24.118: Paradox and Infinity

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1. (a) $P(\text{One red ball}) = \frac{1}{2}$

   (b) $P(\text{Exactly two heads}) = \dfrac{\text{number of ways to get two heads}}{\text{number of possible outcomes of five flips}} = \dfrac{\binom{5}{2}}{2^5} = \dfrac{10}{32}$

   (c) The outcomes are equally likely. A specific ordering of ten coin flips. regardless of the flip results, has probability $\frac{1}{2^{10}}$ of appearing.

   (d) $P(\text{No aces}) = \dfrac{\text{number of ways to get no aces}}{\text{number of possible card hands}} = \dfrac{\binom{48}{5}}{\binom{52}{5}} = \dfrac{35673}{54145} \approx 0.658$

   $P(\text{One ace}) = \dfrac{\text{number of oways to get one ace}}{\text{number of possible card hands}} = \dfrac{\binom{4}{1}\binom{48}{4}}{\binom{52}{5}} = \dfrac{3243}{10829} \approx 0.299$

   $P(\text{At least two aces}) = 1 - (P(\text{No aces}) + P(\text{One ace})) = 1 - \left(\dfrac{35673}{54145} + \dfrac{\binom{4}{1}\binom{48}{4}}{\binom{2257}{54145}}\right) \approx 0.041$

   (e) $P(BBBBBBBBBB) = \dfrac{26!42!}{16!52!}$