# 24.118: Paradox and Infinity

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1. (a) $\forall a.\forall b.\forall c.\ \neg((a*(a*a))+(b*(b*b))=(c*(c*c)))$

   (b) For any $x$ there exists a number $y$ that is greater in value than $x$ and that is only divisible by one and itself (a prime number).

   Define there exists – $\exists x$: $\neg\forall x\neg\Phi$

   Define greater than – $x_2 > x_1$: $\exists y.\ ((x_1 + y = x_2) \wedge \neg(y = 0))$

   Define prime – $Prime(x)$: $\forall y_1.\forall y_2.\ \neg((y_1 * y_2 = x) \wedge (y_1 > 1) \wedge (y_2 > 1))$

   $\forall x.\exists y.\ ((y > x) \wedge Prime(y))$

2. (a) The formula $Pair(n, a, b)$ is expressed in $\mathcal{L}$ as $n = 2^{(a+1)} * 3^{(b+1)}$

   (b) The formula $Inc(n, m)$ is expressed in $\mathcal{L}$ as $\exists p.\exists c.\ ((n = p^{(m+1)} * c) \wedge (Prime(p)) \wedge (c > 0))$

   (c) Define implies – $x \implies y$: $\neg(x \wedge \neg y)$

   The formula $NSeq(n, k)$ is expressed in $\mathcal{L}$ as the conjunction of the following:

   All elements are pairs
   $\forall x.\ Inc(n, x) \implies \exists u.\exists v.\ Pair(x, u, v)$

   Each pair is unique
   $\forall x.\forall y.\ (Inc(n, x) \wedge Inc(n, y) \implies \exists z.\exists z_1.\exists z_2.\ \neg(Pair(x, z, z_1) = Pair(y, z, z_2)))$

   Pairs up to $k$ included
   $\forall c.\ (((k + 1 > c) \wedge (c > 0)) \implies \exists x.\exists y.\ (Inc(n, x) \wedge (Pair(x, c, y))))$

   (d) The formula $Seq(n, k, i, m)$ is expressed in $\mathcal{L}$ as $\exists x.\ (NSeq(n, k) \wedge (x = Pair(i, m)) \wedge (Inc(n, x))$