Fall 2023 - Project

We would like to price a contract paying on the settlement date $T + \Delta$ the payoff amount in USD, defined on the maturity date T as:

$$N. \max \left[0, \left(k - \frac{S(T)}{S(0)}\right) \cdot \left(\frac{L(T, T, T + \Delta)}{L(0, T, T + \Delta)} - k'\right)\right]$$

with:

- N the Notional in USD (e.g. 1 million USD)
- S(t) the EUROSTOXX50 spot price at time t, quantoed from EUR into USD
- $L(t, T, T + \Delta)$ is the 3-month USD forward rate (SOFR) between T and $T + \Delta$, observed at time t
- Δ is a period of 3-month (0.25 years). T the maturity date (e.g. 3 years) and $T + \Delta$ the settlement date (e.g. 3.25 years)
- k, k' given relative strike prices (e.g. both could be 1.00 or ...)

Provide a pricing routine (e.g. Python script) calculating the price of this contract, taking as inputs: the deal terms (T, Δ, k, k') and the relevant market data (interest rates, volatilities, spot prices, correlations).

Explain your precise assumptions and methodology choices clearly in an accompanying write-up.