University of Salzburg | Lecturer: Roland Kwitt

Machine Learning (911.236)

Exercise sheet **E**

Exercise 1. 4 P.

Take a hypothesis class $\mathcal{H}$ with $\mathrm{VC}(H) = d$ and let m (number of samples) be $\geq d$. From Sauer's lemma, we know that, in that case,

$$\tau_{\mathcal{H}}(m) \leq \left(\frac{em}{d}\right)^d = O(m^d) \quad . \tag{1}$$

When proofing this inequality, we proceed as follows:

$$\begin{aligned}
\tau_{\mathcal{H}}(m) &\leq \sum_{i=1}^{d} \binom{m}{i} \\
&\leq \sum_{i=1}^{d} \binom{m}{i} \left(\frac{m}{d}\right)^{d-i} \\
&\leq \sum_{i=1}^{m} \binom{m}{i} \left(\frac{m}{d}\right)^{d-i} \\
&\cdots \text{(fill in)} \\
&\leq \left(\frac{m}{d}\right)^d e^d
\end{aligned}$$

Please provide arguments for **each step** in this inequality chain and **fill in the blanks**, i.e., the $\cdots$ part to arrive at the final statement from Eq. (1)

Total #points: 4 P.