Structured and Hybrid Products Final Project - Notes

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1 Setup

- `EUROSTOXX50` is quantoed from EUR to USD. We can use a exchange rate + GBM framework to model its dynamics, which would result in quantoed stock dynamics as suggested by Proposition 17.4 and 17.5:

$$d\tilde{S}_f = \tilde{S}_f(\alpha_f + \alpha_X + \sigma_f \sigma_X^*)dt + \tilde{S}_f(\sigma_f + \sigma_X)d\overline{W}_t \qquad (\mathbb{P}\text{-measure})$$
$$d\tilde{S}_f = \tilde{S}_f r_d dt + \tilde{S}_f(\sigma_f + \sigma_X)dW_t \qquad (\mathbb{Q}\text{-measure})$$

  If we were to simulate exchange rate and stock price simultaneously, we might make use of Remark 17.2.4 in Bjork and treat the diffusion term as

$$\sigma_f + \sigma_X = \sqrt{\delta_f^2 + \delta_X^2 + 2\delta_f \delta_X \rho_{fX}}$$

  where δ_f, δ_X, ρ_{fX} are idiosyncratic diffusion coefficient for foreign stock and exchange rate (domestic/foreign), and their correlation coefficient.

  To extend the chapter 17 module, we might replace the r_d constant with $r_d(t)$ to allow for term structure.

  Under said assumption,

$$\frac{S(T)}{S(0)} = \exp\left[\int_0^T r_d(s)ds - \frac{1}{2}\int_0^T \sigma_S^2(s)ds + \int_0^T \sigma_S(s)dW_s\right] \qquad (1)$$
$$\text{where} \quad \sigma_S(t,S) = \sigma_f(t,S) + \sigma_X(t,S)$$

  and the volatility is assumed constant. Should at the late stage of the project, that we decide to consider real data, the data source in Yahoo Finance, `^STOXX50E`, denotes the index in EUR instead of USD. This implies that quanto may need to be done manually *i.e.* calibrate on synthesized `^STOXX50E` $\cdot$ USD/EUR exchange rate data.

- The term rate ratio $\dfrac{L(T,T,T+\Delta)}{L(0,T,T+\Delta)}$ is given by

$$\begin{aligned}
\frac{L(T,T,T+\Delta)}{L(0,T,T+\Delta)} &= \frac{1-p(T,T+\Delta)}{\Delta p(T,T+\Delta)} \bigg/ \frac{p(0,T)-p(0,T+\Delta)}{\Delta p(0,T+\Delta)} \\
&= \frac{1-p(T,T+\Delta)}{p(0,T)-p(0,T+\Delta)} \cdot \frac{p(0,T+\Delta)}{p(T,T+\Delta)} \\
&= \frac{\dfrac{1}{p(T,T+\Delta)} - 1}{\dfrac{1}{p(0,T,T+\Delta)} - 1}
\end{aligned}$$

which reduces forward term rates to forward bond prices. Furthermore, from the instantaneous forward rate perspective,

$$\begin{aligned}
\frac{L(T,T,T+\Delta)}{L(0,T,T+\Delta)} &= \frac{\dfrac{1}{p(T,T+\Delta)} - 1}{\dfrac{1}{p(0,T,T+\Delta)} - 1} \\
&= \frac{\exp\left(\displaystyle\int_T^{T+\Delta} f(T,u)du\right) - 1}{\exp\left(\displaystyle\int_T^{T+\Delta} f(0,u)du\right) - 1}
\end{aligned}$$

- *Claim structure.* The T-claim is given by

$$\boldsymbol{\chi}(N,k,k',\Delta) = N \max\left\{0, \left(k - \frac{S(T)}{S(0)}\right)\left(\frac{L(T,T,T+\Delta)}{L(0,T,T+\Delta)} - k'\right)\right\}$$

where a $\max(0,\cdot) \triangleq (\cdot)^+$ rectifier is applied on a product expression. This indicates that the rectifier would only activate if the two component in the product have opposite signs. In particular, the rectifier equals 0 precisely if

$$\begin{aligned}
&k > \frac{S(T)}{S(0)} \text{ and } k' < \frac{L(T,T,T+\Delta)}{L(0,T,T+\Delta)}; \\
\text{OR}\quad &k < \frac{S(T)}{S(0)} \text{ and } k' > \frac{L(T,T,T+\Delta)}{L(0,T,T+\Delta)}
\end{aligned}$$

From another view, the value of the contract increases if both ratio deviates further from k and k', but in the same direction (simultaneously above or below).

2 To Do List

- Important: ascertain whether the final result need to incorporate real data or not.

- If so, figure out how to obtain SOFR forward rate data (Bloomberg? CME?)
- If not, see below simulation modules.

- Deduce the pricing formula for the contract under $\mathbb{Q}$, *optional*.

- Python modules to be constructed:
  - Simulation module for exchange rate.
  - Simulation module for European stock.
  - Simulation module for interest rate (for selected short rate model/forward rate model).
  - Integration module that computes forward term rates from simulated interest rates.
  - Main module that combines above simulation into one pricing scheme.

- Finally, if time, obtain available real-world data to calibrate all parameters, and use a data sample to compute pseudo-realistic price of the contract in interest.