24.118: Paradox and Infinity

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1. (a) $\forall a.\forall b.\forall c.\ \neg((a*(a*a))+(b*(b*b))=(c*(c*c)))$

   (b) For any x there exists a number y that is greater in value than x and that is only divisible by one and itself (a prime number).

   Define there exists – $\exists x$: $\neg\forall x\neg\Phi$

   Define greater than – $x_2 > x_1$: $\exists y.\ ((x_1+y=x_2)\wedge\neg(y=0))$

   Define prime – $Prime(x)$: $\forall y_1.\forall y_2.\ \neg((y_1*y_2=x)\wedge(y_1>1)\wedge(y_2>1))$

   $\forall x.\exists y.\ ((y>x)\wedge Prime(y))$

2. (a) The formula $Pair(n,a,b)$ is expressed in $\mathcal{L}$ as $n=2^{(a+1)}*3^{(b+1)}$

   (b) The formula $Inc(n,m)$ is expressed in $\mathcal{L}$ as $\exists p.\exists c.\ ((n=p^{(m+1)}*c)\wedge(Prime(p))\wedge(c>0))$

   (c) Define implies – $x \implies y$: $\neg(x\wedge\neg y)$

   The formula $NSeq(n,k)$ is expressed in $\mathcal{L}$ as the conjunction of the following:

   All elements are pairs
   $\forall x.\ Inc(n,x) \implies \exists u.\exists v.\ Pair(x,u,v)$

   Each pair is unique
   $\forall x.\forall y.\ (Inc(n,x)\wedge Inc(n,y) \implies \exists z.\exists z_1.\exists z_2.\ \neg(Pair(x,z,z_1)=Pair(y,z,z_2)))$

   Pairs up to k included
   $\forall c.\ (((k+1>c)\wedge(c>0)) \implies \exists x.\exists y.\ (Inc(n,x)\wedge(Pair(x,c,y))))$

   (d) The formula $Seq(n,k,i,m)$ is expressed in $\mathcal{L}$ as $\exists x.\ (NSeq(n,k)\wedge(x=Pair(i,m))\wedge(Inc(n,x))$