

24.118: Paradox and Infinity

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1. The expected value of partying is greater than the expected value of studying, therefore you should choose to party!

$$\begin{aligned}E(\textit{party}) &= V(\textit{party}, \textit{hard})P(\textit{hard}|\textit{party}) + V(\textit{party}, \textit{easy})P(\textit{easy}|\textit{party}) \\&= (-25)(0.2) + (35)(0.8) \\&= 23\end{aligned}$$

$$\begin{aligned}E(\textit{study}) &= V(\textit{study}, \textit{hard})P(\textit{hard}|\textit{study}) + V(\textit{study}, \textit{easy})P(\textit{study}, \textit{easy}) \\&= (18)(0.2) + (18)(0.8) \\&= 18\end{aligned}$$

2. The expected value of studying is greater than the expected value of partying, therefore you should choose to study...

$$\begin{aligned}E(\textit{party}) &= V(\textit{party}, \textit{hard})P(\textit{hard}|\textit{party}) + V(\textit{party}, \textit{easy})P(\textit{easy}|\textit{party}) \\&= (-25)(0.7) + (35)(0.3) \\&= -7\end{aligned}$$

$$\begin{aligned}E(\textit{study}) &= V(\textit{study}, \textit{hard})P(\textit{hard}|\textit{study}) + V(\textit{study}, \textit{easy})P(\textit{study}, \textit{easy}) \\&= (18)(0.2) + (18)(0.8) \\&= 18\end{aligned}$$

3. You can party or study with equal consequence if the professor gives exams such that:

$$P(\text{hard}|\text{party}) = \frac{17}{60} \text{ and } P(\text{hard}|\text{study}) = \frac{1}{5}$$

We know from (2) that expected value of partying when $P(\text{hard}|\text{study}) = \frac{1}{5}$ is 18. So solving for the conditional exam probability for the same expected value gives us what we want.

$$\begin{aligned} 18 &= (-25)(P) + (25)(1 - P) \\ -17 &= -60P \\ P &= \frac{17}{60} \end{aligned}$$

4. According to Evidential Decision Theory you should TwoBox. As long as the probability of Closed having \$1M is nonzero, then the expected value of TwoBox must be greater than OneBox because the \$100 is guaranteed for both situations. Slightly more formally:

$$E(\text{OneBox}) = P(\text{open} = \$100)V(\text{open}) = (1)(100) = 100$$

$$E(\text{TwoBox}) = P(\text{open} = \$100)V(\text{open}) + P(\text{closed} = \$0)V(\text{closed}) + P(\text{closed} = \$1M)V(\text{closed}) = 100 + \alpha \$1M \text{ for some } \alpha > 0$$

5. Even under the predictor condition Evidential Decision Theory supports the OneBox strategy. In fact the OneBox strategy will be favored until the predictor's correctness approaches 50 %, in which case the expected value of TwoBox will become greater.

$$\begin{aligned} E(\text{OneBox}) &= V(\text{full})P(\text{full}|\text{OneBox}) + V(\text{empty})P(\text{empty}|\text{OneBox}) \\ &= (1000000)(0.8) + (0)(0.2) \\ &= 800,000 \end{aligned}$$

$$\begin{aligned} E(\text{TwoBox}) &= V(\text{full})P(\text{full}|\text{TwoBox}) + V(\text{empty})P(\text{empty}|\text{TwoBox}) \\ &= (1000010)(0.2) + (10)(0.8) \\ &= 200,010 \end{aligned}$$