Draft version March 28, 2025
Typeset using LaTeX **default** style in AASTeX7

Going the Distance: Examining Parallax to Identify the Distance of a Star

Raynell Labayog[1]

[1] *San Diego State University*

ABSTRACT

Measuring stellar distances is a fundamental task in astronomy, and parallax is one of the most direct and reliable methods. In this report, we investigate the distance to the white dwarf star EGGR453 using parallax analysis. Observational data was obtained from the Palomar Transient Factory (PTF), while positional and proper motion data was sourced from Gaia DR3. After removing the proper motion component from the observed data, we calculated the parallax factors in both the right ascension and the declination. A least-squares fitting method was applied to model the star's motion over time and extract the best-fit parameters, including the parallax angle. The resulting parallax value of 0.144591 arcseconds corresponds to a distance of approximately 6.9 parsecs. This analysis demonstrates the effectiveness of combining astrometric observations with computational modeling to derive accurate stellar distances.

Keywords: Astrometry (187) — Stellar parallax (1619) — Proper motions (1299) — White dwarf stars (179) — Stellar distances (1587) — Astronomical catalogs (93)

1. INTRODUCTION

Looking up at night, we are faced with thousands and thousands of stars and objects in the sky, depending on where you are observing. An important aspect of astronomy is determining the distance of an object or star relative to the Earth. One way to determine the distance is to find the parallax.

The parallax is the shift in the position of a star resulting from Earth's orbit. As the Earth moves around the Sun we see the star from different angles. Now, with a little trigonometry, we are able to find the parallax angle. A way to do this is to observe the star's position at different times of the year to obtain its observed data. After finding the observed data, we now need the proper motion of the star being observed. For the sake of this paper we will observe the star "EGGR453", and ultimately determine its distance from Earth using parallax measurements.

Using the parallax to find the distance of a star ~~seems straight forward, but it still~~ requires precise measurements from 'EGGR453'. The second section of this paper will cover what data is used and how the data is recorded along with potential uncertainties it may produce. The third section will cover the analysis of the data, along with the use of least-squares fitting to minimize the residual data from the observed data. The use of least-squares fitting is important to the analysis of the data as this gives the best-fit values for the data. One of the values the best-fit parameters give will be the parallax. Once that value is found, all that is needed to do left is to take its inverse, and from that we are able to determine the distance from the star in parsecs.

2. DATA

The data used in this report will come from two sets of data given. The first will contain observation data from the Palomar Transient Factory survey. The other will be proper motion along with R.A and Dec information pertaining to EGGR453. This will come from the data release from the Gaia satellite.

Email: rlabayog3505@sdsu.edu

2.0.1. *Palomar Transient Factory*

Part of the data used in this paper were taken from the Palomar Transient Factory (PTF) observations. The Palomar Transient Factory was a wide-field survey that aimed to explore the transient sky. This data was taken from the Palomar Observatory using the 1.2m (Samuel Oschin telescope) and 1.5m telescopes. On the 1.2m telescope there was a new wide-field survey camera (PTF Survey Camera) installed that was used to preform the bulk of the data collection. The survey camera installed is a 101 megapixel and has a 8.1 square degree FOV.(N. M. Law et al. 2009) The use of two telescopes to collect the data is beneficial since this approach allows for both a high survey throughput along with a flexible follow-up program. What makes this interesting is the data is sent at almost real-time to the Lawrence Berkeley National Laboratory where they are tasked with identifying any optical transients from the images being sent from the Observatory. The survey created a database that included every source detected in each frame (N. M. Law et al. 2009). The data used in this report will be from the survey observations.

2.0.2. *Gaia DR3*

Another part of the data used in this paper will be position and proper motion data of EGGR453. This data will come from Gaia DR3. Gaia is a satellite located at Lagrange Point L2, which is about 1.5 million km from Earth (S. Jordan 2016). The satellite being placed at Lagrange Point L2 allows for the satellite to be 'stationary' since the gravitational forces on the satellite from the Sun and Earth are at equilibrium. Lagrange Point L2 is ideal for data collection, since it allows for uninterrupted observations as these points do not need to worry about a day-night cycle. It also produces less noise in the observations, as there is no atmosphere to worry about. The Gaia satellite first launched in 2013 as part of the Gaia mission conducted by the European Space Agency (ESA). This would be the third and latest data release since its launch into space. The DR3 is a catalog of more than 30 million objects that is a result of the direct observations from the satellite in the first 34 months of its mission (Gaia Collaboration et al. 2023).

3. ANALYSIS

3.1. *Plotting of the Raw Dec vs RA Data*

Taking the data obtained from the PTF survey, it shows the motion of the star EGGR453 across the sky. Figure 1 Shows the raw data for the declination and the right ascension before any techniques are applied to remove any part of the data.

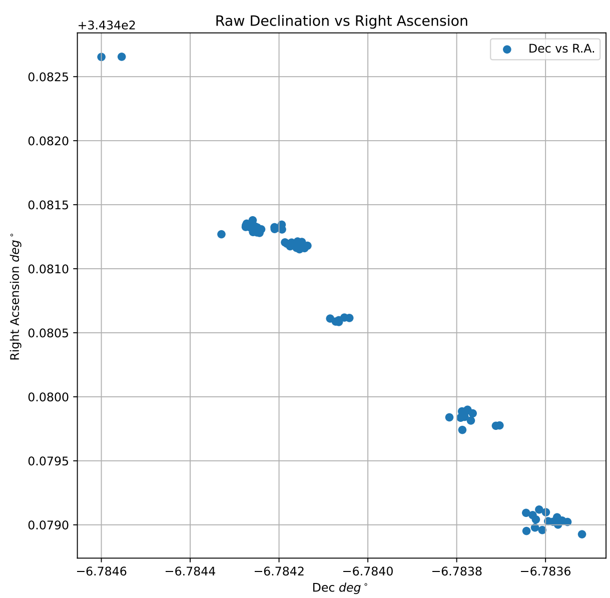

Figure 1. Plotted data of the Declination vs Right Ascension of EGGR453 observed from the Palomar Transient Factory

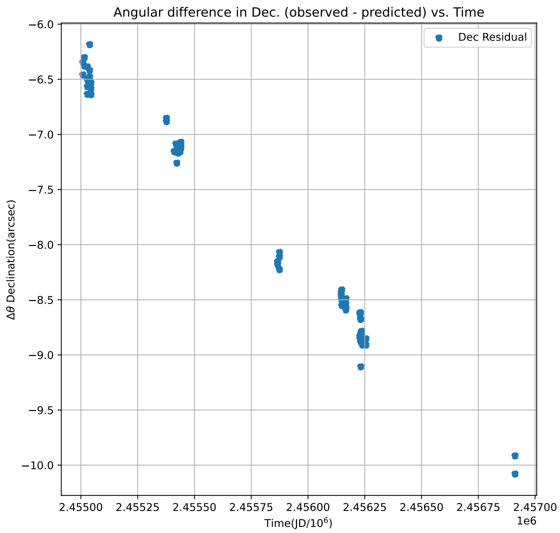

Figure 2. (a) Plotted Graph of the Angular Difference in the Declination vs Time

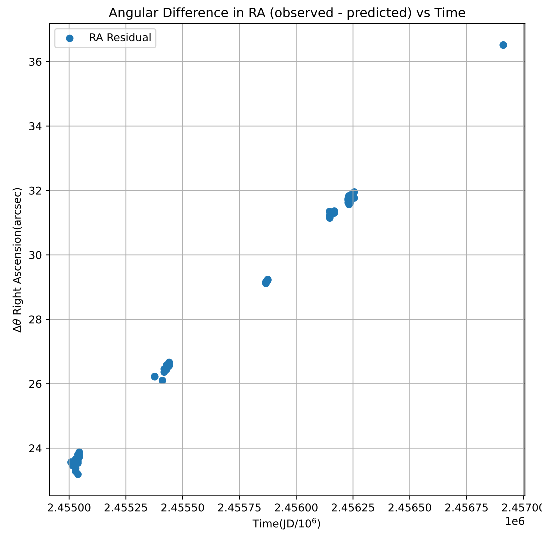

Figure 3. (b) Plotted Graph of the Angular Difference in the Right Ascension vs Time

3.2. *Removing Proper Motion of EGGR453*

This is an important step when it comes to finding the parallax of an object. Removing the proper motion allows us to filter out the star's motion and just measure the motion due to the perspective shift from Earth. This is done by subtracting the predicted values from the observed values. In the case of this paper the observed data will be the table given from the PTF survey. The predicted values will come from Gaia DR3 which gives the position and proper motion of EGGR453. Since the data given by Gaia is just the proper motion with the positional data at JD time 2000, the predicted position still needs to be figured out in order to be subtracted from the observed data. The proper motion given by Gaia is in mas/yr(mili-arc seconds) per year, we need to convert this to degrees as the RA and DEC are recorded in degrees. Once the proper motion has been converted to degrees and get the predicted coordinates from the same epoch time as the observed data.

$$\Delta\theta RA = (\alpha_1 - \alpha_0)\cos(\delta_0) \tag{1}$$

After that has been determined, Equation 1 will result in the angular difference in the Right Ascension. α_1 represents the observed data and α_1 represents the predicted position data, which were data based on the proper motion of EGGR453 given by Gaia DR3. The difference is then multiplied by the cosine of the predicted declination δ_0. The reason for multiplying the cosine is that it translates the coordinate difference into the true angular separation on the sky. The angular difference in the right ascension is plotted over time as seen in Figure 3.

$$\Delta\theta Dec = (\delta_1 - \delta_0) \tag{2}$$

Now that the angular difference is determined for the right ascension, it is time to determine the same thing for the declination. This process is more straightforward than finding the angular difference of the right ascension. From Eq. 2 all that really needs to be done is subtract the observed declination δ_1 to the predicted declination δ_0, and those values will represent the angular difference for the declination. The angular difference of the declination over time can be seen in Figure 2

3.3. *Finding the Parallax factors*

As discussed in the introduction, the parallax is the shift in the position of a star resulting from Earth's orbit. It is important to identify the parallax factors as this quantifies the parallax motion of the star in both the RA and Dec. Based on the notes from van Altena's book (W. F. van Altena 2013) the equations to find the parallax motions in both the RA and Dec can be seen in Eq. 3 and Eq. 4.

$$F_\alpha = R \sin(A - \alpha) \cos D \tag{3}$$

F_α represents the parallax factor for the right ascension. R represents the distance of the Earth to the Sun. R is multiplied by the sine of difference between the RA of the sun and the predicted position of EGGR453 $\sin A - \alpha$. We do this because this reflects the projected direction of the Earth's motion perpendicular to the line of sight to EGGR453. After that the whole product is multiplied by $\cos D$ where D represents the declination of the sun. This term adjusts how much of the Earth's motion projects in the RA direction considering the Sun's varying Dec.

$$F_\delta = R[\sin D \cos \delta - \cos D \sin \delta \cos(A - \alpha)] \tag{4}$$

F_δ represents the parallax factor for the declination. It is set up similar to Eq. 3 using the same variables, but instead accounts for the declination. Basically what the whole expression tells us is how much of Earth's motion appears to shift the star in terms of its declination.

Now that the parallax factors have been determined for RA and Dec, the next step is to create a best-fit model using least squares techniques.

3.4. *Fitting the Data into a Model*

Now that the parallax factors have been determined for the observed data, it is time to create a model that fits the data. To do this, the least squares method is applied. This method involves minimizing the sum of the residuals of the points in the graph. The least squares method allows us to find the best parameters for the model to minimize the difference between the model and the observed data. A way to do this with a data set that contains numerous data points is through matrix algebra.

$$Y = Xp \tag{5}$$

Using Eq. 6, it creates a system of equations where each variable Y, X, p represents a matrix. Y represents the dependent variable which is the positional vector in the RA component. X is the independent variable that creates a model to describe the observed position of EGGR453 over time, taking into account for the proper and parallax motion. p is what we are trying to find as this is the parameters that describes the proper motion, error, and parallax of the star. This is what transforms X into Y.

Since we now have an equation that represents p, with some algebraic manipulation, we can isolate p:

$$Y = Xp \tag{6}$$

$$X^T X p = X^T Y \tag{7}$$

$$(X^T X)^{-1} (X^T X) p = (X^T X)^{-1} X^T Y \tag{8}$$

$$p = (X^T X)^{-1} X^T Y \tag{9}$$

Starting with Eq. 6 we will take the transpose of X and multiply it to both sides of the equation, resulting in Eq. 7. Next we will take the inverse of $X^T X$ and distribute it to both sides, giving us Eq. 8. Recall the matrix rule where multiplying the transpose of a matrix with the inverse will result in an identity matrix. An identity matrix is a square matrix where the diagonal values are all 1 and everything else is 0. This would result in $(X^T X)^{-1}(X^T X)$ canceling out on the left side, and we are left with Eq. 9.

Now that there is a function for determining the parameters, it is time to plot the data and compare the model with the observed data.

3.5. *Plotting the Best Fit Model*

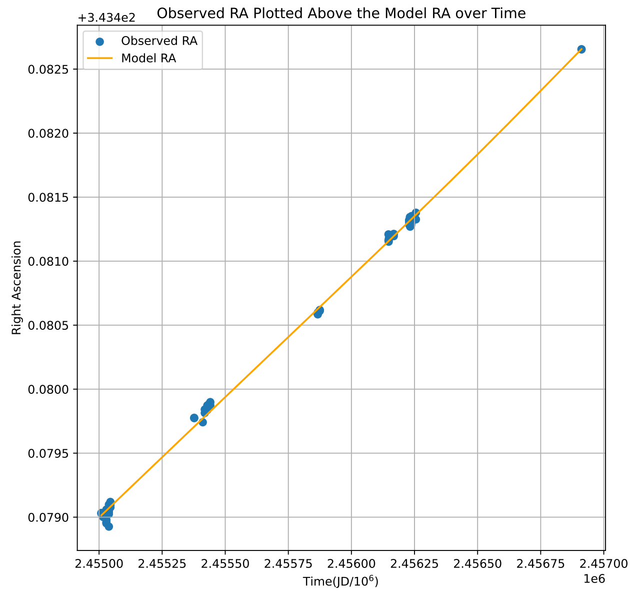

Figure 4. Plotted data of the Observed RA over the Model RA over time

Plotting the observed RA with the model RA as seen in Figure 4. The model appears to be in line with the observed data, although not fully accurate this is really close to the actual plotted data. The best-fit parameters given by the model is as given: RA Offset =-0.46 ± 0.31 arcsec, Proper Motion RA = 2.5 ± 0.029 arcsec/yr, Parallax = 0.144591 ± 0.073 arcsec. The Parallax parameter is what is needed to determine the distance from EGGR453.

$$\pi = 1/d \tag{10}$$

A way to determine the distance from the star would be to use Eq. 10 where π is the parallax angle and d is the distance in parsecs. What we found in the earlier section was the parallax factors, this described how much the star shifted due to the Earth's orbit. The parallax angle, π, is half of the apparent angular shift of a star over six months. So we take the parallax value of 0.144591, enter it into Eq. 10 and we get the result of 6.9 parsecs.

4. CONCLUSION

Determining the distance to nearby stars is one of the most fundamental challenges in astronomy, and the use of parallax provides a ~~reliable~~ geometric method to achieve this. In this report, we analyzed the star EGGR453 using observational data from the Palomar Transient Factory and proper motion data from Gaia DR3. By applying trigonometric relationships and least-squares model fitting, we successfully isolated the star's parallax motion from its proper motion.

The process began with raw astrometric data, in which the apparent movement of the star across the sky was plotted. We then removed the long-term proper motion component using Gaia's precise measurements, allowing us to observe the residual parallax shift caused by Earth's orbit around the Sun. After deriving the parallax factors for both right ascension and declination, we applied a least-squares fitting method to model the star's motion and solve for its best-fit parameters, including the parallax angle. From the model, we obtained a parallax of 0.144591 arcseconds, which corresponds to a distance of approximately 6.9 parsecs from Earth.

In conclusion, this study demonstrates the importance of combining high-precision data with computational methods to extract meaningful astronomical quantities. The approach applied here not only determined the distance to

EGGR453 but also showcased the significance of separating proper motion from parallax to obtain accurate results in stellar astrometry.

REFERENCES

Gaia Collaboration, Vallenari, A., Brown, A. G. A., et al. 2023, A&A, 674, A1, doi: 10.1051/0004-6361/202243940

Jordan, S. 2016,, https://youtu.be/m1ZNSPrH0q8

Law, N. M., Kulkarni, S. R., Dekany, R. G., et al. 2009, PASP, 121, 1395, doi: 10.1086/648598

van Altena, W. F. 2013, Astrometry for Astrophysics: Methods, Models, and Applications (Cambridge University Press)