
[1] https://www.nasa.gov/wp-content/uploads/2021/07/583137main_inverse_square_law_of_light.pdf
The Inverse Square Law of Light,

Appendix

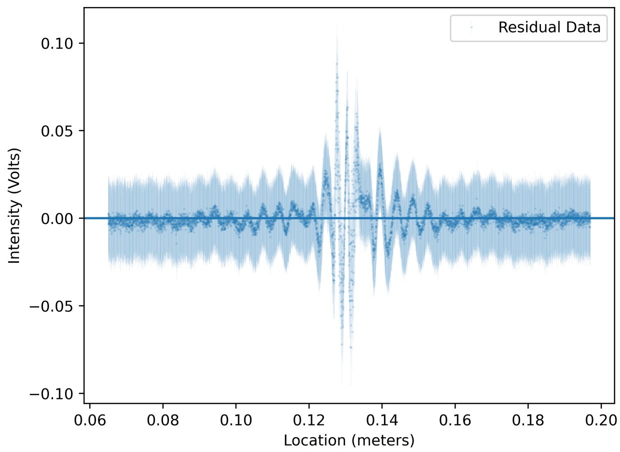
Residual Plots



(a) Residual plot associated with 0.02 mm - Single Slit

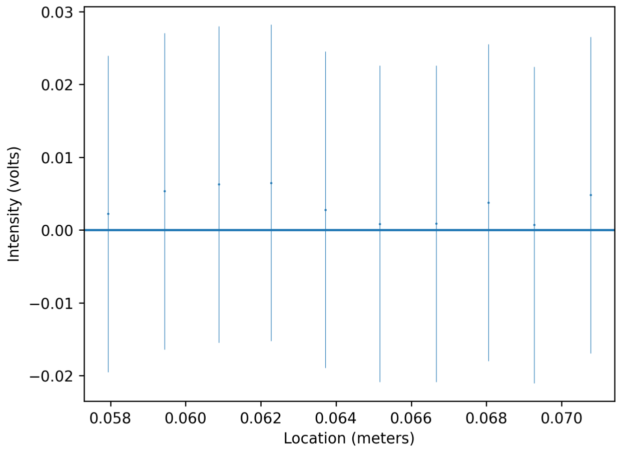


(b) Residual plot associated with 0.04 mm - Single Slit

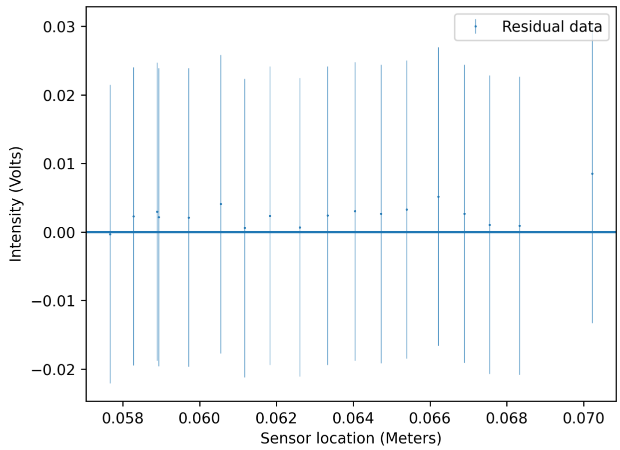


(c) Residual plot associated with 0.08 mm - Single Slit

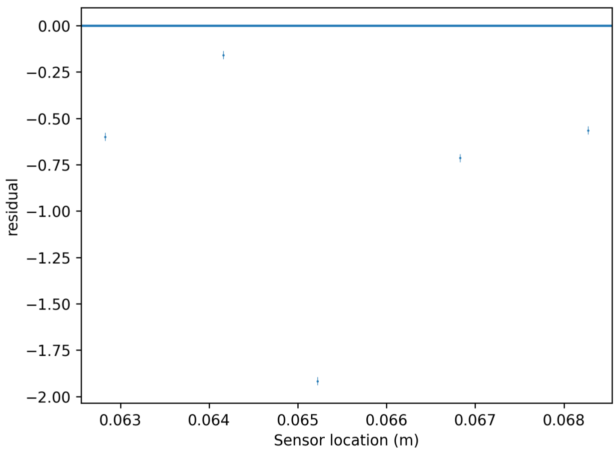
The above graphs show residual plots for the intensity data collected by the light sensor in the single slit experiment. The x-axis for all plots displays the location on the screen in meters and the y-axis show the residuals with respect to the best fitting curve for the data in Volts.



(a) Residual plot associated with double slits of width 0.04 mm separated by 0.25 mm



(b) Residual plot associated with double slits of width 0.04 mm separated by 0.25 mm



(b) Residual plot associated with double slits of width 0.08 mm separated by 0.25 mm

The above graphs show residual plots for the intensity data collected by the light sensor in the double slit experiment. The x-axis for all plots displays the location of the peak in the central envelope in meters and the y axis displays how close to the diffraction curve the peaks of the central envelope are in Volts.

Raw Data

https://q.utoronto.ca/groups/538315/files/folder/InterFerence%20and%20diffraction%20data

Sample Calculations

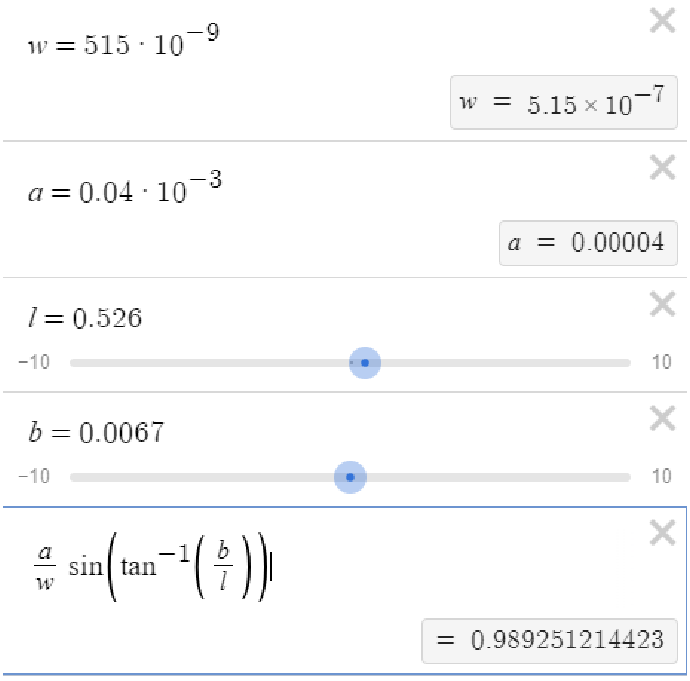


Figure 9: Verification of the Heisenberg uncertainty principle for the slit width of 0.04mm

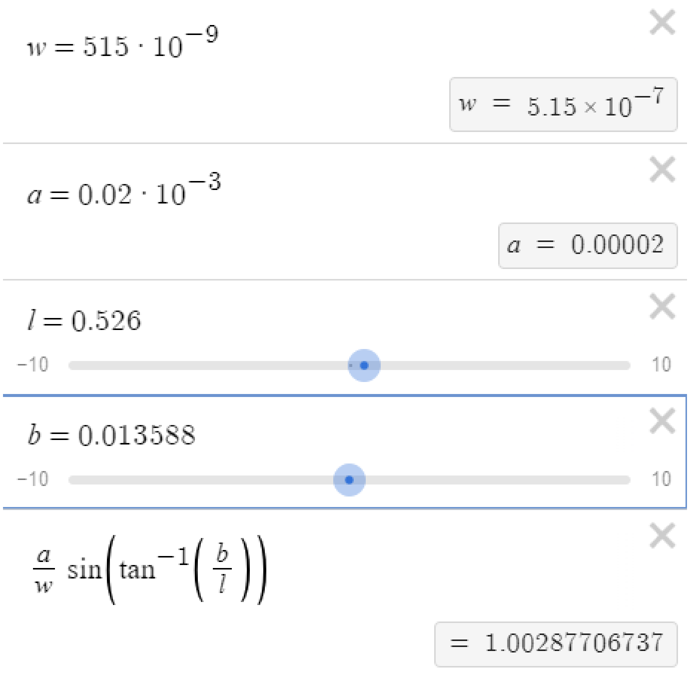


Figure 10: Verification of the Heisenberg uncertainty principle for the slit width of 0.02mm

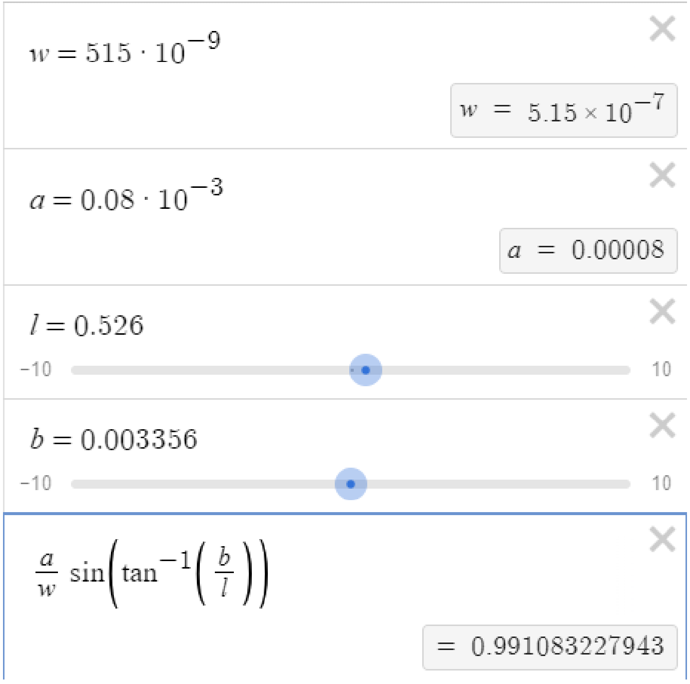


Figure 11: Verification of the Heisenberg uncertainty principle for the slit width of 0.08mm