Radioactive Decay

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Abstract

The purpose of this experiment is to calculate the half-life of a Barium 137 isotope and to statistically analyse the radioactive count of a fiesta plate manufactured before 1970 by measuring the radioactive activity with a Geiger counter. The main result obtained from this experiment was that the half life of the BA-137 isotope is 2.5 ± 0.04 min. The radioactive counts of the Fiesta plate followed the Poisson distribution, with (radioactive) count of 40 being the most frequent. The Poisson Distribution and Gaussian Distribution of the data follow the same pattern.

Introduction

Radioactive decay happens to emission events, whose intensity as a function of time is described by the exponential formula

$$I(t) = I_0 exp(rt) \tag{1}$$

r is the rate of decay, and y_0 is the initial intensity of radioactive decay.[3] Setting $I(t)$ to be equal to half of the radioactive intensity output $I_0\frac{1}{2}$, one can then solve for the half life value. The solution is

$$t_{1/2} = \frac{ln(\frac{1}{2})}{r} \tag{2}$$

To find the half life value, non linear regression will be performed on the exponential model, where its parameter can be used to determine the half life value. Equation (1) can be used.

$$y = b(exp(ax)) \tag{3}$$

By taking the logarithm of the linear equation (3), one obtains another method of non linear regression.

$$ln(y) = ax + ln(b) \tag{4}$$

Accurate measurement of radioactive decay requires the count rate R; Δt is the sample time, N is number of events and $u(N)$ is the uncertainty of the data[3].

$$R = \frac{N}{\Delta t} \pm \frac{u(N)}{\Delta t} \tag{5}$$

Two sources of radiation make up the final reading N_s: the emissions from a source N_T and the background radiation N_b, which is subtracted from the latter.

$$N_s = N_T - N_b \tag{6}$$

The uncertainties of the data $u(N_s)$ are propagated by

$$u(N_s) = \sqrt{u(N_T)^2 + u(N_b)^2} = \sqrt{N_T + N_b} \tag{7}$$

The mean of the background radiation data is used assuming its constant statistics. When performing non linear regression on the linear model (4), taking the logarithm of y will require uncertainty propagation to be accurate for the data $z_i = ln(y_i)$.

$$u(z_i) = \sqrt{\left(\frac{\partial z_i}{\partial y_i} u(y_i)\right)^2} \tag{8}$$

The reduced Chi r^2 metric will be used to assess the goodness of fit for our models and data. Reduced means that the dependence on data degree's of freedom is removed.

$${\chi_{red}}^2 = \frac{1}{N-m} \sum_i^N \left(\frac{y_i - y(x_i)}{u(y_i)}\right)^2 \tag{9}$$

N is number of observations and n is the number of parameters. We will be dealing with two parameters.The error of the calculated parameters can also be used to assess the data and fit quality. It is calculated from the variance ρ of a parameter. Variance of the parameter is given by the curve-fit function from the Python programming language. Lastly, parts of this lab will use statistics. The Poisson and Gaussian probability mass distribution functions are provided for by the Python programming language. Their formula can be found in appendix.

Methods and Procedure

A Geiger counter is linked to a computer for automatic data collection. A radioactive source is then brought close under the Geiger counter for the device to read the radiation counts.

The experiment consists of two parts which differ in radioactive source used. In the first part the radioactive source is a metastable Barium 137 isotope created by flowing acid through a generator containing Cesium 137. The radioactive count was taken every 20 seconds up to 20 minutes.

The second part uses a Fiesta Plate from before 1970. The kitchen-wares contain small traces of Uranium Oxide.[3] The radioactive count was taken every 3 seconds up to 20 minutes. Images of the experiment can be found in the Appendix.

Time was plotted against the device counts of the ionization. The half life is determined through curve fitting to find the parameter values; both the data and its line of best fit were plotted on the same graph. The curve fitting function also calculated the parameter variance. Chi r^2 values were calculated for both parts, histogram was plotted for the Fiesta-ware data. Random number analysis was performed on the Fiesta Plate data using Probability mass and Probability density functions.

Results and Analysis

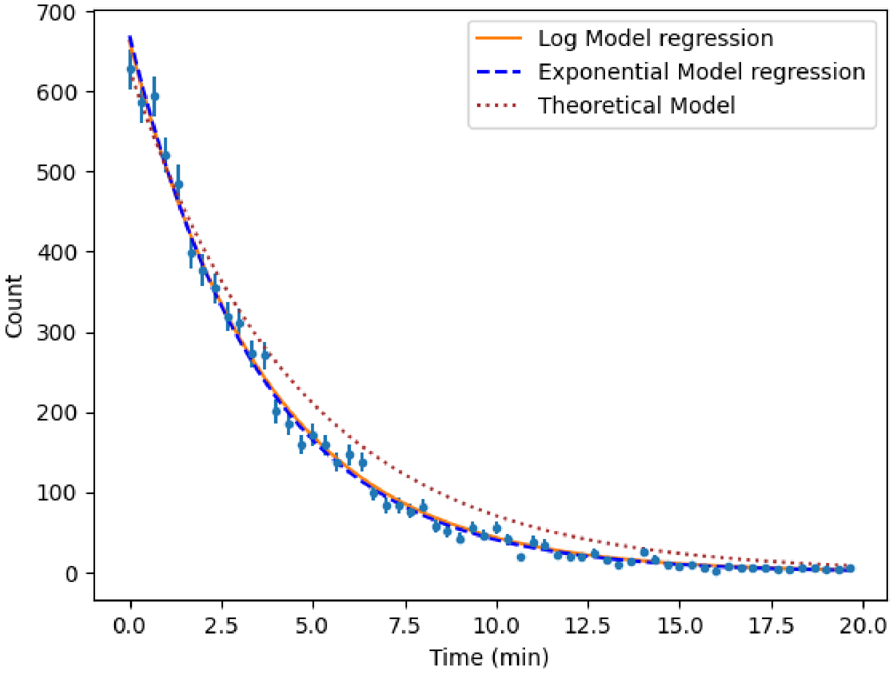
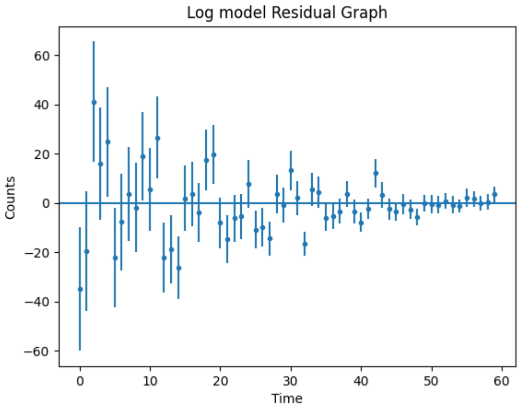


Figure 1: Geiger Counter Data curve fitted using three models. The Solid Line fits the data using the log linear model from Equation 4. The Dashed line fits the same data using the Equation 3 model.The dotted line represents the The data fitted to a theoretical model with the ideal decay rate.

The count data from the Geiger counter for the Ba$-$137 isotope and the associated background radiation data was recorded as text files. [1]. Mean radiation from the background data is subtracted from each count value. The uncertainty for each count is calculated using Equation 7. The data is fitted to the Equation 4 using curve fit. The output parameter received are the decay rate and the log of the initial count value. The value for decay rate received is 0.09 ± 0.001 and value for the initial count is calculated by taking the exponent of the output value which comes out to be 663 ± 11. The error is calculated using uncertainty propagation Equation 8. The half life is calculated using the obtained rate in Equation 2. The value for half life is 2.5 ± 0.04 min. The error is calculation for half life is calculated as shown in the Appendix. The ideal half life for BA$-$137 is 2.6 minutes.[2] The ideal value falls within the error range of the obtained half life proving that the Equation 4 is very effective in fitting the data. Figure 2 shows the residual plot for Equation 4 model. The reduced χ^2 value obtained for this model is 1.25. The value is slightly greater than 1 indicated that the uncertainties were underestimated. The only source of uncertainty considered for this experiment was the statistical uncertainty in the count from the Geiger counter. This uncertainty was calculated using the $\sqrt{N}$ model. The higher the count data the higher the uncertainty. This can be seen in both graphs as the error bars get smaller as the count gets smaller.



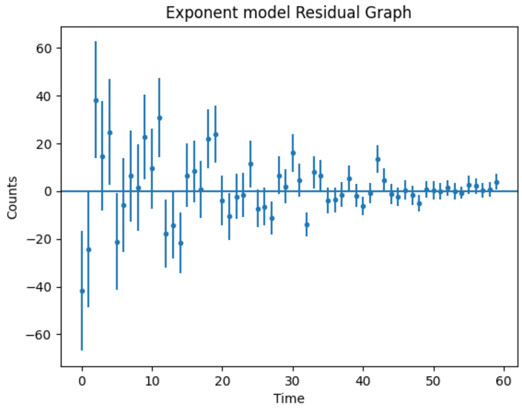


Figure 2: Residual plotted for Equation 4 model.

Figure 3: Residual plotted for Equation 3 model.

The count data fitted to the Equation 3 model is displayed as dashed blue curve in Figure 1. This model gives a very similar curve to the log model. This model outputs the decay rate as 0.09 ± 0.001 which is almost exactly the same as the last model and the half life calculated is the also the same 2.5 ± 0.03 min. The initial intensity received from this model was 669 ± 11. The reduced χ^2 value for this model was 1.15 suggesting that this model was a better fit than the last one. This value is also slightly higher than 1 suggesting that there was maybe another source of uncertainty that was not considered in the experiment.

For the theoretical model the average of all the rates was used as the decay rate and the first count data was used as the initial intensity. The curve for this is displayed as the black dotted line in Figure 1. The half life obtained from this method was 2.2 ± 26 min. The uncertainty for this model was extremely high indicating that this method is not very effective. Th reduced χ^2 value obtained for this model was 12.9. This high χ^2 value supports the residual graph shown in Figure 4.

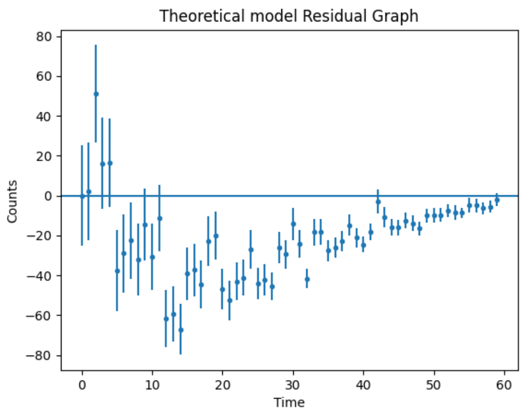


Figure 4: Residual plotted for Theoretical model.

The second part of the experiment deals with random data obtained from the Fiesta plate and its related background data. The count data is filtered using the background data just as before and the histogram for the data is displayed in Figure 5.

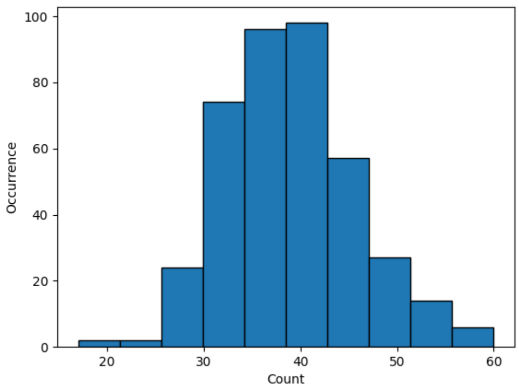


Figure 5: Histogram for the count data for the Fiesta plate. The x axis displays the count value recorded by the Geiger counter and the Y -axis displays the number of times the count occurs in the data

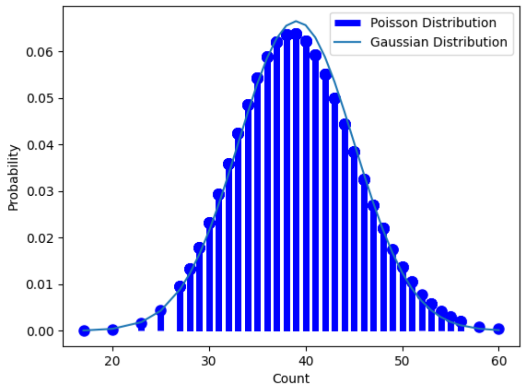


Figure 6: The probability distribution for the count data. The Poisson distribution is displayed by the blue lines. The Gaussian Distribution is displayed as a curve.

The random data is evaluated using the python Poisson function and the output is plotted as blue lines in Figure 6. The same data is analyzed using the Probability Density function and the resulting Gaussian curve is displayed in Figure 6. The mean for the data was calculated as 39.03 and the standard deviation for the data was 6.24. Since, both the Poisson distribution and the Gaussian distribution look the same it is fair to assume that there are enough data points.

Conclusion

In conclusion, the half life of the BA-137 sample is 2.5 ± 0.04 minutes for the logarithm model and the exponent model. Both models are very effective in modelling the Geiger counter data. The slightly smaller χ^2 value for the exponent model suggets that the model is a better fit.

The probability distribution of the radioactive counts for the Fiesta plate follows the Poisson distribution, with radioactive count of around 40 being the most frequent. The Gaussian probability density function looked similar to the Poisson function regarding the Fiesta plate radioactive data, however the Gaussian distribution graph for the background radioactive data was less similar in appearance to its histogram and Poisson graph.

References

[1] https://q.utoronto.ca/courses/337705/files/30422032?wrap=1
https://q.utoronto.ca/courses/337705/files/30422029?wrap=1
Geiger Counter data for BA-137 and related background data
2023, February 28.

[2] https://q.utoronto.ca/courses/337705/files/30422030?wrap=1
https://q.utoronto.ca/courses/337705/files/30422031?wrap=1
Geiger Counter data for the Fiesta plate and related background data
2023, February 28.

[3] https://q.utoronto.ca/courses/337705/files/29613567?wrap=1
LAB manual for Radioactive Decay
2023, February 28.
2023, February 19.

Appendix

Figure 7: Geiger counter used in this lab

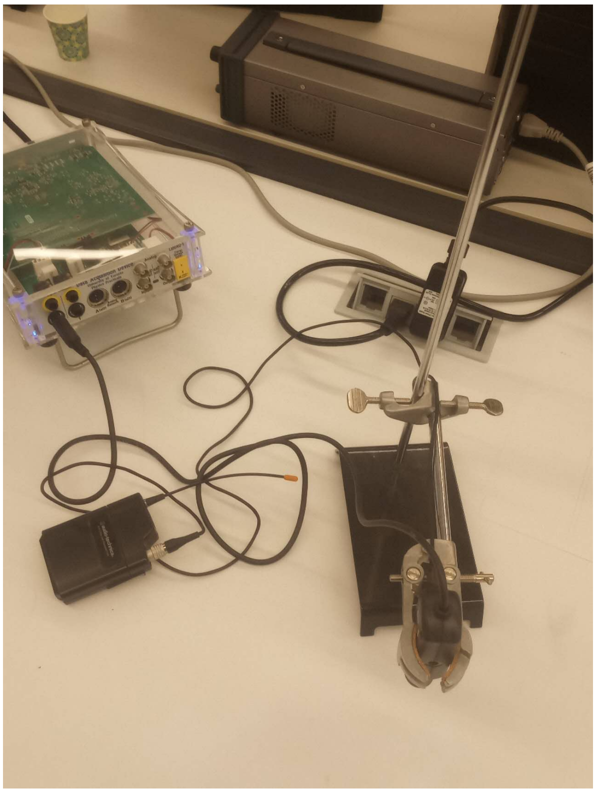


Figure 8: Setup of the Geiger counter and connection to the computer

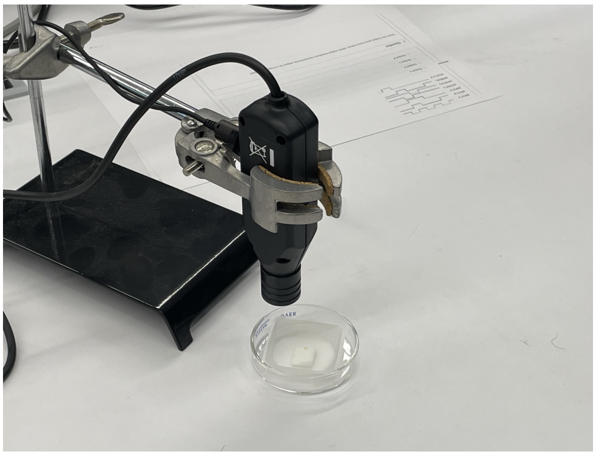


Figure 9: The metastable Barium 137 isotope

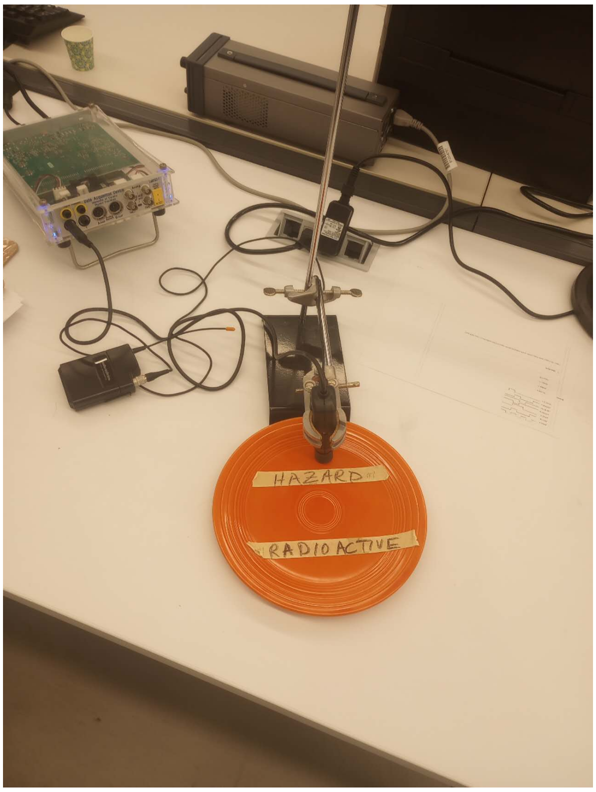


Figure 10: Fiesta plate

Uncertainty Calculations:

Geiger count uncertainty calculation:

For a sample Geiger count of 631 the uncertainty of this count is $\sqrt{631}$. The mean background radiation for the give data is 3.33 and the uncertainty is $\sqrt{3.33}$ So the uncertainty for the count value is given by $\sqrt{631 + 3.33} = 25.1$

Half Life Uncertainty Calculation:

Decay rate obtained from curve fit: $0.09079032 \pm 0.00125184$ The half life is obtained by taking reciprocal of this value and converting the value to minutes:

$$half\ life = \frac{ln(1/2)}{0.09079032} * \frac{20}{60} = 2.544864$$

$$half\ life\ error = \frac{ln(1/2)}{0.09079032^2} * 0.00125184 * \frac{20}{60} = 0.0336523$$

Poisson distribution

$$P_\mu(n) = e^{-\mu}\frac{\mu^n}{\Gamma(n+1)} \tag{10}$$

Gaussian distribution

$$f(x) = \frac{1}{\rho\sqrt{2\pi}} exp\left(\frac{-1}{2}\frac{(x-\mu)^2}{\rho^2}\right) \tag{11}$$

μ is the expected mean of counts per counting interval, and ρ is the standard deviation of counts per counting interval.