

DSP — IDFT in Matrix Notation (\mathbf{W} , \mathbf{K}) + Signal Synthesis

We synthesize discrete-time signals by **IDFT in matrix form** for different values of (N). We explicitly build the matrices (\mathbf{K}) (outer product of indices) and (\mathbf{W}) (Fourier matrix), and plot the synthesized signals.

```
In [7]: import numpy as np
import matplotlib.pyplot as plt

def fourier_matrices(N:int):
    k = np.arange(N).reshape(-1,1)
    mu = np.arange(N).reshape(1,-1)
    K = k * mu
    W = np.exp(1j * 2*np.pi * K / N)
    return K, W

def idft_matrix(x_mu):
    x_mu = np.asarray(x_mu, dtype=complex).reshape(-1,1)
    N = x_mu.shape[0]
    K, W = fourier_matrices(N)
    x_k = (W @ x_mu) / N
    return x_k.flatten(), K, W

def show_matrices(K, W, max_show=8):
    N = K.shape[0]
    m = min(N, max_show)
    print(f"N={N}\nK (top-left {m}x{m}):\n", K[:m,:m])
    Wr = np.round(W.real[:m,:m], 3)
    Wi = np.round(W.imag[:m,:m], 3)
    print(f"\nW (real, top-left {m}x{m}):\n", Wr)
    print(f"\nW (imag, top-left {m}x{m}):\n", Wi)

def plot_signal(x_k, title="Synthesized signal x[k] (IDFT)":
    k = np.arange(len(x_k))
    plt.figure(figsize=(8,3))
    plt.stem(k, x_k.real)
    plt.title(title + " - real part")
    plt.xlabel("k")
    plt.ylabel("Re{x[k]}")
    plt.tight_layout()
    plt.show()

    if np.max(np.abs(x_k.imag)) > 1e-9:
        plt.figure(figsize=(8,3))
        plt.stem(k, x_k.imag)
        plt.title(title + " - imaginary part")
        plt.xlabel("k")
        plt.ylabel("Im{x[k]}")
        plt.tight_layout()
        plt.show()
```

1) Show K and W for N=4

```
In [8]: K4, W4 = fourier_matrices(4)
        show_matrices(K4, W4, max_show=4)
```

N=4

K (top-left 4x4):

```
[[0 0 0 0]
 [0 1 2 3]
 [0 2 4 6]
 [0 3 6 9]]
```

W (real, top-left 4x4):

```
[[ 1.  1.  1.  1.]
 [ 1.  0. -1. -0.]
 [ 1. -1.  1. -1.]
 [ 1. -0. -1.  0.]]
```

W (imag, top-left 4x4):

```
[[ 0.  0.  0.  0.]
 [ 0.  1.  0. -1.]
 [ 0.  0. -0.  0.]
 [ 0. -1.  0.  1.]]
```

2) Variants — IDFT synthesis for the given (x_{μ})

```
In [9]: variants = [
        [6, 2, 4, 3, 4, 5, 0, 0, 0, 0],
        [10, 5, 6, 6, 2, 4, 3, 4, 5, 0, 0, 0, 0],
        [6, 2, 4, 3, 4, 5, 0, 0, 0, 0],
        [6, 2, 4, 3, 4, 5, 0, 0, 0],
        [6, 4, 4, 5, 3, 4, 5, 0, 0, 0, 0],
        [7, 2, 4, 3, 4, 5, 0, 0, 0, 0],
        [6, 8, 2, 4, 3, 4, 5, 0, 0, 0],
        [6, 2, 4, 4, 4, 5, 0, 0, 0, 0],
        [6, 5, 4, 3, 4, 5, 0, 0, 0, 0],
        [6, 2, 4, 3, 4, 4, 0, 0, 0, 0],
    ]

    for i, xmu in enumerate(variants, start=1):
        xk, K, W = idft_matrix(xmu)
        print(f"\n=== Variant {i} — N={len(xmu)} ===")
        show_matrices(K, W, max_show=8)
        plot_signal(xk, title=f"Variant {i}: N={len(xmu)}")
```

=== Variant 1 – N=10 ===

N=10

K (top-left 8x8):

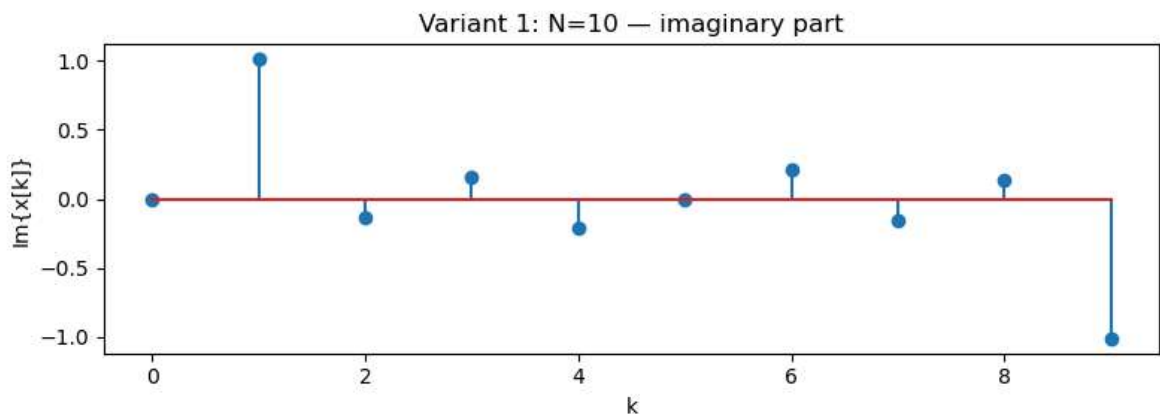
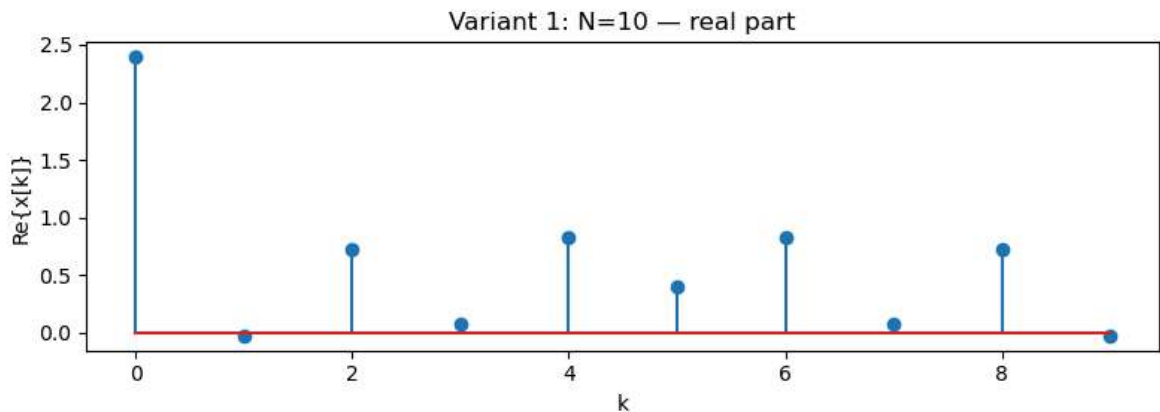
```
[[ 0  0  0  0  0  0  0  0]
 [ 0  1  2  3  4  5  6  7]
 [ 0  2  4  6  8 10 12 14]
 [ 0  3  6  9 12 15 18 21]
 [ 0  4  8 12 16 20 24 28]
 [ 0  5 10 15 20 25 30 35]
 [ 0  6 12 18 24 30 36 42]
 [ 0  7 14 21 28 35 42 49]]
```

W (real, top-left 8x8):

```
[[ 1.    1.    1.    1.    1.    1.    1.    1. ]
 [ 1.    0.809 0.309 -0.309 -0.809 -1.   -0.809 -0.309]
 [ 1.    0.309 -0.809 -0.809  0.309  1.    0.309 -0.809]
 [ 1.   -0.309 -0.809  0.809  0.309 -1.    0.309  0.809]
 [ 1.   -0.809  0.309  0.309 -0.809  1.   -0.809  0.309]
 [ 1.   -1.    1.   -1.    1.   -1.    1.   -1. ]
 [ 1.   -0.809  0.309  0.309 -0.809  1.   -0.809  0.309]
 [ 1.   -0.309 -0.809  0.809  0.309 -1.    0.309  0.809]]
```

W (imag, top-left 8x8):

```
[[ 0.    0.    0.    0.    0.    0.    0.    0. ]
 [ 0.    0.588 0.951 0.951 0.588 0.   -0.588 -0.951]
 [ 0.    0.951 0.588 -0.588 -0.951 -0.    0.951 0.588]
 [ 0.    0.951 -0.588 -0.588 0.951 0.   -0.951 0.588]
 [ 0.    0.588 -0.951 0.951 -0.588 -0.    0.588 -0.951]
 [ 0.    0.   -0.    0.   -0.   -0.   -0.    0. ]
 [ 0.   -0.588 0.951 -0.951 0.588 -0.   -0.588 0.951]
 [ 0.   -0.951 0.588 0.588 -0.951 0.    0.951 -0.588]]
```



=== Variant 2 - N=13 ===

N=13

K (top-left 8x8):

```
[[ 0  0  0  0  0  0  0  0]
 [ 0  1  2  3  4  5  6  7]
 [ 0  2  4  6  8 10 12 14]
 [ 0  3  6  9 12 15 18 21]
 [ 0  4  8 12 16 20 24 28]
 [ 0  5 10 15 20 25 30 35]
 [ 0  6 12 18 24 30 36 42]
 [ 0  7 14 21 28 35 42 49]]
```

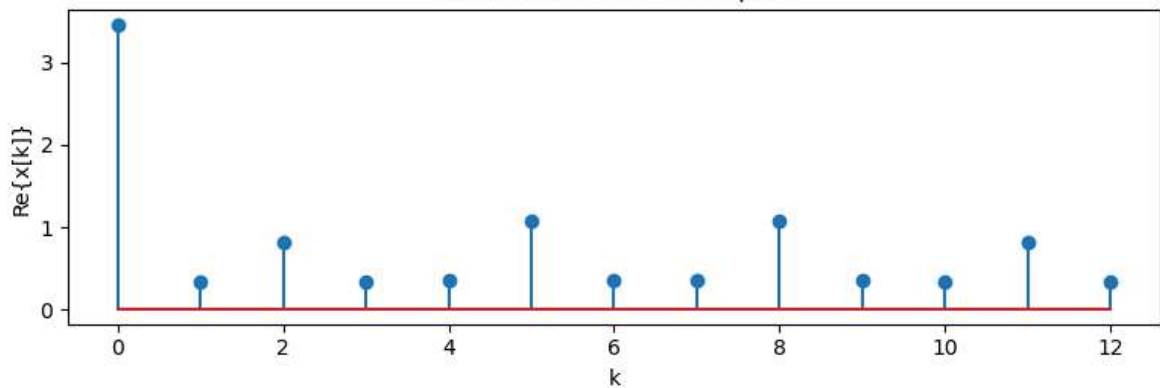
W (real, top-left 8x8):

```
[[ 1.    1.    1.    1.    1.    1.    1.    1. ]
 [ 1.    0.885 0.568 0.121 -0.355 -0.749 -0.971 -0.971]
 [ 1.    0.568 -0.355 -0.971 -0.749 0.121 0.885 0.885]
 [ 1.    0.121 -0.971 -0.355 0.885 0.568 -0.749 -0.749]
 [ 1.   -0.355 -0.749 0.885 0.121 -0.971 0.568 0.568]
 [ 1.   -0.749 0.121 0.568 -0.971 0.885 -0.355 -0.355]
 [ 1.   -0.971 0.885 -0.749 0.568 -0.355 0.121 0.121]
 [ 1.   -0.971 0.885 -0.749 0.568 -0.355 0.121 0.121]]
```

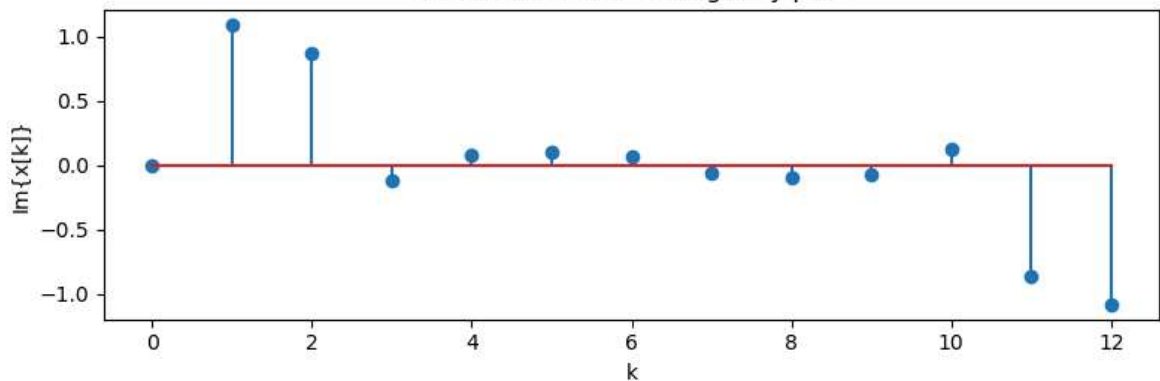
W (imag, top-left 8x8):

```
[[ 0.    0.    0.    0.    0.    0.    0.    0. ]
 [ 0.    0.465 0.823 0.993 0.935 0.663 0.239 -0.239]
 [ 0.    0.823 0.935 0.239 -0.663 -0.993 -0.465 0.465]
 [ 0.    0.993 0.239 -0.935 -0.465 0.823 0.663 -0.663]
 [ 0.    0.935 -0.663 -0.465 0.993 -0.239 -0.823 0.823]
 [ 0.    0.663 -0.993 0.823 -0.239 -0.465 0.935 -0.935]
 [ 0.    0.239 -0.465 0.663 -0.823 0.935 -0.993 0.993]
 [ 0.   -0.239 0.465 -0.663 0.823 -0.935 0.993 -0.993]]
```

Variant 2: N=13 — real part



Variant 2: N=13 — imaginary part



=== Variant 3 - N=10 ===

N=10

K (top-left 8x8):

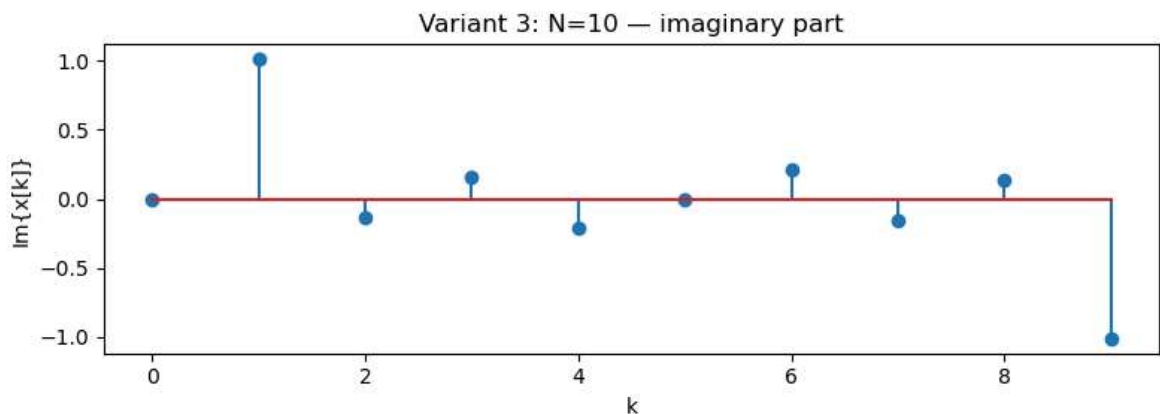
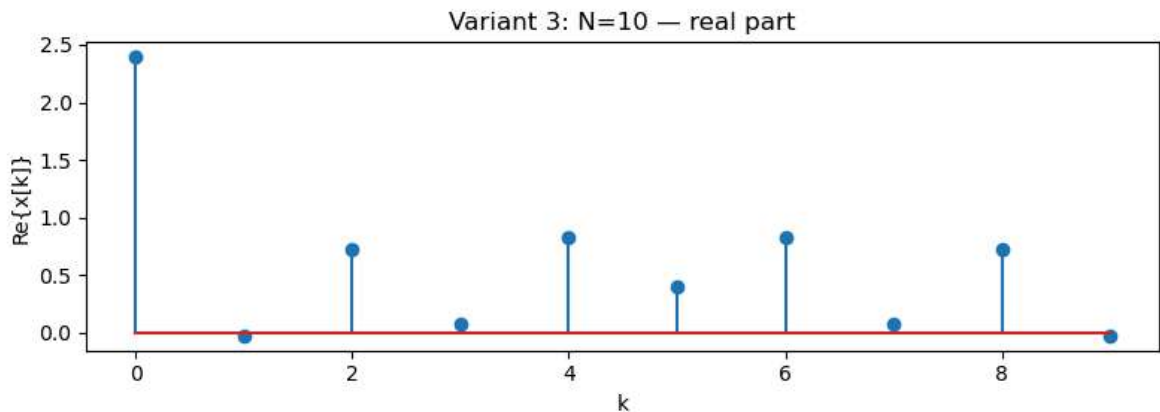
```
[[ 0  0  0  0  0  0  0  0]
 [ 0  1  2  3  4  5  6  7]
 [ 0  2  4  6  8 10 12 14]
 [ 0  3  6  9 12 15 18 21]
 [ 0  4  8 12 16 20 24 28]
 [ 0  5 10 15 20 25 30 35]
 [ 0  6 12 18 24 30 36 42]
 [ 0  7 14 21 28 35 42 49]]
```

W (real, top-left 8x8):

```
[[ 1.    1.    1.    1.    1.    1.    1.    1. ]
 [ 1.    0.809 0.309 -0.309 -0.809 -1.   -0.809 -0.309]
 [ 1.    0.309 -0.809 -0.809  0.309  1.    0.309 -0.809]
 [ 1.   -0.309 -0.809  0.809  0.309 -1.    0.309  0.809]
 [ 1.   -0.809  0.309  0.309 -0.809  1.   -0.809  0.309]
 [ 1.   -1.    1.   -1.    1.   -1.    1.   -1. ]
 [ 1.   -0.809  0.309  0.309 -0.809  1.   -0.809  0.309]
 [ 1.   -0.309 -0.809  0.809  0.309 -1.    0.309  0.809]]
```

W (imag, top-left 8x8):

```
[[ 0.    0.    0.    0.    0.    0.    0.    0. ]
 [ 0.    0.588 0.951 0.951 0.588  0.   -0.588 -0.951]
 [ 0.    0.951 0.588 -0.588 -0.951 -0.    0.951 0.588]
 [ 0.    0.951 -0.588 -0.588 0.951  0.   -0.951 0.588]
 [ 0.    0.588 -0.951 0.951 -0.588 -0.    0.588 -0.951]
 [ 0.    0.   -0.    0.   -0.   -0.   -0.    0. ]
 [ 0.   -0.588 0.951 -0.951 0.588 -0.   -0.588 0.951]
 [ 0.   -0.951 0.588 0.588 -0.951  0.    0.951 -0.588]]
```



=== Variant 4 – N=9 ===

N=9

K (top-left 8x8):

```
[[ 0  0  0  0  0  0  0  0]
 [ 0  1  2  3  4  5  6  7]
 [ 0  2  4  6  8 10 12 14]
 [ 0  3  6  9 12 15 18 21]
 [ 0  4  8 12 16 20 24 28]
 [ 0  5 10 15 20 25 30 35]
 [ 0  6 12 18 24 30 36 42]
 [ 0  7 14 21 28 35 42 49]]
```

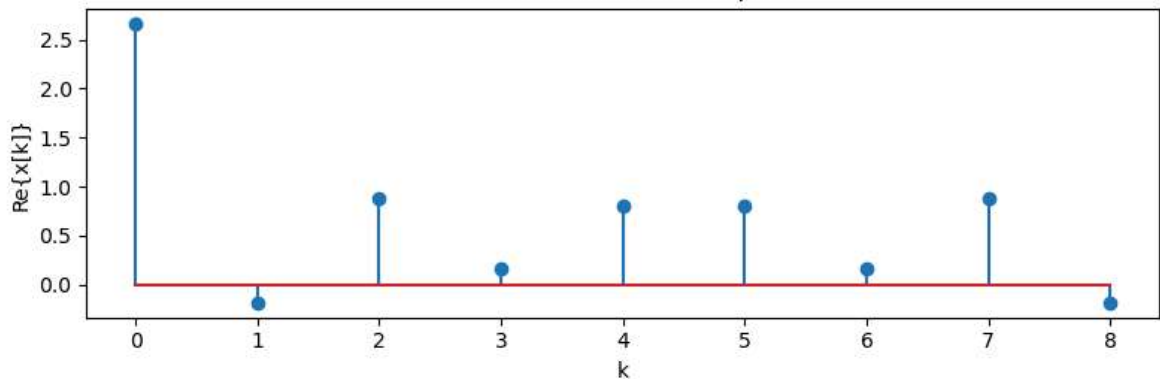
W (real, top-left 8x8):

```
[[ 1.    1.    1.    1.    1.    1.    1.    1. ]
 [ 1.    0.766 0.174 -0.5  -0.94 -0.94 -0.5  0.174]
 [ 1.    0.174 -0.94 -0.5   0.766 0.766 -0.5  -0.94 ]
 [ 1.   -0.5  -0.5   1.   -0.5 -0.5   1.   -0.5 ]
 [ 1.   -0.94  0.766 -0.5   0.174 0.174 -0.5  0.766]
 [ 1.   -0.94  0.766 -0.5   0.174 0.174 -0.5  0.766]
 [ 1.   -0.5  -0.5   1.   -0.5 -0.5   1.   -0.5 ]
 [ 1.    0.174 -0.94 -0.5   0.766 0.766 -0.5  -0.94 ]]
```

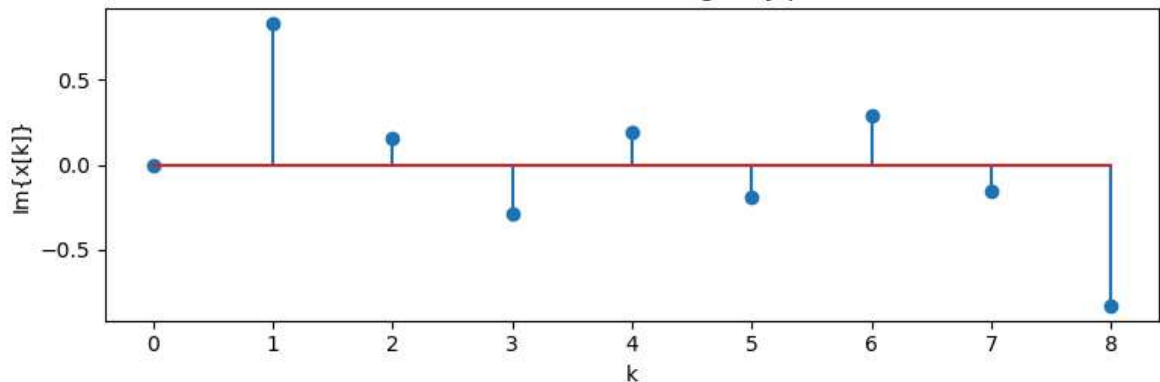
W (imag, top-left 8x8):

```
[[ 0.    0.    0.    0.    0.    0.    0.    0. ]
 [ 0.    0.643 0.985 0.866 0.342 -0.342 -0.866 -0.985]
 [ 0.    0.985 0.342 -0.866 -0.643 0.643 0.866 -0.342]
 [ 0.    0.866 -0.866 -0.    0.866 -0.866 -0.    0.866]
 [ 0.    0.342 -0.643 0.866 -0.985 0.985 -0.866 0.643]
 [ 0.   -0.342 0.643 -0.866 0.985 -0.985 0.866 -0.643]
 [ 0.   -0.866 0.866 -0.   -0.866 0.866 -0.   -0.866]
 [ 0.   -0.985 -0.342 0.866 0.643 -0.643 -0.866 0.342]]
```

Variant 4: N=9 — real part



Variant 4: N=9 — imaginary part



=== Variant 5 - N=11 ===

N=11

K (top-left 8x8):

```
[[ 0  0  0  0  0  0  0  0]
 [ 0  1  2  3  4  5  6  7]
 [ 0  2  4  6  8 10 12 14]
 [ 0  3  6  9 12 15 18 21]
 [ 0  4  8 12 16 20 24 28]
 [ 0  5 10 15 20 25 30 35]
 [ 0  6 12 18 24 30 36 42]
 [ 0  7 14 21 28 35 42 49]]
```

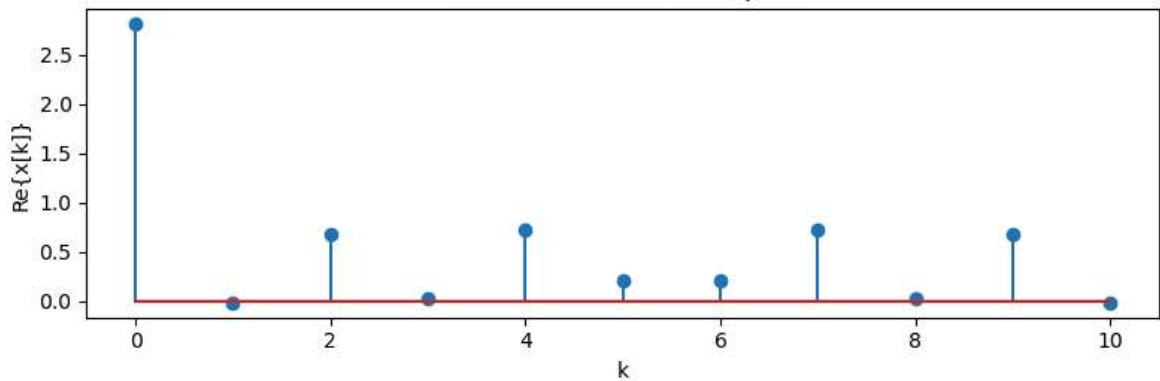
W (real, top-left 8x8):

```
[[ 1.    1.    1.    1.    1.    1.    1.    1. ]
 [ 1.    0.841 0.415 -0.142 -0.655 -0.959 -0.959 -0.655]
 [ 1.    0.415 -0.655 -0.959 -0.142 0.841 0.841 -0.142]
 [ 1.   -0.142 -0.959 0.415 0.841 -0.655 -0.655 0.841]
 [ 1.   -0.655 -0.142 0.841 -0.959 0.415 0.415 -0.959]
 [ 1.   -0.959 0.841 -0.655 0.415 -0.142 -0.142 0.415]
 [ 1.   -0.959 0.841 -0.655 0.415 -0.142 -0.142 0.415]
 [ 1.   -0.655 -0.142 0.841 -0.959 0.415 0.415 -0.959]]
```

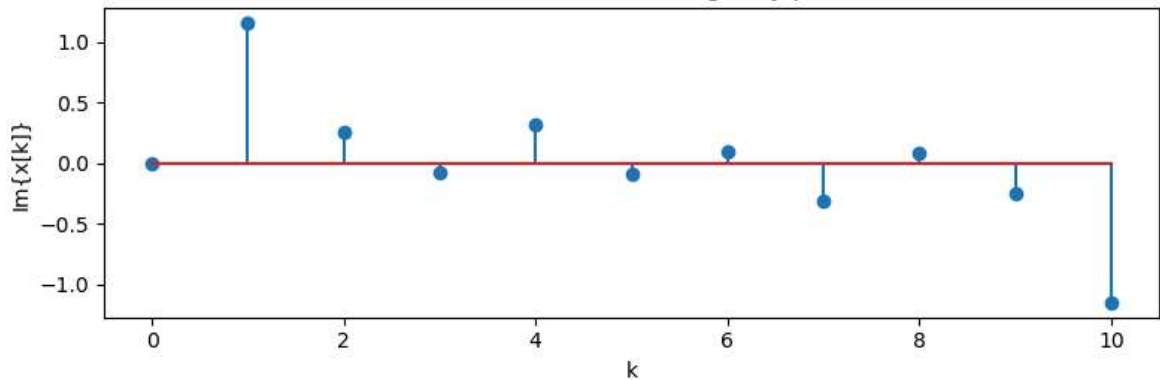
W (imag, top-left 8x8):

```
[[ 0.    0.    0.    0.    0.    0.    0.    0. ]
 [ 0.    0.541 0.91 0.99 0.756 0.282 -0.282 -0.756]
 [ 0.    0.91 0.756 -0.282 -0.99 -0.541 0.541 0.99 ]
 [ 0.    0.99 -0.282 -0.91 0.541 0.756 -0.756 -0.541]
 [ 0.    0.756 -0.99 0.541 0.282 -0.91 0.91 -0.282]
 [ 0.    0.282 -0.541 0.756 -0.91 0.99 -0.99 0.91 ]
 [ 0.   -0.282 0.541 -0.756 0.91 -0.99 0.99 -0.91 ]
 [ 0.   -0.756 0.99 -0.541 -0.282 0.91 -0.91 0.282]]
```

Variant 5: N=11 — real part



Variant 5: N=11 — imaginary part



=== Variant 6 - N=10 ===

N=10

K (top-left 8x8):

```
[[ 0  0  0  0  0  0  0  0]
 [ 0  1  2  3  4  5  6  7]
 [ 0  2  4  6  8 10 12 14]
 [ 0  3  6  9 12 15 18 21]
 [ 0  4  8 12 16 20 24 28]
 [ 0  5 10 15 20 25 30 35]
 [ 0  6 12 18 24 30 36 42]
 [ 0  7 14 21 28 35 42 49]]
```

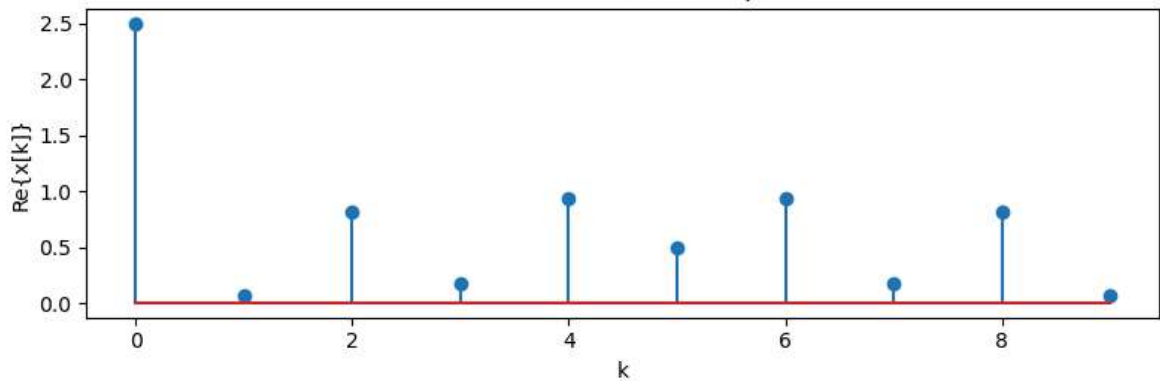
W (real, top-left 8x8):

```
[[ 1.    1.    1.    1.    1.    1.    1.    1. ]
 [ 1.    0.809 0.309 -0.309 -0.809 -1.   -0.809 -0.309]
 [ 1.    0.309 -0.809 -0.809  0.309  1.    0.309 -0.809]
 [ 1.   -0.309 -0.809  0.809  0.309 -1.    0.309  0.809]
 [ 1.   -0.809  0.309  0.309 -0.809  1.   -0.809  0.309]
 [ 1.   -1.    1.   -1.    1.   -1.    1.   -1. ]
 [ 1.   -0.809  0.309  0.309 -0.809  1.   -0.809  0.309]
 [ 1.   -0.309 -0.809  0.809  0.309 -1.    0.309  0.809]]
```

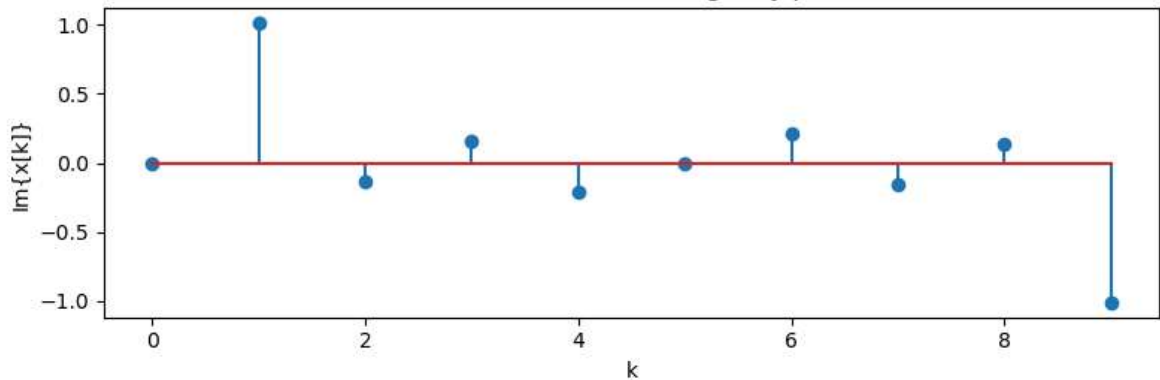
W (imag, top-left 8x8):

```
[[ 0.    0.    0.    0.    0.    0.    0.    0. ]
 [ 0.    0.588 0.951 0.951 0.588 0.   -0.588 -0.951]
 [ 0.    0.951 0.588 -0.588 -0.951 -0.    0.951 0.588]
 [ 0.    0.951 -0.588 -0.588 0.951 0.   -0.951 0.588]
 [ 0.    0.588 -0.951 0.951 -0.588 -0.    0.588 -0.951]
 [ 0.    0.   -0.    0.   -0.   -0.   -0.    0. ]
 [ 0.   -0.588 0.951 -0.951 0.588 -0.   -0.588 0.951]
 [ 0.   -0.951 0.588 0.588 -0.951 0.    0.951 -0.588]]
```

Variant 6: N=10 — real part



Variant 6: N=10 — imaginary part



=== Variant 7 – N=10 ===

N=10

K (top-left 8x8):

```
[[ 0  0  0  0  0  0  0  0]
 [ 0  1  2  3  4  5  6  7]
 [ 0  2  4  6  8 10 12 14]
 [ 0  3  6  9 12 15 18 21]
 [ 0  4  8 12 16 20 24 28]
 [ 0  5 10 15 20 25 30 35]
 [ 0  6 12 18 24 30 36 42]
 [ 0  7 14 21 28 35 42 49]]
```

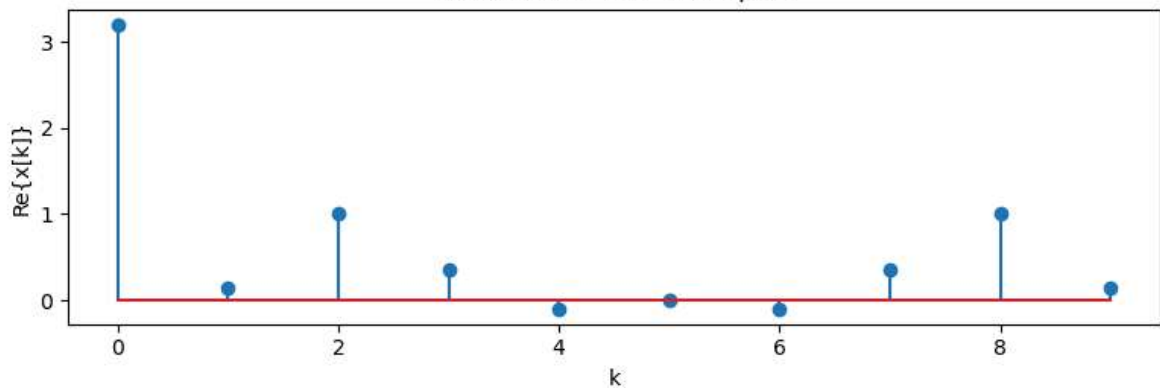
W (real, top-left 8x8):

```
[[ 1.    1.    1.    1.    1.    1.    1.    1. ]
 [ 1.    0.809 0.309 -0.309 -0.809 -1.   -0.809 -0.309]
 [ 1.    0.309 -0.809 -0.809 0.309 1.    0.309 -0.809]
 [ 1.   -0.309 -0.809 0.809 0.309 -1.    0.309 0.809]
 [ 1.   -0.809 0.309 0.309 -0.809 1.   -0.809 0.309]
 [ 1.   -1.    1.   -1.    1.   -1.    1.   -1. ]
 [ 1.   -0.809 0.309 0.309 -0.809 1.   -0.809 0.309]
 [ 1.   -0.309 -0.809 0.809 0.309 -1.    0.309 0.809]]
```

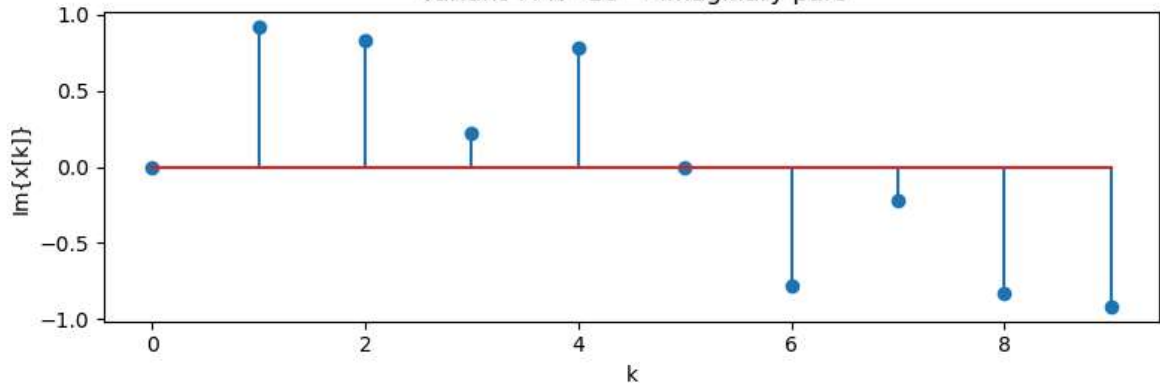
W (imag, top-left 8x8):

```
[[ 0.    0.    0.    0.    0.    0.    0.    0. ]
 [ 0.    0.588 0.951 0.951 0.588 0.   -0.588 -0.951]
 [ 0.    0.951 0.588 -0.588 -0.951 -0.    0.951 0.588]
 [ 0.    0.951 -0.588 -0.588 0.951 0.   -0.951 0.588]
 [ 0.    0.588 -0.951 0.951 -0.588 -0.    0.588 -0.951]
 [ 0.    0.   -0.    0.   -0.   -0.   -0.    0. ]
 [ 0.   -0.588 0.951 -0.951 0.588 -0.   -0.588 0.951]
 [ 0.   -0.951 0.588 0.588 -0.951 0.    0.951 -0.588]]
```

Variant 7: N=10 — real part



Variant 7: N=10 — imaginary part



=== Variant 8 - N=10 ===

N=10

K (top-left 8x8):

```
[[ 0  0  0  0  0  0  0  0]
 [ 0  1  2  3  4  5  6  7]
 [ 0  2  4  6  8 10 12 14]
 [ 0  3  6  9 12 15 18 21]
 [ 0  4  8 12 16 20 24 28]
 [ 0  5 10 15 20 25 30 35]
 [ 0  6 12 18 24 30 36 42]
 [ 0  7 14 21 28 35 42 49]]
```

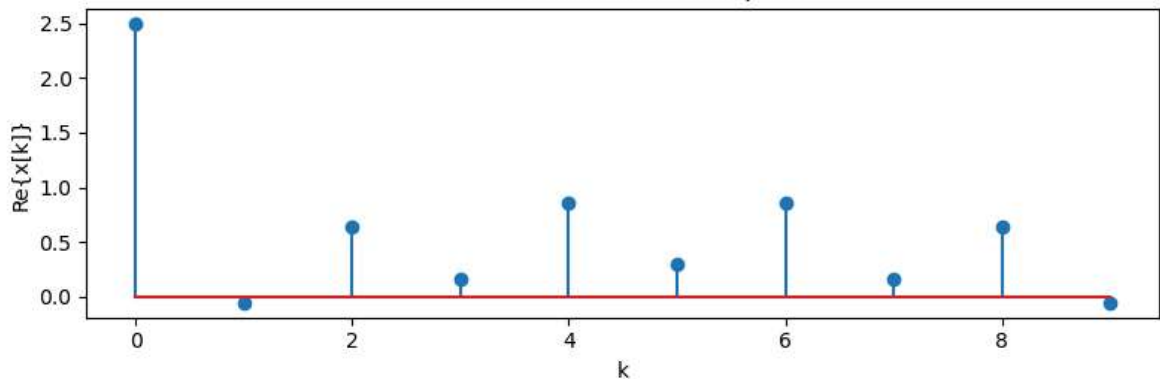
W (real, top-left 8x8):

```
[[ 1.    1.    1.    1.    1.    1.    1.    1. ]
 [ 1.    0.809 0.309 -0.309 -0.809 -1.   -0.809 -0.309]
 [ 1.    0.309 -0.809 -0.809  0.309  1.    0.309 -0.809]
 [ 1.   -0.309 -0.809  0.809  0.309 -1.    0.309  0.809]
 [ 1.   -0.809  0.309  0.309 -0.809  1.   -0.809  0.309]
 [ 1.   -1.    1.   -1.    1.   -1.    1.   -1. ]
 [ 1.   -0.809  0.309  0.309 -0.809  1.   -0.809  0.309]
 [ 1.   -0.309 -0.809  0.809  0.309 -1.    0.309  0.809]]
```

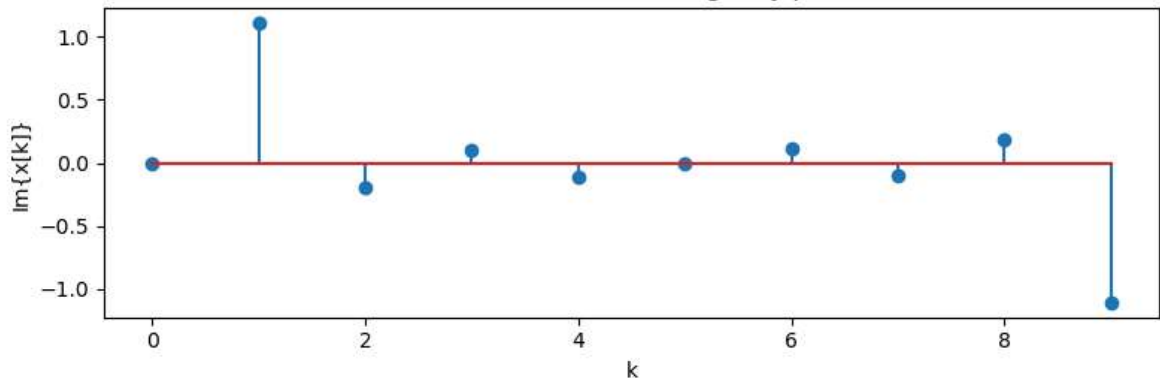
W (imag, top-left 8x8):

```
[[ 0.    0.    0.    0.    0.    0.    0.    0. ]
 [ 0.    0.588 0.951 0.951 0.588 0.   -0.588 -0.951]
 [ 0.    0.951 0.588 -0.588 -0.951 -0.    0.951 0.588]
 [ 0.    0.951 -0.588 -0.588 0.951 0.   -0.951 0.588]
 [ 0.    0.588 -0.951 0.951 -0.588 -0.    0.588 -0.951]
 [ 0.    0.   -0.    0.   -0.   -0.   -0.    0. ]
 [ 0.   -0.588 0.951 -0.951 0.588 -0.   -0.588 0.951]
 [ 0.   -0.951 0.588 0.588 -0.951 0.    0.951 -0.588]]
```

Variant 8: N=10 — real part



Variant 8: N=10 — imaginary part



=== Variant 9 - N=10 ===

N=10

K (top-left 8x8):

```
[[ 0  0  0  0  0  0  0  0]
 [ 0  1  2  3  4  5  6  7]
 [ 0  2  4  6  8 10 12 14]
 [ 0  3  6  9 12 15 18 21]
 [ 0  4  8 12 16 20 24 28]
 [ 0  5 10 15 20 25 30 35]
 [ 0  6 12 18 24 30 36 42]
 [ 0  7 14 21 28 35 42 49]]
```

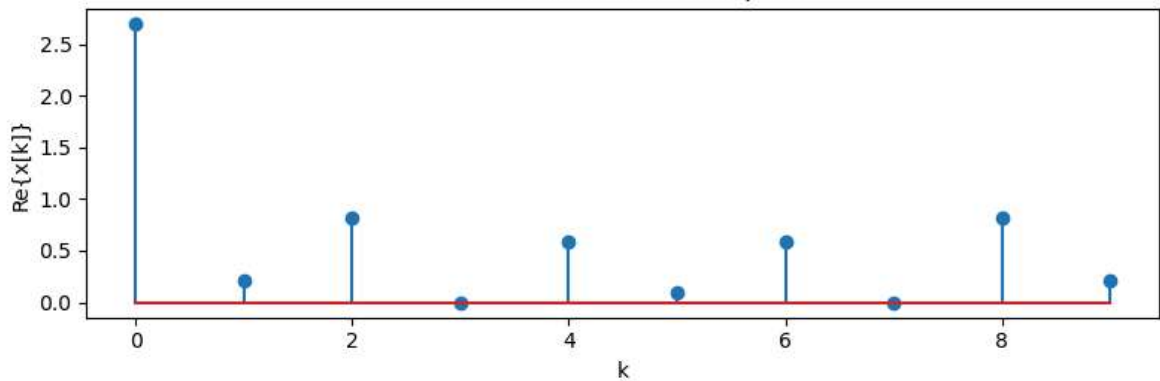
W (real, top-left 8x8):

```
[[ 1.    1.    1.    1.    1.    1.    1.    1. ]
 [ 1.    0.809 0.309 -0.309 -0.809 -1.    -0.809 -0.309]
 [ 1.    0.309 -0.809 -0.809 0.309 1.    0.309 -0.809]
 [ 1.   -0.309 -0.809 0.809 0.309 -1.    0.309 0.809]
 [ 1.   -0.809 0.309 0.309 -0.809 1.   -0.809 0.309]
 [ 1.   -1.    1.   -1.    1.   -1.    1.   -1. ]
 [ 1.   -0.809 0.309 0.309 -0.809 1.   -0.809 0.309]
 [ 1.   -0.309 -0.809 0.809 0.309 -1.    0.309 0.809]]
```

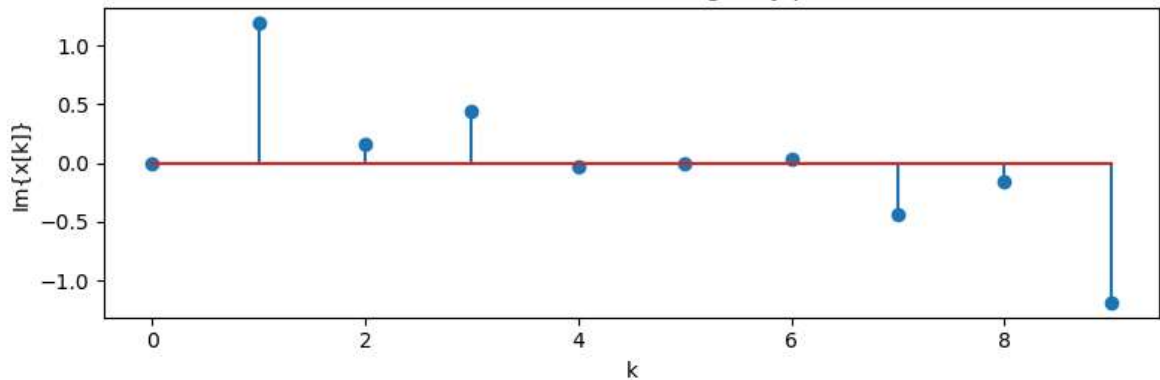
W (imag, top-left 8x8):

```
[[ 0.    0.    0.    0.    0.    0.    0.    0. ]
 [ 0.    0.588 0.951 0.951 0.588 0.   -0.588 -0.951]
 [ 0.    0.951 0.588 -0.588 -0.951 -0.    0.951 0.588]
 [ 0.    0.951 -0.588 -0.588 0.951 0.   -0.951 0.588]
 [ 0.    0.588 -0.951 0.951 -0.588 -0.    0.588 -0.951]
 [ 0.    0.   -0.    0.   -0.   -0.   -0.    0. ]
 [ 0.   -0.588 0.951 -0.951 0.588 -0.   -0.588 0.951]
 [ 0.   -0.951 0.588 0.588 -0.951 0.    0.951 -0.588]]
```

Variant 9: N=10 — real part



Variant 9: N=10 — imaginary part



=== Variant 10 — N=10 ===

N=10

K (top-left 8x8):

```
[[ 0  0  0  0  0  0  0  0]
 [ 0  1  2  3  4  5  6  7]
 [ 0  2  4  6  8 10 12 14]
 [ 0  3  6  9 12 15 18 21]
 [ 0  4  8 12 16 20 24 28]
 [ 0  5 10 15 20 25 30 35]
 [ 0  6 12 18 24 30 36 42]
 [ 0  7 14 21 28 35 42 49]]
```

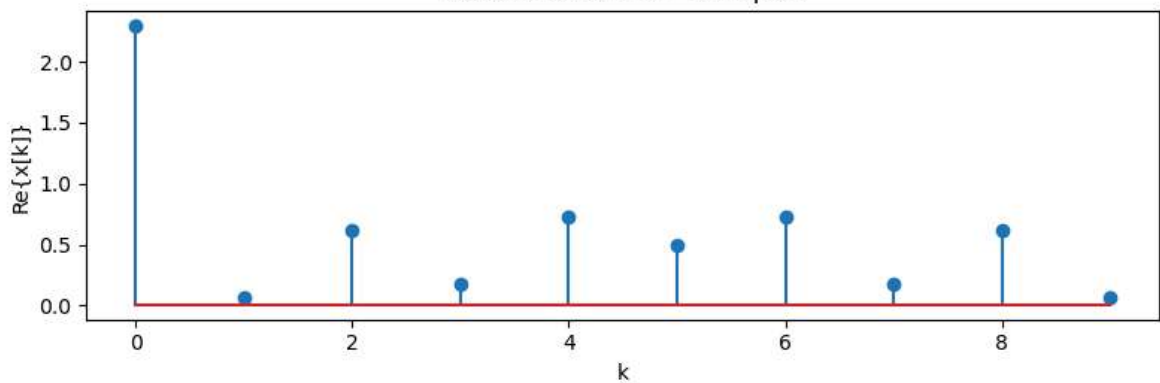
W (real, top-left 8x8):

```
[[ 1.    1.    1.    1.    1.    1.    1.    1. ]
 [ 1.    0.809 0.309 -0.309 -0.809 -1.   -0.809 -0.309]
 [ 1.    0.309 -0.809 -0.809  0.309  1.    0.309 -0.809]
 [ 1.   -0.309 -0.809  0.809  0.309 -1.    0.309  0.809]
 [ 1.   -0.809  0.309  0.309 -0.809  1.   -0.809  0.309]
 [ 1.   -1.    1.   -1.    1.   -1.    1.   -1. ]
 [ 1.   -0.809  0.309  0.309 -0.809  1.   -0.809  0.309]
 [ 1.   -0.309 -0.809  0.809  0.309 -1.    0.309  0.809]]
```

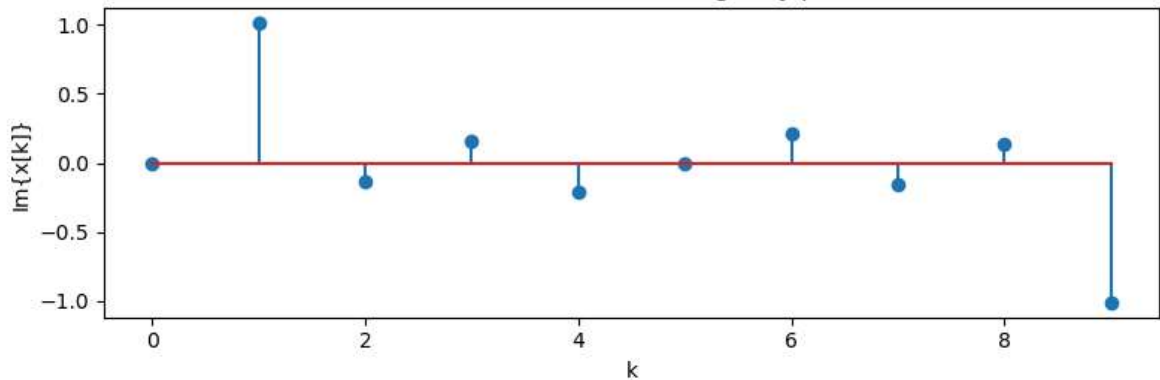
W (imag, top-left 8x8):

```
[[ 0.    0.    0.    0.    0.    0.    0.    0. ]
 [ 0.    0.588 0.951 0.951 0.588 0.   -0.588 -0.951]
 [ 0.    0.951 0.588 -0.588 -0.951 -0.    0.951 0.588]
 [ 0.    0.951 -0.588 -0.588 0.951 0.   -0.951 0.588]
 [ 0.    0.588 -0.951 0.951 -0.588 -0.    0.588 -0.951]
 [ 0.    0.   -0.    0.   -0.   -0.   -0.    0. ]
 [ 0.   -0.588 0.951 -0.951 0.588 -0.   -0.588 0.951]
 [ 0.   -0.951 0.588 0.588 -0.951 0.    0.951 -0.588]]
```

Variant 10: N=10 — real part



Variant 10: N=10 — imaginary part



3) Playground — try your own vector x_{μ}

```
In [10]: xmu_custom = [8, 2, 4, 0, 0, 0, 0, 0] # N=8 example
         xk, K, W = idft_matrix(xmu_custom)
         show_matrices(K, W, max_show=8)
         plot_signal(xk, title=f"Custom: N={len(xmu_custom)}")
```

N=8

K (top-left 8x8):

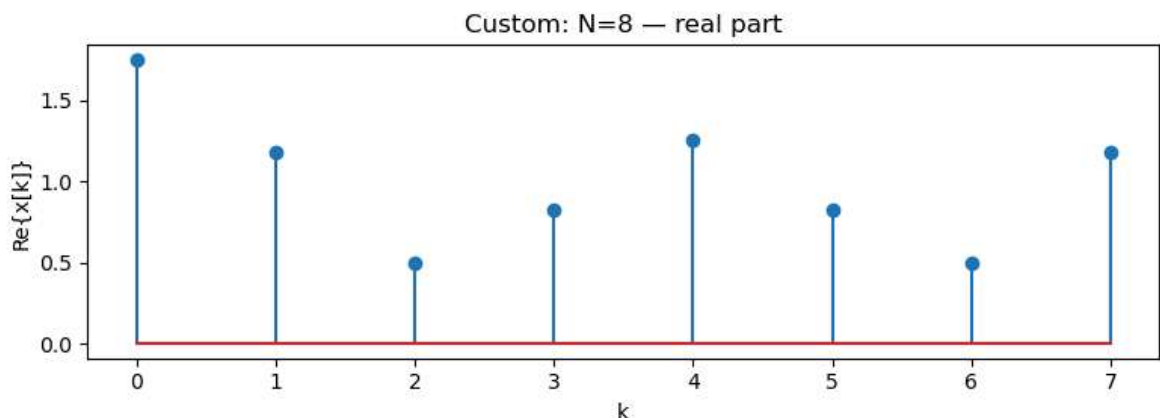
```
[[ 0  0  0  0  0  0  0  0]
 [ 0  1  2  3  4  5  6  7]
 [ 0  2  4  6  8 10 12 14]
 [ 0  3  6  9 12 15 18 21]
 [ 0  4  8 12 16 20 24 28]
 [ 0  5 10 15 20 25 30 35]
 [ 0  6 12 18 24 30 36 42]
 [ 0  7 14 21 28 35 42 49]]
```

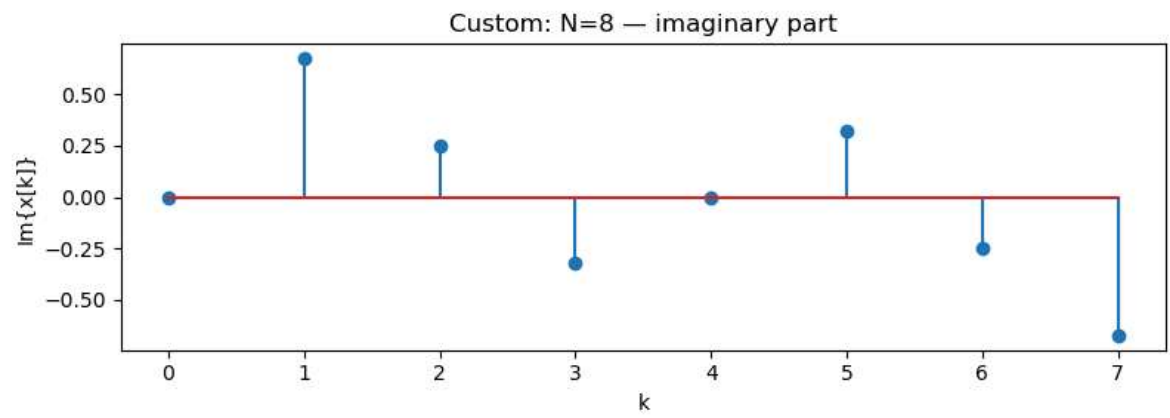
W (real, top-left 8x8):

```
[[ 1.    1.    1.    1.    1.    1.    1.    1.]
 [ 1.    0.707 0.   -0.707 -1.   -0.707 -0.    0.707]
 [ 1.    0.   -1.   -0.    1.    0.   -1.   -0.]
 [ 1.   -0.707 0.    0.707 -1.    0.707 0.   -0.707]
 [ 1.   -1.    1.   -1.    1.   -1.    1.   -1.]
 [ 1.   -0.707 0.    0.707 -1.    0.707 -0.   -0.707]
 [ 1.   -0.   -1.    0.    1.   -0.   -1.   -0.]
 [ 1.    0.707 0.   -0.707 -1.   -0.707 -0.    0.707]]
```

W (imag, top-left 8x8):

```
[[ 0.    0.    0.    0.    0.    0.    0.    0.]
 [ 0.    0.707 1.    0.707 0.   -0.707 -1.   -0.707]
 [ 0.    1.    0.   -1.   -0.    1.    0.   -1.]
 [ 0.    0.707 -1.    0.707 0.   -0.707 1.   -0.707]
 [ 0.    0.   -0.    0.   -0.    0.   -0.    0.]
 [ 0.   -0.707 1.   -0.707 0.    0.707 -1.    0.707]
 [ 0.   -1.    0.    1.   -0.   -1.    0.    1.]
 [ 0.   -0.707 -1.   -0.707 0.    0.707 1.    0.707]]
```





In []: